Solution to Quiz 1

Section A (50%):

1 (6%): [$D\rho/Dt = 0$]

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{v}) = 0$$

$$\nabla \cdot \vec{v} = 0$$

2 (4%): $\vec{v} \cdot \vec{n} = 0$

[fixed boundary]

3 (2%): $\frac{\partial \rho}{\partial t} + \vec{v} \cdot \nabla \rho = 0$

4 (6%): [incompressible] [linear relationship between viscous stress and rate of strain] [isotropic]

$\tau_{xx} = 4\mu x$

$\tau_{xy} = 0$

$F_x = 4\mu x n_x A$

5 (9%):

$Re = 1.6 \times 10^9$ $\quad (Re = \frac{UL}{\nu})$

$Fr = 0.25$ $\quad (Fr = \frac{U}{\sqrt{gL}})$

$U = 100 \ m/s$

$U = \sqrt{10} \ m/s$

$g = 1 \times 10^4 \ m/s^2$

$\nu = \sqrt{10} \times 10^{-8} \ m^2/s$

6. (3%): $2.8 \times 10^4 \ N/m^2$ $\quad (P_{bubble} = P_{atmosphere} + \rho g H + \frac{2\Sigma}{R})$

7 (6%):

(a) [true]

(b) [true]

(c) [false]

(d) [true]

Section A. (contd)

8 (6%): Ideal fluid flow and conservative body force

$\omega_1 = \Gamma_1 / A_1$

$A_2 = 2A_1$

$\Gamma_2 = \Gamma_1$

$\omega_2 = \Gamma_1 / 2A_1$

9 (4%): $U_2 = 10\ m/s$

$P_2 = 50005\ N/m^2$

10 (4%): $A=2$ $(\nabla \times \vec{v} = 0)$

$B=4$ $(\nabla \cdot \vec{v} = 0)$

Section B (50%)

1.(20%):

(a) (i) Continuity equation: $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$

(ii) N-S equation.

$$x. \quad v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \frac{\mu}{\rho}\frac{\partial^2 u}{\partial y^2}$$

$$y. \quad u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \frac{\mu}{\rho}\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right)$$

(b) At $y=0$, $u=0$, $v=0$

At $y=H$: $u=0$, $v=0$

(c) $\frac{\partial u}{\partial x} = 0 \Rightarrow \frac{\partial v}{\partial y} = 0 \xrightarrow{v=0 \text{ at } y=0,H} v=0$ everywhere

(d) $$v=0 \Rightarrow \left.\begin{cases} \frac{\partial p}{\partial y} = 0 \Rightarrow p = p(x) \\ -\frac{1}{\rho}\frac{\partial p}{\partial x} + \frac{\mu}{\rho}\frac{\partial^2 u}{\partial y^2} = 0 \end{cases}\right\} \Rightarrow \underset{\uparrow \atop \text{function of } x}{\frac{dp}{dx}} = \mu \underset{\uparrow \atop \text{function of } y}{\frac{d^2 u}{dy^2}}$$

$\therefore \frac{dp}{dx}$ is a constant

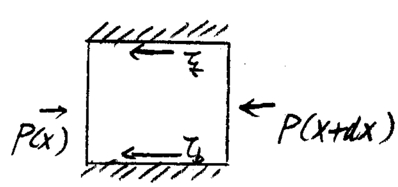
(e) (i) $u = \frac{1}{2}\frac{C}{\mu}(Hy - y^2)$

(ii) u reaches maximum value at $y = H/2$

(iii) $u_{max} = \frac{1}{8}\frac{C}{\mu}H^2$

(f) $\tau_t = \tau_b = \frac{1}{2}CH$, in positive x direction (stress on plate)

(g)



$(\tau_t + \tau_b)\,dx = [P(x) - P(x+dx)]\,H$

$\Rightarrow \tau_t + \tau_b = CH$

Symmetric flow $\Rightarrow \tau_t = \tau_b = \frac{1}{2}CH$

Section B (contd.)

2. (30%): (a)–(c): 10%, (d): 12%, (e): 8%

(a) ρ: ML^{-3}, μ: $ML^{-1}T^{-1}$, U: LT^{-1}

L: L, D: L, F: MLT^{-2}

(b) $\Pi_1 = \frac{D}{L}$, $\Pi_2 = \frac{F}{\rho U^2 L^2}$, $\Pi_3 = \frac{\rho U L}{\mu}$

(c) $L_m = 2m$, $U_m = 5\ m/s$

(d) (i) $-\rho U_m h + \rho \alpha U_m H + \rho \frac{7}{6} U_m (h-H) = 0 \Rightarrow \alpha = \frac{1}{6}$

(ii) $P_A + \frac{1}{2}\rho U_m^2 = P_B + \frac{1}{2}\rho (\frac{7}{6}U_m)^2 \Rightarrow P_A - P_B = \frac{13}{72}\rho U_m^2$

(iii) $-\rho U_m^2 h + \rho \alpha^2 U_m^2 H + \rho (\frac{7}{6})^2 U_m^2 (h-H) = (P_A - P_B)h + F_x$

$\Rightarrow \frac{F_x}{\rho U_m^2 H} = -\frac{1}{4} \Rightarrow F_x = -2500\ N/m$

$\Rightarrow$ Drag $= -F_x = 2500\ N/m$

(iv) Total force on the mode: Drag $\cdot L_m = 5000\ N$

Similitud of Π_2 and $\Pi_3 \Rightarrow F_s = F_m$

$\therefore$ Total force on the full-scale submarine: $5000\ N$

(e)
$$\begin{cases} -\rho U_m H + \rho \alpha U_m H + (Q_t + Q_b) = 0 \\ -\rho U_m^2 H + \rho \alpha^2 U_m^2 H + (Q_t + Q_b) U_m = (P_A - P_B) H + F_x \end{cases}$$

$$\left.\begin{array}{l} P_A = P_B \\ F_x = -2500\ N/m \text{ or } \frac{F_x}{\rho U_m^2 H} = -\frac{1}{4} \end{array}\right\} \Rightarrow \alpha = \frac{1}{2}$$