FULL NAME____

DEPARTMENT OF MECHANICAL ENGINEERING

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

2.20 Marine Hydrodynamics

Fall 2008 Quiz I

6 IMPORTANT INSTRUCTIONS:

1. **This packet has 7 pages – verify that you have these**!!!
2. **Part A**:
   Fill in the required answers where indicated by _______________.
   When a list of options, [. . .] [. . .] [. . .], is provided select (by circling) all the options which apply.
3. **Part B**:
   Write your solutions in the exam book provided.

4. **Unless otherwise indicated**, use:

gravitational acceleration	$g = 10\ \mathrm{m/s^2}$
water density	$\rho = 10^3\ \mathrm{kg/m^3}$
water kinematic viscosity	$\nu = 10^{-6}\ \mathrm{m^2/s}$
air density	$\rho = 1\ \mathrm{kg/m^3}$
air kinematic viscosity	$\nu = 10^{-5}\ \mathrm{m^2/s}$

5. For numerical answers, you MUST give the proper SI units (**kg, m, s** for example, write **5 m/s** <u>not</u> just **5**).
6. There are many problems and each individual answer is only worth a few points. DO NOT spend a disproportionate amount of time on any one problem.

SECTION A [50%]

1. A fluid flow is incompressible *only* if **[ρ =constant] [$\partial\rho/\partial t$ =0] [$D\rho/Dt$ =0] [$\partial\rho/\partial t$ =0 and $D\rho/Dt$ =0] [either $\partial\rho/\partial t$ =0 or $D\rho/Dt$ =0]**. For a general (compressible) flow, the equation expressing conservation of mass in terms of density ρ and velocity $\vec{v}$ is______________. This has been derived assuming **[inviscid fluid] [Newtonian fluid] [constant density] [irrotational flow] [rotational flow] [no mass can be created/destroyed] [volume has to stay the same]**.

2. In a steady flow, injecting dye at some point in the fluid visualizes **[pathline] [streamline] [streakline]**. In the flow becomes unsteady, the dye will mark **[pathline] [streamline] [streakline]**. If a particle is released at the same point in an unsteady flow, its time exposed photograph will mark **[pathline] [streamline] [streakline]**.

3. The relationship between viscous stress and velocity gradient $\tau_{ij} = \mu\left(u_{i,j} + u_{j,i}\right)$ is obtained under the following assumption(s): **[steady] [incompressible] [linear relationship between viscous stress and rate of strain] [inviscid] [isotropic] [conservative force] [none of above].** In a two-dimensional flow satisfying above assumption(s), the velocity is given by $\vec{v}(x,y) = 2\cdot\left[\left(x^2+y^2\right)\vec{i} + 2xy\,\vec{j}\right]$, then stress components τ_{xx} = ______________ , τ_{xy} = _____________. By symmetry, τ_{yx} = _________________. A small flat surface of area A located in the flow at *(x, y)* has a unit normal vector given by $\vec{n} = (n_x, n_y)$. The viscous force on this surface in the *x* direction is given by F_x = ___________.

4. A small temperature probe moves with velocity $\vec{U}(\vec{x},t)$ under water. The temperature field in the water is $T(\vec{x},t)$. The temperature change that the probe measures is ____________________. If for a certain $\vec{U}(\vec{x},t)$, the temperature measured by the probe is found to be a constant (in time), $\vec{U}(\vec{x},t)$ and $T(\vec{x},t)$ must satisfy the condition _______________.

5. It is well known that wires "whistle" in the wind. Frequency of the sound *f* depends on wind speed *U*, wire diameter *D* and kinematic viscosity *ν*. The independent dimensionless parameter(s) governing this problem is (are) ____. However, it was found that in the frequency interval where whistling occurs, viscosity plays no role. In that case, if we increase the wind speed twice, the pitch (sound frequency) will be **[increased] [decreased]** by a factor of ____________.

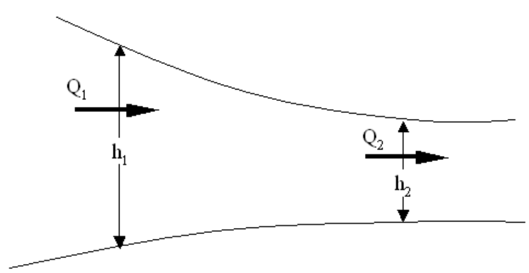
6. A diver is releasing air bubbles in the water as he dives under a ship. If the surface tension on the air-water interface is *Σ*, the pressure inside of a spherical air bubble of radius *R* is **[larger] [smaller]** than the pressure just outside the bubble by Δp = _________. As the diver surfaces, he sprays the water around him. The ratio between the pressure jump across the interface of the water droplet of radius *R* in the air, and the Δp found before is **[smaller than 1] [equal to 1] [larger than 1] [cannot tell].**

7. Express the following in terms of fluid velocity $\vec{v}\left(\vec{x},t\right)$, body velocity $\vec{U}\left(\vec{x},t\right)$, surface unit normal vector $\vec{n}$ and surface tangent vector $\vec{t}$. No slip boundary condition on the body states that _____________ and is an example of **[kinematic] [dynamic]** boundary condition for **[inviscid] [viscous]** flow. No flux boundary condition is expressed as ___________ is an example of **[kinematic] [dynamic]** boundary condition. **[Pressure continuity] [Shear stress continuity]** is/are (an) example(s) of **[kinematic] [dynamic]** boundary condition(s).

8. Kelvin theorem says that under the assumption of **[compressible] [rotational] [ideal]** flow, the rate of change of circulation $\frac{d\Gamma}{dt}=0$ for any **[line] [closed curve] [material line] [closed material curve] [line fixed in space]**. Under these conditions, a thin vortex tube has at time t_1 a cross-section area A_1, length tube L_1, and circulation Γ_1. If the vorticity ω_1 is uniform across the cross-section, the value of ω_1 should be $\omega_1=$________________. At a later time t_2, the length of the tube becomes $L_2=2L_1$. Now, the tube cross-section area is $A_2=$_______________, the circulation is $\Gamma_2=$_______________, and the uniform vorticity in the tube is $\omega_2=$_______________. (*Please write all your answers in terms of A_1, L_1 and Γ_1*)

9. Consider the steady flow in a water tunnel of circular cross section. The tunnel has a contraction ratio of 5 so that the radius at the upstream station is $R_1=5m$ and the radius in the test section is $R_2=1m$. Assuming ideal uniform one-dimensional (longitudinal velocity U only) fluid flow and uniform pressure in the cross section, if $p_1=10^5 N/m^2$ and $U_1=1m/s$ at the upstream station, then at the test section, $U_2=$ ________m/s and $p_2=$_____________ N/m^2.

10. The volume flux of air between two streamlines at two sections is Q_1 and Q_2.



For $h_1/h_2=3$, and:

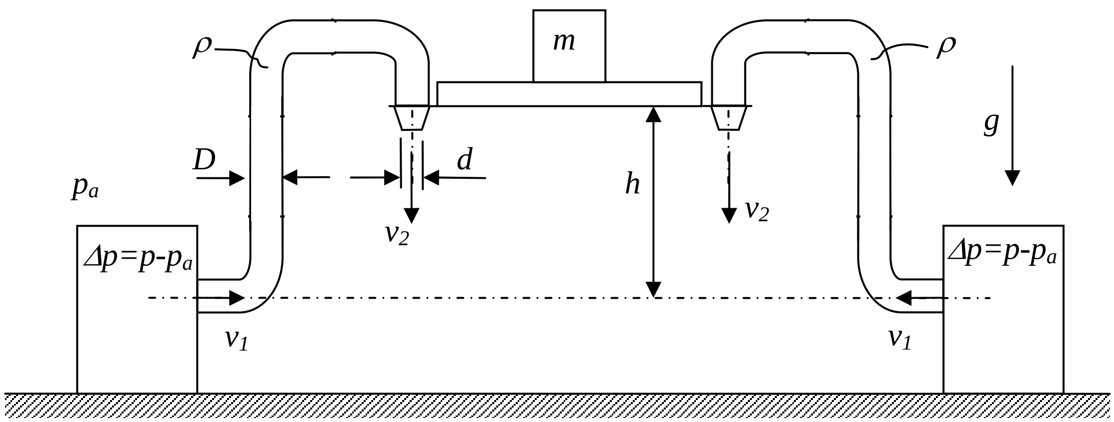
a. steady incompressible flow, then: Q_1/Q_2 = [1/3], [1], [3], [can't tell]

b. unsteady incompressible flow then: Q_1/Q_2 = [1/3], [1], [3], [can't tell]

c. steady compressible flow then: Q_1/Q_2 = [1/3], [1], [3], [can't tell]

SECTION B [50%] (Read through ALL problems before you proceed)

1. [25%] You have designed a very simple cargo lift for Navy submarines which is used in emergencies if all other systems fail. Lift is being provided by fluid reactive force, i.e. jets of water shooting downwards from the cargo platform, as seen on the figure below. Water is supplied from large pressurized water tanks and transported up to the loading platform by fire hoses. In order to put the design into reality, you have to calculate a few things first. You assume that the flow is ideal and that velocity profile across the hose is uniform.



a. [4%] What is the relationship between the velocity of the fluid at the exit from the tank and at the exit from the nozzle?

b. [7%] What is the lift (force in the vertical direction) per water hose in terms of water density ρ, diameter of the exit nozzle d and velocity on the exit v_2? Water hose is horizontal as it leaves the tank. (Hint: use a control volume that is the inside of the entire hose.)

c. [3%] If your design load that needs to be lifted is 500kg, what should be the exit velocity v_2 if there are 4 hoses that lift the platform?

d. [6%] For that load, what should be the pressure difference Δp between the pressure inside of the tank and the atmospheric pressure p_a if you want to lift the load to the height h of 3m? You can assume that the water tanks are large and that there is no significant fluid motion in them.

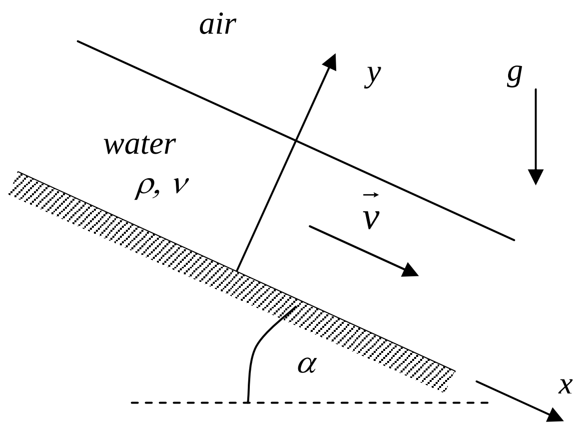
(Note: the following part of the problem can be solved without solving the first one). You are a bit worried about your assumption of ideal flow. In order to test that, you want to make a small scale model of the design to see what is the pressure difference needed if we have real viscous fluid. You notice that the dimensional parameters defining the problem are pressure difference Δp, density of the water ρ, gravity g, mass of the load m, lifting height h, diameters of the hose and the nozzle D and d, volume flux Q and kinematic viscosity of the water ν.

e. [2%] How many independent non-dimensional parameters define this problem?

f. [3%] It can be shown that two non-dimensional parameters are $\frac{Q}{d\nu}$ and $\frac{gd^5}{Q^2}$ which correspond to Reynolds and Froude numbers. If you build your model in the laboratory and properly scale it down (geometrically), can you achieve dynamic similitude between the model and the prototype using in both cases water as the lifting force? Why?

g. (Extra credit: [5%] Write down all non-dimensional parameters and show that two of them are equal to the ones in part (e).)

2. [25%] As the water from your lifting device flows down the hull, it forms a thin film. A simplified sketch of the flow is given below. In the following, you can assume that the flow is viscous, steady, two-dimensional and developed and use the coordinate system given on the sketch.



a. [3%] Write down the boundary condition(s) that the flow has to satisfy. Assume that the air above is inviscid and that it *cannot exert any shear force* on the fluid.

b. [8%] Obtain the simplified Navier-Stokes equations in x and y directions.

c. [2%] What is the driving force that forces the fluid to flow down the hull in this case?

d. [7%] Solve Navier-Stokes equation to get the velocity profile. What is the velocity U of the water on the top of the film $y = h$?

e. [2%] Calculate the shear stress τ_w on the hull.

f. [3%] You observe some dust particles spinning in the flow as water carries them down. What is their angular velocity at height $y = h/2$?

g. (Extra Credit: [4%] Using control volume, relate ν, g, α and U)

(END OF QUIZ 1)

Solutions to Quiz 1

PART A

1) [3%]

- $\frac{D\rho}{Dt} = 0$
- $\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{v}) = 0$
- no mass can be created / destroyed

2) [5%]

- pathline, streamline, streakline
- streakline
- pathline

3) [9%]

- incompressible, linear relationship between viscous stress and rate of strain, isotropic
- $\vec{v}(x,y) = 2\left[(x^2+y^2)\vec{i} + 2xy\vec{j}\right]$

$$\tau_{xx} = 2\mu \frac{\partial u}{\partial x} = 8\mu x$$

$$\tau_{xy} = \tau_{yx} = \mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) = 8\mu y$$

- $F_x = A \cdot \tau_{ij} n_j = A\,(\tau_{xx} n_x + \tau_{xy} n_y) = 8A\mu\,(x n_x + y n_y)$

4) $\bullet$ $\frac{DT}{Dt} = \frac{\partial T}{\partial t} + (\vec{U}\vec{\nabla})T$
[3%]

$\bullet$ $\frac{\partial T}{\partial t} = -(\vec{U}\vec{\nabla})T$

5) $\bullet$ $St = \frac{fD}{U}$, $Re = \frac{UD}{\nu}$
[6%]

$\bullet$ increased by a factor of 2

6) $\bullet$ larger
[3%]

$\bullet$ $\Delta p = \frac{2\Sigma}{R}$

$\bullet$ equal to 1

7) $\bullet$ $\vec{v} = \vec{U}$
[8%] $\bullet$ kinematic , viscous flow

$\bullet$ $\vec{v} \cdot \vec{n} = \vec{U} \cdot \vec{n}$

$\bullet$ kinematic

$\bullet$ Pressure continuity , shear stress continuity

$\bullet$ dynamic

8) [6%]

- ideal
- closed material curve
- $\omega_1 = \frac{\Gamma_1}{A_1}$
- $A_2 = \frac{1}{2} A_1$
- $\Gamma_2 = \Gamma_1$
- $\omega_2 = \frac{2\Gamma_1}{A_1}$

9) [4%] $U_1 = 0,2 \text{ m/s}$ ← correction

- $U_2 = U_1 \frac{R_1^2}{R_2^2} = 5 \text{ m/s}$
- $p_2 = p_1 + \frac{1}{2} \rho U_1^2 \left(1 - \frac{R_1^4}{R_2^4}\right) = 87520 \text{ N/m}^2$

10) [3%]

a) $Q_1/Q_2 = 1$

b) $Q_1/Q_2 = 1$

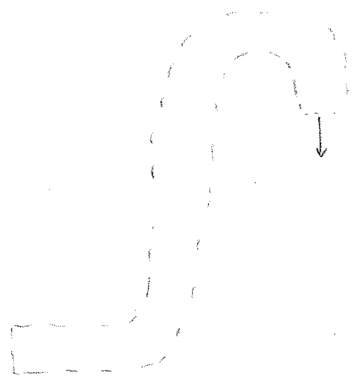
c) can't tell

PART B

(1) [25%]

a) $v_1 D^2 = v_2 d^2$ — continuity

b)



– momentum conservation

$$\oiint_S \rho \vec{v}(\vec{v}\cdot\vec{n})\,dS = \vec{F}_{TOT}$$

$$\Rightarrow \left(F_{HOSE}\right)_z = -\left(F_{TOT}\right)_z = \underline{\rho v_2^2 \frac{d^2\pi}{4}}$$

c) $m = 500\,kg \Rightarrow F_G = 5000\,N$

$$F_G = 4\cdot\left(F_{HOSE}\right)_z \Rightarrow v_2 = \sqrt{\frac{F_G}{\rho d^2 \pi}}$$

for $d = 5\,cm \Rightarrow \underline{v_2 = 25{,}2\ m/s}$

d) - Bernoulli eq. from inside of the tank to the exit

$$p = p_a + \rho g h + \frac{1}{2}\rho v_2^2$$

$$\Rightarrow \quad \Delta p = \rho g h + \frac{1}{2}\rho v_2^2 = 3,48 \cdot 10^5 \, N/m^2 \approx 3,5 \, atm$$

e)

$$\Delta p, \rho, g, m, h, D, d, Q, \nu \Rightarrow N = 9$$

$$J = N - K = 9 - 3 = \underline{6 \;\; \Pi\text{-parameters}}$$

f) - scaling factor : $\lambda = \frac{d_p}{d_M}$

$$\frac{Q}{d\nu} = \text{const} \Rightarrow \frac{Q_p}{d_p \cdot \cancel{\nu}} = \frac{Q_M}{d_M \cdot \cancel{\nu}} \Rightarrow Q_M = Q_p \cdot \frac{d_M}{d_p}$$

$$\Rightarrow \boxed{Q_M = \frac{Q_p}{\lambda}}$$

$$\frac{g d^5}{Q^2} = \text{const} \Rightarrow \frac{\cancel{g} \cdot d_p^5}{Q_p^2} = \frac{\cancel{g} d_M^5}{Q_M^2} \Rightarrow Q_M = Q_p \left(\frac{d_M}{d_p}\right)^{\frac{5}{2}}$$

$$\Rightarrow \boxed{Q_M = Q_p \, \lambda^{-\frac{5}{2}}}$$

- Dynamic similitude is <u>NOT</u> possible to achieve if $\lambda \neq 1$

g) (EXTRA CREDIT)

- dimensionally independent set : $\{Q, d, \rho\}$

$$\Pi_1 = \frac{h}{d} \quad ; \quad \Pi_2 = \frac{D}{d} \quad , \quad \Pi_3 = \frac{m}{\rho d^3}$$

Π_4 : $\Delta p \, Q^{x_1} d^{x_2} \rho^{x_3} = \Pi_4$

$$M L^{-1} T^{-2} L^{3x_1} T^{-x_1} L^{x_2} M^{x_3} L^{-3x_3} = M^0 L^0 T^0$$

M : $1 + x_3 = 0 \Rightarrow x_3 = -1$

L : $-1 + 3x_1 + x_2 - 3x_3 = 0 \Rightarrow x_2 = 4 \quad \Rightarrow \boxed{\Pi_4 = \frac{\Delta p \, d^4}{\rho Q^2}}$

T : $-2 - x_1 = 0 \Rightarrow x_1 = -2$

Π_5 : $g \, Q^{x_1} d^{x_2} \rho^{x_3} = \Pi_4$

$$L T^{-2} L^{3x_1} T^{-x_1} L^{x_2} M^{x_3} L^{-3x_3} = M^0 L^0 T^0$$

M : $x_3 = 0$

L : $1 + 3x_1 + x_2 - 3x_3 = 0 \Rightarrow x_2 = 5 \quad \Rightarrow \boxed{\Pi_5 = \frac{g d^5}{Q^2}}$

T : $-2 - x_1 = 0 \Rightarrow x_1 = -2$

Π_6 : $\nu \, Q^{x_1} d^{x_2} \rho^{x_3} = \Pi_6$

$$L^2 T^{-1} L^{3x_1} T^{-x_1} L^{x_2} M^{x_3} L^{-3x_3} = M^0 L^0 T^0$$

M : $x_3 = 0$

L : $2 + 3x_1 + x_2 - 3x_3 = 0 \Rightarrow x_2 = 1 \quad \Rightarrow \Pi_6' = \frac{\nu d}{Q}$

T : $-1 - x_1 = 0 \Rightarrow x_1 = -1 \quad \Rightarrow \boxed{\Pi_6 = \Pi_6'^{-1} = \frac{Q}{\nu d}}$

(2) a) $\vec{v} = 0 \quad @ \; y = 0$ - no slip
[25%]

$$\tau_{xy}\Big|_{y=h} = \mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)\Big|_{y=h} = 0$$ - no shear stress on the free surface

b)

- from continuity (developed flow $\Rightarrow \frac{\partial \vec{v}}{\partial x} \equiv 0$)

$$\underbrace{\cancel{\frac{\partial u}{\partial x}}}_{\substack{=0 \\ \text{(developed)}}} + \frac{\partial v}{\partial y} = 0 \Rightarrow v \neq f(y) \underset{\text{slip}}{\overset{\text{no}}{\Rightarrow}} \underline{v = 0}$$

- Navier-Stokes

$$x: \quad \rho \underbrace{\cancel{\frac{Du}{Dt}}}_{=0} = -\frac{\partial p}{\partial x} + \mu\left(\underbrace{\cancel{\frac{\partial^2 u}{\partial x^2}}}_{=0} + \frac{\partial^2 u}{\partial y^2}\right) + \rho g \sin\alpha$$

$$y: \quad \underbrace{\rho \cancel{\frac{Dv}{Dt}}}_{=0} = -\frac{\partial p}{\partial y} + \underbrace{\cancel{\mu \nabla^2 v}}_{=0} - \rho g \cos\alpha$$

$$\Rightarrow \frac{\partial p}{\partial y} = -\rho g \cos\alpha \Rightarrow \frac{\partial}{\partial y}\left(\frac{\partial p}{\partial x}\right) = 0$$

$$\frac{\partial p}{\partial x}\Big|_{y=h} = 0 \Rightarrow p \neq p(x)$$

$$\therefore \quad \boxed{\begin{aligned} x: \quad & \frac{d^2 u}{dy^2} = -\frac{\rho g}{\mu}\sin\alpha \\ y: \quad & \frac{dp}{dy} = -\rho g \cos\alpha . \end{aligned}}$$

c) Gravity is the driving force

d)

$$u(y) = -\frac{\rho g y^2}{2\mu}\sin\alpha + C_1 y + C_2$$

- Boundary conditions:

- $u(0) = 0 \Rightarrow \quad C_2 = 0$

- $\tau_{xy}\Big|_{y=h} = \mu\left(\frac{\partial u}{\partial y} + \underbrace{\cancel{\frac{\partial v}{\partial x}}}_{=0}\right)\Big|_{y=h} = \mu \frac{\partial u}{\partial y}\Big|_{y=h} = 0$

$$\Rightarrow \quad 0 = -\frac{\rho g h}{\mu}\sin\alpha + C_1 \Rightarrow \boxed{C_1 = \frac{\rho g h}{\mu}\sin\alpha}$$

$$\Rightarrow \boxed{u(y) = \frac{g h^2}{\nu}\sin\alpha\left[\frac{y}{h} - \frac{1}{2}\left(\frac{y}{h}\right)^2\right]}$$

$$U = u(h) = \frac{g h^2}{2\nu}\sin\alpha$$

e)

$$\tau_w = \mu \frac{\partial u}{\partial y}\Big|_{y=0} = \rho g h^2 \sin\alpha \cdot \frac{1}{h} = \rho g h \sin\alpha$$

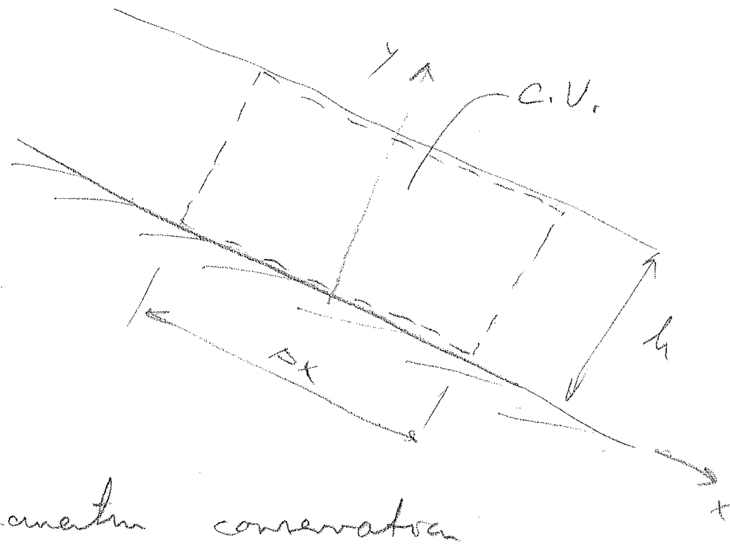
f)

$$\vec{\omega} = \nabla \times \vec{v} = \left(\underbrace{\cancel{\frac{\partial v}{\partial x}}}_{=0} - \frac{\partial u}{\partial y}\right)\vec{k}$$

$$\omega_z\Big|_{y=\frac{h}{2}} = -\frac{g h^2}{\nu}\sin\alpha\left[\frac{1}{h} - \frac{y}{h^2}\right]_{y=\frac{1}{2}h} = -\frac{g h}{2\nu}\sin\alpha$$

$$\boxed{\vec{\Omega} = \frac{1}{2}\vec{\omega} = -\frac{g h}{4\nu}\sin\alpha\,\vec{k}}$$

g) (EXTRA CREDIT)



- momentum conservation

$$\underbrace{\oint\!\!\oint \rho \vec{v}(\vec{v}\cdot\vec{n})\,dS}_{=0} = \sum \text{Forces} \Rightarrow \sum \text{Forces} = 0$$

$$\Rightarrow \underbrace{\iiint_V \vec{F}_B\,dV}_{\text{body force}} \underbrace{- \oint\!\!\oint_S p\vec{n}\,dS + \oint\!\!\oint_S \tau_{ij} n_j\,dS}_{\text{surface forces}} = 0$$

- x-component ($\tau_{xx}=0$, $\frac{dp}{dx}=0$)

$$\rho g \sin\alpha \cdot h\,\Delta x - \underbrace{\cancel{\int_0^h \Delta p\,dy}}_{=0} + \underbrace{\int_{x_1}^{x_2} \tau_{xy}\Big|_{y=0} dx}_{=\tau_w \Delta x} + \underbrace{\cancel{\int_{x_1}^{x_2} \tau_{xy}\Big|_{y=h} dx}}_{=0} = 0$$

$$\Rightarrow \boxed{\rho g h \sin\alpha = \tau_w}$$ gravity force balances shear stress

$\tau_w = \frac{2\mu}{h} \cdot U$

$$\boxed{U = \frac{g h^2}{\nu} \sin\alpha}$$