Outline:

- **Important concepts**
    - Lagrangian, Eulerian description
    - Pathline, streamline, streakline

Lagrangian/ Eulerian

The Lagrangian description describes the entire flow by recording the detailed histories of each fluid particle. A neutrally buoyant probe is an example of a Lagrangian measuring device.

The Eulerian description is a *field* description. Rather than following each fluid particle we can record the evolution of the flow properties at every point in space as time varies. A probe fixed in space is an example of an Eulerian measuring device.

Pathline, Streamline, Streakline

Streamline: A line everywhere tangent to the fluid velocity $\vec{v}$ at a given instant (flow snapshot). It is a strictly Eulerian concept.

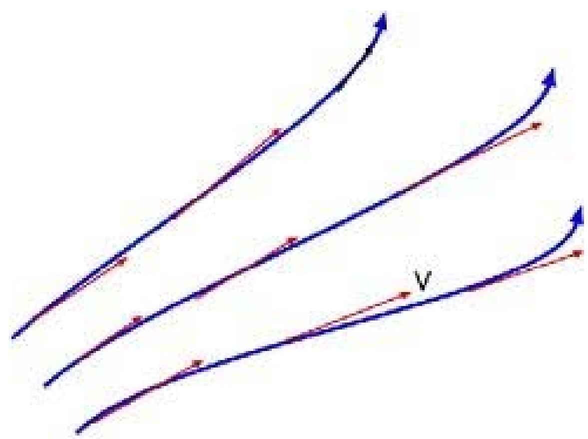
Streakline: Instantaneous locus of all fluid particles that have passed a given point (snapshot of certain fluid particles).

Pathline: The trajectory of a given particle P in time. The photograph analogy would be a long time exposure of a marked particle. It is a strictly Lagrangian concept.

A researcher has a large number of flotation devices each equipped with a battery and a light bulb. Explain how he/she would determine the streamlines, pathlines and streaklines near the surface of a stream with some unknown currents that vary with time.

Streamline

- Line everywhere tangent to instantaneous fluid velocity vectors (snapshot of the flow)
- Strictly an Eulerian concept (theoretically very useful)

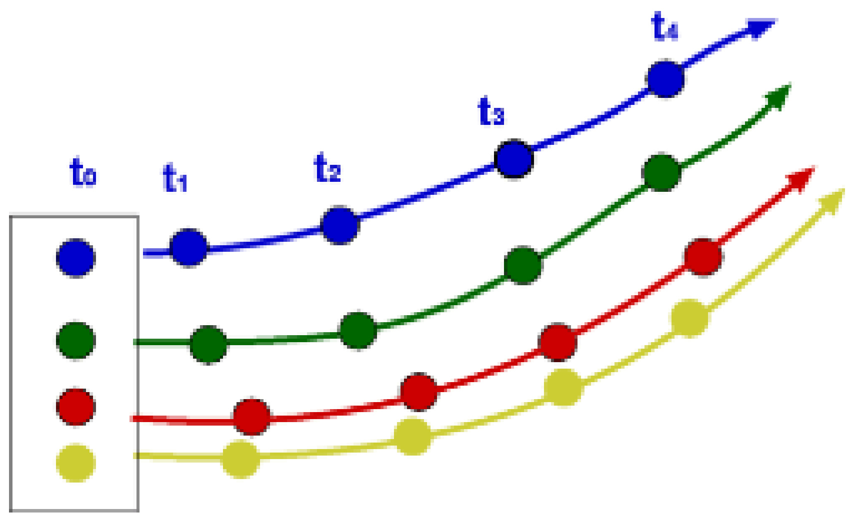


- Can a streamline cross itself?

NEVER!!

Pathline

- Trajectory of a given particle in time (like a long time exposure photo)
- Strictly a Lagrangian concept (useful for tracking, say, radioactive particles)

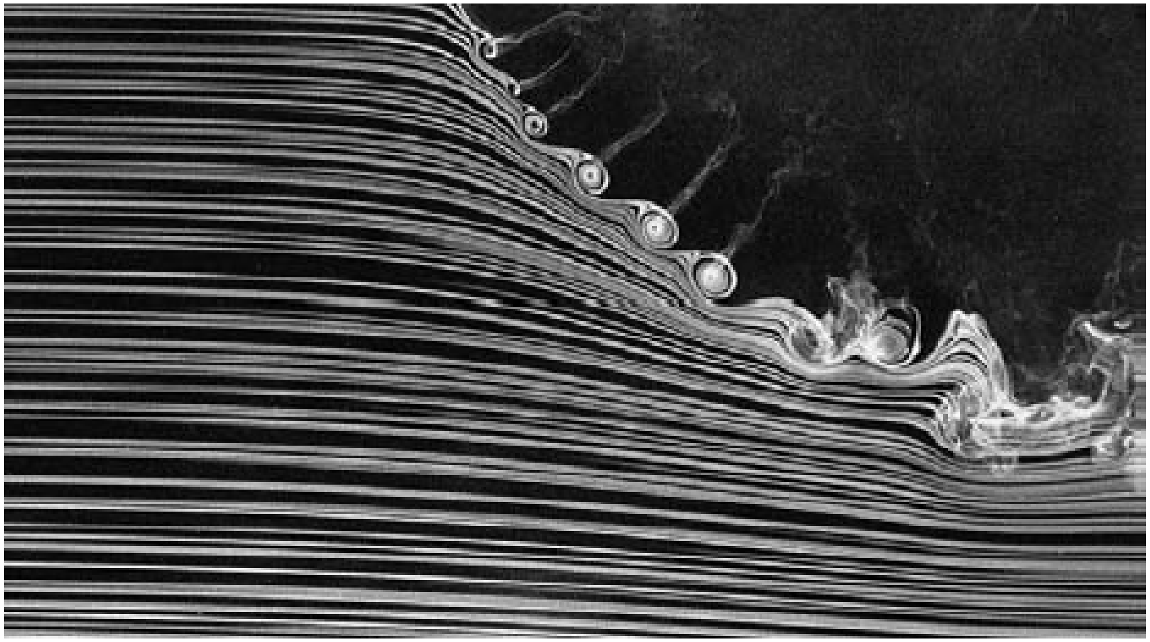


- Can a trajectory cross itself?

YES

Streakline

- Instantaneous position of particles passing a particular *fixed* point
- "simplest" experimental concept (injected dye/smoke)



- Can a streakline cross itself?

NEVER!! (it's a material line)

Steady/Unsteady Flow

- If the flow is *steady*

Streamline = pathline= streakline

-> Comparison of streamlines using streaklines

- If unsteady, lines generally completely different

A note on streamlines & pathlines

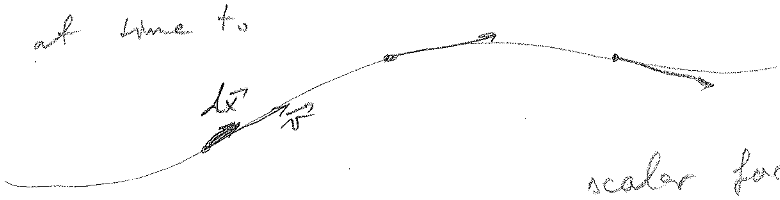
- How to obtain the equations for streamlines & pathlines?

- Streamlines - use the fact that streamline is parallel with velocity at every point in space

$$\vec{v}(x,y,z;t) = \big(u(x,y,z;t),\ v(x,y,z;t),\ w(x,y,z;t) \big)$$

velocity components

$d\vec{x} = (dx, dy, dz)$ - differential element of a streamline



scalar factor

$$\vec{v} \parallel d\vec{x} \Rightarrow \lambda \cdot \vec{v} = d\vec{x} \Rightarrow \begin{cases} \lambda u = dx \\ \lambda v = dy \\ \lambda w = dz \end{cases}$$

$$\Rightarrow \boxed{\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}}$$ differential equation for a streamline

Note that here time is merely a parameter

- Pathline – connects the positions of a certain particle as we follow it in space

$$\begin{cases} \dfrac{d\vec{x}_p(t)}{dt} = \vec{v}(\vec{x}=\vec{x}_p, t) & \text{– velocity of a particle} \\ \vec{x}_p(t_0) = \vec{x}_{p_0} & \text{– initial position} \end{cases}$$

$$\frac{dx_p}{dt} = u(\vec{x}=\vec{x}_p, t)$$

$$\frac{dy_p}{dt} = v(\vec{x}=\vec{x}_p, t)$$

$$\frac{dz_p}{dt} = w(\vec{x}=\vec{x}_p, t)$$

differential equation of a pathline (for a particle P.)