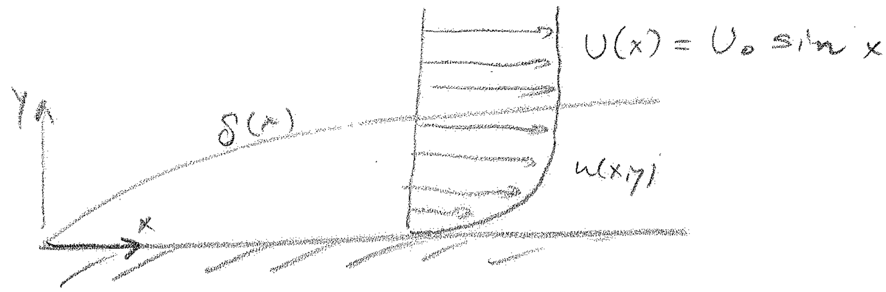
Turbulent Boundary Layer, Model Testing, Wave Forces

Outline:

- Turbulent Boundary Layer using Von Karman Momentum Integral Method
- Ship Model Testing
- Roughness scaling
- Wave Forces on a body

- Turbulent flow over flat plate using VK MIE



- Velocity profile in the turbulent B.L. can be assumed to follow 1/7 law

$$\Rightarrow \quad \frac{u(x,y)}{U(x)} = \left(\frac{y}{\delta}\right)^{1/7}$$

★Q: Find $\delta(x)$!!

- Use Von Karman Momentum Integral Equation

$$\frac{\tau_w}{\rho} = \frac{d}{dx}\left(U^2\theta\right) + \delta^* \cdot U \cdot \frac{dU}{dx} = U^2\frac{d\theta}{dx} + (2\theta + \delta^*)U\frac{dU}{dx}$$

- Calculate $\theta(x)$ and $\delta^*(x)$ as functions of $\delta(x)$

$$\eta = \frac{y}{\delta}$$

$$\delta^*(x) = \int_0^\delta \left(1 - \frac{u}{U}\right) dy = \delta\int_0^1 \left(1 - \eta^{1/7}\right) d\eta = \delta\left[\eta - \frac{7}{8}\eta^{\frac{8}{7}}\right]_0^1 =$$

$$= \delta\left(1 - \frac{7}{8}\right) = \underline{\underline{\frac{\delta(x)}{8}}}$$

$$\theta(x) = \int_0^\delta \frac{u}{U}\left(1 - \frac{u}{U}\right) dy = \delta\int_0^1 \eta^{\frac{1}{7}}\left(1 - \eta^{\frac{1}{7}}\right) d\eta = \delta\left[\frac{7}{8}\eta^{8/7} - \frac{7}{9}\eta^{9/7}\right]_0^1 =$$

$$= \delta\left[\frac{7}{8} - \frac{7}{9}\right] = \underline{\underline{\frac{7}{72}\delta(x)}}$$

- for $(1/7)^{th}$ law, τ_w is given by empirical formula

$$\frac{\tau_w}{\rho U^2} = 0{,}0227 \left(\frac{U\delta}{\nu} \right)^{-1/4}$$

- Substitute everything into VK MTE

$$\frac{\tau_w}{\rho} = U^2 \frac{d\theta}{dx} + (2\theta + \delta^*) U \frac{dU}{dx}$$

$$0{,}0227\, U^2 \left(\frac{U\delta}{\nu} \right)^{-1/4} = U^2 \frac{1}{8} \frac{d\delta}{dx} + \frac{25}{72} \delta U \frac{dU}{dx}$$

$$\frac{0{,}0227}{\nu} U^{7/4} \delta^{-1/4} = \frac{1}{8} U^2 \frac{d\delta}{dx} + \frac{25}{72} \delta U \frac{dU}{dx}$$

$$\boxed{\frac{0{,}0227}{\nu} (U_0 \sin x)^{7/4} \delta^{-1/4} = \frac{1}{8} (U_0 \sin x)^2 \frac{d\delta}{dx} + \frac{25}{72} \delta U_0^2 \sin x \cos x}$$

ODE for $\delta(x)$

(can be solved numerically to get $\delta(x)$)

- Ship Model Testing

sailing at $U_S = 10\,m/s$

The resistance of a $L_S = 300\,m$ tanker is to be determined by towing a $L_M = 3\,m$ model in a towing tank. The ship's wetted surface is $S_S = 20000\,m^2$.

a) Determine the speed at which the model should be towed

b) The measured drag is 5 N. What is the predicted full-scale drag?

- define $\lambda \equiv \dfrac{L_S}{L_M} = \dfrac{300\,m}{3\,m} = 100$ → length ratio

a) $(Fr)_S = (Fr)_M$ → Froude similitude HAS to be achieved

$$\frac{U_S}{\sqrt{g L_S}} = \frac{U_M}{\sqrt{g L_M}} \Rightarrow U_M = U_S \cdot \sqrt{\frac{L_M}{L_S}} = \frac{U_S}{\sqrt{\lambda}} = \frac{10\,m/s}{\sqrt{100}} = \underline{\underline{1\,m/s}}$$

b) $D_M = 5\,N$ → Model drag

$S_M = \dfrac{S_S}{\lambda^2} = 2\,m^2$ → Wetted surface scales with the square of the length

$C_{D_M} = \dfrac{D_M}{\frac{1}{2}\rho U_M^2 \cdot S_M} = \dfrac{5}{\frac{1}{2}\cdot 1000 \cdot 1^2 \cdot 2} = 0{,}005$ → Model drag coefficient

$$C_{D_M} = C_R(Fr) + C_f(Re_M)$$

$$Re_M = \frac{U_M \cdot L_M}{\nu} = \frac{1 \cdot 3}{10^{-6}} = 3 \cdot 10^6$$

$$C_f(Re_M) = \frac{0{,}075}{(\log_{10} Re_M - 2)^2} = \frac{0{,}075}{(6{,}477 - 2)^2} = \underline{\underline{0{,}00374}}$$

$$\Rightarrow C_e(Fr) = C_{Dm} - C_f(Re_m) = 0{,}005 - 0{,}00374 = \underline{0{,}00126}$$

$$C_{DS} = C_e(Fr) + C_f(Re_s) + \Delta C_f$$

$\Delta C_f = 0{,}0004$ - roughness allowance

$$Re_s = \frac{U_s \cdot L_s}{\nu} = \frac{10 \cdot 300}{\nu} = 3 \cdot 10^9$$

$$C_f(Re_s) = \frac{0{,}075}{(\log Re_s - 2)^2} = 0{,}00134$$

$$\Rightarrow C_{DS} = 0{,}00126 + 0{,}00134 + 0{,}000 = \underline{0{,}00264}$$

$D_s = \frac{1}{2} \rho U_s^2 \cdot S_s \cdot C_{Ds}$ → Drag on a ship

$$D_s = \frac{1}{5} \cdot 1000 \cdot 10^2 \cdot 20000 \cdot 0{,}00264 = \underline{2{,}64 \cdot 10^6 N}$$

* What is the power required to propel the ship?

$$P_S = D_s \cdot U_s = 2{,}64 \cdot 10^6 N \cdot 10 m/s = \underline{26{,}4 MW}$$

- Roughness / smoothness of prototype/model for ship model testing

$$\lambda = \frac{L_p}{L_m} > 1$$ - geometrical scale of the model

$$Fr_m = Fr_p \Rightarrow \frac{U_m}{\sqrt{gL_m}} = \frac{U_p}{\sqrt{gL_p}} \Rightarrow \frac{U_m}{U_p} = \sqrt{\frac{L_m}{L_p}} = \frac{1}{\sqrt{\lambda}}$$

$$Re_m = \frac{U_m \cdot L_m}{\nu} \; ; \; Re_p = \frac{U_p \cdot L_p}{\nu}$$

$$\Rightarrow Re_m = \frac{U_m \cdot L_m}{U_p \cdot L_p} \cdot Re_p = \frac{1}{\lambda\sqrt{\lambda}} Re_p = \lambda^{-\frac{3}{2}} Re_p$$

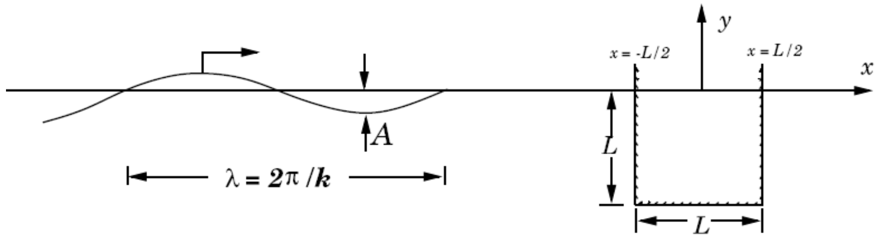
$$\Rightarrow \underline{Re_m < Re_p}$$

$$\frac{k}{\delta} = \frac{k}{l} \cdot \frac{l}{\delta} \propto \frac{k}{l} Re^{1/5}$$

for turbulent B.L. $\frac{\delta}{l} \propto Re^{-1/5}$

$$\frac{(k/\delta)_m}{(k/\delta)_p} = \frac{(k/l)_m \cdot Re_m^{1/5}}{(k/l)_p Re_p^{1/5}} = \frac{(k/l)_m}{(k/l)_p} \lambda^{-\frac{3}{10}}$$

A deep water wave of amplitude A and wave number $k = 2\pi/\lambda$ is incident upon a stationary long (two-dimensional) square barge of dimension L:



(a) Assuming $A/L \ll 1$ and $kL \ll 1$ and ignoring both viscous and diffraction effects, calculate the horizontal wave force F_x on the barge using a Froude-Krylov approximation.
(b) An alternative approach is to use Morison's equation and write:

$$F_x = C_m \rho L^2 \frac{dU(t)}{dt}$$

where dU/dt is the (Eulerian) fluid acceleration at the center of the waterplane ($x = y = 0$). By comparing the resulting formulas from (a) and (b), find an expression for C_m as a function of the dimensionless parameter kL. What is the asymptotic value of C_m for $kL \ll 1$?

a) $kL \ll 1 \Rightarrow \frac{L}{\lambda} \ll 1 \Rightarrow$ ignore diffraction

$\frac{A}{L} \ll 1 \Rightarrow K_c \ll 1 \Rightarrow$ ignore viscous forces

- Froude-Krylov

$$p_d = \rho g A e^{ky} \cos(kx - \omega t)$$

$$\frac{\partial p_d}{\partial x} = -\rho g A k e^{ky} \sin(kx - \omega t)$$

$$F_{FK,x} = -\int_{-L}^{0} \left(-\rho g A k e^{ky} \sin(kx - \omega t)\Big|_{x=0}\right) \cdot \overbrace{L \cdot 1}^{\text{cross-section}} \, dy$$

$$= -\rho g A k L \sin(\omega t) \cdot \frac{1}{k} e^{ky}\Big|_{-L}^{0} = \underline{-\rho g A L (1 - e^{-kL}) \sin \omega t}$$

b) $F_M = C_m \rho L^2 \cdot 1 \cdot \frac{dU}{dt}$

$$U = A\omega \cos(kx - \omega t) \Rightarrow \frac{dU}{dt}\Big|_{x=0} = -A\omega^2 \sin \omega t$$

$$\Rightarrow F_M = -C_m \cdot \rho L^2 \cdot A\omega^2 \sin \omega t$$

$$F_M = F_K \Rightarrow -C_m \rho L^2 A \omega^2 \cancel{\sin \omega t} = -\rho g A L (1 - e^{-kL}) \sin \omega t$$

$$\Rightarrow C_m = \frac{g}{\omega^2 L}(1 - e^{-kL}) \underset{(\text{deep water})}{\overset{\omega^2 = gk}{=}} \underline{\frac{1}{kL}(1 - e^{-kL})}$$

- for $kL \to 0$, $1 - e^{-kL} \sim 1 - (1 - kL) = kL \Rightarrow \boxed{C_m \simeq 1}$