Marine Hydrodynamics Homework #1(b) Solutions
September 16, 2009

Question 1: Basic Flow Concepts

1. (a) In an incompressible flow, the density can change as long as the temporal change of density is balanced by the spacial changes.
2. (c) A steady flow is defined as a flow in which $\frac{\partial}{\partial t} \equiv 0$.
3. Continuum, shear stress

Question 2: Stream-, Path-, and Streaklines

1. The streamline can be calculated by:

$$\vec{V} = \frac{d\vec{x}}{dt} \qquad \rightarrow \qquad \frac{dx}{u} = dt = \frac{dy}{v} \qquad \rightarrow \qquad \frac{dx}{y} = \frac{dy}{t^3 y^2 \cos(3x)}$$

$$t^3 \cos(3x) dx = \frac{dy}{y}$$

$$\frac{t^3 \sin(3x)}{3} = ln|y| + \mathbb{C}$$

The pathline can be calculated by:

$$\frac{\partial \vec{x}_p}{\partial t} = \vec{V}$$

$$\frac{\partial x_p}{\partial t} = y_p \qquad \qquad \frac{\partial y_p}{\partial t} = t^3 y^2 \cos(3x_p)$$

2. Ba 14
   (a) Streamline, pathline, and streakline because for steady flow, they are equivalent
   (b) Streakline
   (c) Pathline , steamline, steakline
   (d) Pathline
3. Ba 15
   (a) streakline
   (b) pathline

Question 3: Flow Kinematics

1. Ba 6

$$\vec{V}_{ROV} = 2xt\hat{i} + 4y^2\hat{j} - 3t\hat{k}$$
$$S = 2x\cos(at)$$

Rate of Change of salinity calculated by:

$$\begin{aligned} \frac{DS}{Dt} &= \frac{\partial S}{\partial t} + \vec{V}_{ROV} \cdot \nabla S \\ &= -2ax\sin(at) + 2xt\left[2\cos(at)\right] \\ &= 4xt\cos(at) - 2ax\sin(at) \end{aligned}$$

2. Ba 7

The material derivative is derived by following the motion of a single particle in the flow. So when the material derivative is written as:

$$\frac{Df}{Dt} = \frac{\partial f}{\partial t} + \vec{v}_p \cdot \nabla f$$

the velocity $\vec{v}_p$ is the velocity of that particle being followed. In these problems, you need to consider which particle is being followed in the calculation.

   (a) The particle being followed is the fish which is moving with velocity $\vec{u}$:

$$\frac{Df}{Dt}|_{Fish} = \frac{\partial f}{\partial t} + \vec{u} \cdot \nabla f$$

   (b) In this case, the particle is an egg which is neutrally buoyant (behaves like a fluid particle) and moves with velocity $\vec{v}$:

$$\frac{Df}{Dt}|_{Egg} = \frac{\partial f}{\partial t} + \vec{v} \cdot \nabla f$$

   (c) Here, the egg is stuck and no longer moves ($\vec{v}_p = \vec{0}$):

$$\frac{Df}{Dt}|_{Rock} = \frac{\partial f}{\partial t} + \vec{0} \cdot \nabla f = \frac{\partial f}{\partial t}$$

3. Ba 8

Given the following:

$$\begin{aligned} \text{Fixed Probe: } \dot{\theta} &= bt \\ \text{Dropped Probe: } \frac{D\theta}{Dt} &= -aW + bt \\ \text{Known that: } \theta &= \theta(z,t) \end{aligned}$$

The fixed probe case is a special case of the falling probe case.

From the fixed probe:

$$\frac{D\theta_{Fixed}}{Dt} = \frac{\partial\theta_{Fixed}}{\partial t} + \vec{0}\cdot\nabla\theta_{Fixed} = \frac{\partial\theta_{Fixed}}{\partial t} = bt \qquad \rightarrow \qquad \theta_{Fixed} = 1/2bt^2 + g(z)$$

From the falling probe:

$$\frac{D\theta_{Fall}}{Dt} = \frac{\partial\theta_{Fall}}{\partial t} + (-W)\frac{\partial\theta_{Fall}}{\partial z} = -aW + bt$$

By comparing the two solutions, it becomes apparent that the time derivative of the fixed probe is the same as the time derivative of the falling probe.

$$\frac{D\theta_{Fall}}{Dt} = bt - W\frac{\partial\theta_{Fall}}{\partial z} = bt - aW$$

$$-W\frac{\partial\theta_{Fall}}{\partial z} = -aW \qquad \rightarrow \qquad \theta_{Fall} = az + f(t)$$

Since both probes are making measurements on the same system, $\theta_{Fall} = \theta_{Fixed}$, which implies

$$az + f(t) = 1/2bt^2 + g(z) \qquad \rightarrow \qquad \theta = az + 1/2bt^2 + \mathbb{C}$$

If the fluid velocity is given by $\vec{v} = u\hat{i} + v\hat{j} + w\hat{k}$, then the neutrally boyant temperature probe which drifts with the fluid will experience a time rate of change:

$$\begin{aligned} \frac{D\theta}{Dt} &= \frac{\partial\theta}{\partial t} + \vec{v}\cdot\nabla\theta \\ &= bt + aw \end{aligned}$$