2.25

9 Vorticity and Circulation

9.1 Vorticity $\vec{\omega} = \nabla \times \vec{v}$ and its physical significance. Vortex lines and vortex tubes. The solenoidal nature of vortex lines. Examples.

9.2 Circulation Γ: its definition and relationship to vorticity. Vorticity as a "source" of circulation (Biot-Savart's law).

9.3 Kelvin's theorem on circulation in barotropic flows in irrotational force fields:

$$\frac{D\Gamma_c}{Dt} = \nu \oint_c \nabla^2 \vec{v} \cdot d\vec{l}$$

For inviscid flows:

$$\frac{D\Gamma_c}{Dt} = 0$$

Examples: steady sink vortex, tornado or cyclone, bathtub vortex.

9.4 Three vortex theorems for inviscid, barotropic flow in an irrotational force field (corollaries of Kelvin's theorem):

(a) Vortex lines move with the fluid.

Examples: behavior of vortex rings; instability of shear layer or vortex sheet; secondary flow induced in bends.

(b) Once irrotational($\vec{\omega} = 0$), a fluid particle will remain so forever.

Consequence: Potential flows.
Example: solution of steady sink vortex based on constraint that $\vec{\omega} = 0$.

(c) For vortex "tube",

$$\frac{D}{Dt}\left(\frac{\omega}{\rho L}\right) = 0$$

Example: accelerating inviscid flow with transverse velocity gradient.

9.5* Vorticity transport equation in differential form. The effect on vorticity of vortex line stretching and turning; the role of kinematic viscosity as the diffusivity of vorticity.

Read: Fay, pp 271-276
e.g. Potter & Foss pp. 179-184, 250-262, 390-392

Problems: Shapiro & Sonin 10.3, 10.4, (10.5, 10.7),10.11