# MASSACHUSETTS INSTITUTE OF TECHNOLOGY DEPARTMENT OF MECHANICAL ENGINEERING CAMBRIDGE, MASSACHUSETTS 02139

## 2.29 NUMERICAL FLUID MECHANICS—SPRING 2019

Course Website: <a href="http://web.mit.edu/2.29/www/">http://web.mit.edu/2.29/www/</a>

Tuesday	Thursday	HOMEWORK
11 - 12:30	, 11 - 12:30	&
Rm 1-150 (Rec: 1-135)	Rm 1-150	QUIZZES
5-Feb		
5-Feb		Rec. 1 – Intro. to MATLAB
	7-Feb	HW1 Posted
12-Feb		
	14-Feb	
19-Feb (no class)		
	21-Feb	HW1 Due / HW2 Posted
26-Feb		
	28-Feb	
5-Mar		
	7-Mar	HW2 Due / HW3 Posted
12-Mar		
	14-Mar	Quiz 1
19-Mar		
	21-Mar	HW3 Due / HW4 Posted
25 - 29 Mar: Spri	ng Vacation	
2-Apr		
	4-Apr	
9-Apr		
	11-Apr	HW4 Due / HW5 Posted
16-Apr Holiday		
	18-Apr	
23-Apr		Quiz 2
	25-Apr	
30-Apr		HW 5 Due /HW 6 Posted
	2-May	
7-May		
	9-May	HW6 Due
14-May		
	16-May	Project Presentations:
	(perhaps 17-May)	4-7:00pm (TBD), Details TBD

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#### 2.29 NUMERICAL FLUID MECHANICS— SPRING 2019

#### **Subject Summary**

Introduction to numerical methods and MATLAB: errors, condition numbers and roots of equations. Navier-Stokes. Direct and iterative methods for linear systems. Finite differences for elliptic, parabolic and hyperbolic equations. Fourier decomposition, error analysis and stability. High-order and compact finite-differences. Finite volume methods. Time marching methods. Navier-Stokes solvers. Grid generation. Finite volumes on complex geometries. Finite element methods. Spectral methods. Boundary element and panel methods. Turbulent flows. Boundary layers. Lagrangian Coherent Structures. Subject includes a final research project.

#### **Syllabus and References**

Lecture Number	Tuesday 11 - 12:30 Rm 1-150	Thursday 11 - 12:30 Rm 1-150	Material covered in Lecture	References CC: Chapra & Canale and other references when mentioned
1	05-Feb		Introduction to Numerical Fluid Mechanics: Models to simulations, Error types. Approximation and round-off errors. Number representations.	CC: PT 1.1-1.3, 1.1 - 1.2, 3.1-3.4
1	05-165		Errors of numerical operations. Recursion.	Ferziger/Peric, ch 1-2
	05-Feb		Recitation - MATLAB Review	
2		7-Feb	Truncation errors, Taylor Series and Error analysis. Error propagation and Estimation. Condition numbers.  Roots of non-linear equations – Introduction and Bracketing Methods.	CC: 3.1 - 3.4 CC: 4.1 - 4.4
3	12-Feb		Roots of non-linear equations, Bracketing Methods: Bisection/False Position. Open Methods: Open-Point Iteration/Newton-Raphson/Secant methods, Extension to systems of equations	CC: 5.1 -5.4, CC: 6.1 - 6.5
	12-Feb		Recitation – Review selected from: Navier-Stokes Equations and their approximations. Conservation Laws, Material Derivative, Reynolds Transport Theorem, Constitutive equations. Compressible and incompressible flows, vorticity, Euler's equations, potential flows and (boundary) integral equations.	Ferziger/Peric, ch 1 Cohen/Kundu ch4 White ch 4
4		14-Feb	Systems of Linear Equations: Motivations and Plans, Direct Methods, Gauss Elimination	CC: 9.1 - 9.8
	No class		Tuesday with Monday schedule of classes to be held	
5		21-Feb	Systems of linear equations. Gaussian elimination (special cases, multiple right hand sides). LU decomposition and factorization, Pivoting. Error analysis for linear systems. Operations counts	CC: 9.1 - 9.8 CC: 10.1 -10.3 Ferziger/Peric, ch 5
6	26-Feb		Systems of linear equations. Special Matrices: LU Decompositions, Tridiagonal systems, General Banded Matrices, Symmetric, positive-definite Matrices. Introduction to iterative Methods	CC: 10.1 -10.3 CC: 11.1
	26-Feb		Recitation - Examples	
7		28-Feb	Systems of linear equations. Iterative Methods: Jacobi's method, Gauss-Seidel iteration, Convergence, Successive Over-Relaxation Methods, Gradient Methods, Stop Criteria	CC: 11.1-11.3

8	05-Mar	End of systems of linear equations: Gradient methods, Pre-conditioning. Krylov Methods. Finite-Differences (FD): Classification of PDEs and examples, Error Types and discretization properties. Consistency, Truncation error, Error equation, Stability, Convergence	CC: 14, CC: PT 8.1-2, CC: 23, 18 Ferziger/Peric, ch 3
	05-Mar	Recitation - Examples	
9	07-Mar	FD schemes: Finite difference based on Taylor Series for higher order accuracy differences. Taylor tables or method of undetermined coefficients	CC: 23, 18 Ferziger/Peric, ch 3 Lomax et al, ch 3
10	12-Mar	Polynomial approximations and higher-order (Newton, Lagrange, Hermite and Pade Schemes), Iterative improvements and extrapolations, Boundary conditions, Non-Uniform Grids, Grid refinement	CC: 23, 18 Ferziger/Peric, ch 3 Lomax et al, ch 3
	12-Mar	Recitation - Examples	
	14-Mar	Quiz 1	
11	19-Mar	Fourier Error Analysis, Introduction to Stability: Heuristic, Energy and Von Neumann methods, Hyperbolic PDEs, Characteristics	Lapidus/Pinder ch 4.5 Ferziger/Peric, ch 3 Lomax et al, ch 3
	19-Mar	Recitation – Examples	
12	21-Mar	Stability, CFL condition and Von Neumann stability, Hyperbolic/Elliptic/ Parabolic equations revisited, Crank-Nicholson, ADI methods	CC: 29, 30 Lapidus/Pinder, ch 4.5 Ferziger/Peric, ch 3 Lomax et al, ch 3
	25-29 Mar: Spring Vac.		
13	02-Apr	Finite Volume (FV) Methods: Introduction, Approximations and basic elements of a FV scheme, Surface integrals and volume integrals, Examples	CC: 29.4 Ferziger/Peric, ch 4 Lomax, ch 5 Cebici et al, ch 5.6
	02-Apr	Recitation – Examples	,
14	04-Apr	FV Methods: Interpolations and differentiations, Special advection schemes (Donor Cell, Flux-corrected transport, WENO).	CC: 29.4 Ferziger/Peric, ch 4 Lomax, ch 5 Cebici et al, ch 5.6
15	09-Apr	Solutions of the Navier Stokes Eqs.: Introduction, incompressible and compressible, convective and viscous terms, pressure term	Ferziger/Peric, ch 7 Cebeci et al, ch 11 Fletcher, ch 17
	09-Apr	Recitation – Examples using the FV method	
16	11-Apr	Solutions of the Navier Stokes Eqs.: Pressure Correction and Projection Methods, Fractional step Methods	Ferziger/Peric, ch 7 Cebeci et al, ch 11 Fletcher, ch 17
	16-Apr No Class	Holiday – Patriots Day	
17	18-Apr	Solutions of the Navier Stokes Eqs.: Vorticity, Artificial compressibility and other methods, Boundary Conditions.  Methods for Unsteady Problems. Time Marching Methods: Introduction.	Ferziger/Peric, ch 6 & 7 Lomax et al, ch 6 CC: 25
	23-Apr	Quiz 2	
18	<b>25-Apr</b> Drop date	Methods for Unsteady Problems. Time Marching Methods. Ordinary differential equations (ODEs). Initial value problems (IVPs). Euler's method. Runge-Kutta methods.	Ferziger/Peric, ch 6 Lomax et al, ch 6 CC: 25.1 - 25.5
19	30-Apr	Time Marching Cont'd: Higher order ODEs, Stiffness and multistep methods.	CC: 25-26 Ferziger/Peric, ch 6 Lomax et al, ch 6
	30-Apr	Recitation – Special Topics: e.g. ANSYS-Fluent, OpenFOAM	2 2 2 2 2 2 3 2 3 2 3 2 3 2 3 2 3 2 3 2
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20	02-May	Grid Generation and complex geometries.	Ferziger/Peric, ch 8 Cebeci et al, ch 9 Fletcher, ch 13 Thompson et al, 1985
21	07-May	Finite Volume on complex geometries. Finite Element methods: Introduction. Fluid Applications	Ferziger/Peric, ch 8 CC: 31.1 - 31.4 Fletcher, ch 5 Wendt, ch 10 Lohner, ch 4 and ch 8
	07-May	Recitation - – Special Topics: e.g., Inviscid Flow equations: Boundary Element methods. Panel methods. Machine Learning for solving PDEs	Cebeci et al, ch 6 Ferziger/Peric, ch 7 Kundu et al, ch 6
22	9-Ma <b>y</b>	Finite Element methods, cont'd: Continuous Galerkin and (Hybridizable) Discontinuous Galerkin Methods. Spectral Methods.	Cebeci et al, ch 6 Wendt, Ch 10 Lohner, Ch 4 and Ch 8 Hesthaven/Warburton Ern/Guermond
23	14-May	Finite Element methods, end. Turbulent flows: Introduction and models	Ferziger/Peric, ch 9 Cebeci et al, ch 3 Kundu et al, ch 4
	14-May	Recitation – Special Topics: e.g. Boundary Layer equations, ODEs – Boundary value problems. Machine Learning for solving PDEs	Cebeci et al, ch 7 CC: 27.1 - 27.3
24	16-May	Turbulent flows: models and numerical simulations	Ferziger/Peric, ch 9 Versteeg/Malalasekra,
	16-May	Final Project Presentations (Part I): 4-7:00pm in room TBD (we may not need all that time but we have the room reserved if needed).	
	17-May	Final Project Presentations (Part II): 4-6:00pm in room TBD (we may not need that day at all).	