On implementing level-set methods for single-phase fluid flows

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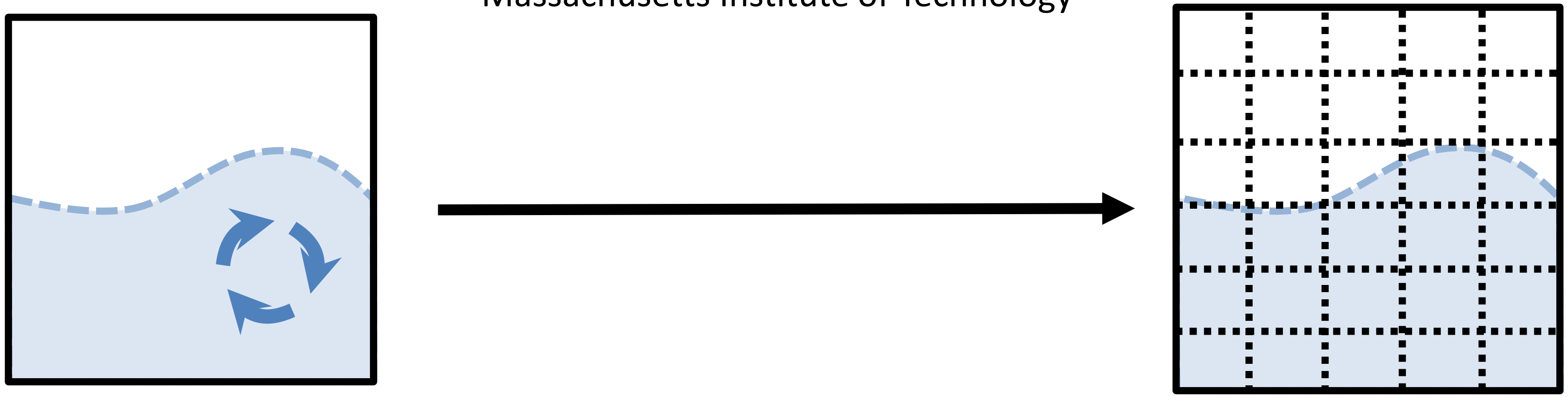

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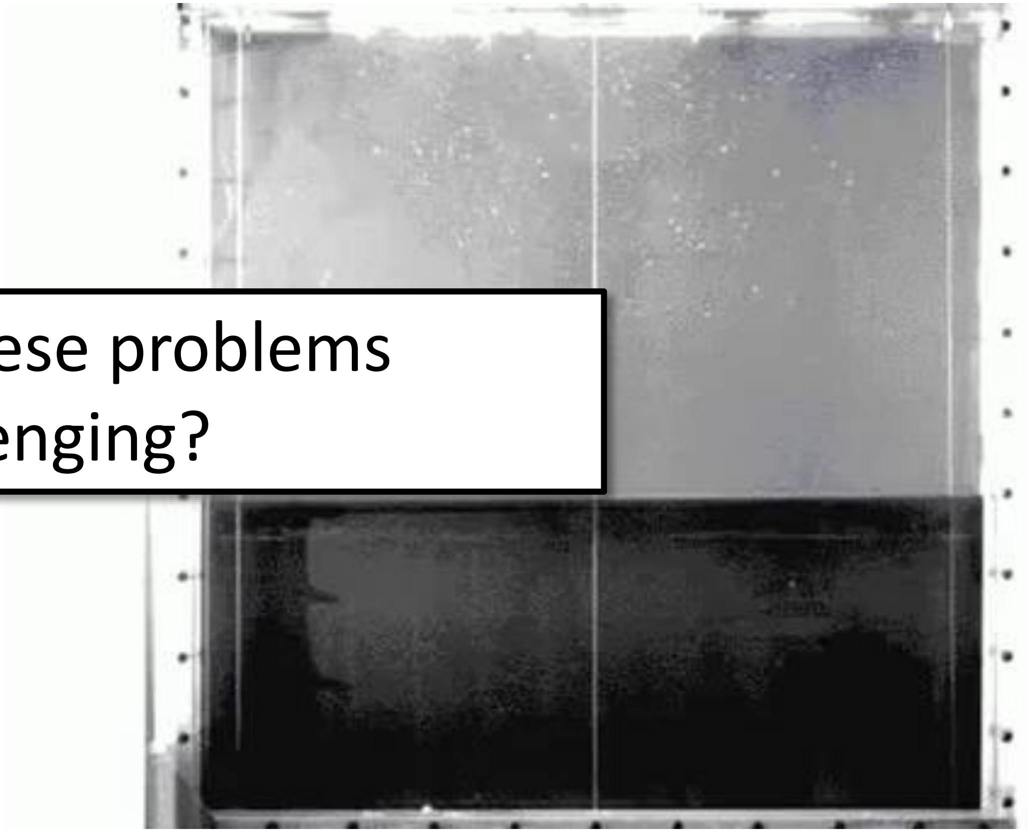
How do we model materials undergoing arbitrarily large deformations?

- Geomechanics
- Ocean Engineering
- Fluid-Structure Interaction
  - Sloshing
  - Aeroelasticity
  - Biomechanics
- Hyper-velocity Impact

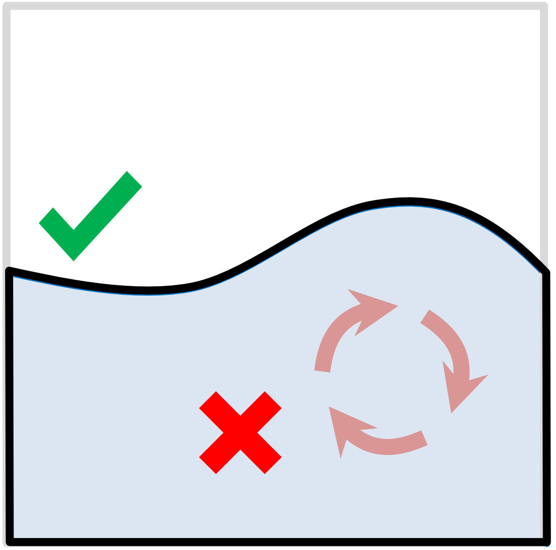
Why are these problems challenging?



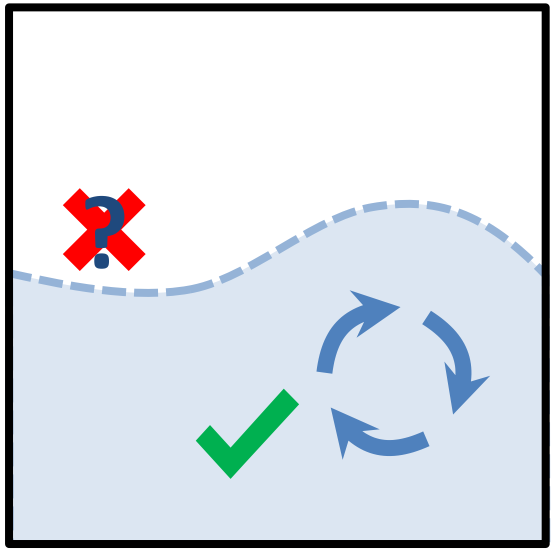
Source: SPH [illegible] (Youtube)

Simple Lagrangian and Eulerian methods cannot guarantee accuracy for these problems

- Lagrangian Methods
  - Computation points move with material
  - Follows deforming boundaries
  - Lose accuracy with large deformations

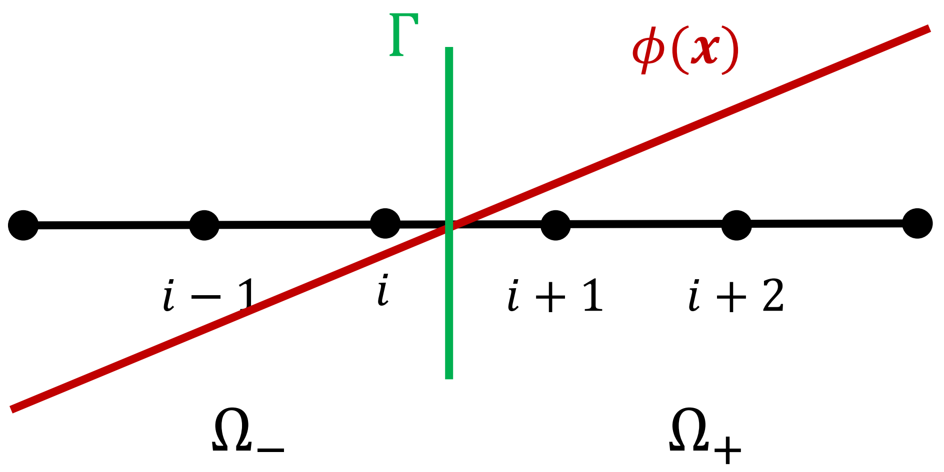


- Eulerian Methods
  - Computation points move with spatial frame
  - Tracks of deforming boundaries
  - Advection avoids mesh entanglement



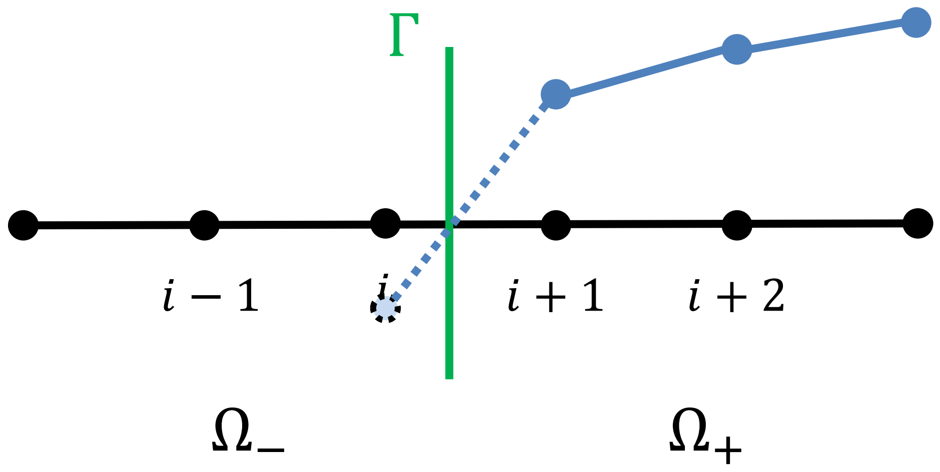
What is a level-set method?

- Level-set methods use signed distance function, ϕ, to 'track' boundary, Γ
- Define Ω_+ and Ω_- domains for fluid and gas (or vacuum)
- Advect ϕ with material motion
  - Impose jump conditions across boundary through time
  - Need to 'reinitialize' periodically
  - Ensures 'sharp' interface



For ‘free surfaces’ enforce $p = 0$ on Γ

- How do we enforce boundary conditions on Γ?
  - Use ‘Ghost Fluid Method’ summarized in Gibou et al. (2018)
  - Use solution on one side of boundary to estimate properties on other side



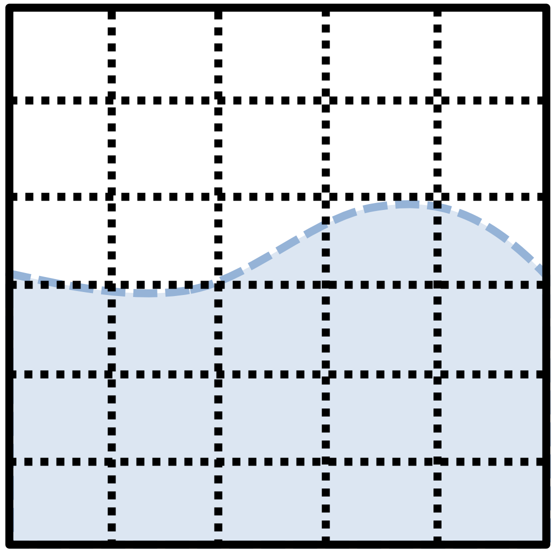
Discretize Navier-Stokes eq'ns using FV method

- Begin with incompressible Navier-Stokes equations

$$\frac{\partial \rho u_i}{\partial t} + \frac{\partial \rho u_i u_j}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \frac{\partial \tau_{ij}}{\partial x_j} + \rho g_i$$

$$\frac{\partial u_i}{\partial x_i} = 0$$

- Discretize into set of finite volumes



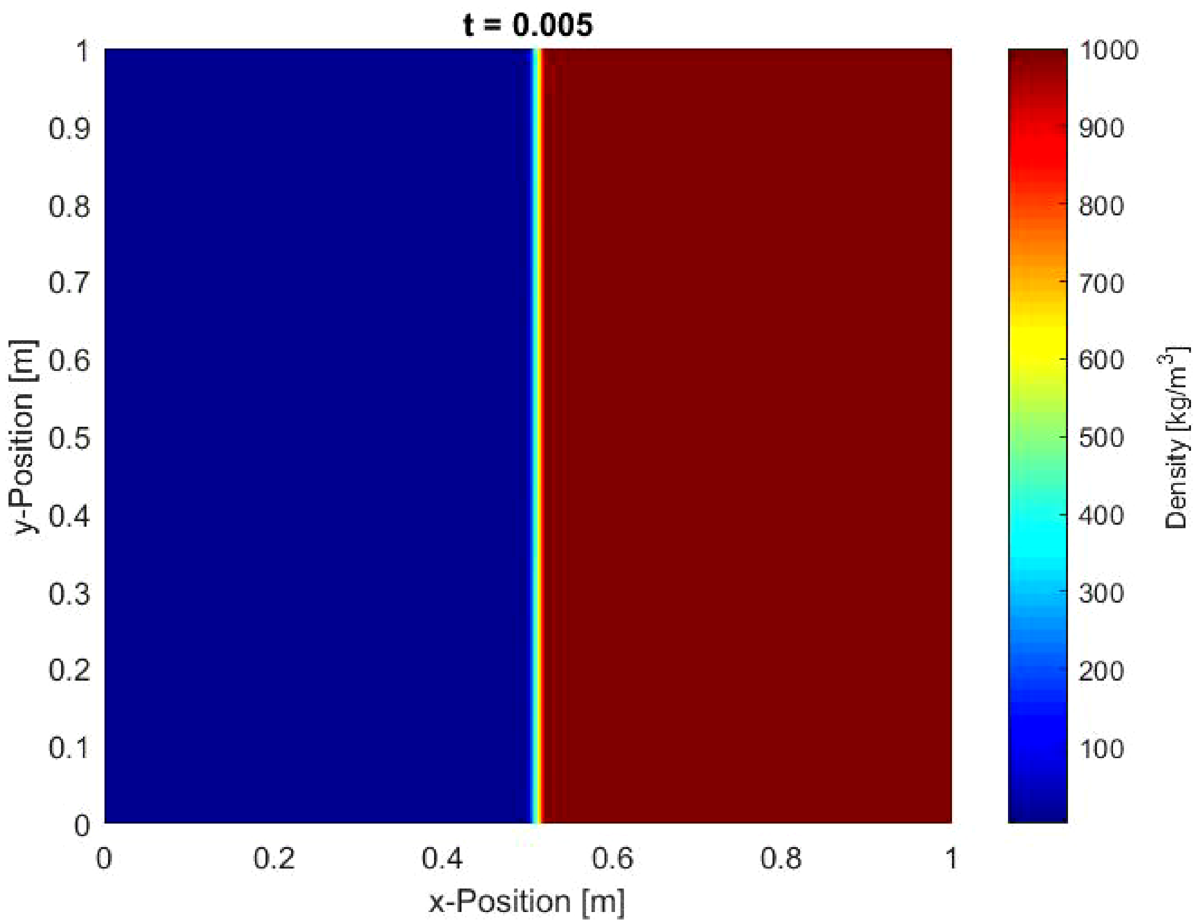
$$\frac{d}{dt}\int_{\Omega_i} \rho u_i dv = -\int_{\partial\Omega_i} \rho u_i u_j n_j + p n_i - \tau_{ij} n_j dS + \int_{\Omega_i} \rho g_i dv$$

$$\int_{\partial\Omega_i} u_i n_i dS = 0$$

Test Case: Lock-Exchange with two fluids

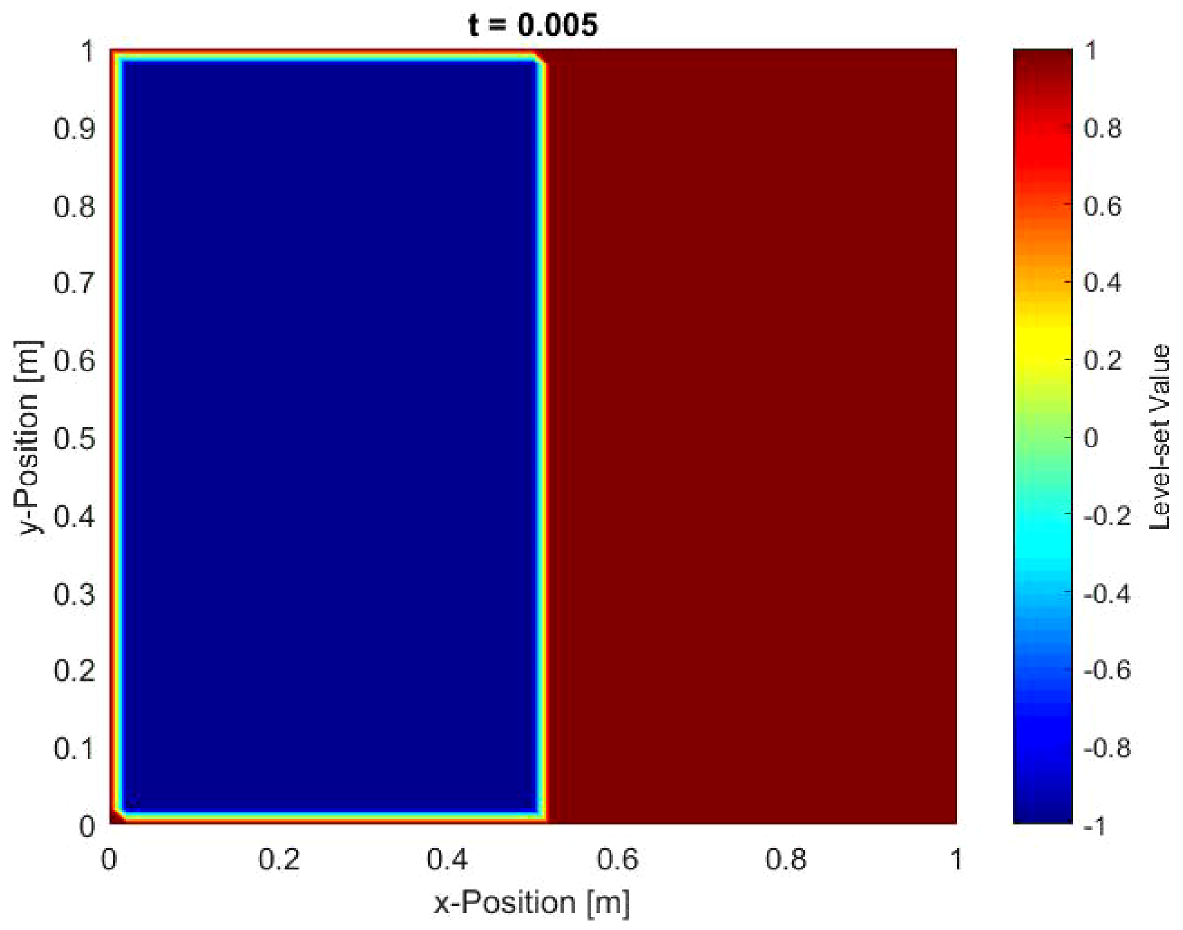
- Confirms implementation ‘works’
  - Unknown source of oscillations, need to investigate further

- Numerical diffusion from low-order advection scheme dominates solution at long time-scales



Level-set Problem: Lock-Exchange with single fluid

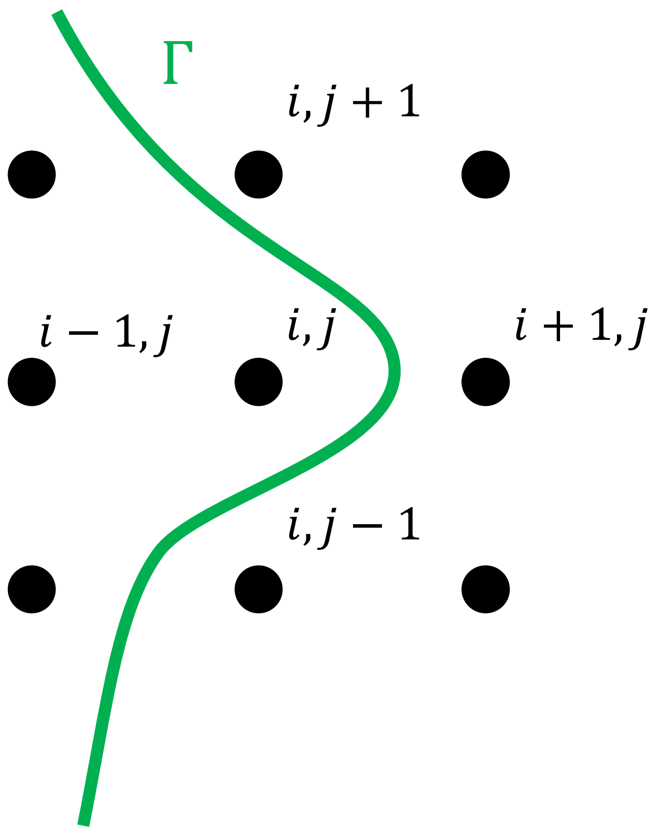
- First several time-steps match test case with two fluids
- As level-set field ϕ is advected, spurious oscillation develop near boundary
  - Unknown cause
  - Stability in first several steps suggests coupled to topology of boundary Γ



What's going wrong?

- Still investigating source of error
  - Is it intrinsic to my N-S solver?
  - Is it a coupling issue with boundaries of domain?
  - Is it a problem with GFM?

- I have several theories, but no solutions yet



Questions?

- For the audience: Can anyone identify what's wrong?

- References:

  - Gibou, Frederic, Ronald Fedkiw, and Stanley Osher. "A review of level-set methods and some recent applications." *Journal of Computational Physics* 353 (2018): 82-109.
  - Carrica, P. M., R. V. Wilson, and F. Stern. "An unsteady single-phase level set method for viscous free surface flows." *International Journal for Numerical Methods in Fluids* 53.2 (2007): 229-256.
  - Frolkovič, Peter, and Karol Mikula. "Flux-based level set method: A finite volume method for evolving interfaces." *Applied numerical mathematics* 57.4 (2007): 436-454.