Electrified Droplet Dynamics using the Level-Set Method

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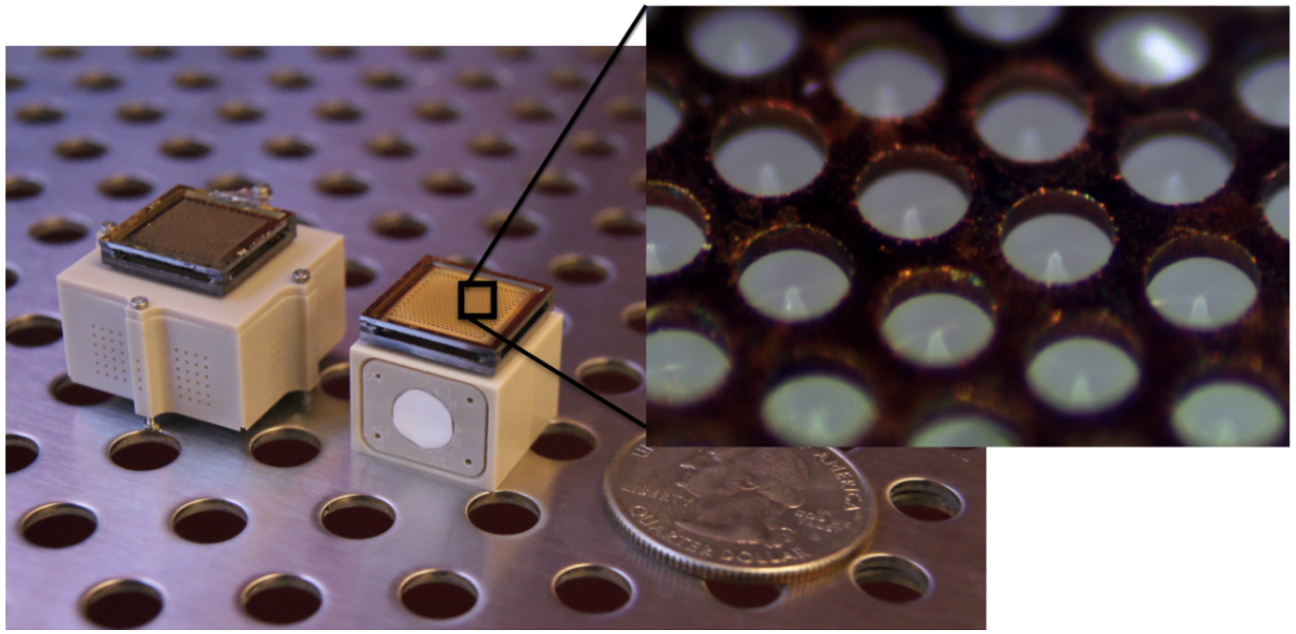
Motivation: Electrospray Propulsion



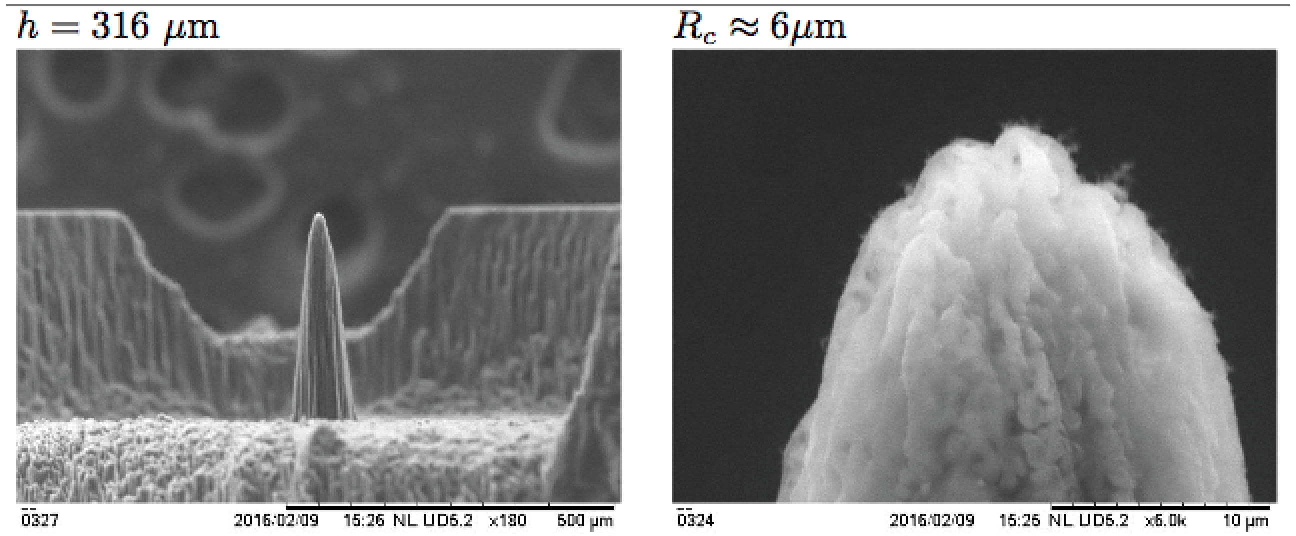
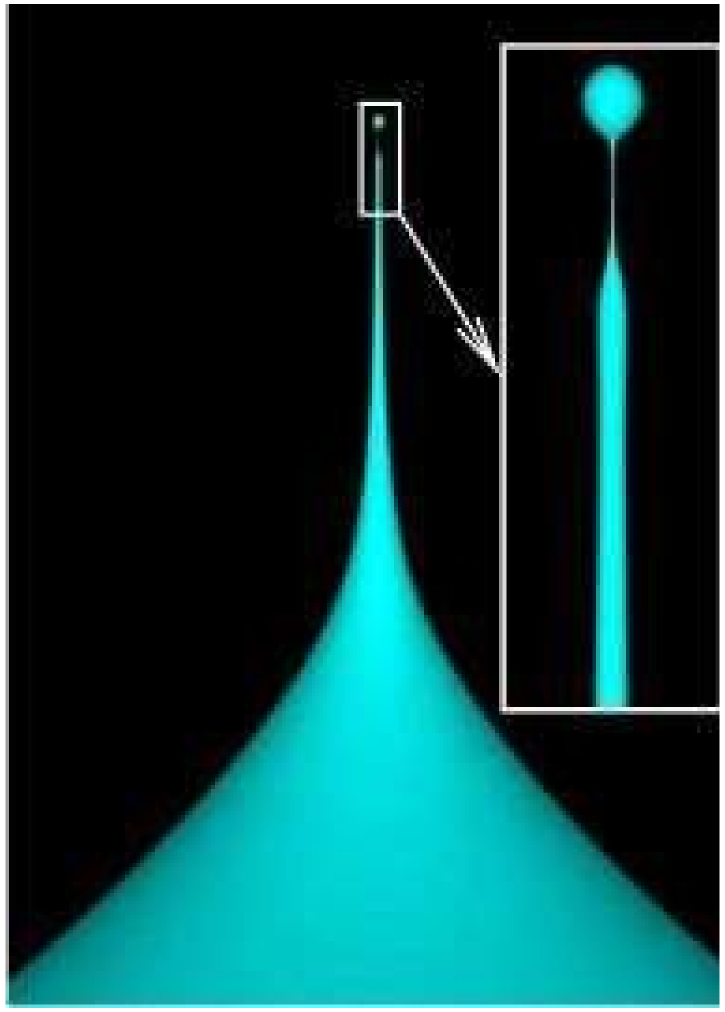


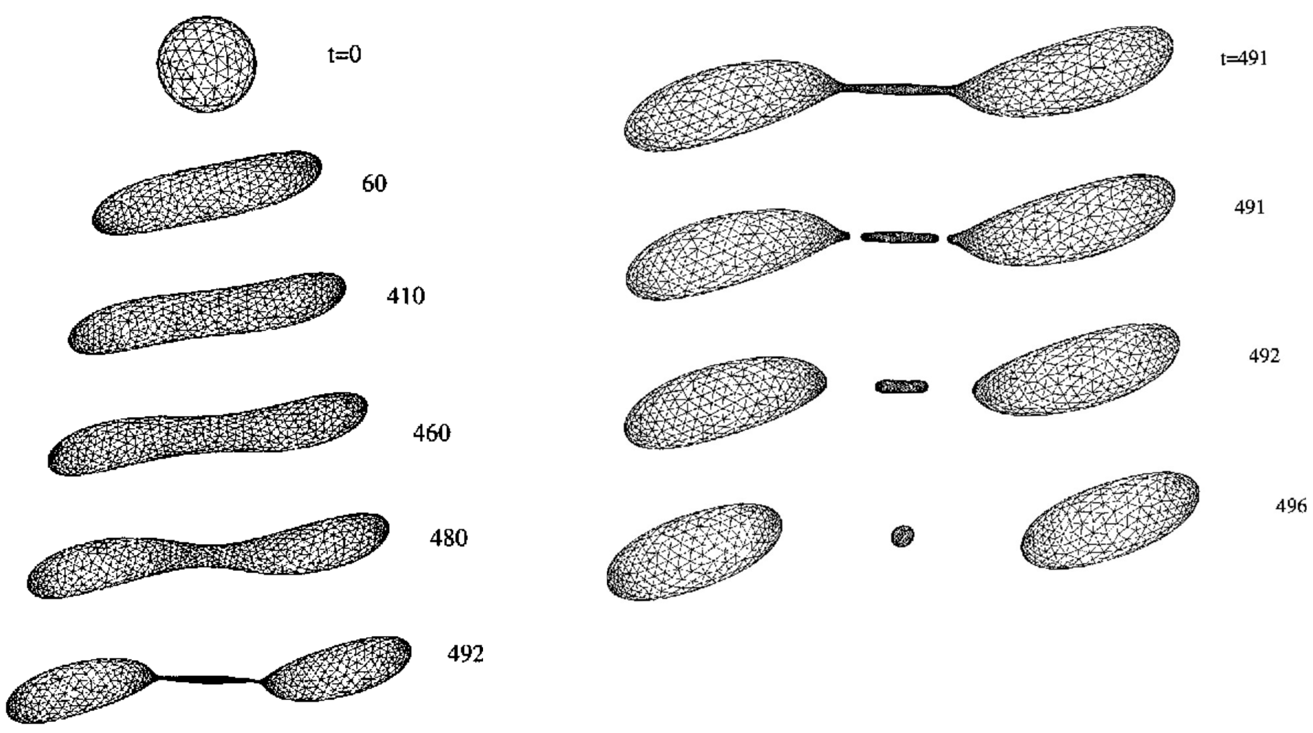
Image credits: MIT SPL Lab

https://www.youtube.com/watch?v=9iwjrx_NSAc



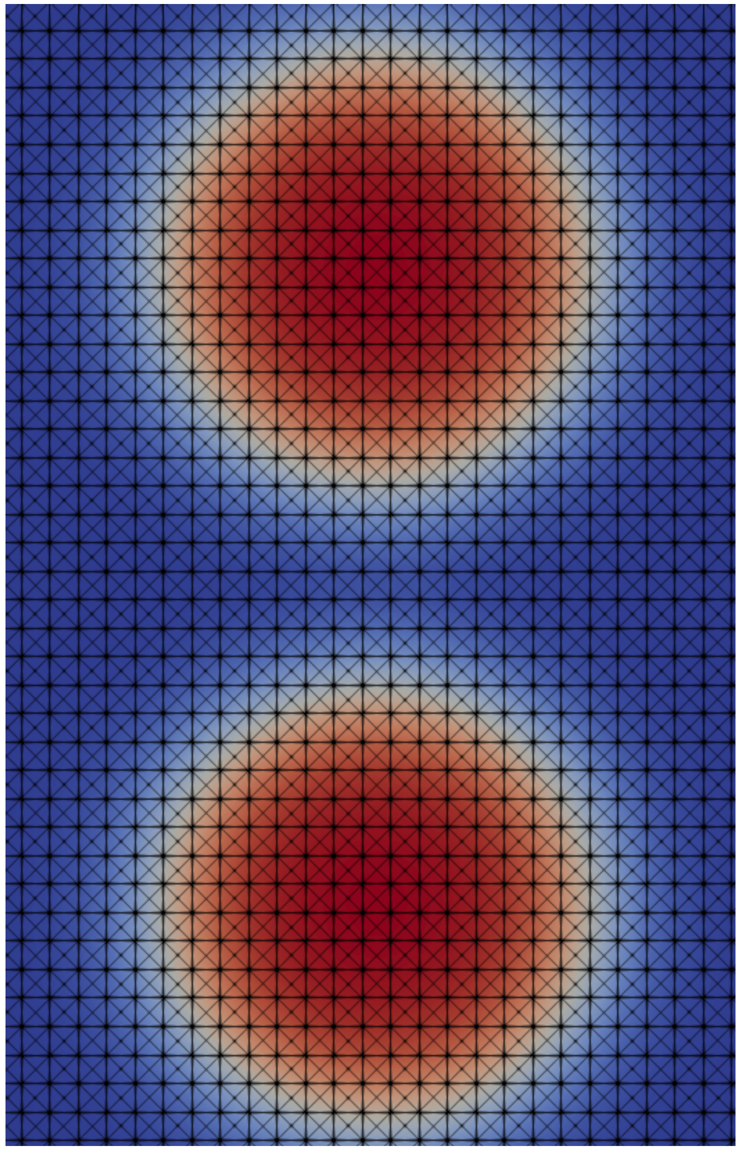
Main challenges: Capillary Phenomena

- Very challenging with a moving mesh.

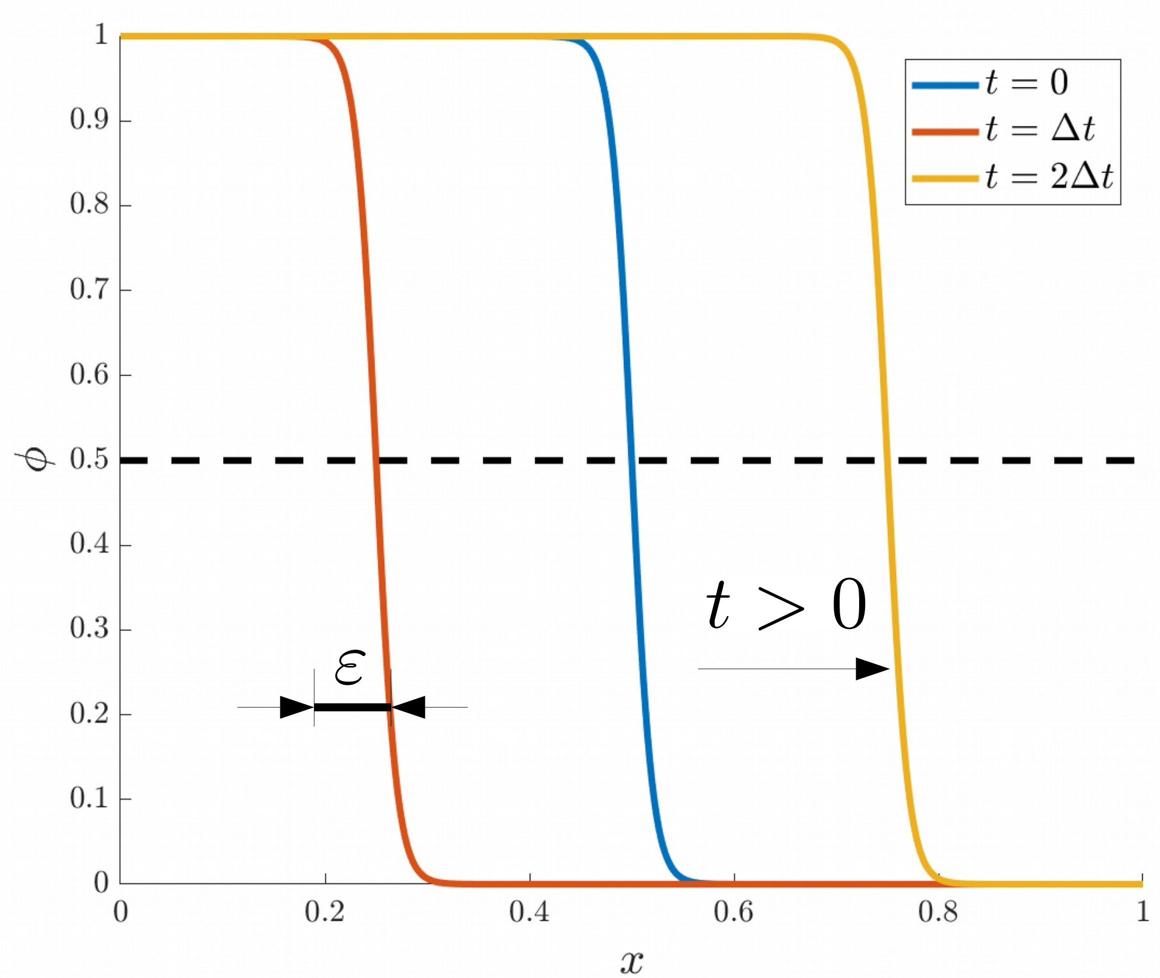


Cristini, Vittorio, Jerzy Blawzdziewicz and Michael Loewenberg. “An adaptive mesh algorithm for evolving surfaces: simulation of drop breakup and coalescence.” (2001).

- Static mesh. Interface embedded between grid-points.



Level Set Overview



Advect solution with velocity field (calculated from Navier-Stokes).

$$\frac{\partial \phi}{\partial t} + u\frac{\partial \phi}{\partial x} = 0$$

Problem:

$$\phi(x,t) = \phi_0(x-ut) + \delta(x-ut)$$

How do we stabilize the PDE?

$$\frac{\partial \phi}{\partial t} + u\frac{\partial \phi}{\partial x} = \varepsilon\frac{\partial^2 \phi}{\partial x^2} - \frac{\partial \phi(1-\phi)}{\partial x}$$

If $\phi_0(x) = \frac{1}{1+e^{-x/\varepsilon}}$ then $\phi(x,t) = \phi_0(x-ut)$

In 2D,3D ⟶ $$\frac{\partial \phi}{\partial t} + \nabla\cdot(\vec{u}\phi) = \varepsilon\nabla\cdot((\nabla\phi\cdot\vec{n})\vec{n}) - \nabla\cdot(\phi(1-\phi))\vec{n}$$

Incremental Navier-Stokes Equations with Surface Tension and Electric Fields

$$\frac{1}{\Delta t}(\rho\vec{u}_*^{n+1} - \rho\vec{u}^n) + \nabla\cdot(\rho\vec{u}_*^{n+1}\vec{u}) = -\nabla p^n + \nabla\cdot\left(\mu\left(\nabla\vec{u}_*^{n+1} + (\nabla\vec{u}^{n+1})^T\right)\right)$$

$$+\sigma\kappa^{n+1}\delta(\phi - \mathbf{1/2}) - \frac{\epsilon_0}{2}\left(\frac{(\epsilon\vec{E}^{n+1}\cdot\vec{n})^2}{\epsilon_{in}\epsilon_{out}} + (\vec{E}^{n+1}\cdot\vec{t})^2\right)(\epsilon_{in} - \epsilon_{out})\delta(\phi - \mathbf{1/2})$$

Surface Tension

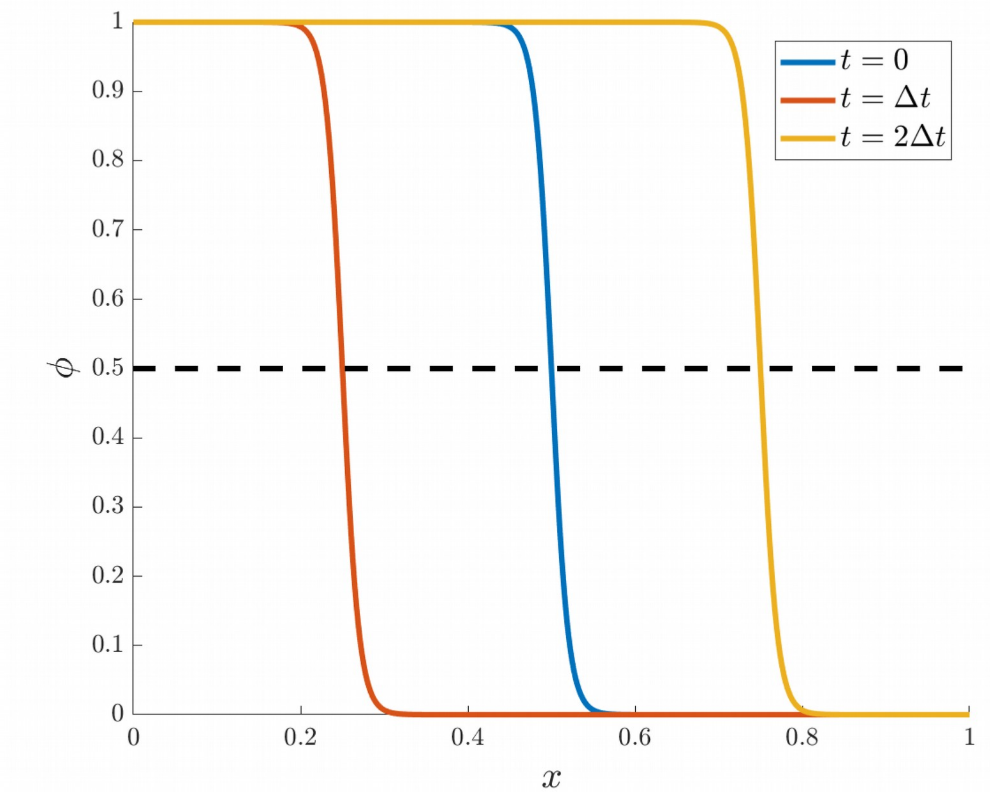
Electric Stress

$$\delta(\phi - \mathbf{1/2}) \approx \nabla\phi$$

$$\rho = \phi\rho_{in} + (1-\phi)\rho_{out}$$

$$\mu = \phi\mu_{in} + (1-\phi)\mu_{out}$$

...



Algorithm (Modified from Olsson et al. 2007)

For $n = 1$ to $n = N$:

Level set update

1. Find provisional Level Set with previous velocity:

$$\frac{\phi_*^{n+1} - \phi_*^n}{\Delta t} + \nabla \cdot (\vec{u}^{n-1}\phi_*^{n+1}) = 0$$

2. Calculate normal vector:

$$\vec{\mathrm{n}} = \nabla\phi_*^{n+1}$$

3. Stabilize Level Set:

$$\frac{\phi^{n+1} - \phi_*^{n+1}}{\Delta t} + \nabla \cdot (\phi^{n+1}(1-\phi^{n+1})\vec{n}) = \varepsilon\nabla \cdot \left((\nabla\phi^{n+1} \cdot \vec{n})\vec{n}\right)$$

4. Compute curvature:

$$\kappa^{n+1} = \nabla \cdot \vec{n}$$

Solve Electric problem

5. Solve Weighted Poisson:

$$\nabla \cdot \left(\epsilon_r\epsilon_0\vec{E}^{n+1}\right) = 0$$

Solve Fluid Problem

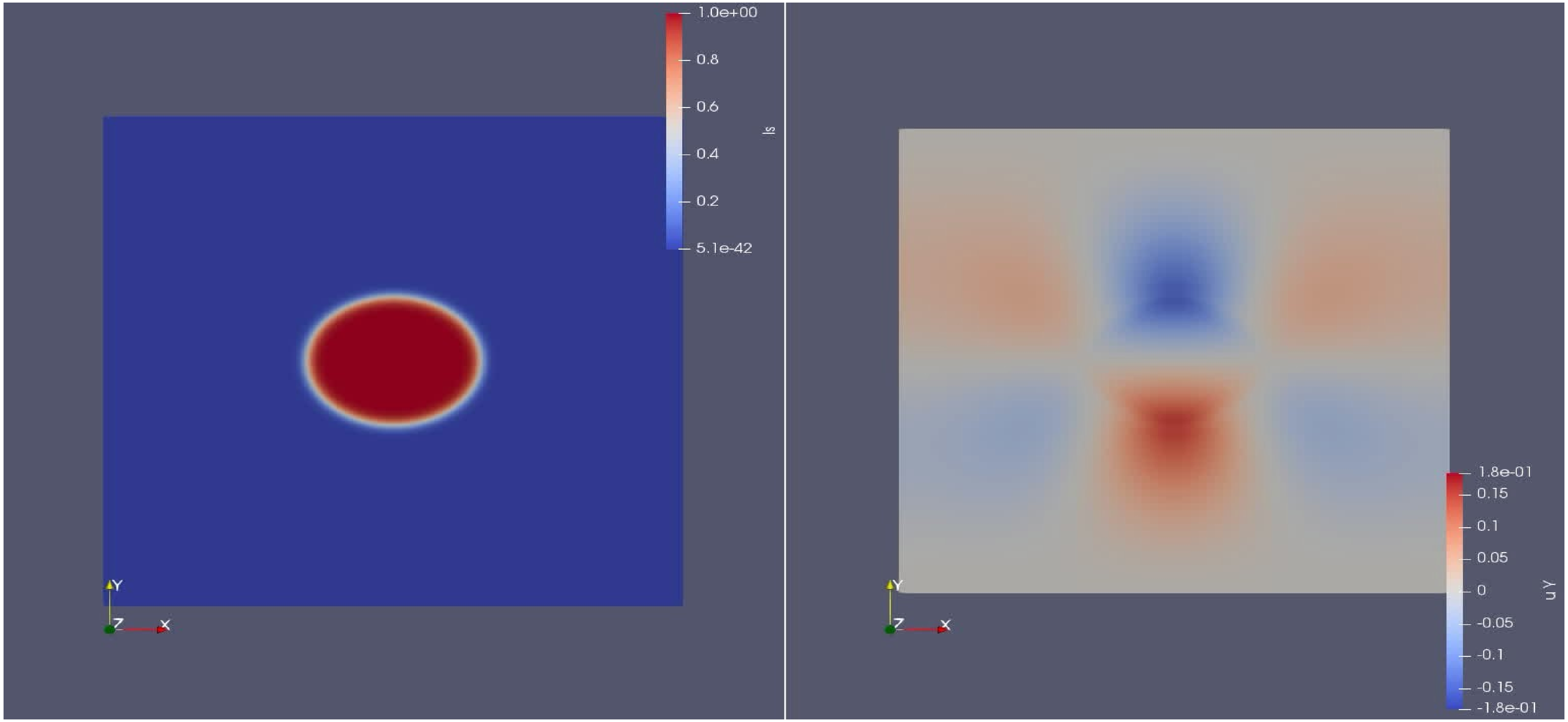
6. Solve Incremental Pressure Navier-Stokes:

- Find $\vec{u}_*^{n+1}$
- Calculate pressure p^{n+1}
- Correct velocity $\vec{u}_*^{n+1}$

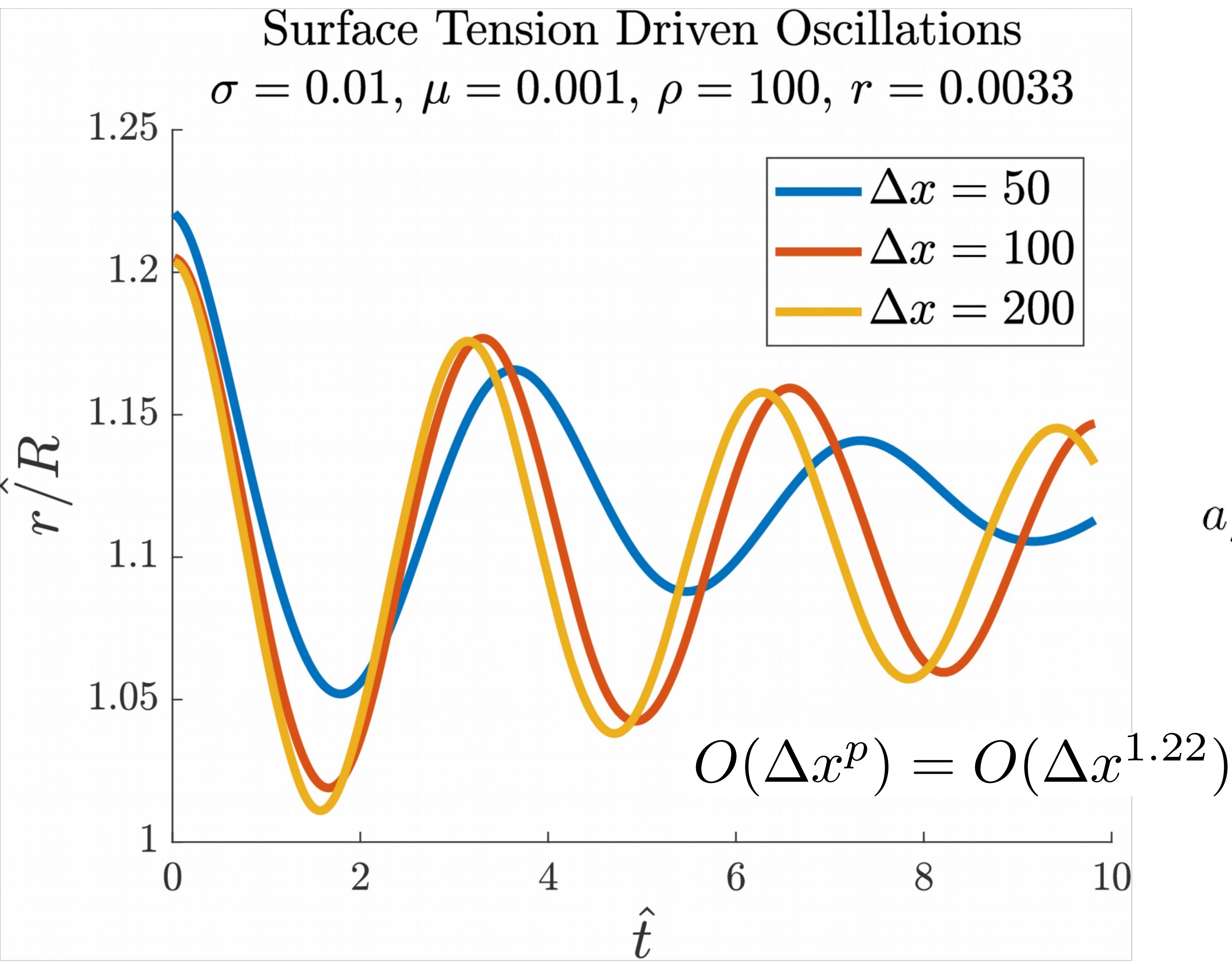
Perturbed droplet under Surface Tension

Level Set

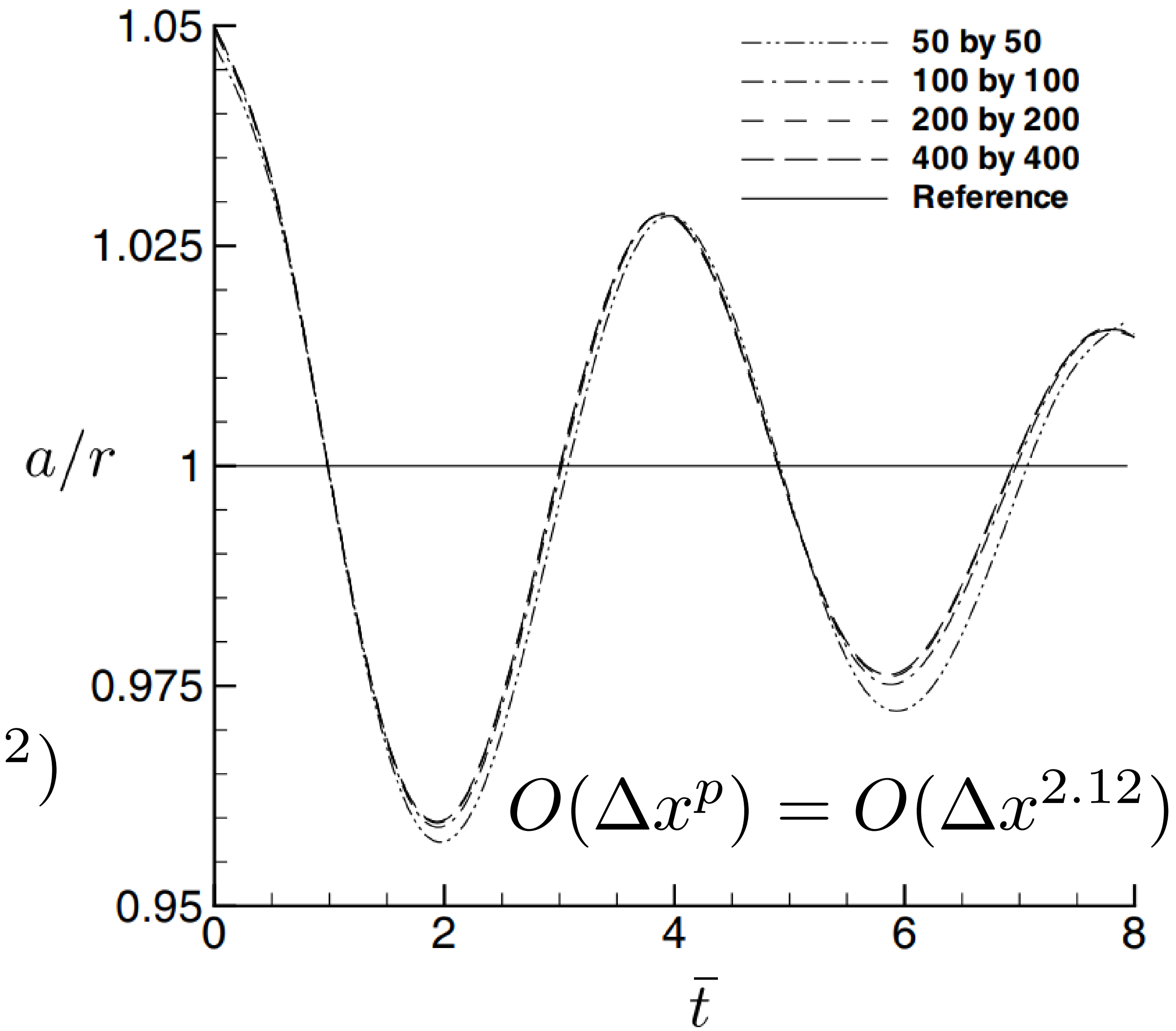
Velocity y direction



Validation

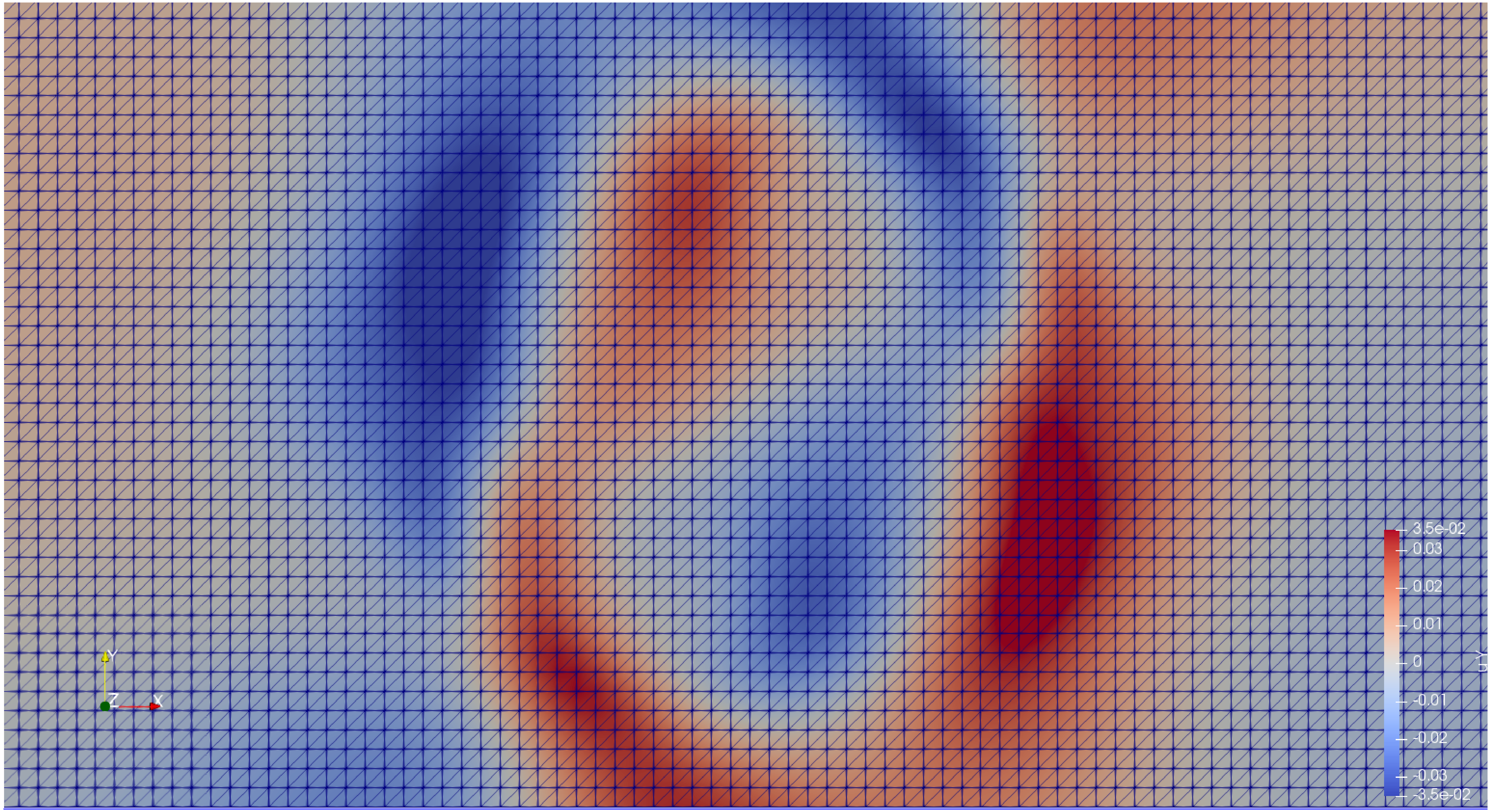


Erik Bjørklund. The level-set method applied to droplet dynamics in the presence of an electric field, Computers & Fluids, Volume 38, Issue 2, 2009.



Mesh directionality influence

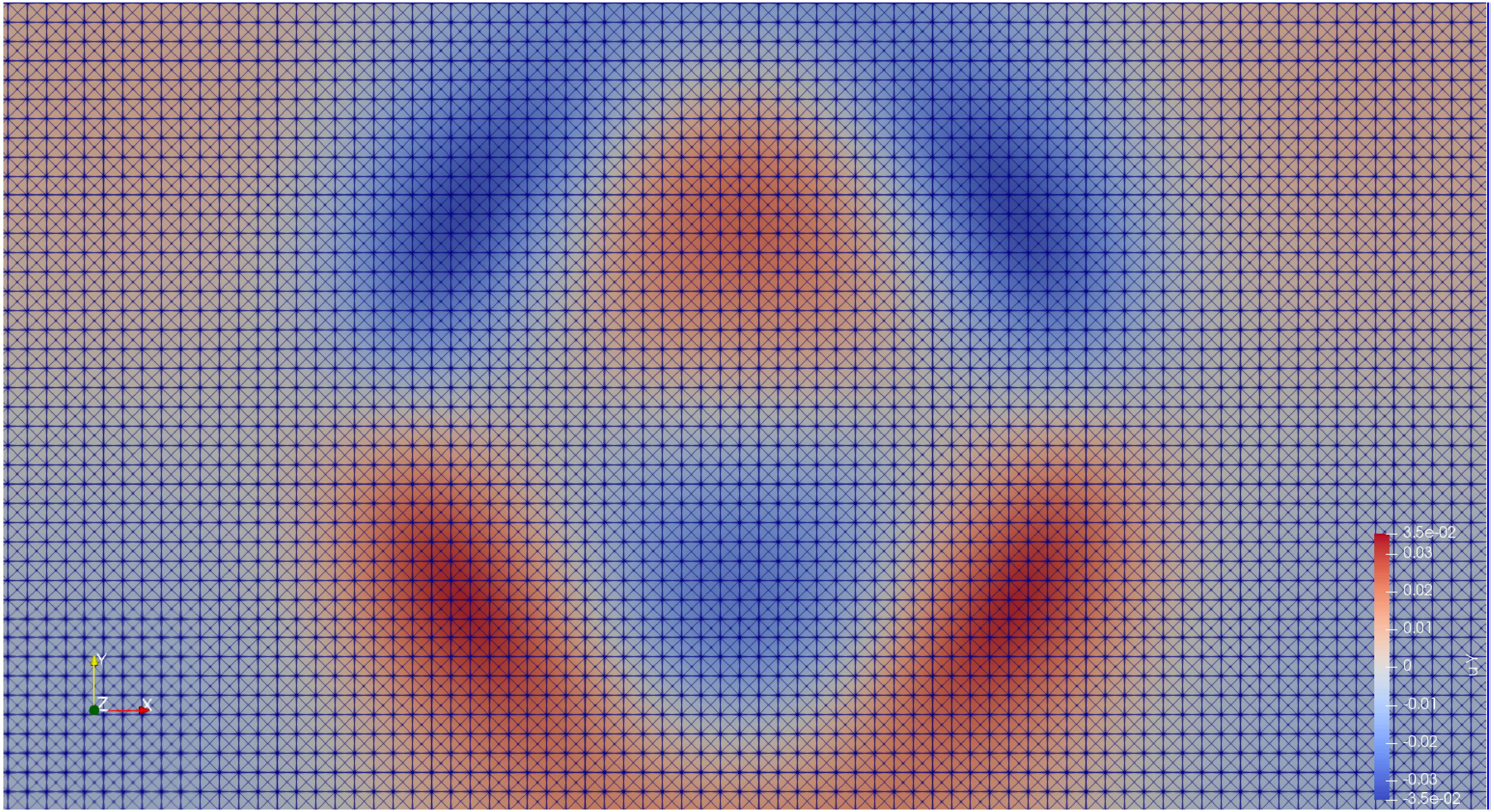
Velocity y direction



IS Units $\sigma = 0.01,\ \rho = 100,\ \mu = 0.001\ E = 1.28 \times 10^5$

Mesh directionality influence

Velocity y direction



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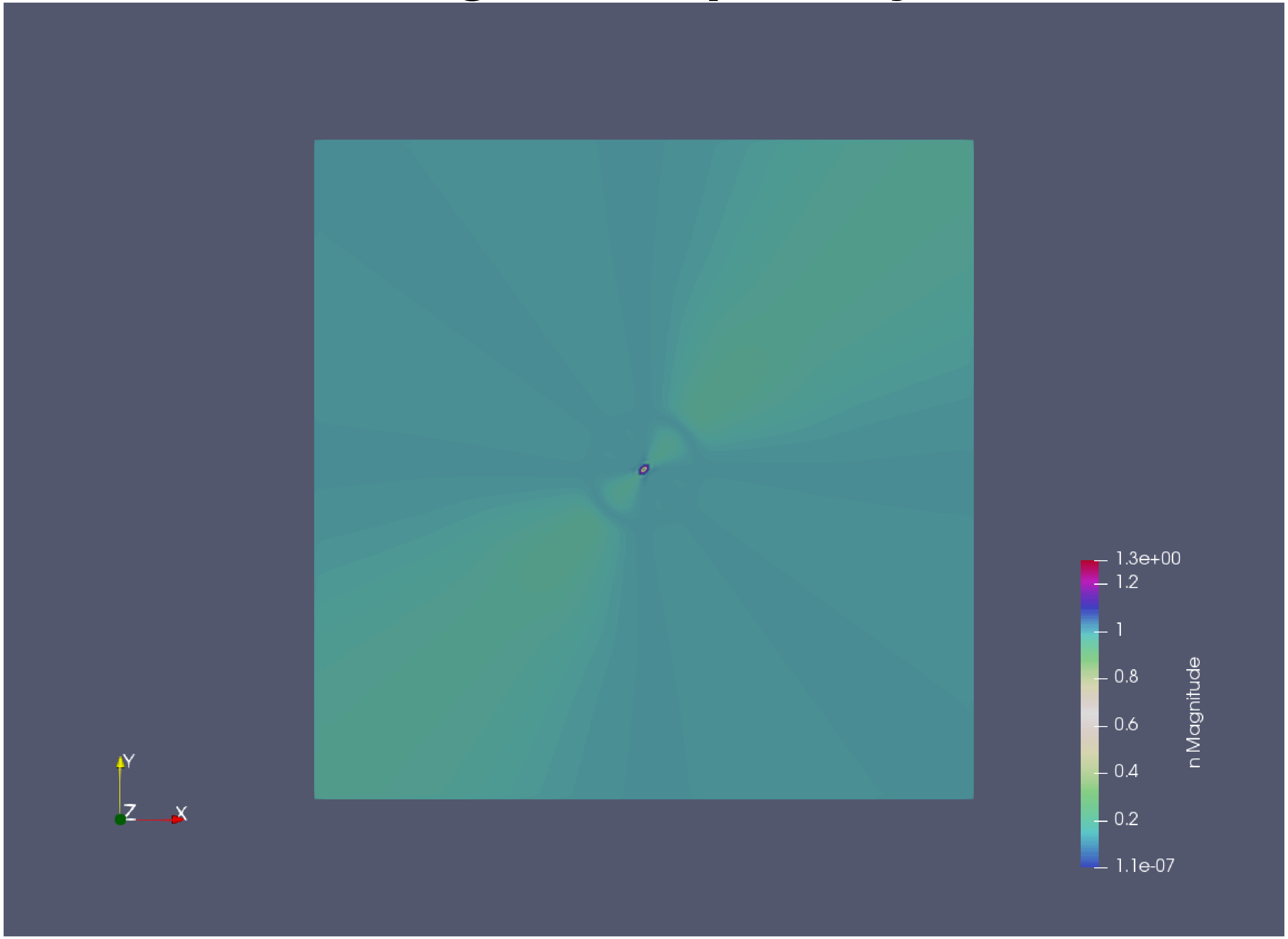
Mesh directionality influence

Normal vector magnitude (non-symmetric mesh)



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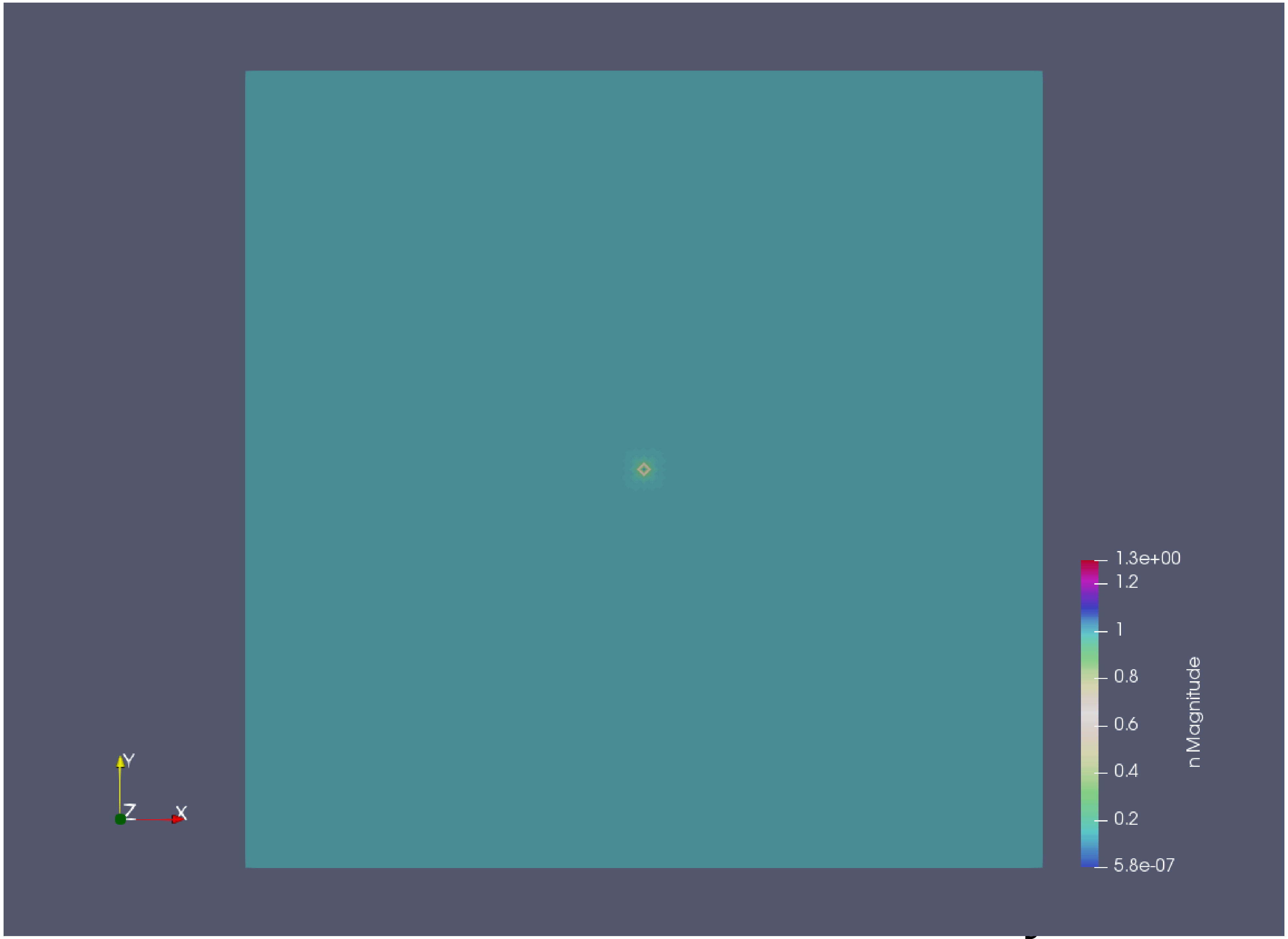
Normal vector magnitude (symmetric mesh)



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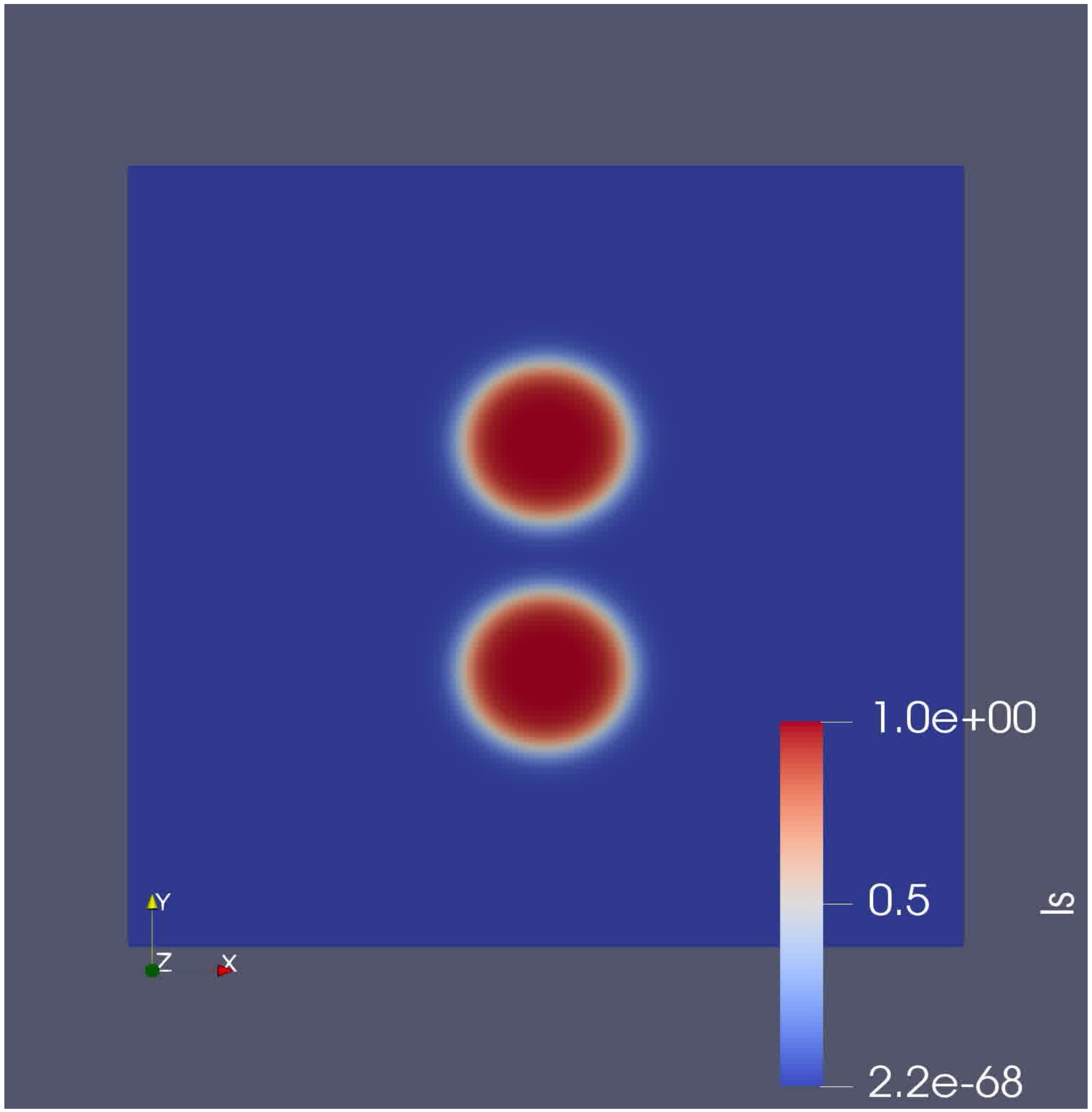
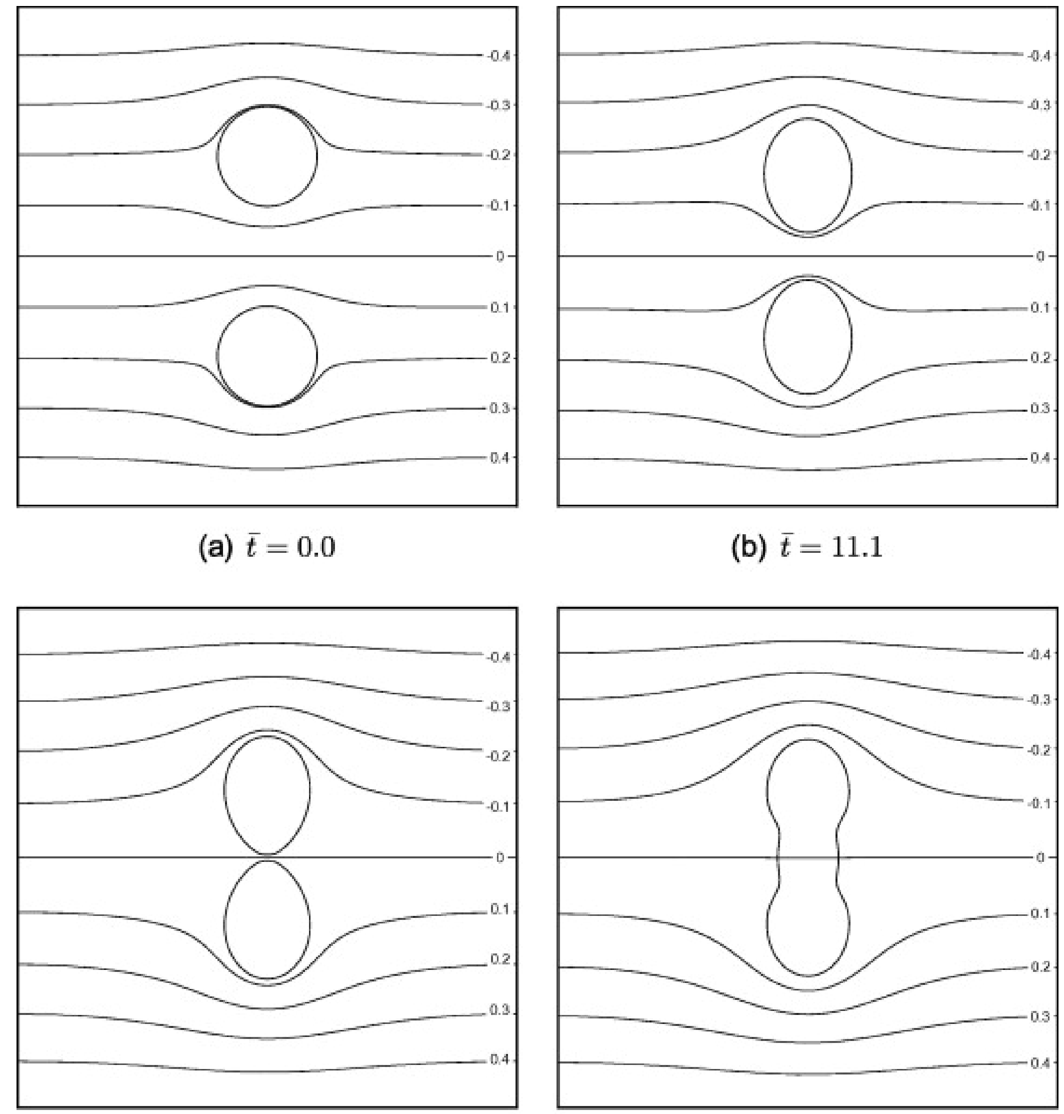
Droplet Coalescence

Level Set





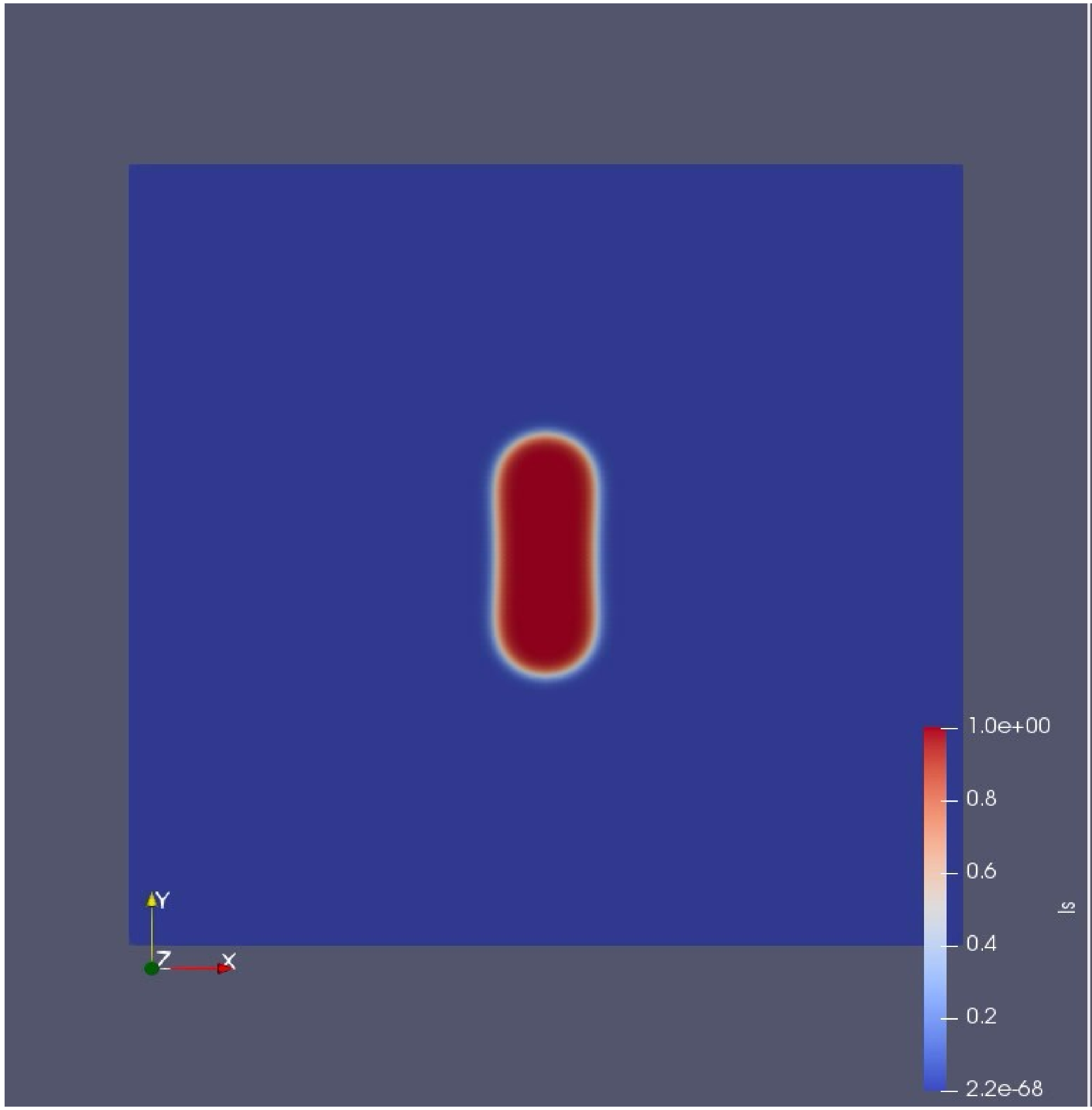
(a) $\bar{t} = 0.0$ (b) $\bar{t} = 11.1$

(c) $\bar{t} = 14.4$ (d) $\bar{t} = 14.8$

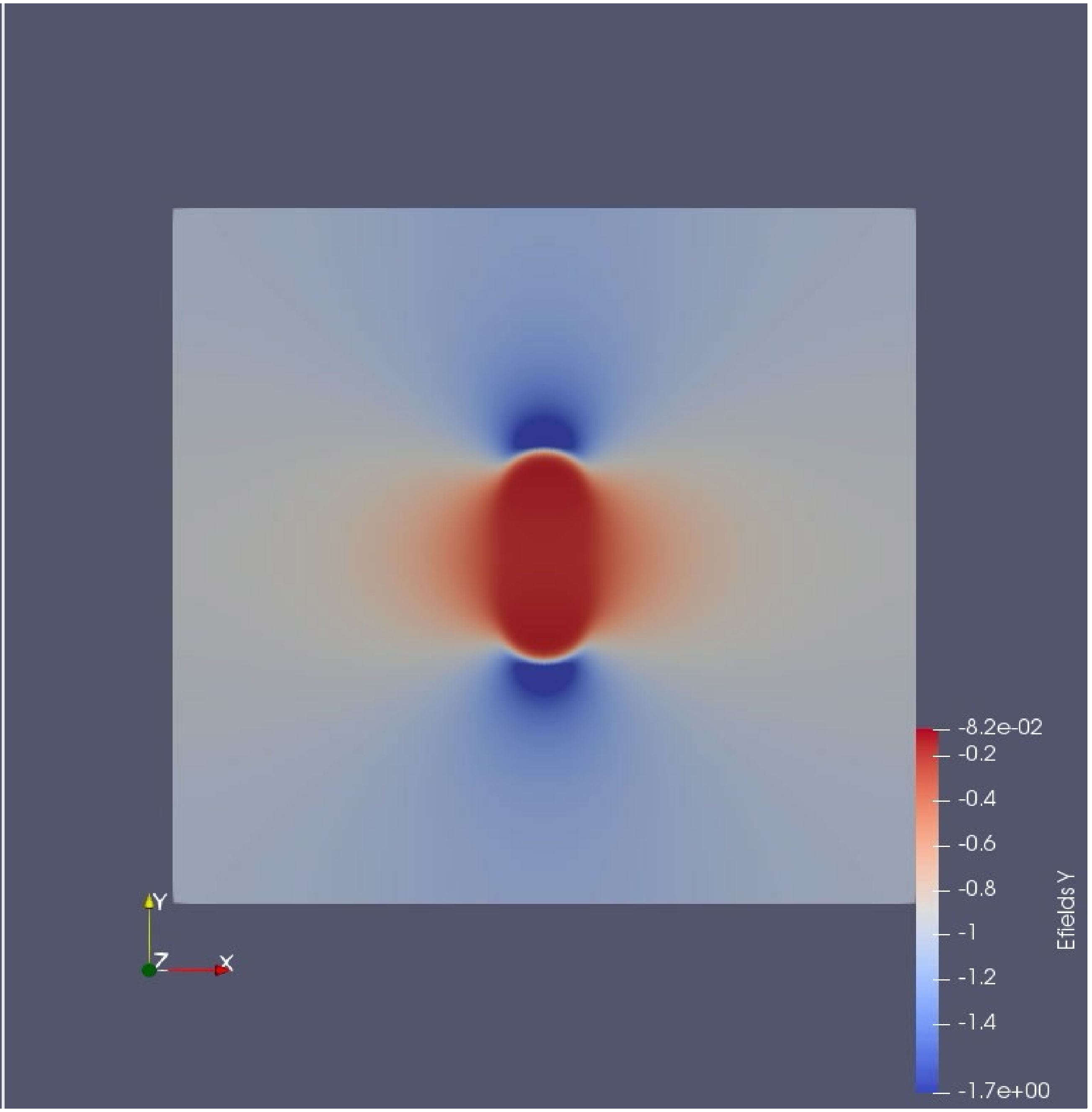
Erik Bjørklund. The level-set method applied to droplet dynamics in the presence of an electric field, Computers & Fluids, Volume 38, Issue 2, 2009.

Droplet Breakup Trial

Level Set



Magnitude of Electric Field



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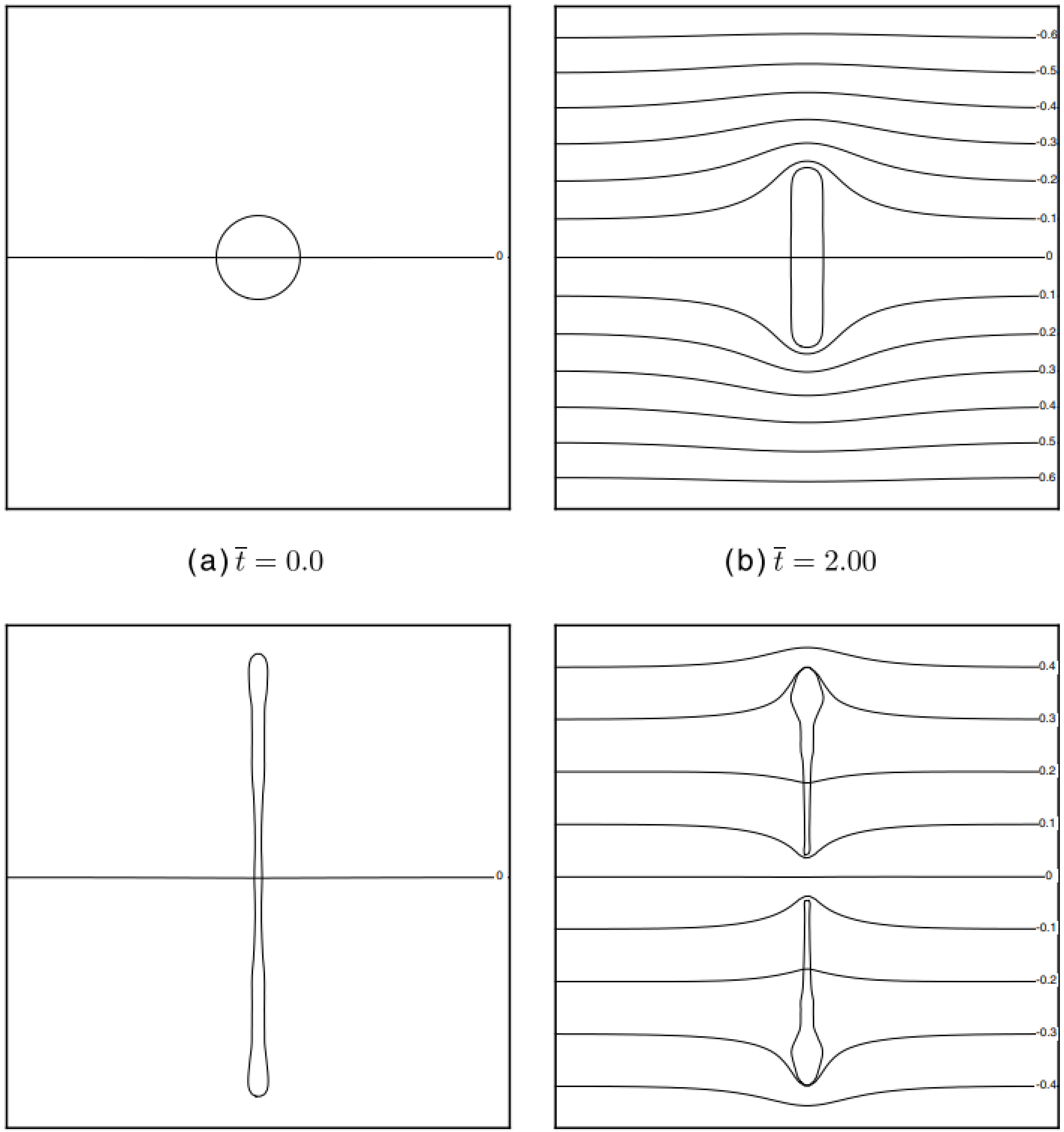

(a) $\bar{t} = 0.0$ (b) $\bar{t} = 2.00$ (c) $\bar{t} = 4.00$ (d) $\bar{t} = 5.00$

Erik Bjørklund. The level-set method applied to droplet dynamics in the presence of an electric field, Computers & Fluids, Volume 38, Issue 2, 2009.

Conclusions and future work

- Method implemented follows literature trends, except the breakup droplet.

-First validation tests show linear convergence, which agrees with the scheme implemented.

-Directionality of shape functions (mesh) matter for accuracy, specifically when solutions are expected to be symmetric.

-Future work: Validation of the implementation of the electric stresses. Exploration of the trade-off ε vs accuracy of stresses.