A Literature Review of Variable Fidelity Methods and their Use in Airfoil Optimization

Laura Yenchesky

2.29 Final Project, Spring 2019

Outline

Aerodynamic Analysis

- What are aerodynamic coefficients for a given surface?
  - C_l Lift coefficient
  - C_d Drag coefficient

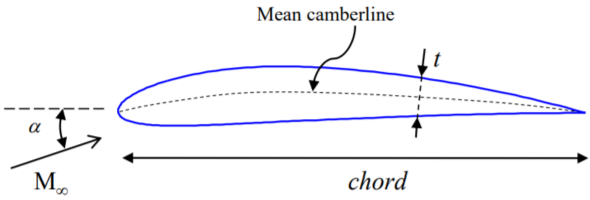

Figure: Sample NACA Airfoil

Aerodynamic Shape Optimization

Objective: use a search algorithm for the design of aerodynamic surfaces and adhere to appropriate constraints

History

- Conjugate-gradient method was rst for 2D airfoil shapes (Hicks et al. 1974)
- Steepest-gradient method for 3D transonic wing design (Hicks and Henne 1978)
- Gradient-based and gradient-free approaches in use now

Gradient-Free vs Gradient-Based

Gradient-Free Approaches

- best for problems with a few design variables
- explore a search space
- exploit design as it approaches the global optimum
- successful in non-smooth design spaces
- requires large number of model evaluations (esp. in large design space)

Gradient-Free vs Gradient-Based

Gradient-Free Approaches

- best for problems with a few design variables
- explore a search space
- exploit design as it approaches the global optimum
- successful in non-smooth design spaces
- requires large number of model evaluations (esp. in large design space)

Gradient-Based Approaches

- applicable to problems with large number of design variables
- requires substantial amount of samples to ensure good accuracy
- cost of gradient calculation can be made nearly independent of number of design variables (with use of adjoint approach)
- considered current state of the art

Surrogate Modeling

- mathematical approximation that mimics the deterministic computationally expensive response or behavior of an original system
- improves global accuracy over entire domain
- approximates to the optimum to locally improve the current design

Surrogate Modeling

Challenges

- accuracy requirements
- computational e ciency
- grid deformations

Surrogate Modeling

Challenges

- accuracy requirements
- computational e ciency
- grid deformations

Existing Categories

- Data Fit Models
- Reduced-Order Models
- Variable Fidelity Models

Variable Fidelity Optimization

- replace a computationally expensive model with a cheap surrogate model
- high- delity model f
- low- delity model c
- # of evaluations of f < # of evaluations of c

Variable Fidelity Optimization

- replace a computationally expensive model with a cheap surrogate model
- high- delity model f
- low- delity model c
- # of evaluations of f $<$ # of evaluations of c
- convergence can be guaranteed with proper local search methods
- correction methods reduce prediction error
- reduces computation e ort signi cantly at extremes of ight envelopes

Outline

Model Setup in Numerical Example

Example VFM High-Fidelity Model Setup

- Geometry: NACA Airfoil
- Flow Equations: steady RANS equations with turbulence model by Spalart and Allmaras
- Grid Generation: Structured curvilinear body- tted C-topology (400,000 mesh cells and 1000 iteration limit)
- Numerical Solver: upwind-biased second-order Roe ux scheme performed in FLUENT; convergence by L^2 norm

Low-Fidelity Model: Coarser mesh and relaxed convergence criteria (32,000 cells and 100 iteration limit)

Model Construction in Numerical Example

Correction Method: Output Space Mapping (linear transformations or mappings)

$$\tilde{X}_d = \text{design variable}$$

$$\tilde{S}_{al} = \text{vector of uncertain variables}$$

$$x = \left[\tilde{X}_d \; \tilde{S}_{al} \right]^T$$

$$C_{l;f}; C_{d;f} = \text{high delty lift and drag}$$

$$f(x) = [C_{l;f}(x) \;\; C_{d;f}(x)]^T$$

$$C_{l;c}; C_{d;c} = \text{low delty lift and drag}$$

$$s(x) = A(x) \quad c(x) = [a_l(x) C_{l;c}(x) + d_1 \quad a_d(x) C_{d;c}(x) + d_d]^T$$

Model Construction in Numerical Example

Response correction parameters

Model Construction in Numerical Example

Least-square optimal solution to the linear regression

Model Construction in Numerical Example

Design variable vector $\tilde{x}_d$ with NACA shape parameters $m; p; t{=}c$

$$\tilde{x}_d = [\, m \; p \; t{=}c \quad]^T$$

$$
\begin{array}{ccc}
0{:}0 & m & 0{:}05 \\
0{:}3 & p & 0{:}7 \\
0{:}08 & t{=}c & 0{:}14 \\
0 & & 2 \\
0{:}7 & M_1 & 0{:}8
\end{array}
$$

with NACA 2412

Results

Results

Total cost / # (design variables)^2

$$N = n^2 + 3n + 2$$

$$N = N_f + N_c = r$$

where r is ratio of high- to low- delity simulation times

Outline

Correction Methods

- Space mapping (used in example)
  - simple to implement
- Multi-level optimization
  - uses multiple models so that each iteration of the algorithm requires a smaller number of model evaluations
  - considered more e cient than SM by Leifsson
- Shape-preserving response prediction
  - works at pressure distribution level (rather than aerodynamic forces directly)
- Weight gradients
  - adjust in uence of linear and multiplicative corrections

Other Approaches in VFM

Data Fusion Techniques

- Kriging
  - method of interpolating values with a Gaussian process
- Co-Kriging
  - uses information from other variables
  - predicts 2500   2 cases in 0.023 seconds
  - picks up viscous phenomena from high delity samples
- Co-Kriging POD
  - data: orthonormal set of basis functions to linear subspace
- Direct Gradient Enhanced Kriging (GEK)
  - incorporates gradients into Kriging
- Generalized Hybrid Bridge Function (GHBF)
  - exploits prediction value in low delity data
- Upgrade key points from low to high delity

Summary

Aerodynamic opt! gradient-based ! surrogate! variable delity

- Relatively low computational cost (less than 30% in provided example)
- Similar results to high- delity
- E ective correction and data fusion techniques

Future E orts

- Development of tool boxes that minimize hand coding
- Identi cation of best practices for data fusion and correction methods

For Further Reading I

Yondo, et al.

A Review of Surrogate Modeling Techniques for Aerodynamic Analysis and Optimization: Current Limitations and Future Challenges in Industry

Advances in Evolutionary and Deterministic Methods for Design, Optimization and Control in Engineering and Sciences, Computational Methods

Springer International Publishing AG 2019

Leisson, L and Koziel, S

Aerodynamic shape optimization by variable- delity computational uid dynamics models: A review of recent progress

Journal of Computational Science, 10 (2015) 45-54.

Martins, J and Kennedy, G

Enabling Large-scale Multidisciplinary Design Optimization through Adjoint Sensitivity Analysis

57th AIAA Aerospace Sciences Meeting, AIAA SciTech Forum, 2019

For Further Reading II

Likeng, et al.
Research on multi- delity aerodynamic optimization methods
Chinese Journal of Aeronautics, 2013, 26(2): 279-286

Zhang, et al.
Variable Fidelity Methods and Surrogate Modeling of Critical Loads on X-31 Aircraft
51st AIAA Aerospace Sciences Meeting including the New Horizons Forum and Aerospace Exposition, 2013

Leifsson, L and Koziel, S.
Low-Cost Robust Airfoil Optimization by Variable-Fidelity Models and Stochastic Expansions
51st AIAA Aerospace Sciences Meeting including the New Horizons Forum and Aerospace Exposition, 2013

Han, et all.
Improving variable- delity surrogate modeling via gradient-enhanced kriging and a generalized hybrid bridge function
Aerospace Science and Technology 25 (2013) 177-189