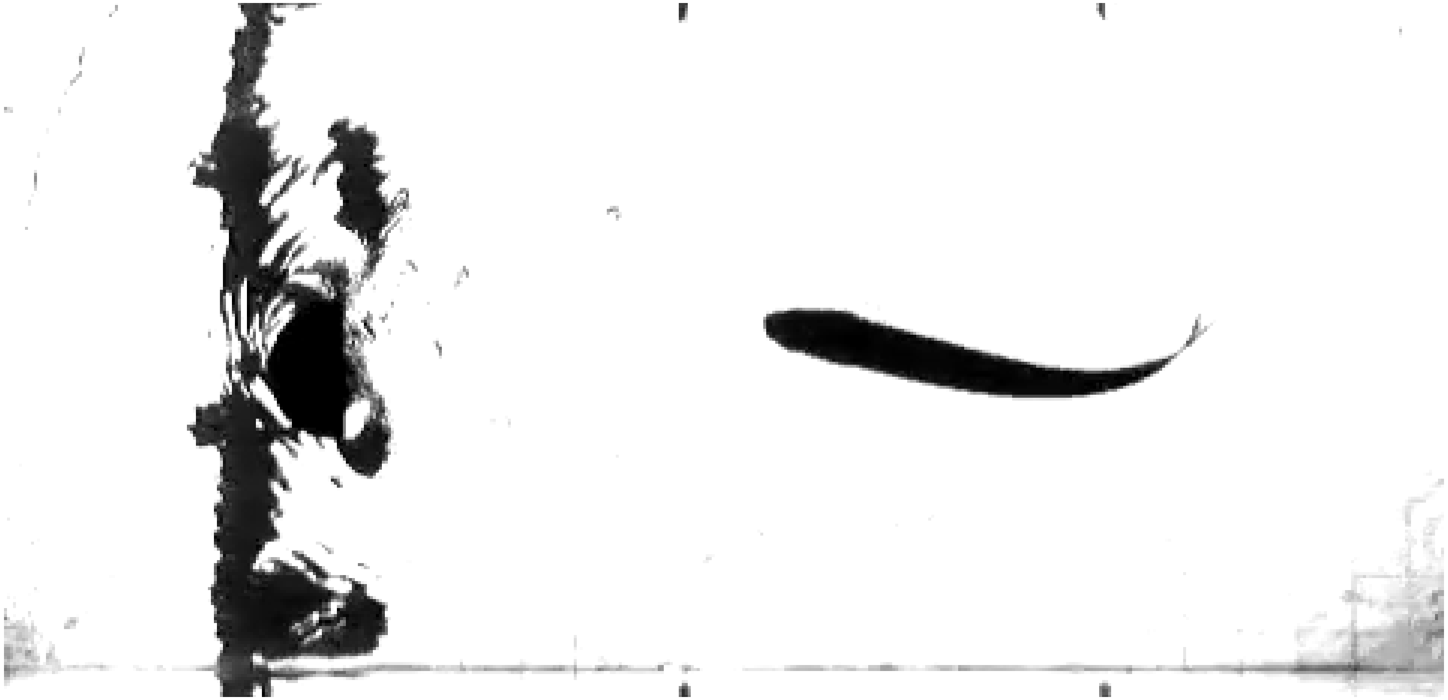
Teaching a Fish to Swim

A hybrid approach for fast fish simulation



Fish gotta swim, bird's gotta fly...

I've gotta sit and wonder why, why, why...

(Beal 2015)

John Z. Zhang | johnz@mit.edu

Outline

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1. Background on Fish Locomotion

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2. Differentiable Projective Dynamics

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3. Hybrid Approach to FSI

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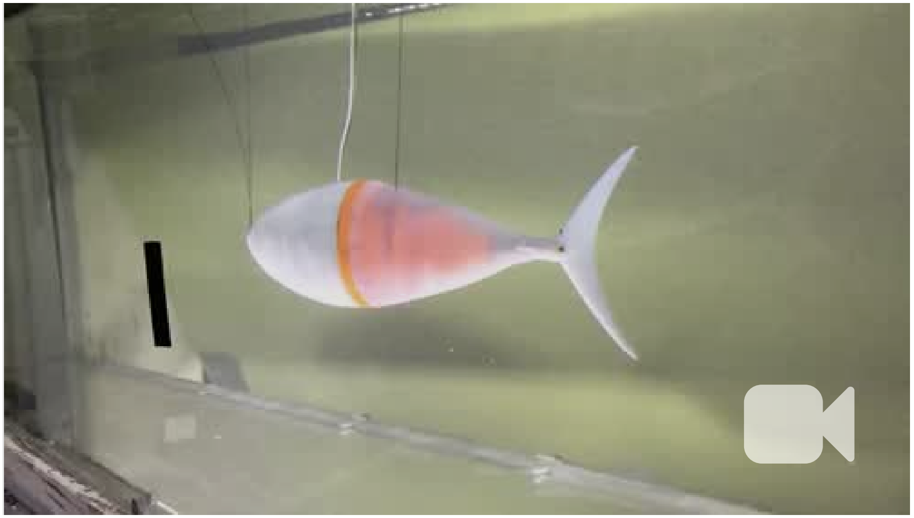
7. Future Directions

Yellowfin Tuna ~2 m

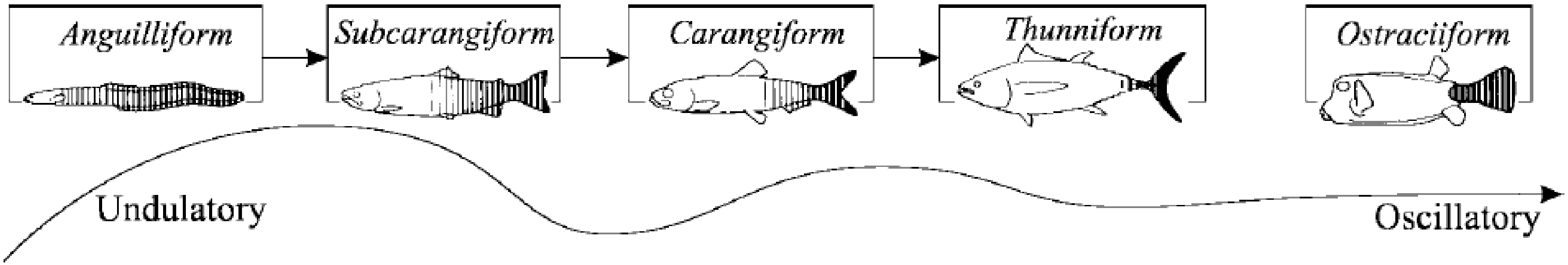
Humpback Whale ~ 15 m

Schooling Fish

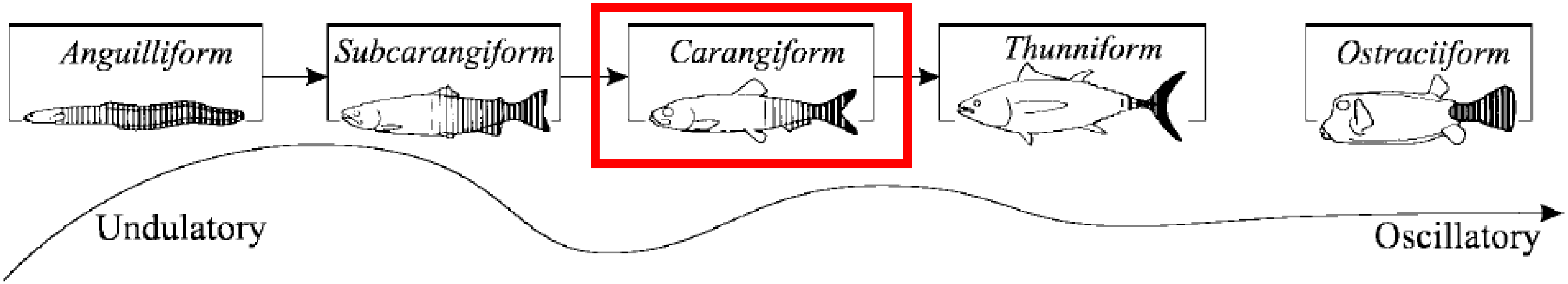
Tunabot 2019

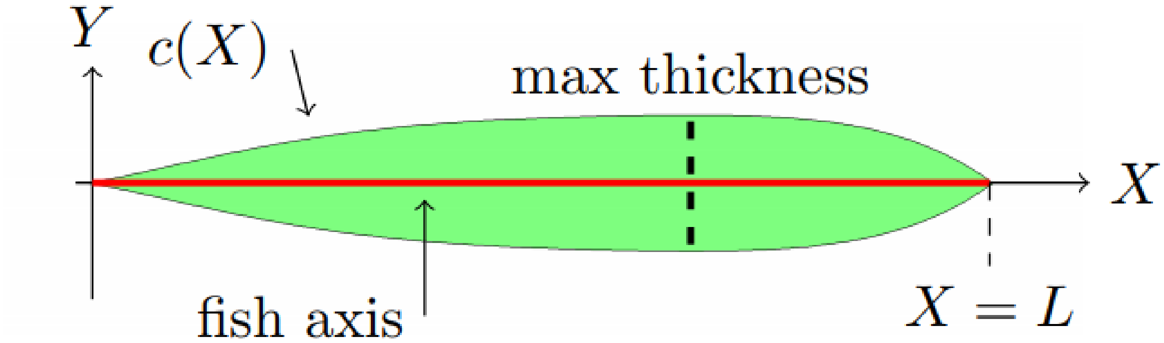


Gifs from *Our Planet* | *High Seas*



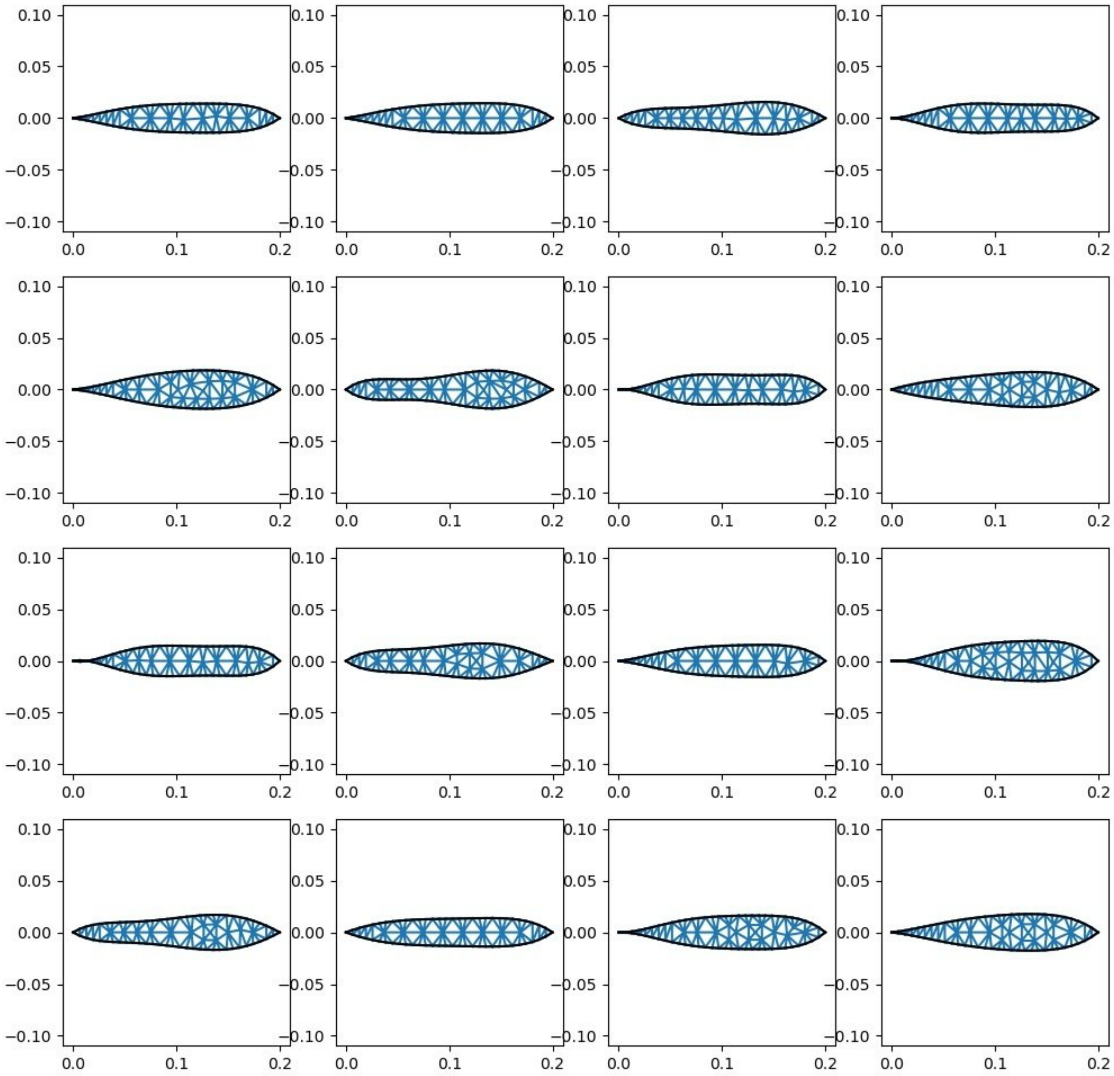
(Curatolo & Teresi 2015)





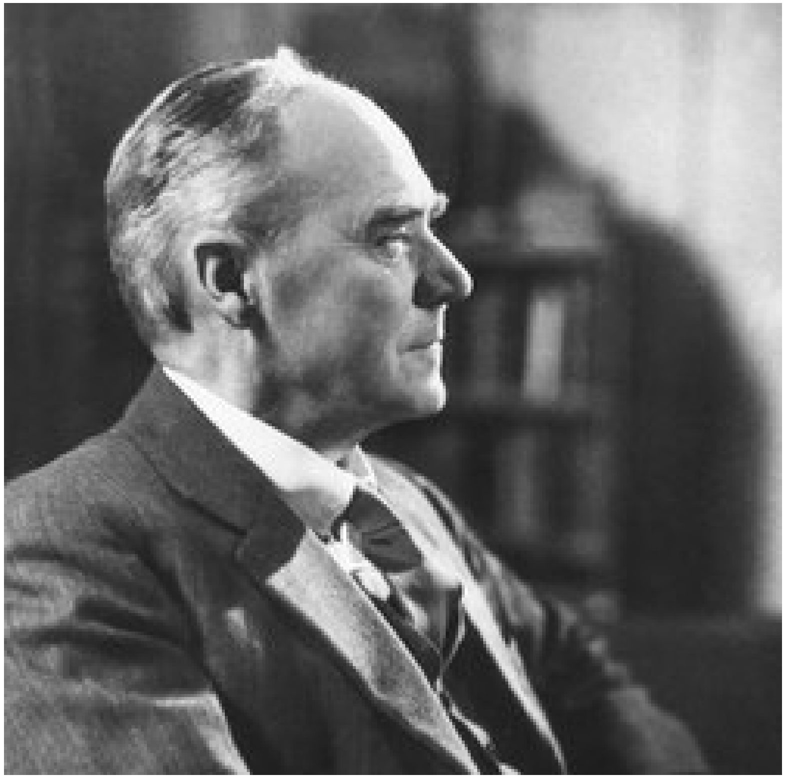
$$Y = a_1 X^5 + a_2 X^4 + a_3 X^3 + a_4 X^2 + a_5 X$$

(Curatolo & Teresi 2015)


0.10
0.05
0.00
−0.05
−0.10
0.0
0.1
0.2

Resistive Force Theory (RFT)

(1952)



Sir Geoffrey Ingram Taylor

(1886-1975)

Elongated Body Theory (EBT)

(1960)

Sir James Lighthill

(1924-1998)

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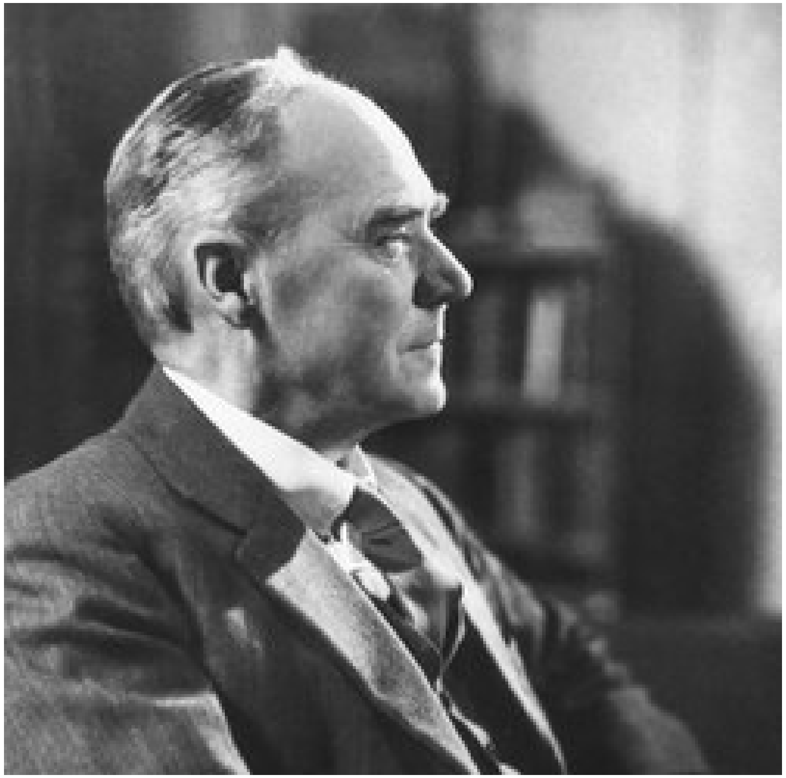

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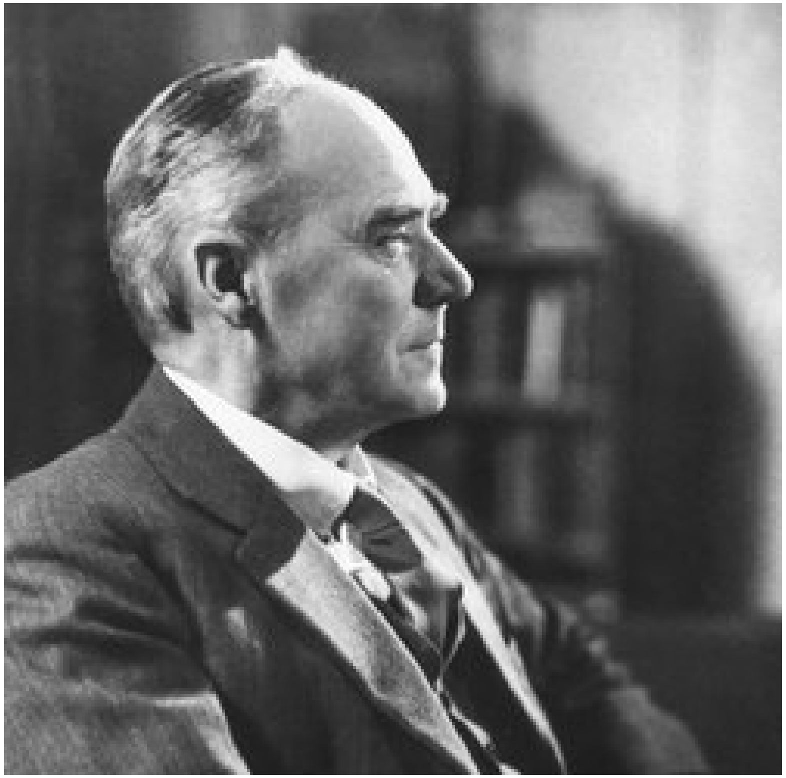

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Inertia Dominant

Ming et al. 2018

Resistive Theory

$$F_s(s,t) = -\frac{1}{2}\rho H(s) C_D v^2(s,t)$$

Elongated Body (Reactive) Theory

$$F_a(s,t) = -\left(\frac{\partial}{\partial t} + U\frac{\partial}{\partial x}\right)\left[V(s,t)m(s)\right]$$

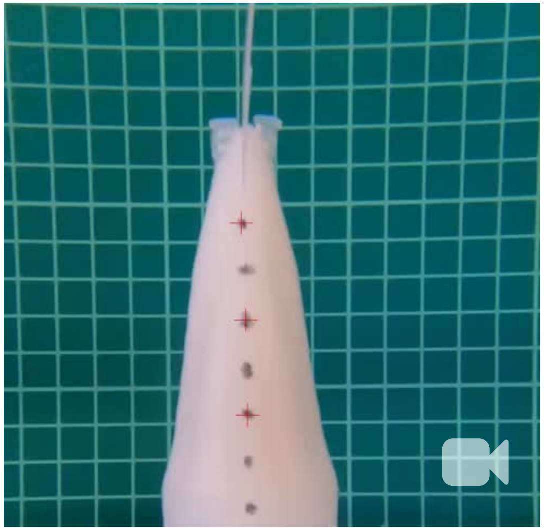
Combined Theory

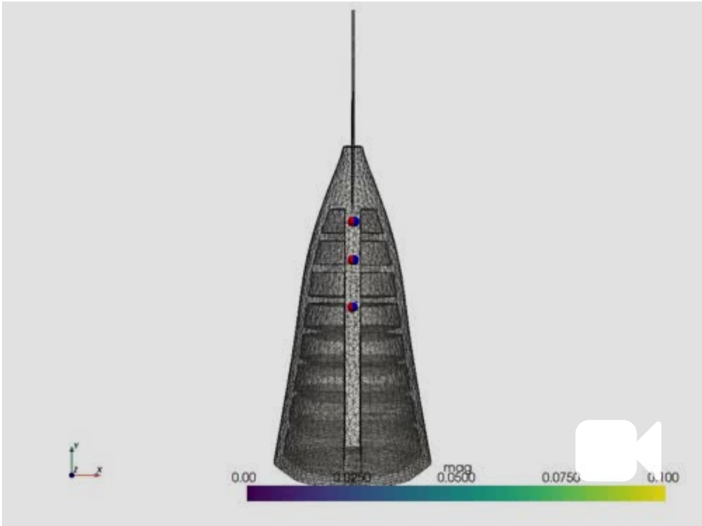
$$F_y(s,t) = C_a(s)F_a(s,t) + C_s(s)F_s(s,t)$$

Problem

How to simulate hydrodynamics **accurately** and **efficiently**?

How can the simulation be used for **computational design**?





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efficient, but inaccurate

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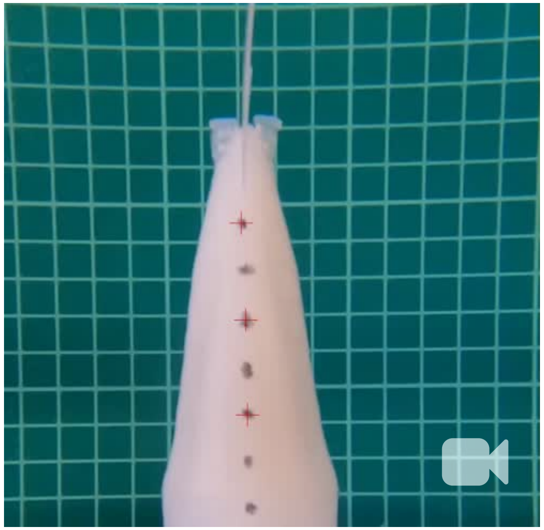

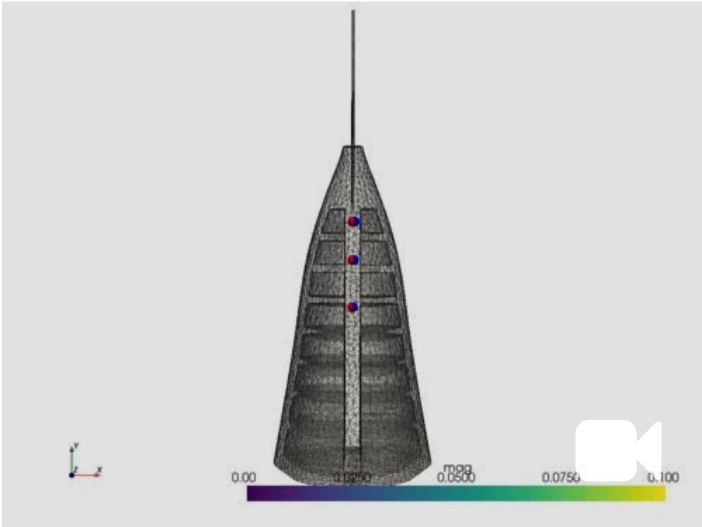

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FSI

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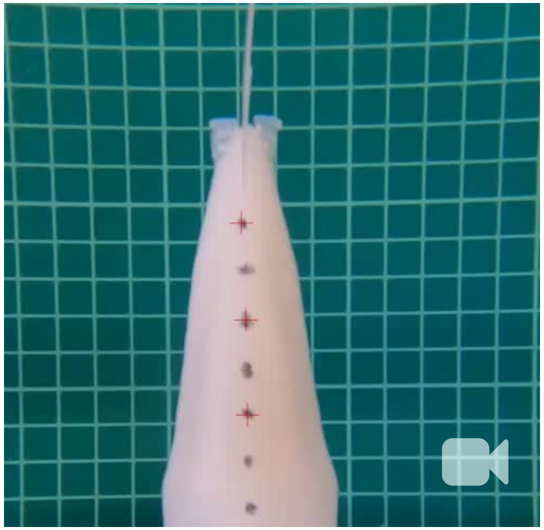

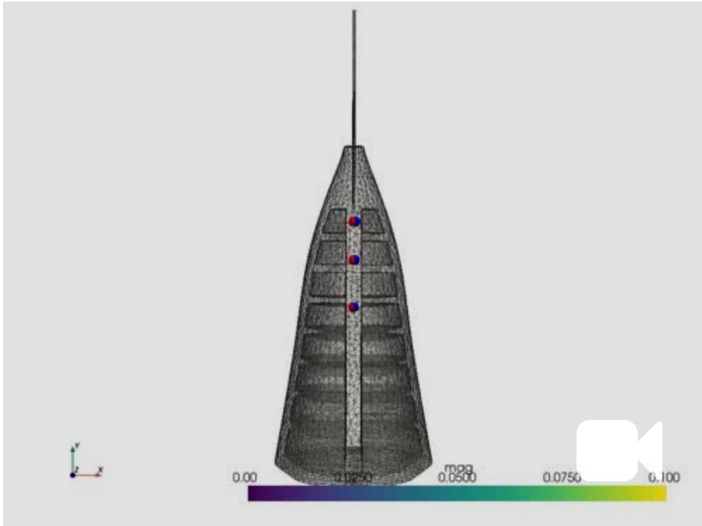

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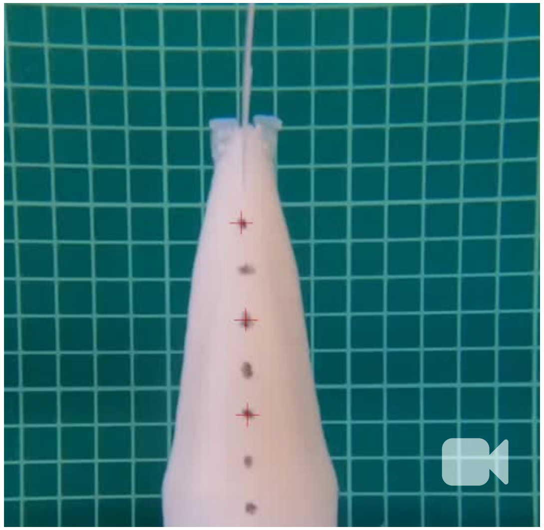

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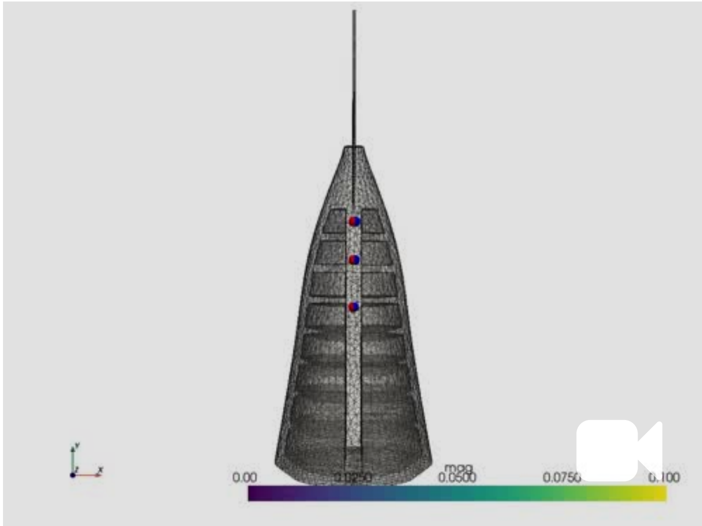
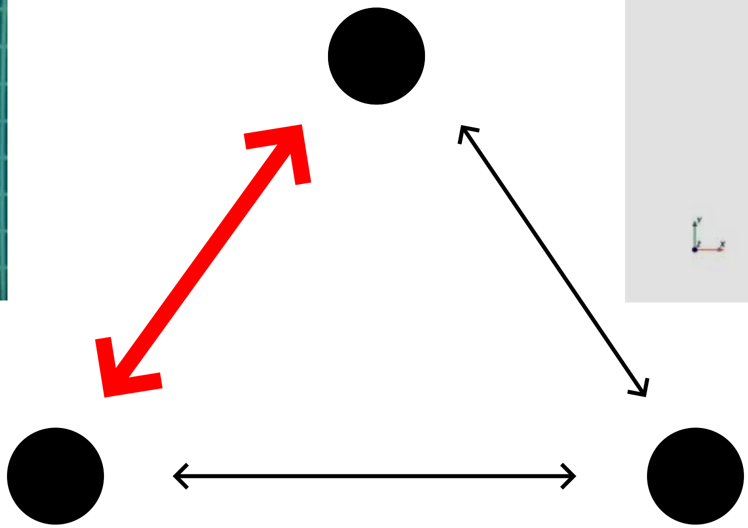
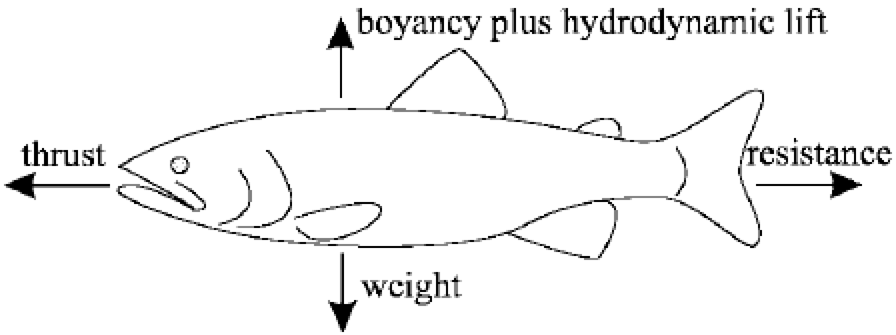
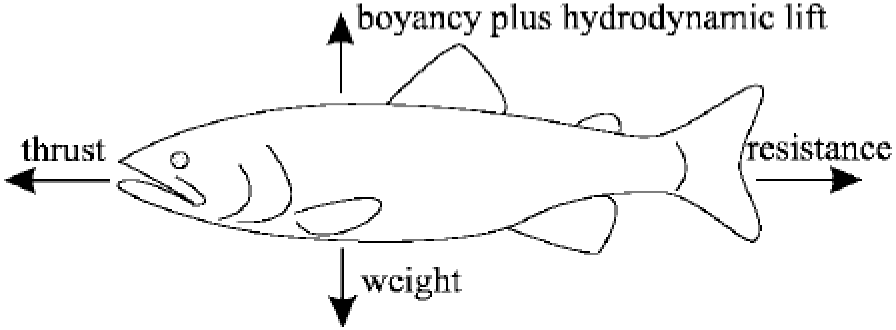






F_T F_D

$$m\dot{v} = F_\mathrm{T} - F_\mathrm{D}$$

F_{T}  F_{D}

$$m\dot{v} = F_{\mathrm{T}} - F_{\mathrm{D}}$$

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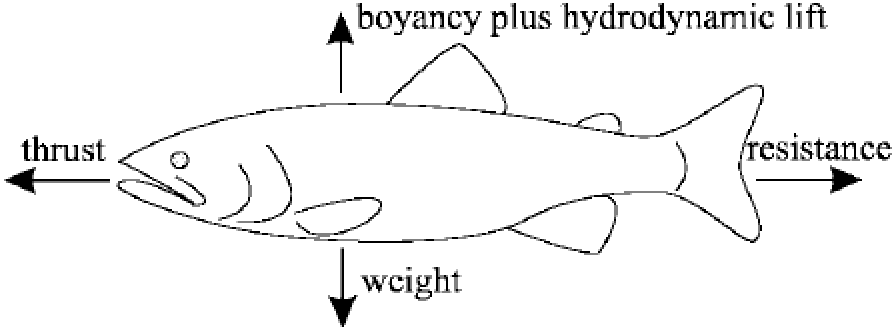
F_{T}



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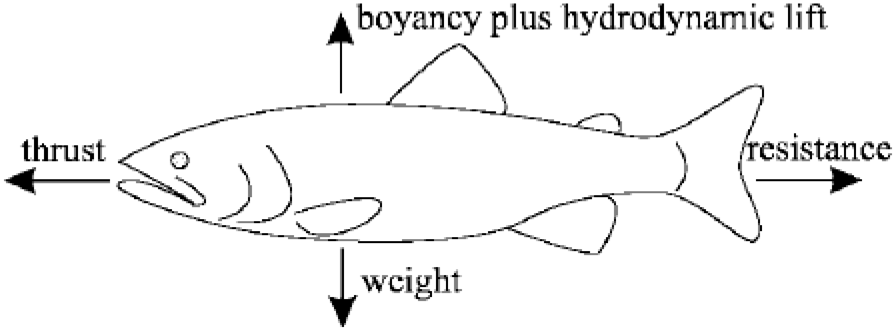
F_{T}  F_{D}

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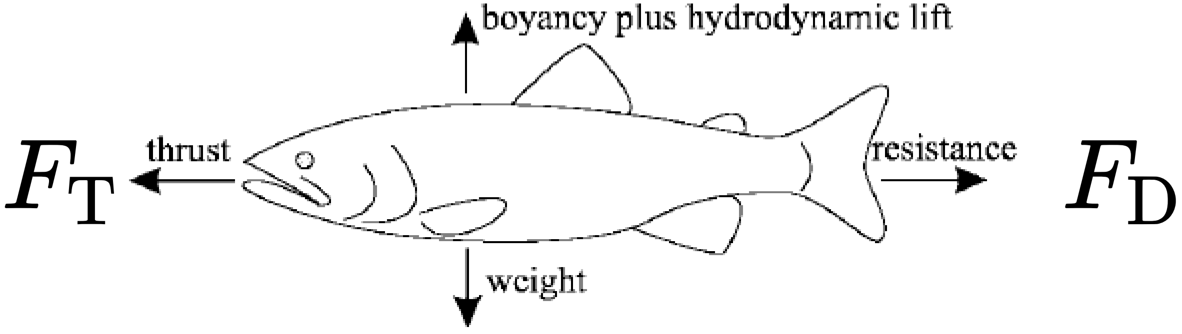
$$v_{\mathrm{max}} = \sqrt{\frac{2F_{\mathrm{T}}}{\rho A C_d}}$$



$$m\dot{v} = F_{\mathrm{T}} - F_{\mathrm{D}} \qquad F_{\mathrm{T}} = \hat{f}_t(\text{actuation}, \text{shape})$$

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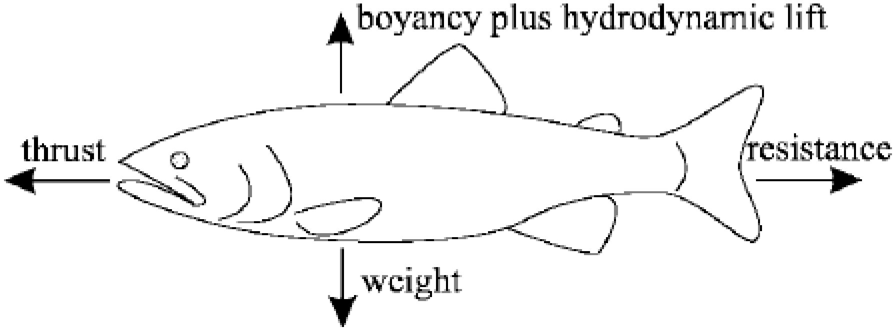

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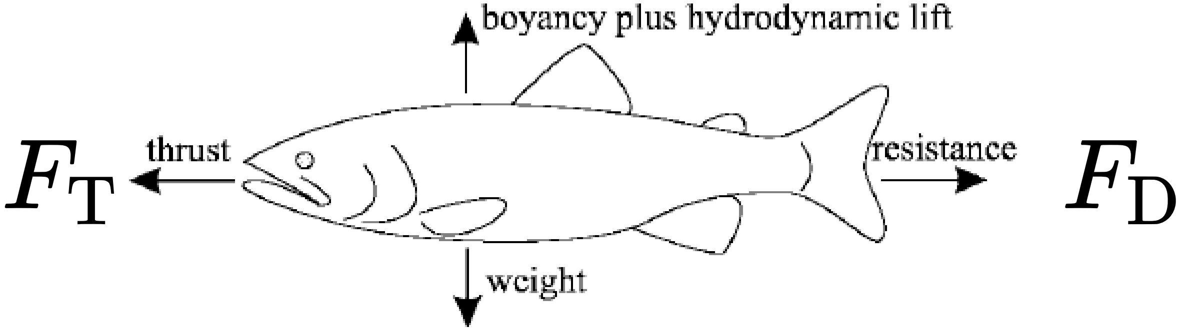

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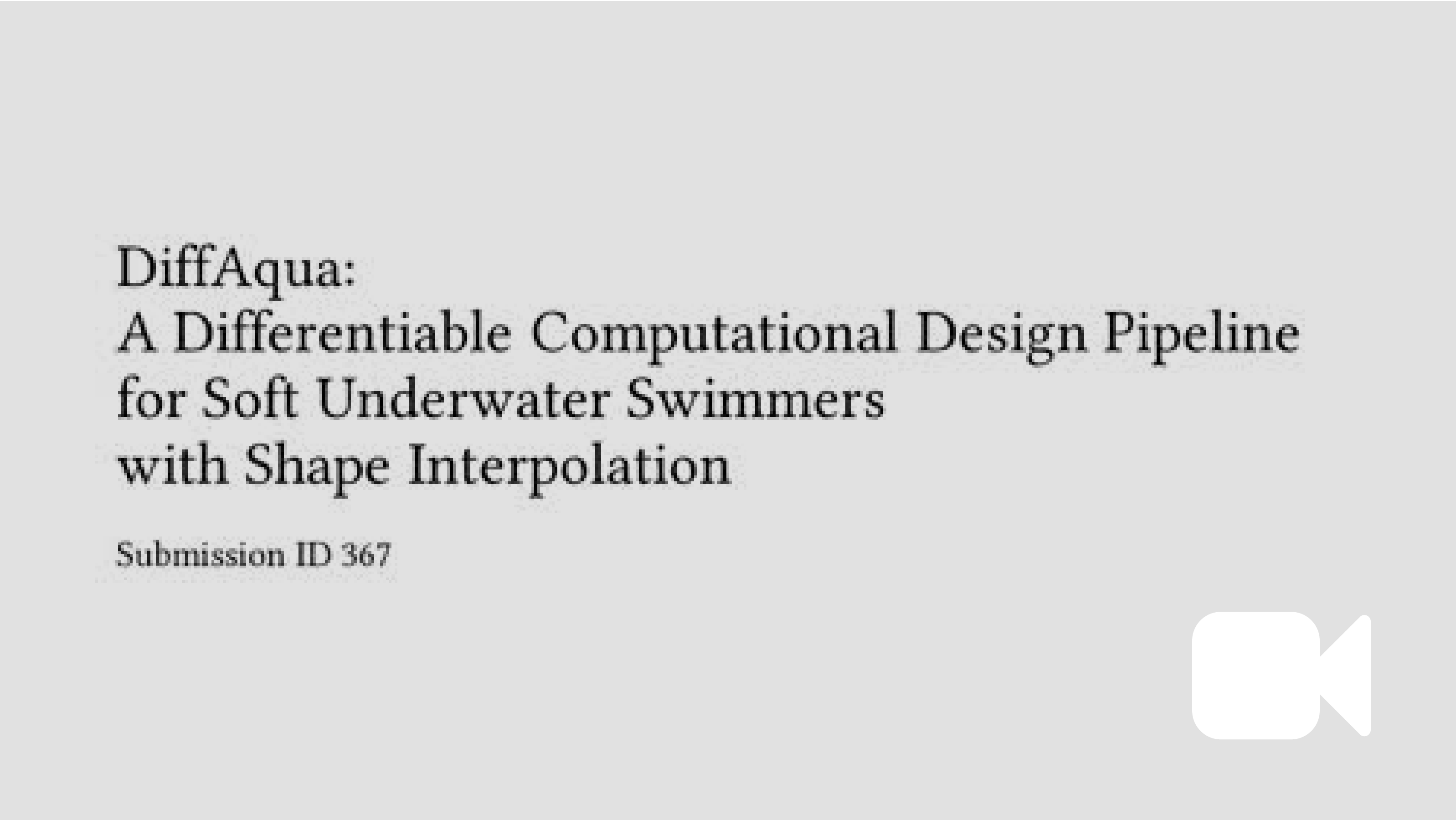
given an initial design, can we optimize this further?

$$v_{\max} = \sqrt{\frac{2F_{\mathrm{T}}}{\rho A C_d}}$$

DiffPD: Existing framework for differentiable simulation of soft bodies from MIT CDFG



(Du 2021, Ma 2021)

Projective Dynamics in 2D

$\mathbf{q} \in \mathbb{R}^{2m}$ $\dot{\mathbf{q}} \in \mathbb{R}^{2m}$ m is the number of particles

position velocity

(Kavan PBA 2014)

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Discrete time: $t_1, t_2, \ldots$

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Internal forces: $\mathbf{f}_{\text{int}} = -\nabla E_{\text{int}}(\mathbf{q})$

(Kavan PBA 2014)

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Muscle fibers: $E(\mathbf{q}) = \frac{w}{2}|(1-a)\mathbf{Fm}|^2$

(Kavan PBA 2014)

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Implicit Euler:

(Kavan PBA 2014)

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Implicit Euler:

$$\mathbf{q}_{n+1} = \mathbf{q}_n + h\dot{\mathbf{q}}_{n+1}$$

$$\dot{\mathbf{q}}_{n+1} = \dot{\mathbf{q}}_n + h\mathbf{M}^{-1}[\mathbf{f}_{\text{int}}(\mathbf{q}_{n+1}) + \mathbf{f}_{\text{ext}}]$$

(Kavan PBA 2014)

Projective Dynamics in 2D (cont'd)

Must solve nonlinear system of equations,

equivalent to optimization problem.

$$\min_{\mathbf{q}_{i+1}} \frac{1}{2h^2}\left|\mathbf{M}^{\frac{1}{2}}(\mathbf{q}_{i+1}-\mathbf{y})\right|^2 + E_{\text{int}}(\mathbf{q}_{i+1})$$

(Bouaziz 2014)

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Using Newton-Raphson

$$\mathbf{A}_{\text{N}} = \frac{1}{h^2}\mathbf{M} + \nabla^2 E_{\text{int}}(\mathbf{q}_{i+1})$$

(Bouaziz 2014)

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In Projective Dynamics, we decouple nonlinear material model from linear systems of equations

Minimize by alternating between a local and **global step**

(Bouaziz 2014)

Projective Dynamics in 2D (cont'd)

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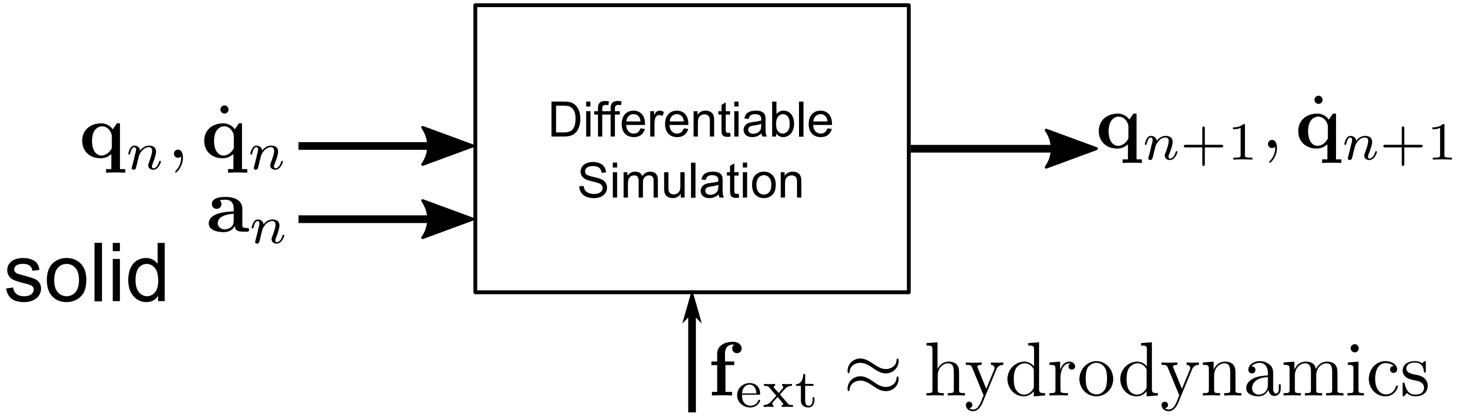
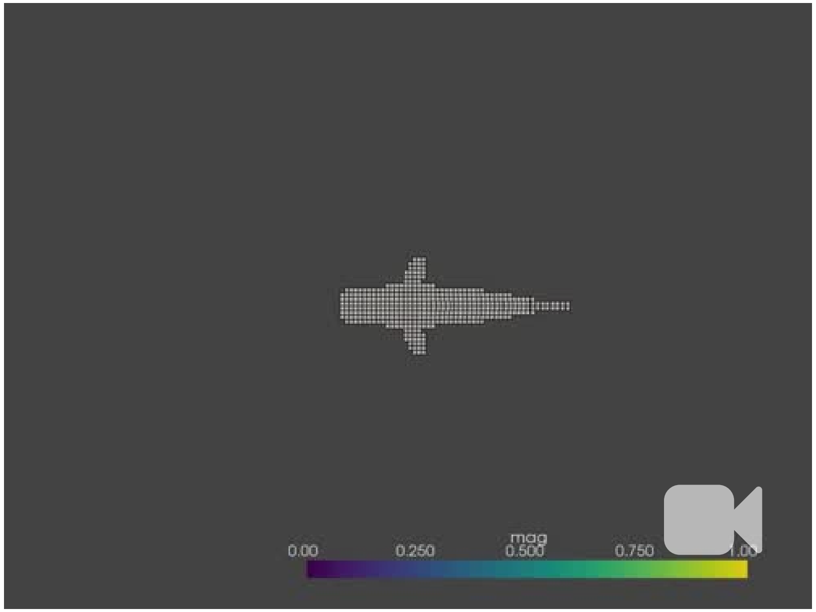
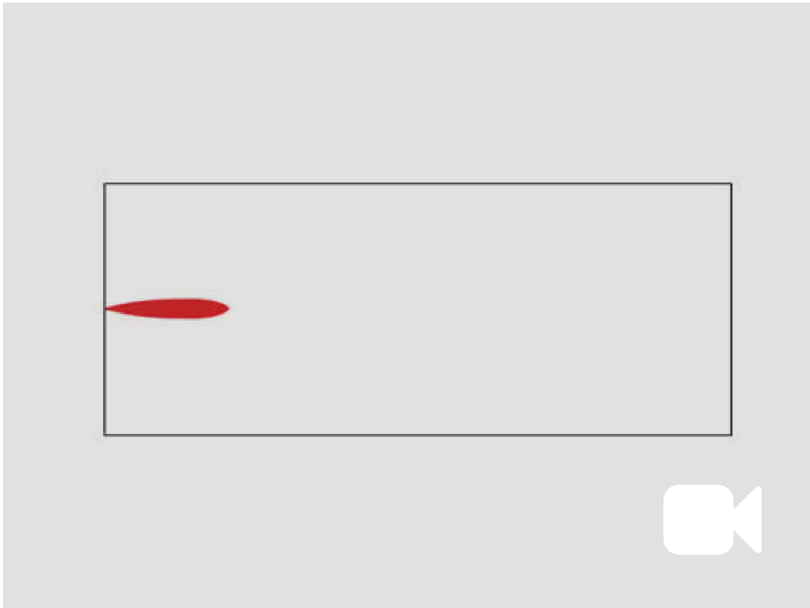
$$\mathbf{A}\mathbf{q} = \mathbf{b}$$ Prefactor! $\mathbf{b}$ changes on each time step

(Bouaziz 2014)

Projective Dynamics in 2D (cont'd)





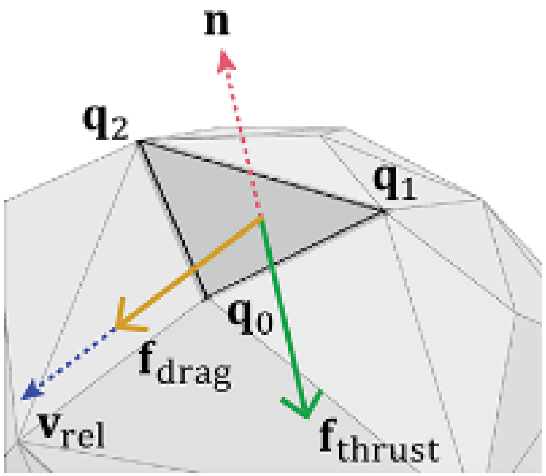


Min et al. 2019

$$\mathbf{f}_{\mathrm{drag}} \propto C_d(\Phi)|\mathbf{v}_{\mathrm{rel}}|^2\mathbf{d} \qquad \mathbf{d} = \frac{\mathbf{v}_{\mathrm{rel}}}{|\mathbf{v}_{\mathrm{rel}}|}$$

$$\mathbf{f}_{\mathrm{thrust}} \propto C_t(\Phi)|\mathbf{v}_{\mathrm{rel}}|^2\mathbf{n}$$

$$\Phi = \frac{\pi}{2} - \cos^{-1}(\mathbf{n}\cdot\mathbf{v}_{\mathrm{rel}})$$



Min et al. 2019

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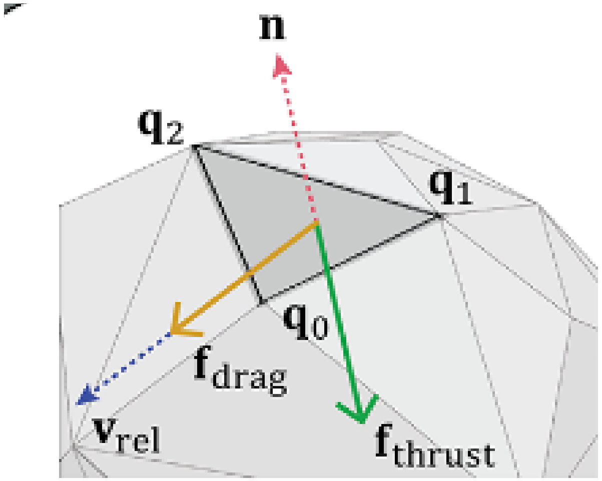

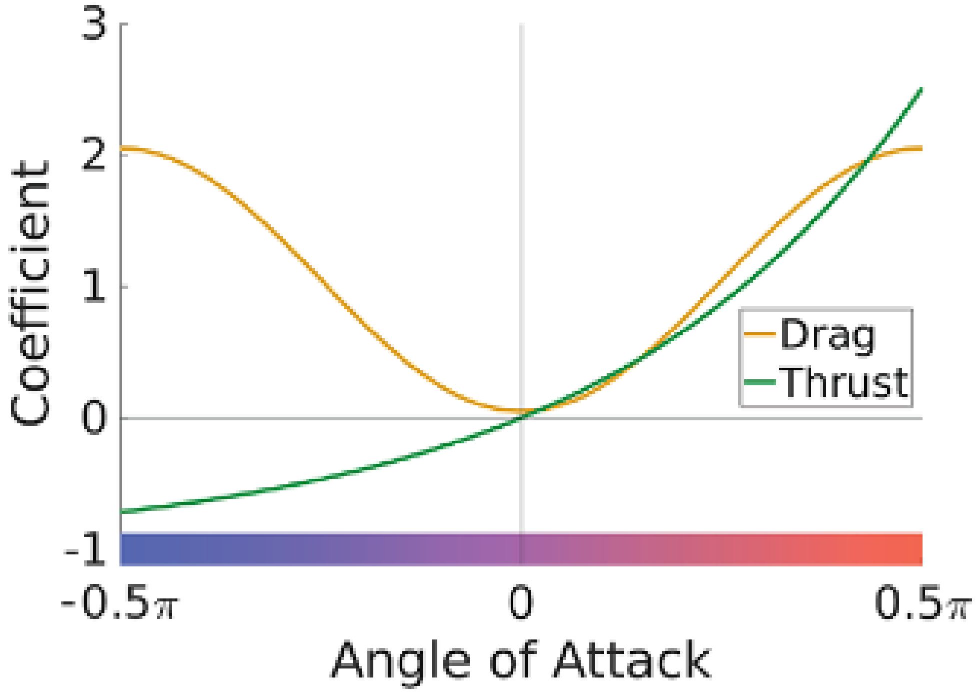

Min et al. 2019

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$$\mathbf{f}_{\text{thrust}} = -\frac{1}{2}\rho A C_t(\Phi)|\mathbf{v}_{\text{rel}}|^2\mathbf{n}$$

Min et al. 2019

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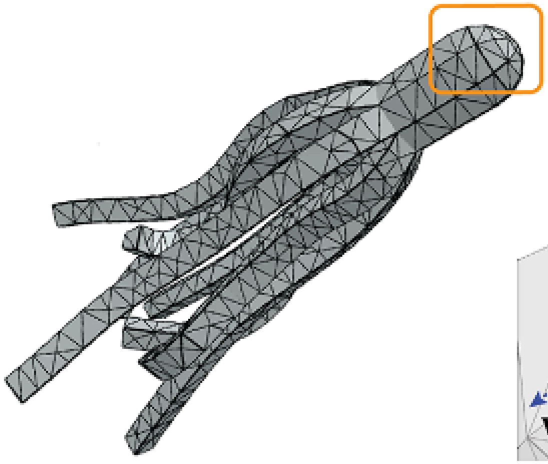

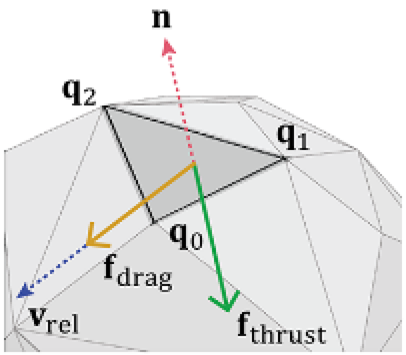

Min et al. 2019

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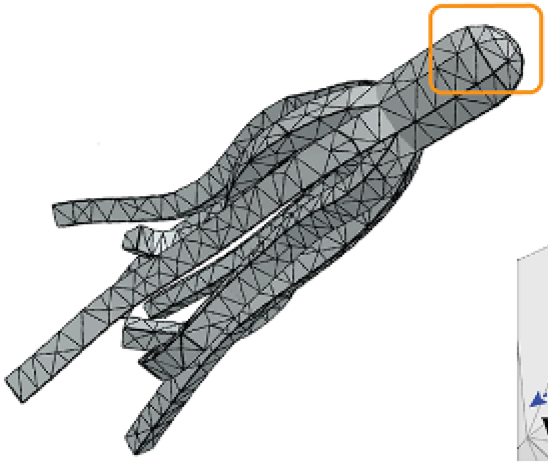

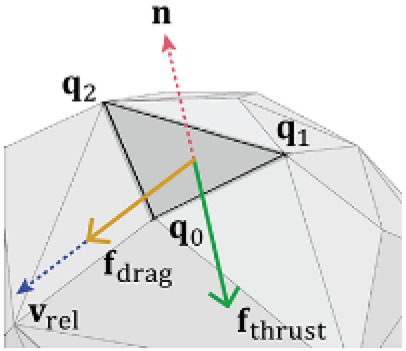

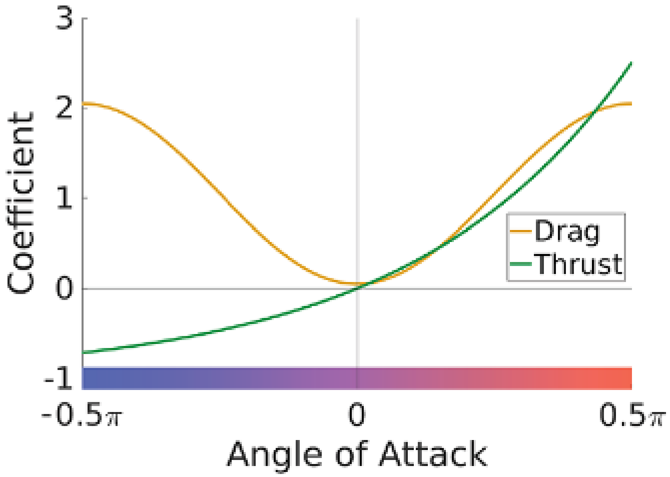

Min et al. 2019

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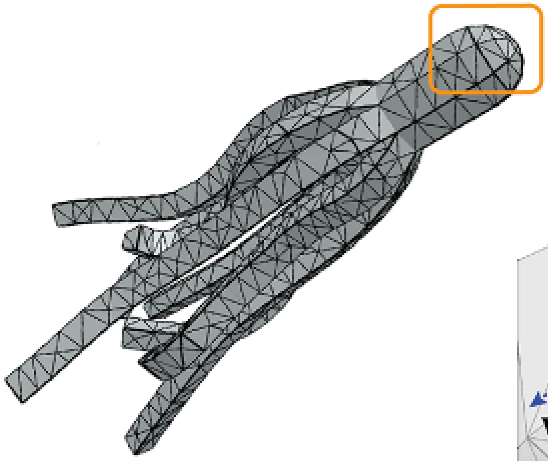

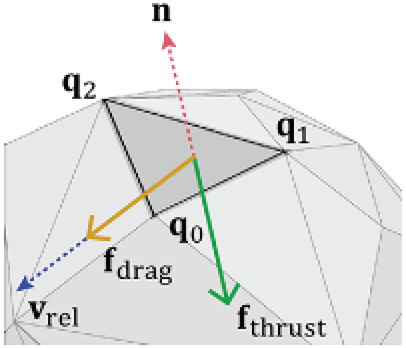

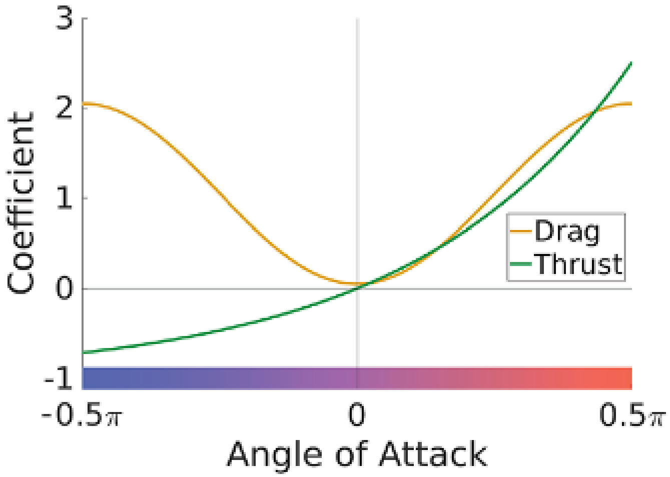

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Min et al. 2019

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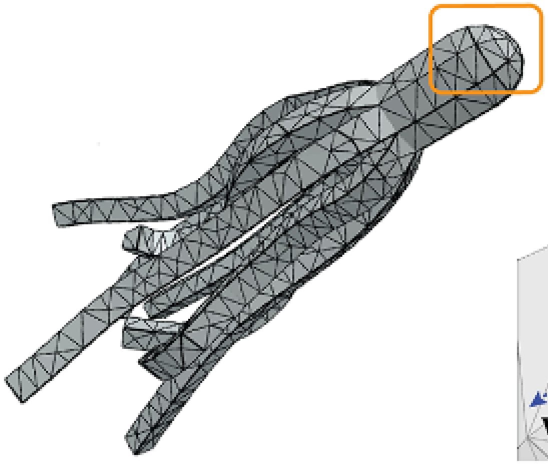

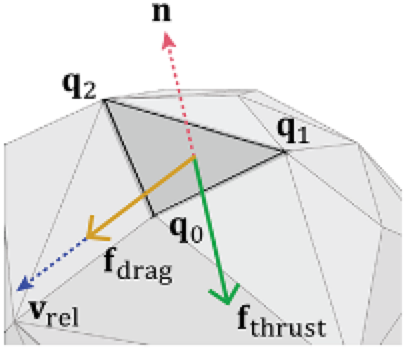

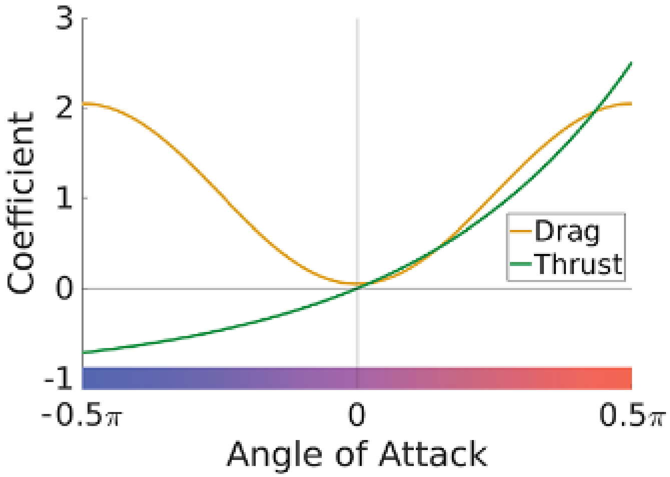

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Min et al. 2019

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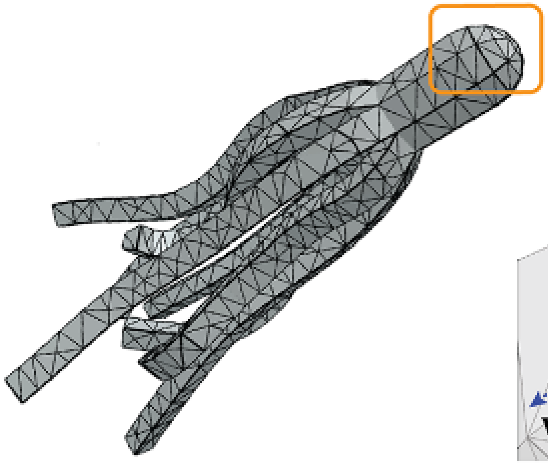

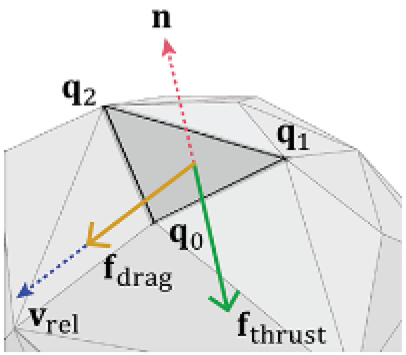

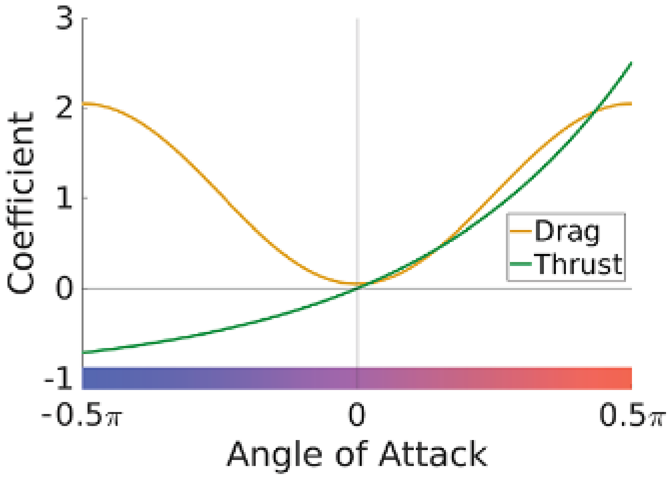

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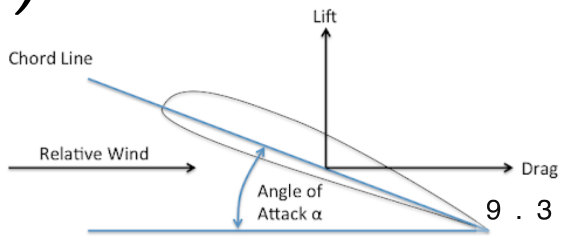
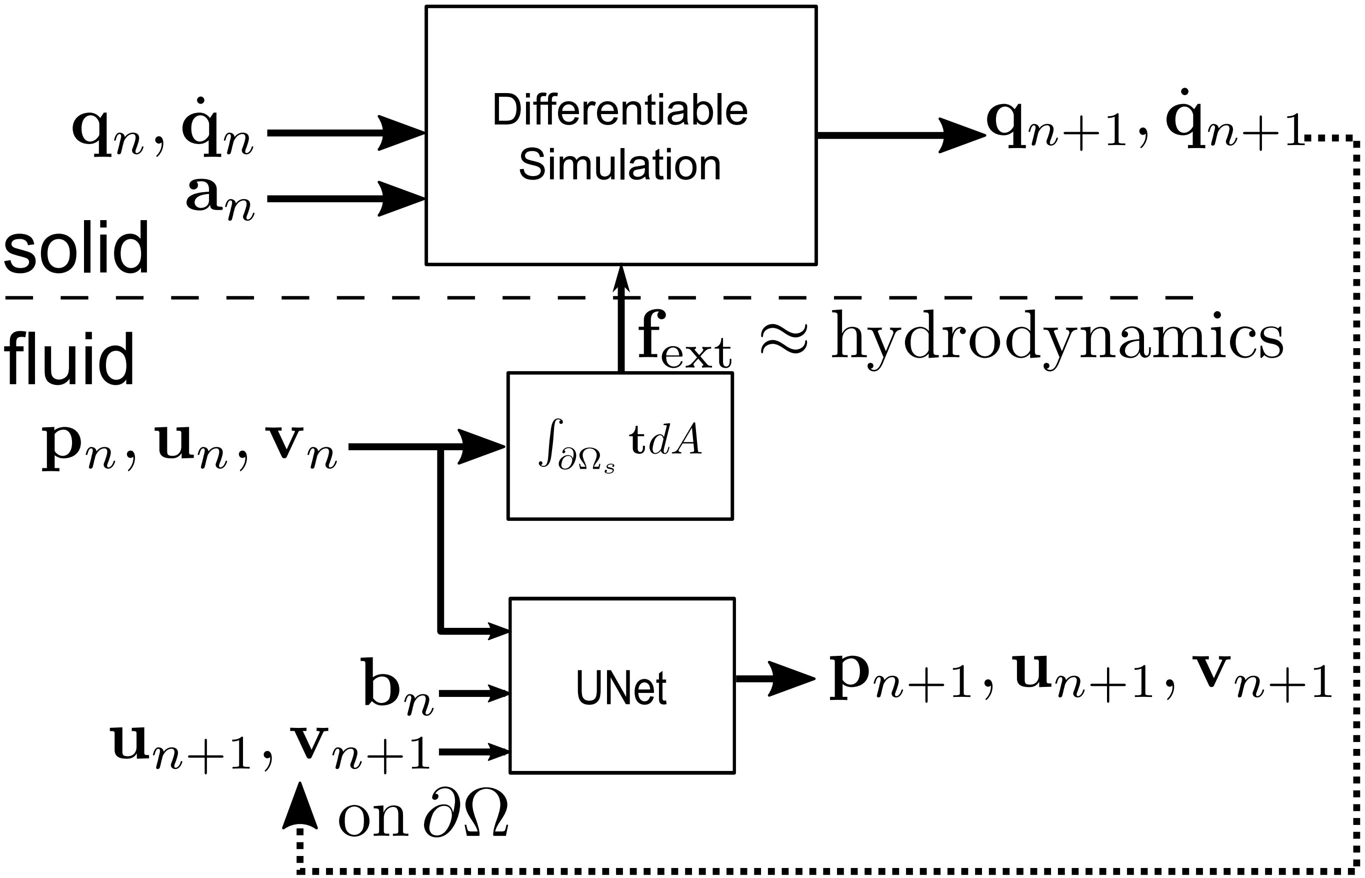


Proposed Solution



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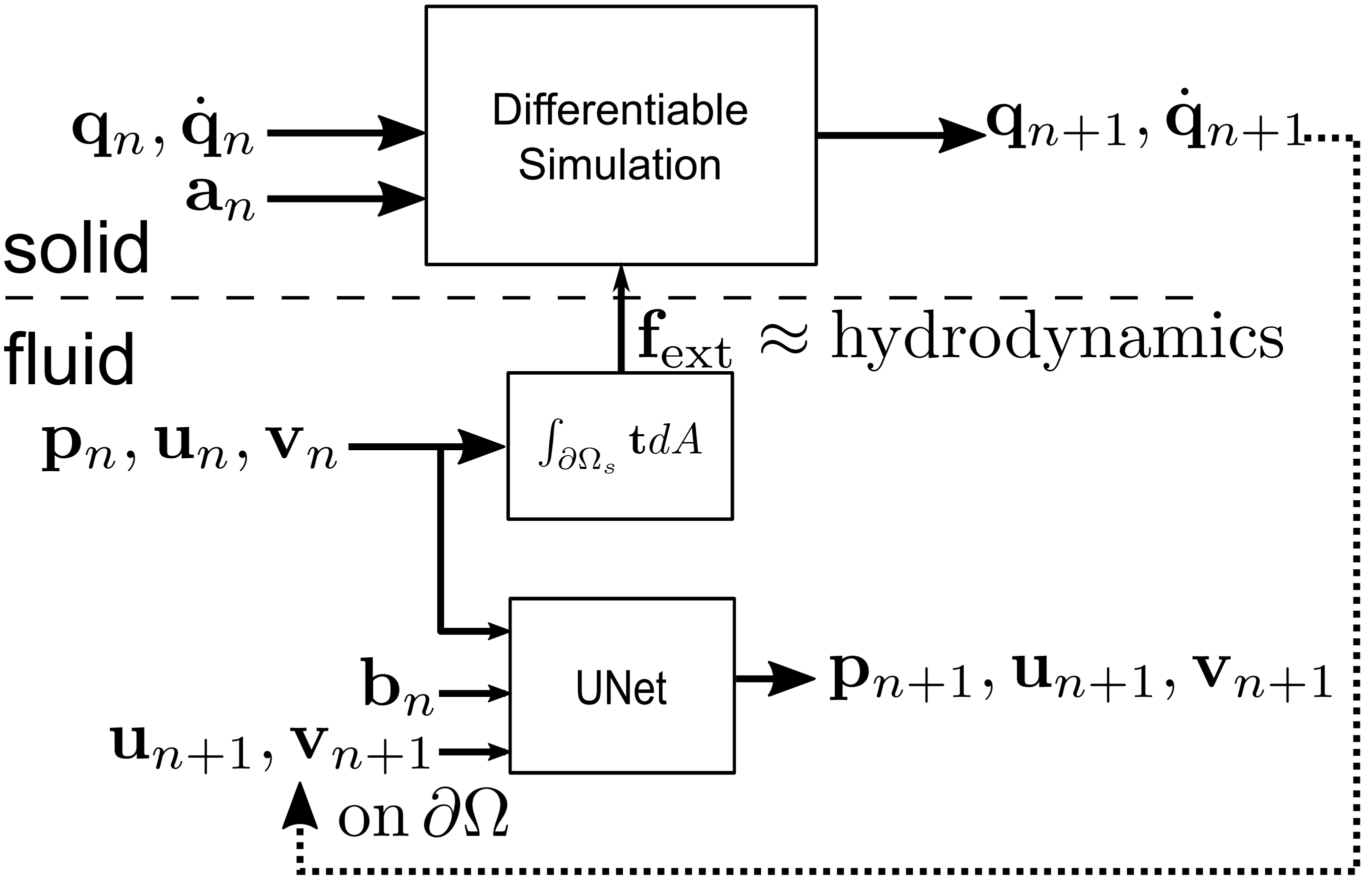


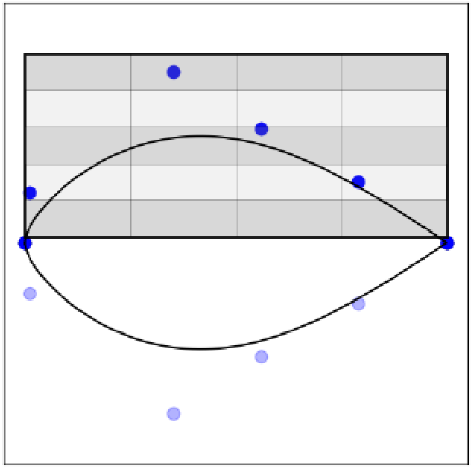
$$\mathbf{f}_{\text{pressure}} = \int_{\partial\Omega} p\mathbf{n}\, dl$$

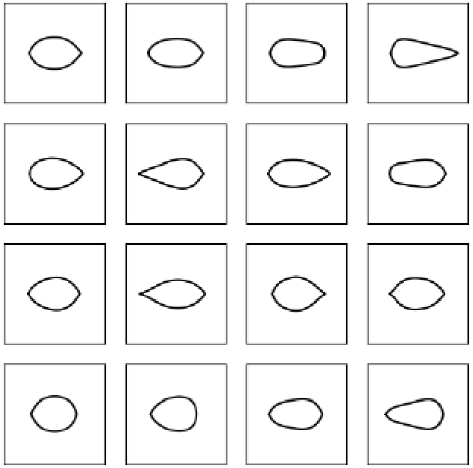
$$\mathbf{f}_{\text{viscous}} = \int_{\partial\Omega} \mu\mathbf{n} \times \omega\, dl \quad \omega = \nabla \times \mathbf{u}$$

Numerical investigation of minimum drag profiles in laminar flow using deep learning surrogates

Li-Wei Chen, Berkay Alp Cakal, Xiangyu Hu, Nils Thuerey

September 2020





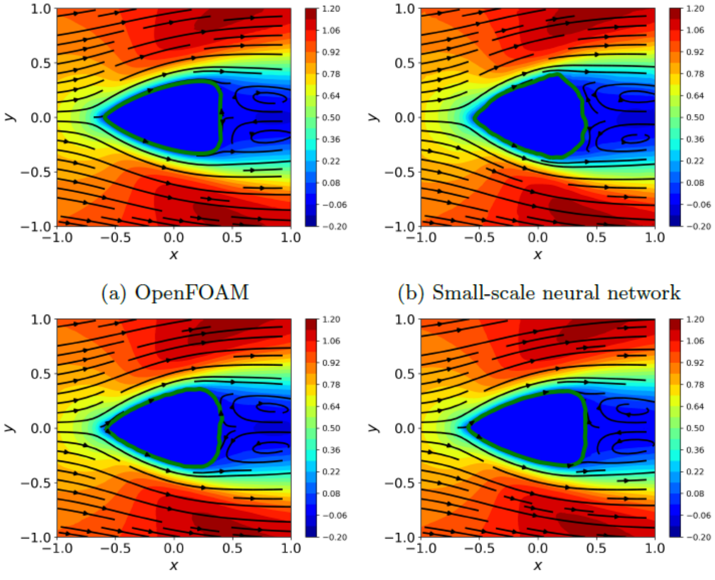
Name	# of flowfields	Re	NN models
Dataset-1	2500	1	small, medium & large
Dataset-40	2500	40	small, medium & large
Dataset-Range	3028	0.5-42.5	large

Table 1: Three datasets for training the neural network models.

Numerical investigation of minimum drag in laminar flow using deep learning surr

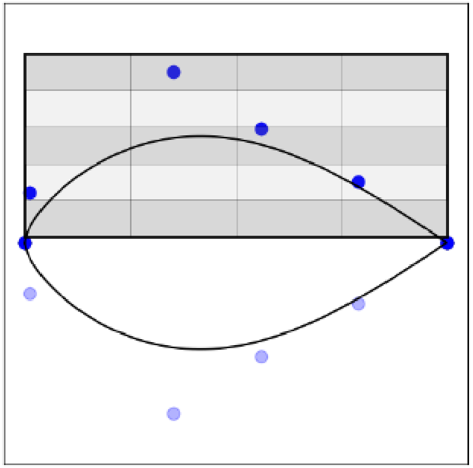
Li-Wei Chen, Berkay Alp Cakal, Xiangyu Hu, Nils T

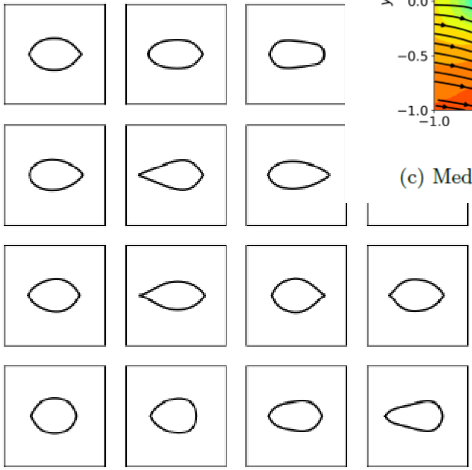
September 2020



(a) OpenFOAM (b) Small-scale neural network

(c) Medium-scale neural network (d) Large-scale neural network





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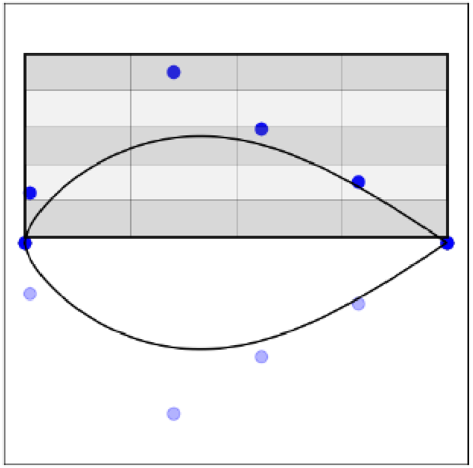

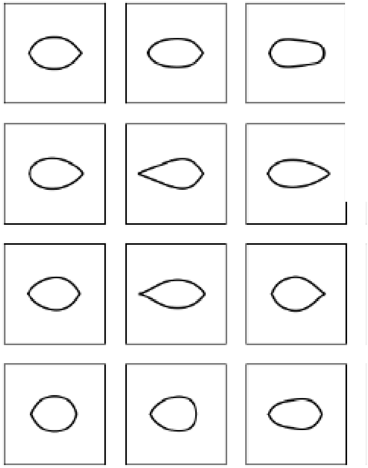

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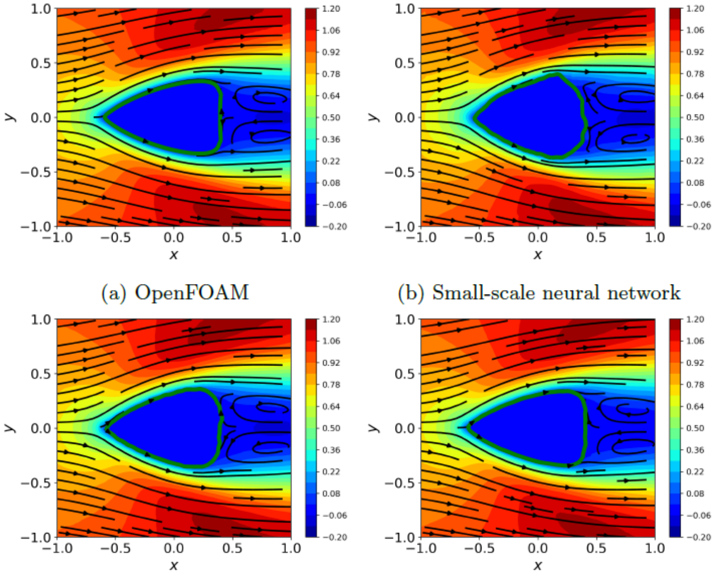

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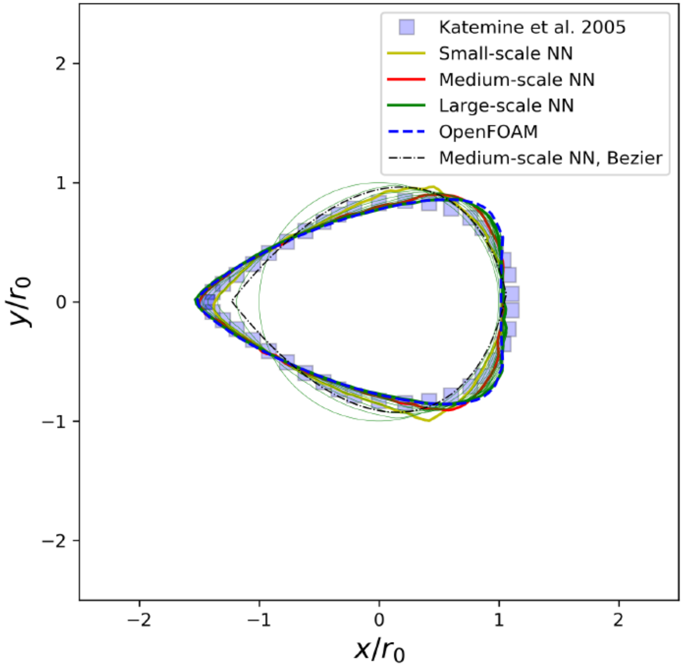
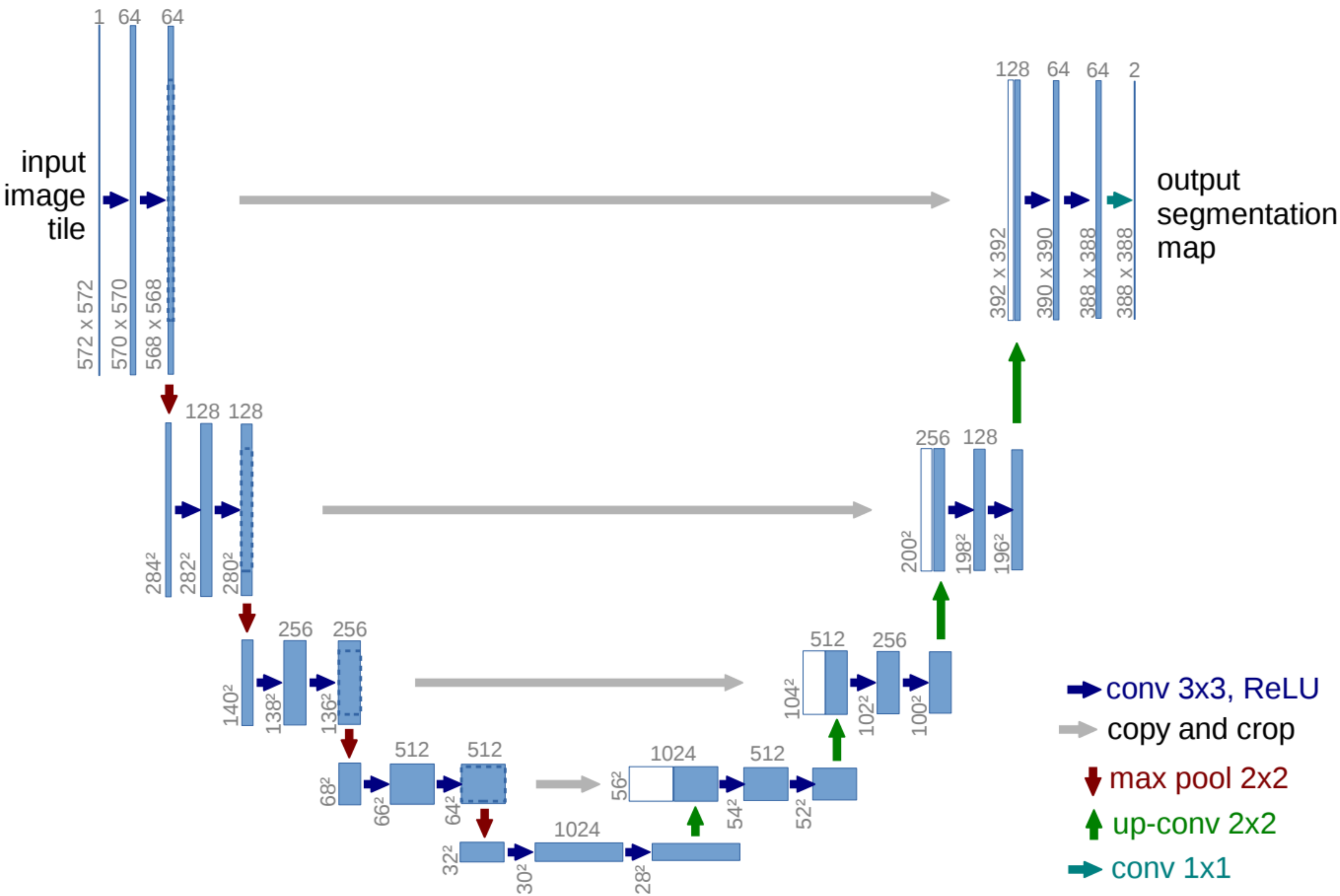
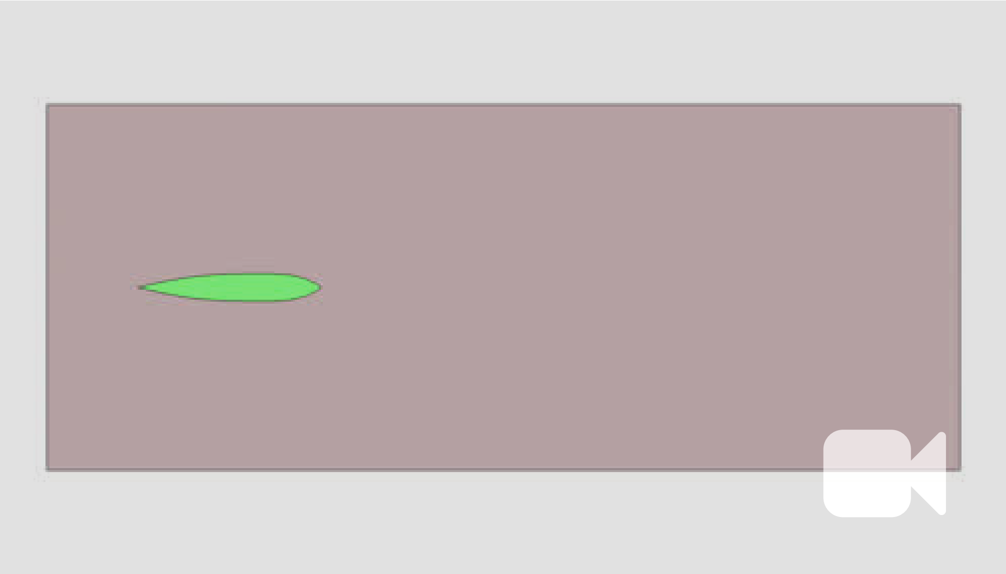


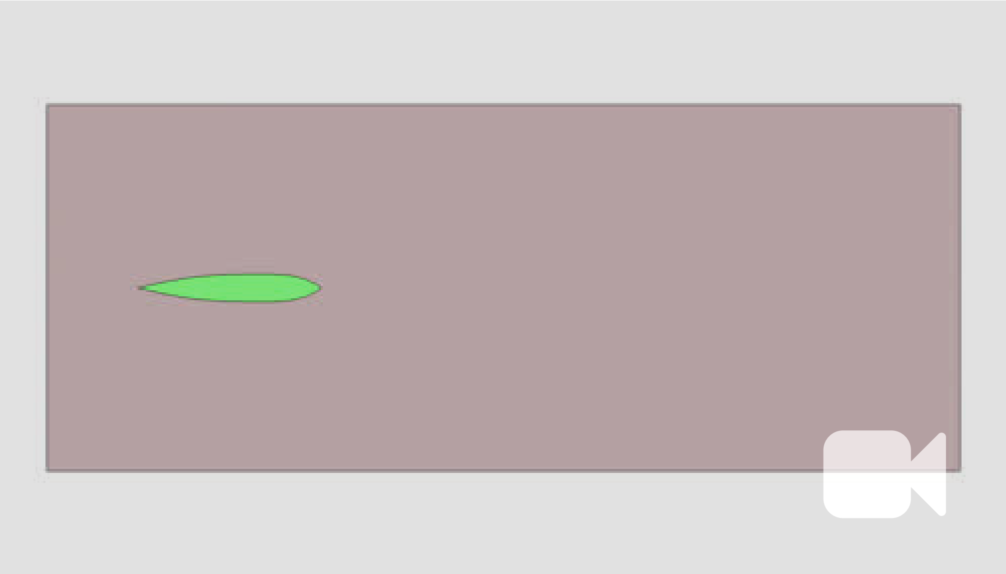


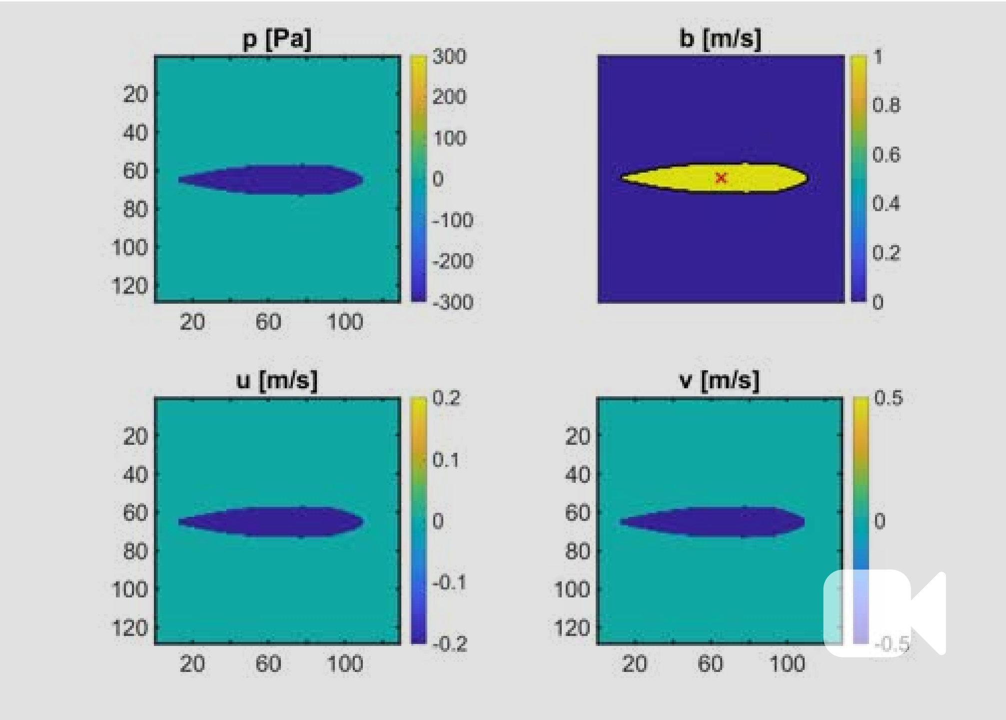
(Ronneberger 2015)



COMSOL FSI
Modified from

(Curatolo & Teresi 2015)





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↓

Post Processed training data in MATLAB

128 x 128 image sequence

Using COMSOL for FSI dataset

Using COMSOL for FSI dataset

Fluid

Using COMSOL for FSI dataset

Fluid

$$\underbrace{\rho \frac{\partial \mathbf{u}}{\partial t} + \rho(\mathbf{u} \cdot \nabla)\mathbf{u}}_{\frac{D\mathbf{u}}{DT}} + \nabla p + \underbrace{\nabla \cdot \tau}_{\eta \nabla^2 \mathbf{u}} = \underbrace{\mathbf{F}}_{=\mathbf{0}}$$

Using COMSOL for FSI dataset

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Using COMSOL for FSI dataset

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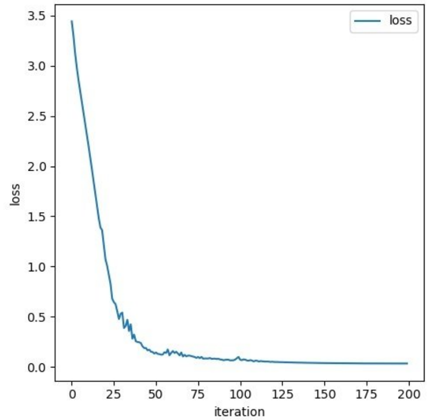
$$\nabla\cdot\mathbf{u} = 0$$

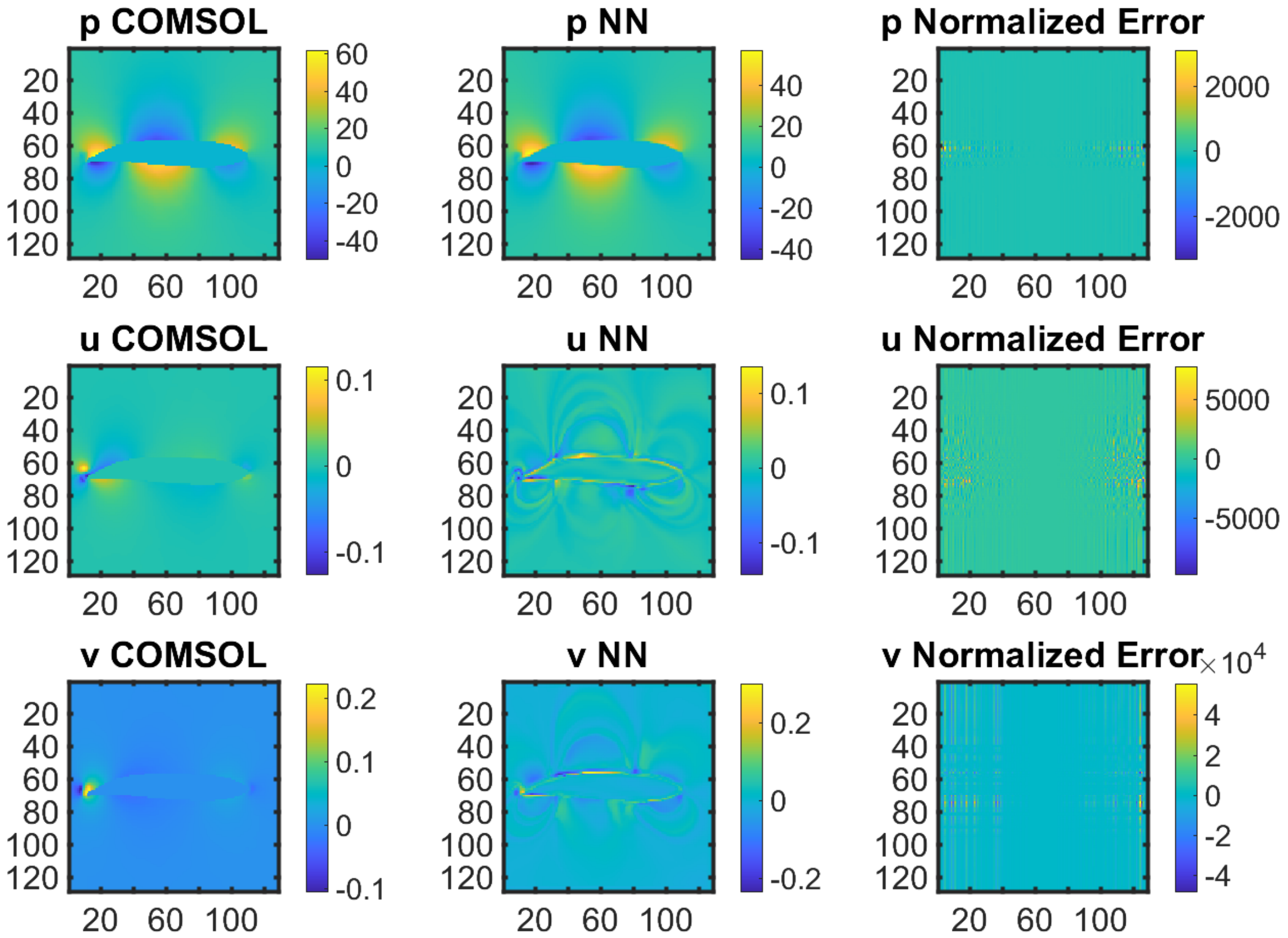
Structure Boundary Condition

$$\mathbf{t} = -\mathbf{n}\cdot(-p\mathbf{I} + \underbrace{\tau}_{\eta\nabla^2\mathbf{u}})$$

Preliminary Results

after 200 epochs



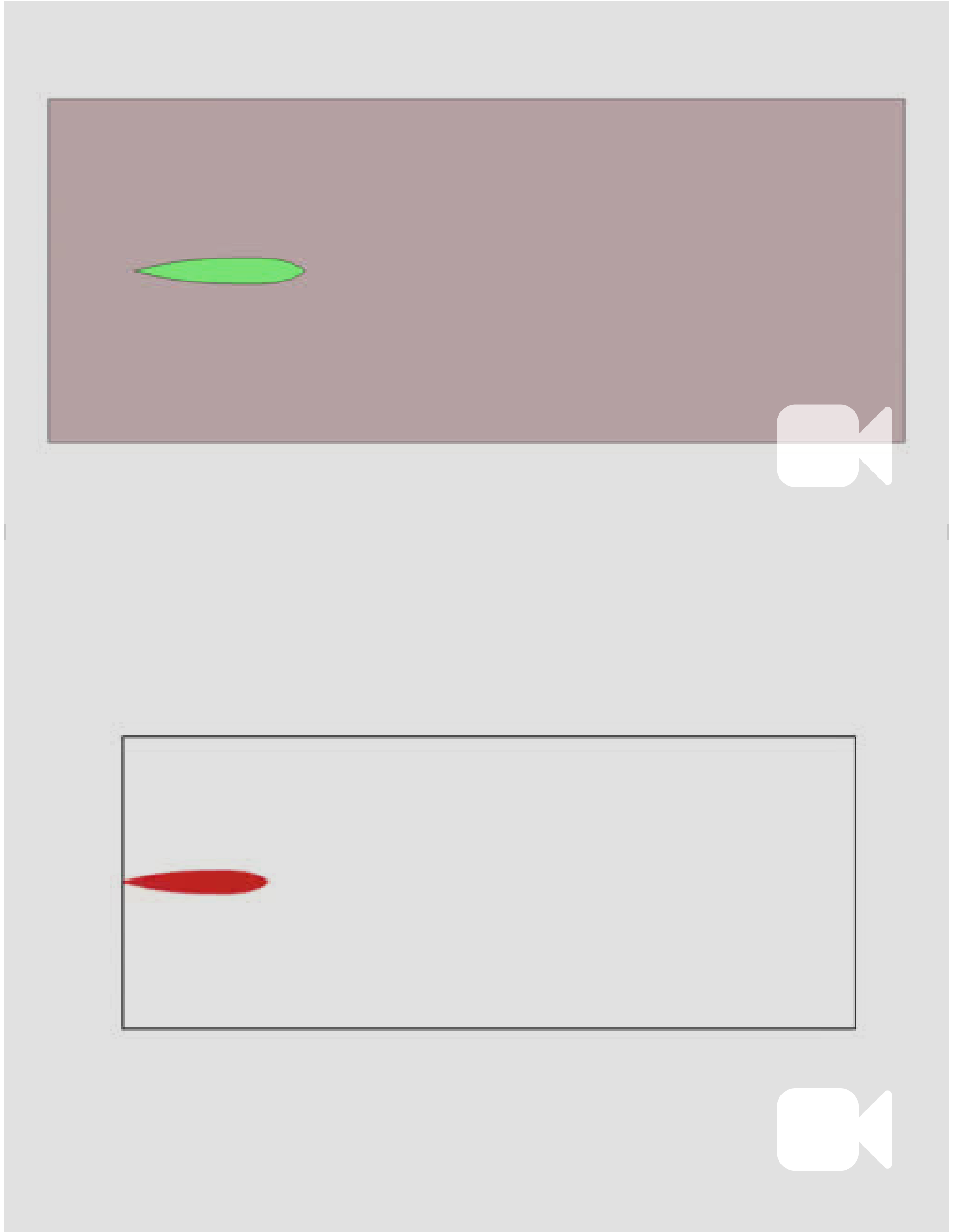


0.58%

66%

156%

The End Goal



Discussion

Parameter	COMSOL FSI	DiffPD
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frames	200	200
machine	local desktop	google cloud platform
run time	> 2 hours	5.4 seconds

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Use NN as a more sophisticated differentiable look up table.

Training takes time, but afterwards simulation will run fast.

Use the hybrid simulator for control, design optimization, fun, etc.

Future Directions

- Find a big computer and train with more data
- Integrate NN output around fish body and use output of network as surrogate hydrodynamics
- Physics-informed Neural Network similar to Raissi 2018 and Wandel 2021
- Consider using just the vorticity instead of the velocities for 2D
- Extend to 3D
- Compare with physical experiments

So long and thanks for all the fish!

Many thanks to Aaron, Courbin, Lisa, Pierre!