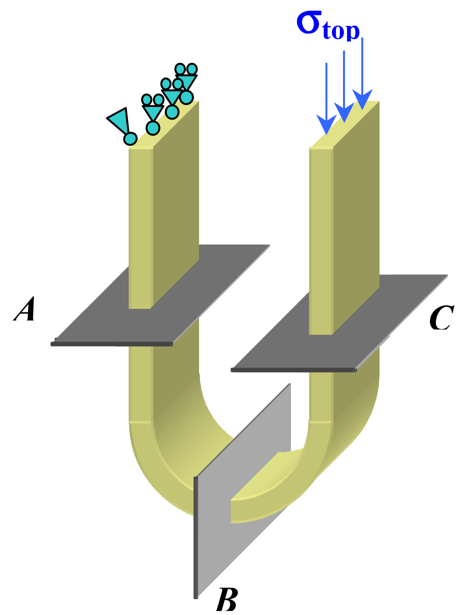
2.31 Project 4
Getting ready for shells

Team 6: Roberts, Schmidt
Team 7: Sobhani, Wolf

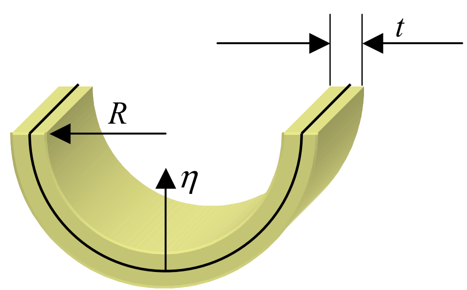
Due Wed, Nov 7 at 9:30 am



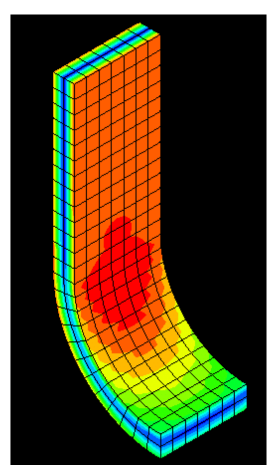
Often when you deal with shell structures additional complications arise from the fact that the shell surface is not flat, and you need to account for initial curvature effects. In elemental beam theory you might have looked at the case of a curved beam; if you haven't, I have attached at the end of this document a simple presentation of the theory.

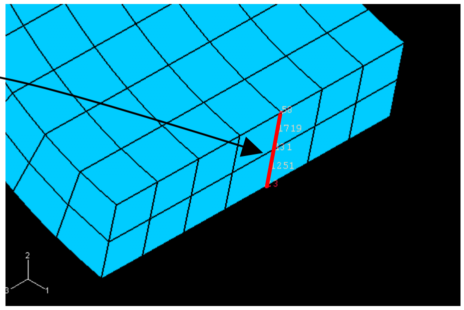
In Assignment 9, you have calculated the bending moment per unit dept, M_{bend} as well as the membrane axial stress, σ_{m_axial} at section B. Using the theory for curved beams in bending, evaluate the following additional quantities at section B:

1. $\sigma_{b\_axial}(\eta)$: the <u>profile</u> of axial stress associated with the bending moment as a function of η: distance from the center line: the center line is at radius $R+t/2$ and <u>does not</u> correspond to the neutral axis (see theory).
2. σ_{top_axial}: the axial stress on the top surface ($\eta = t/2$): this is the superposition of σ_{m_axial} and $\sigma_{b\_axial}(t/2)$.
3. σ_{bot_axial}: the axial stress on the bottom surface ($\eta = -t/2$): this is the superposition of σ_{m_axial} and $\sigma_{b\_axial}(-t/2)$.



Compare your theoretical predictions with the FE results.
In order to obtain the profiles of axial stress across section B you will have to learn to use the Display Group Tool. Create a Display Group that contains only the left part of the U-channel; let's say you call this display group Leftg. Use the Display Group manager to plot only the Leftg group. ⟶
If you need help with this feature, stop by during office hours, or make an appointment to see me (e-mail).





Now you can obtain the stress profiles along η on section B by creating a path along the midplane. Obtain profiles for the axial and transverse shear stress along the path. Compare the FE and theoretical values.

Both teams should conduct this study and write a short report on the results.

For the presentations, I would like to have:

<u>Team 7 (</u>Sobhani, Wolf) presenting and discussing Assignment 9 (stresses and forces at sections A and C) + showing the class how you create and work with display groups.
<u>Team 6</u> (Roberts, Schmidt) giving a brief overview of the theory for curved beams in bending, and discussing the results of the study on the forces and stresses at section B.

6.6 Bending Stresses in Curved Beams

In this section we consider briefly the theory of pure bending of an initially curved bar within the elastic range. Referring to Fig. 6.24a, consider a short portion of a curved bar acted upon by couples of moment M in the plane of initial curvature. Such bending moment which tends to decrease the initial curvature will be considered as *positive*. Each cross-section of the bar is assumed to have an axis of symmetry which lies in the plane of initial curvature. The locus of the centroids then is a plane curve called the *center line* of the bar and its radius of curvature is denoted by R.

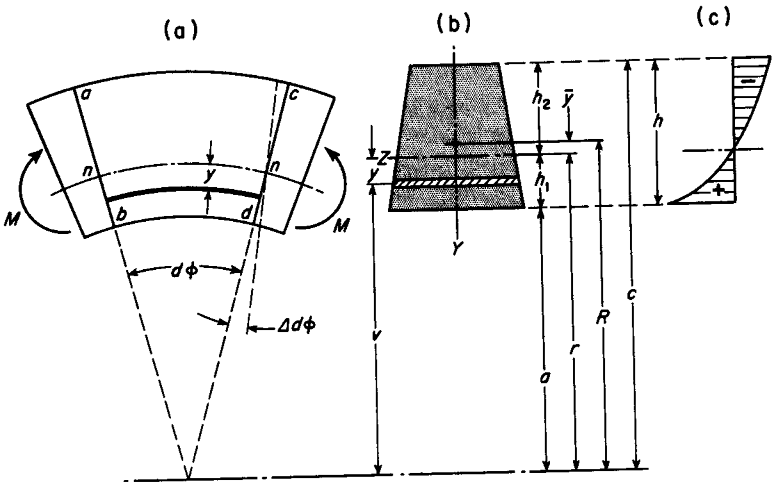


FIG. 6.24

In discussing the stress distribution produced by pure bending of such a curved bar, we make the same assumptions as in the case of straight bars, namely, that transverse cross-sections of the bar, originally plane and normal to the center line, remain so after bending. Let ab and cd denote two neighboring cross-sections of the bar and let $d\phi$ denote the small angle between them before bending. As a result of bending, the cross-section cd rotates with respect to ab. Let $\Delta d\phi$ denote the small angle of rotation. Due to this rotation, the longitudinal fibers on the convex side of the bar are compressed and the fibers on the concave side are extended. If n-n denotes the neutral surface, the extension of any fiber at the distance y from this surface is $y(\Delta d\phi)$ and the corresponding unit elongation is

$$\epsilon = \frac{y(\Delta d\phi)}{(r - y)d\phi}, \tag{a}$$

where r denotes the radius of the neutral surface and the denominator in eq. (a) is the length of the fiber between the adjacent cross-sections before bending. Assuming that there is no lateral pressure between the longitudinal fibers,* the bending stress at a distance y from the neutral surface is

$$\sigma = E\epsilon = \frac{Ey(\Delta d\phi)}{(r - y)d\phi}. \tag{b}$$

Eq. (b) shows that the stress distribution is no longer linear as in the case of straight bars, but that it follows a hyperbolic law as shown in Fig. 6.24c. From the condition that the sum of the normal forces distributed over the cross-section is zero, it can be concluded that the neutral axis is displaced from the centroid of the cross-section towards the center of curvature of the bar.

In the case of a *rectangular* cross-section, the shaded area (Fig. 6.24c) in tension must equal that in compression; hence the greatest bending stress acts on the concave side. In order to make the stresses in the most remote fibers in tension and in compression equal, it is necessary to use sectional shapes which have the centroid nearer the concave side of the bar.

Eq. (b) contains two unknowns, the radius r of the neutral surface and the angle $\Delta d\phi$ which represents the angular displacement due to bending. To determine them, we must use two equations of statics. The first equation is based on the condition that the sum of the normal forces distributed over a cross-section is equal to zero. The second equation is based on the condition that the moment of these normal forces is equal to the bending moment M. Thus

$$\int \sigma dA = \frac{E(\Delta d\phi)}{d\phi} \int \frac{ydA}{r - y} = 0, \tag{c}$$

$$\int \sigma y \, dA = \frac{E(\Delta d\phi)}{d\phi} \int \frac{y^2 dA}{r - y} = M. \tag{d}$$

The integration in both equations is extended over the total area of the cross-section.

Eq. (c) enables one to determine r and, in turn, the distance $\bar{y}$ (considered a positive quantity) from the centroidal axis to the neutral axis of the cross-section. Let v represent the distance from the center of curvature to any element dA; then $y = r - v$, and eq. (c) can be written

$$\int \frac{(r - v)dA}{v} = 0,$$

from which

$$r = \frac{A}{\int \frac{dA}{v}}, \tag{6.13}$$

*The exact theory shows that there is a certain radial pressure but that it has no substantial effect on the stress σ and can be neglected.

or

$$\bar{y} = R - \frac{A}{\int \frac{dA}{v}}, \tag{6.14}$$

where R is the initial radius of curvature of the center line of the bar.

Eq. (d) may be used to obtain a formula for the fiber stresses in terms of the bending moment. The integral in eq. (d) is first simplified as follows:

$$\int \frac{y^2 dA}{r - y} = - \int \left(y - \frac{ry}{r - y} \right) dA = - \int y \, dA + r \int \frac{y dA}{r - y}. \tag{e}$$

The first integral on the right side of eq. (e) represents the moment of the cross-sectional area with respect to the neutral axis, and the second, as is seen from eq. (c), is equal to zero. Hence

$$\int \frac{y^2 dA}{r - y} = -[A(-\bar{y})] = A\bar{y}. \tag{f}$$

Eq. (d) then becomes

$$\frac{E(\Delta d\phi)}{d\phi} = \frac{M}{A\bar{y}}.$$

Substituting this in eq. (b),

$$\sigma = \frac{My}{A\bar{y}(r - y)}. \tag{g}$$

The stresses in the most remote fibers which are the maximum stresses in the bar are

$$\sigma_{\max} = \frac{Mh_1}{A\bar{y}a} \quad \text{and} \quad \sigma_{\min} = - \frac{Mh_2}{A\bar{y}c}, \tag{6.15}$$

in which h_1 and h_2 are the distances from the neutral axis to the most remote fibers, and a and c are the inner and outer radii of the bar.

So far we have considered the case of pure bending where the bar is subjected to end couples only. In a more general case when a curved bar is bent by transverse forces acting in its plane of symmetry, the forces acting upon the portion of the bar to one side of any cross-section may be reduced to a couple and a force applied at the centroid of the cross-section. The moment of this couple equals that of the external forces with respect to the centroidal axis of the cross-section. The stresses produced by the couple are then obtained as explained above. The force is resolved into two components, a longitudinal force N in the direction of the tangent to the center line of the bar and a shearing force V in the plane of the cross-section. The longitudinal force produces tensile or compressive stresses uniformly distributed over the cross-section and equal to N/A. To get the total axial stress acting in any fiber, this uniform stress is added algebraically to the stress caused by the couple. The transverse force V produces shearing stresses and the distribution of these stresses over the cross-section can be taken the same as for a straight bar.

EXAMPLE 1. Determine the numerical value of the ratio $\sigma_{max}/\sigma_{min}$ for the case of a curved beam of rectangular cross-section in pure bending if $R = 5$ in. and $h = 4$ in.

SOLUTION. From eqs. (6.15)

$$\frac{\sigma_{max}}{\sigma_{min}} = \frac{h_1 c}{h_2 a}, \tag{h}$$

where $h_1 = h/2 - \bar{y}$ and $h_2 = h/2 + \bar{y}$ (see Fig. 6.24b). To calculate $\bar{y}$, we use eq. (6.14) in which

$$\int \frac{dA}{v} = \int_a^c \frac{b\,dv}{v} = b \ln\left(\frac{c}{a}\right) = b \ln\left(\frac{7}{3}\right) = 0.847\,b.$$

Thus

$$\bar{y} = R - \frac{bh}{0.847b} = 5 - 4.72 = 0.28 \text{ in.}$$

Then $h_1 = 2 - 0.28 = 1.72$ in. and $h_2 = 2 + 0.28 = 2.28$ in. With these values of h_1 and h_2, eq. (h) becomes

$$\frac{\sigma_{max}}{\sigma_{min}} = \frac{1.72 \times 7}{2.28 \times 3} = 1.76.$$