2.31 Project 6
Getting ready for nonlinearity

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Due November 21

We are going to start by modeling the same channel considered for Assignment 10, but this week we will use beam elements. The geometry, boundary conditions, and loading are the same (σ_{top}= 100 MPa, R=2m, H=4 m, W=0.5 m, t=0.5 m). This week, lets make the material stiffer, and change the Young's modulus to 200 GPa. Will this change the resulting stress? Why? (if you are not sure, rerun your continuum model).

Model the channel using quadratic beam elements (B32: Timoshenko shear flexible beams) with a seed of 0.5. Think carefully about the appropriate boundary conditions on the supported end of the beam.

Make sure you click on the ✦During analysis option in the section integration, so as to have stress contours available in the visualization module. As a default, the stress will be available at the four outer corners of the cross section (points 1,3,21,25 on Figure 6-2 of Handout 12). You can require additional points for the stress output: looking at Figure 6-2, figure out what is the number of the section point in the middle of the section, then in the in the Step module, use the Field Output Request Manager to edit the requests: right underneath the variable list, change the Output at shell, beam, and layered section points to ✦Specify, and in the field enter the numbers for the four corners and of the middle point. This saves stress values at all 5 section points to the .odb file. Also, in the same panel, add Section Forces and Moments to the required forces/reactions output quantities. Now run the job and compare the results of the beam model to the results for the continuum model and of the shell model that you obtained in Assignment 10. I would like to see comparisons for the axial stress, and for section forces and moments. For the beam elements, the axial section force is SF1 and it represents the total axial force on the beam, SF2 is the total transverse shear force in the local 2-direction, and SF3 is the total transverse shear force in the local 1-direction (SF2 and SF3 are not available for Euler-Bernoulli beams). The Moments are SM1 : bending moment about the local 1-axis, SM2: bending moment about the local 2-axis, and SM3: the twisting moment. The stress components are S11, the axial stress, and S12, the shear stress. The S12 component only accounts for the shear stress due to torsion, not for the shear stress from the transverse shear force, so it will not be of much use to us here. Get ready to give a presentation of this work to the class to help everybody understand better the output from beam elements.
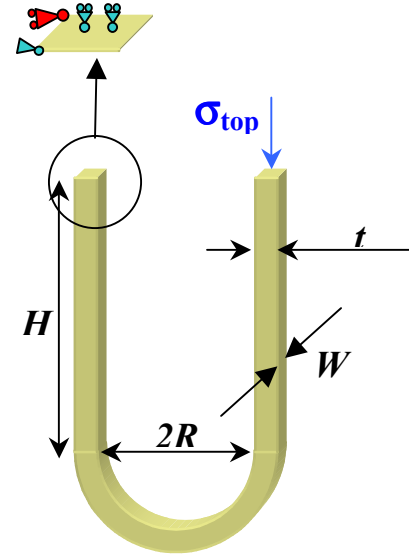
Now to get ready for nonlinearity, I would like you to review what you learned in 2.002 on elastic-plastic beam bending. To help refresh your memory and provide fresh material for your nightmares, I am attaching a 2.002 handout on elastic-plastic beam bending.
I will give a brief review of this stuff on Wed before your presentation.
In the following calculations you should focus on the flat left part of the beam, which, in the fully loaded condition, is subjected to a constant bending moment, M_{loaded}=112.5MNm, and neglect the effects of the superposed axial force.
Assume that the material is elastic-perfectly plastic with a yield stress σ_y=4GPa.
What is the elastic limit moment for this beam? What will be the extension of the elastic core in the fully loaded condition? What will be the curvature of the beam in the fully loaded configuration? What permanent curvature will the beam spring back to when we unload it? How many integration points through the thickness do you think we will need in a beam model or a shell model to capture this behavior properly? How many quadratic continuum elements through the thickness would you use for a continuum model of the structure?
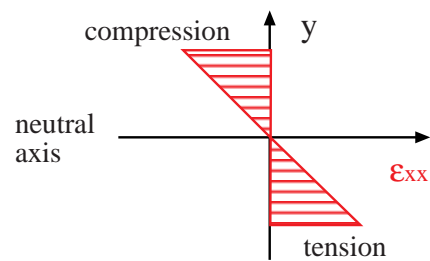Get ready to discuss these issues in your presentation.

Beam Bending -Review

--Plane sections remain plane.

y

x

M   ρ   M
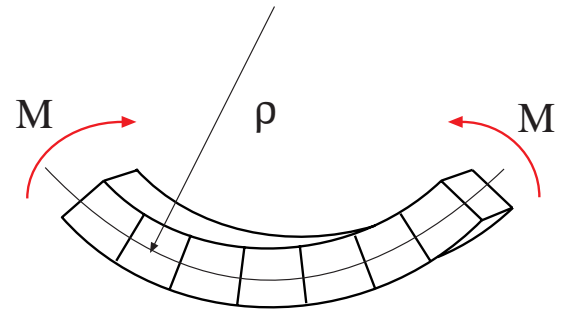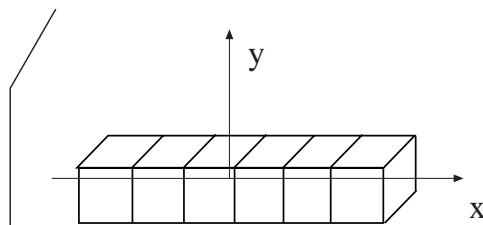
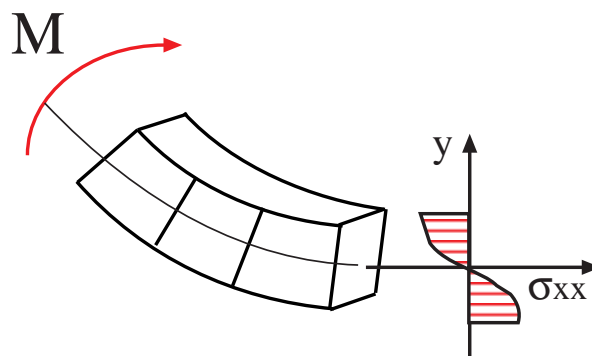ρ : radius of curvature; $k = 1/\rho$:curvature; M: moment

Compatibility : $\varepsilon_{xx} = -y/\rho$

Independent of material behavior

compression   y

neutral
axis

ε_{xx}

tension

Equilibrium: $M = \int_A -\sigma_{xx}\, y \, dA$

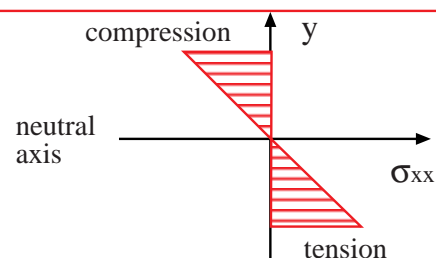*Independent of material behavior*

M

y

σ_{xx}

Elastic beam bending

elastic material behavior : $\boxed{\sigma_{xx} = E\ \varepsilon_{xx}}$
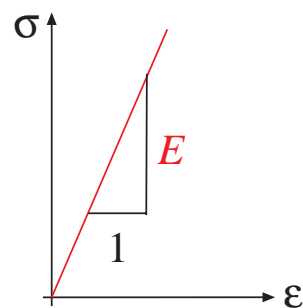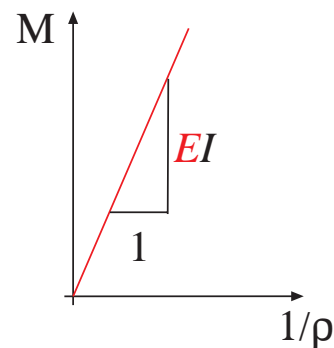

$\sigma_{xx} = E\ \varepsilon_{xx} = -E\ (y/\rho)$ $\longrightarrow$ $\sigma_{xx} = -M\ y/I$

linear stress distribution

$M = \displaystyle\int_A -\sigma_{xx}\ y\ dA = E\ /\rho \int_A y^2\ dA \longrightarrow M = EI\ /\ \rho$

I : second moment of area

linear moment-curvature relationship

$I = \displaystyle\int_A y^2\ dA$

for rectangular beams $\longrightarrow$ $I = BH^3/12$
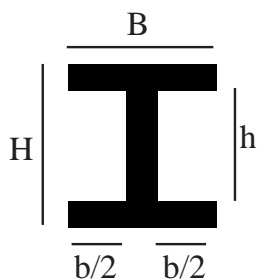
for I-beams $\longrightarrow$ $I = (BH^3 - bh^3)/12$

Elastic -plastic beam bending

elastic-perfectly plastic material behavior

σ

σ_y

E

1

ε

nonlinear Moment-curvature relationship

M_p

$M \to M_p$: fully plastic regime: $|\sigma| = \sigma_y$ at all fibers of beam

σ_y y

σ_{xx}

M_e

$M > M_e$: elastic-plastic regime: $|\sigma| = \sigma_y$ at outer fibers of beam

σ_y y

σ_{xx}

EI

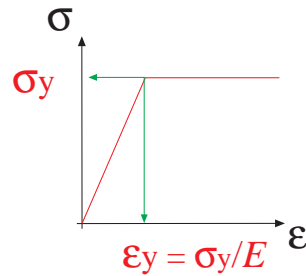$M < M_e$: elastic regime: $|\sigma| < \sigma_y$ at all points

y

σ_{xx}

1

$1/\rho$
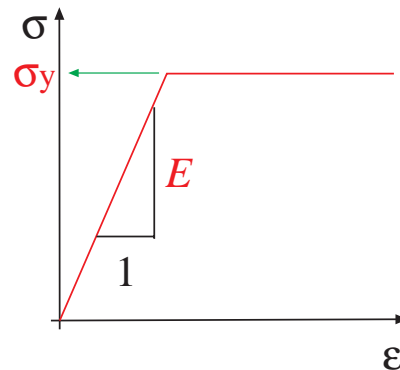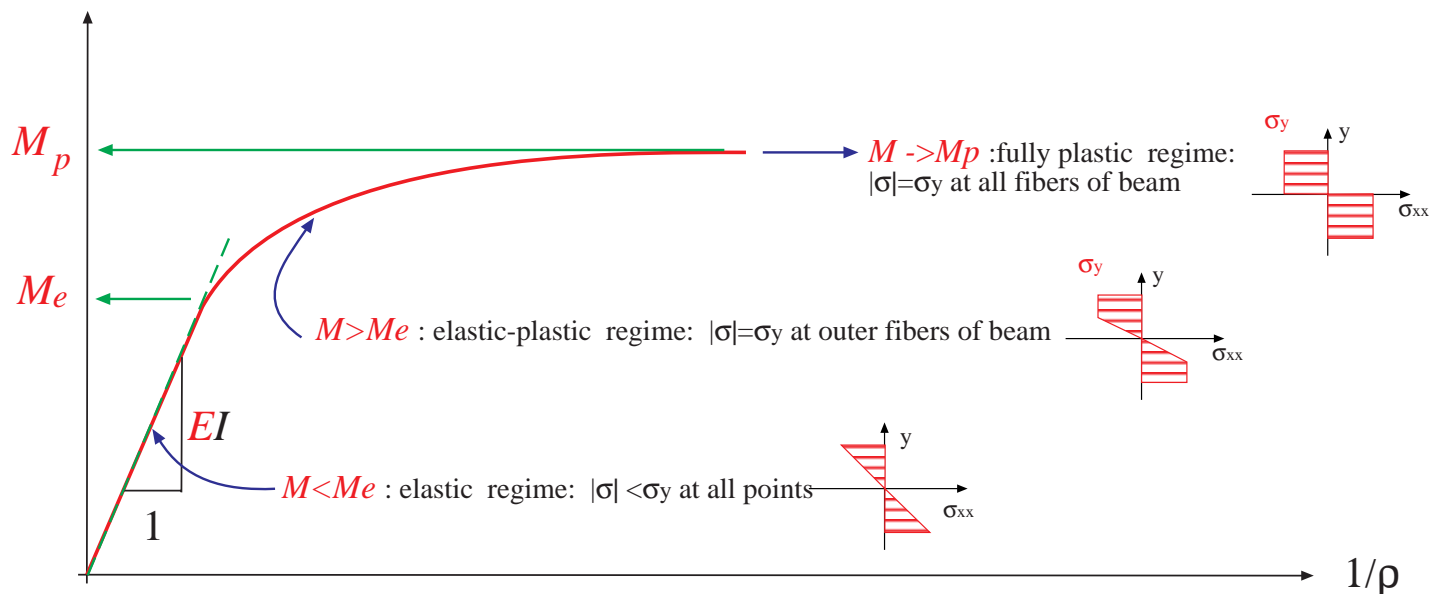
In the elastic regime $\sigma_{xx} = -M\,y/I \;\to\; |\sigma_{max}| = M\,y_{max}/I$

Yielding initiates at the outer fibers of the beam when $|\sigma_{max}| = \sigma_y$. This corresponds to a bending moment M_e :

$$M_e = (\sigma_y\, I)/y_{max}$$
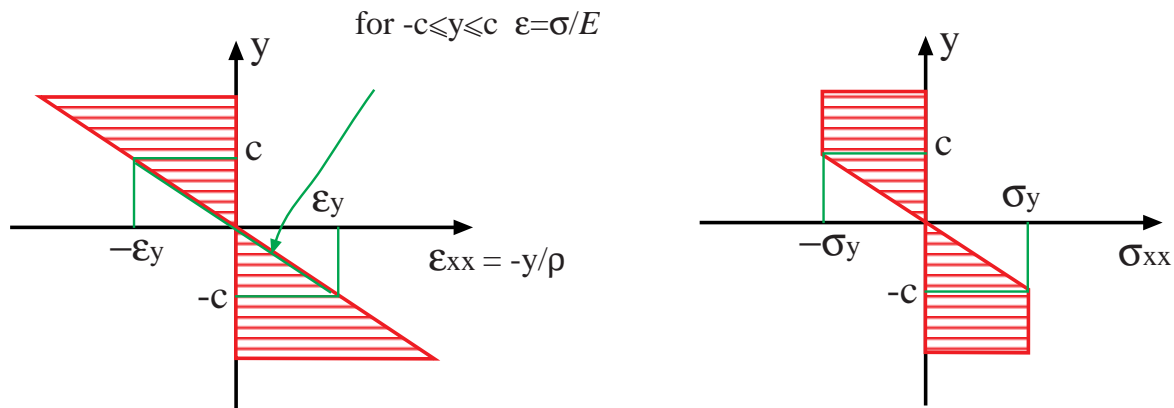
for rectangular beams : $M_e = (\sigma_y\, BH^2)/6$

Elastic -plastic beam bending

elastic-perfectly plastic material behavior

$\varepsilon_y = \sigma_y/E$

For $M > Me$ the beam is in the elastic-plastic regime: the core of the beam (between y=- c and y=c) is in the elastic regime, while the outer fibers are in the plastic regime ($\sigma = \sigma_y$).

for $-c \leqslant y \leqslant c$ $\varepsilon = \sigma/E$

$\varepsilon_{xx} = -y/\rho$

For $|y| = c$, $\varepsilon = -\sigma_y/E \longrightarrow -y/\rho = -c/\rho = -\sigma_y/E \longrightarrow$ the extension of the elastic region is given by:
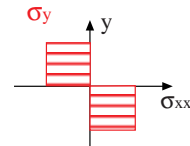
$$c = \rho\,(\sigma_y/E)$$

The moment-curvature relationship is then given by:

For a rectangular beam :

$$M = \int_A -\sigma_{xx}\, y\, dA = E/\rho \int_{-c}^{c} y^2\, dA + \int_{-y_{max}}^{-c} -\sigma_y\, y\, dA + \int_{c}^{y_{max}} -\sigma_y\, y\, dA$$

$$M = \sigma_y\,(3H^2 - 4c^2)\,B/12$$

For very large curvatures c-->0 and the moment approaches the limit moment Mp, where the entire section is in the plastic regime $\longrightarrow$
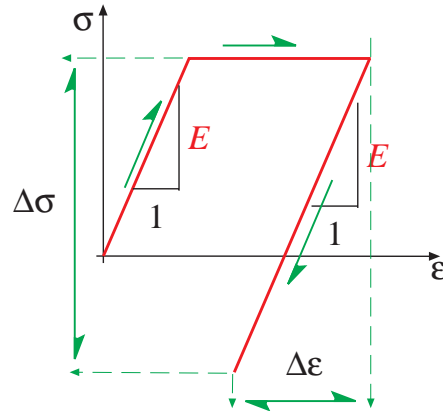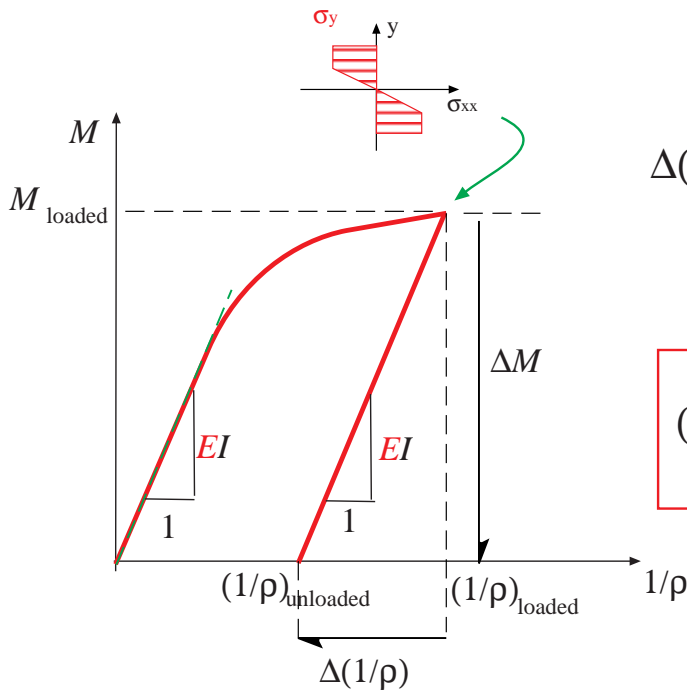

$$M\,p = \int_A -\sigma_y\, y\, dA$$

for rectangular beams : $Mp = (\sigma_y\, BH^2)/4 = 1.5\ Me$

material behavior: <u>elastic unloading</u>
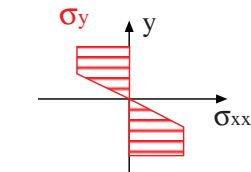
$$\Delta\sigma = E \, \Delta\varepsilon$$

<u>The beam unloads elastically</u>

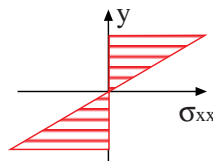$\Delta(1/\rho)=\Delta M \,/\, EI = -M_{\text{ loaded}} \,/EI$

$$(1/\rho)_{\text{unloaded}} =(1/\rho)_{\text{loaded}}-M_{\text{ loaded}} \,/EI$$

$M_{\text{ loaded}}$

ΔM

EI

1

EI

1

$(1/\rho)_{\text{unloaded}}$ $(1/\rho)_{\text{loaded}}$ $1/\rho$

$\Delta(1/\rho)$
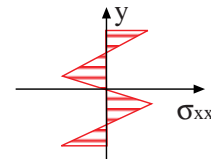
<u>Stress distribution upon unloading</u>

stress distribution in the loaded configuration
σ_{loaded}

Change in stress upon unloading
$\Delta\sigma=(M_{\text{loaded}} \, y)/I$

Residual stress distribution
$\sigma_{residual}$ $M =0$