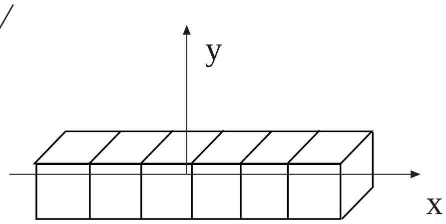
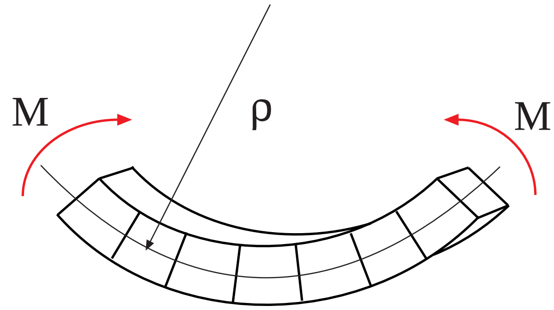
Beam Bending -Review

--Plane sections remain plane.

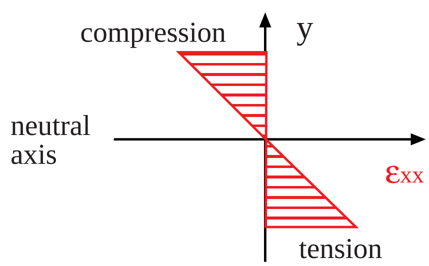




ρ : radius of curvature; $\kappa = 1/\rho$:curvature; *M*: moment

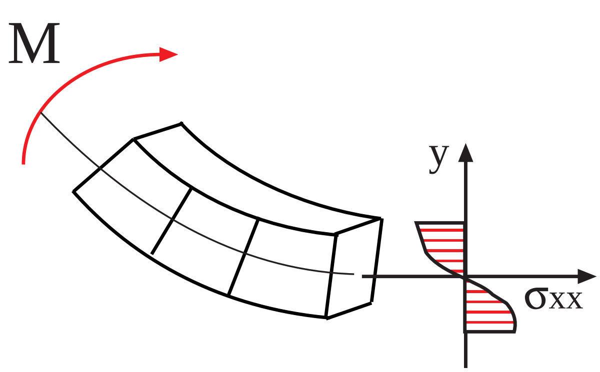
Compatibility : $\varepsilon_{xx} = -y/\rho$

Independent of material behavior



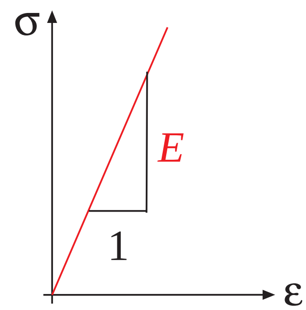
Equilibrium: $M = \int_A -\sigma_{xx}\, y\, dA$

Independent of material behavior



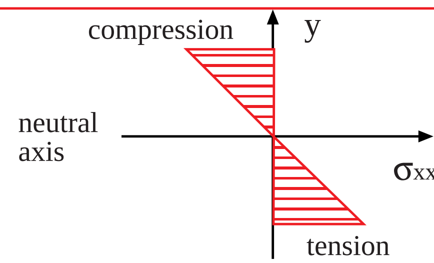
Elastic beam bending

elastic material behavior : $\sigma_{xx} = E \ \varepsilon_{xx}$



$\sigma_{xx} = E \ \varepsilon_{xx} = -E \ (y/\rho)$

$\sigma_{xx} = -M \ y/I$

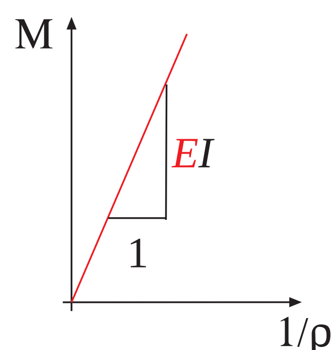


linear stress distribution

$M = \int_A -\sigma_{xx} \ y \ dA = E \ /\rho \int_A y^2 \ dA$

I : second moment of area

$M = EI / \rho$



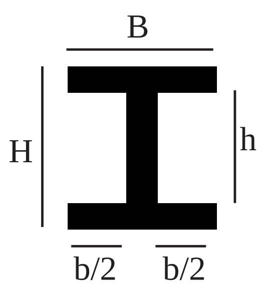
linear moment-curvature relationship

$I = \int_A y^2 \ dA$

for rectangular beams

$I = BH^3/12$

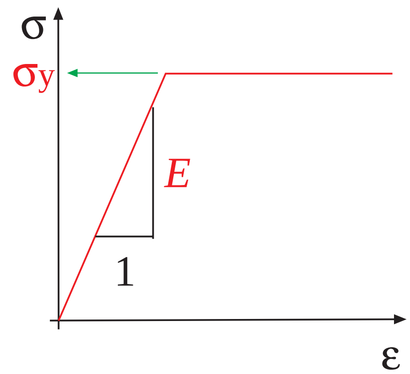
for I-beams



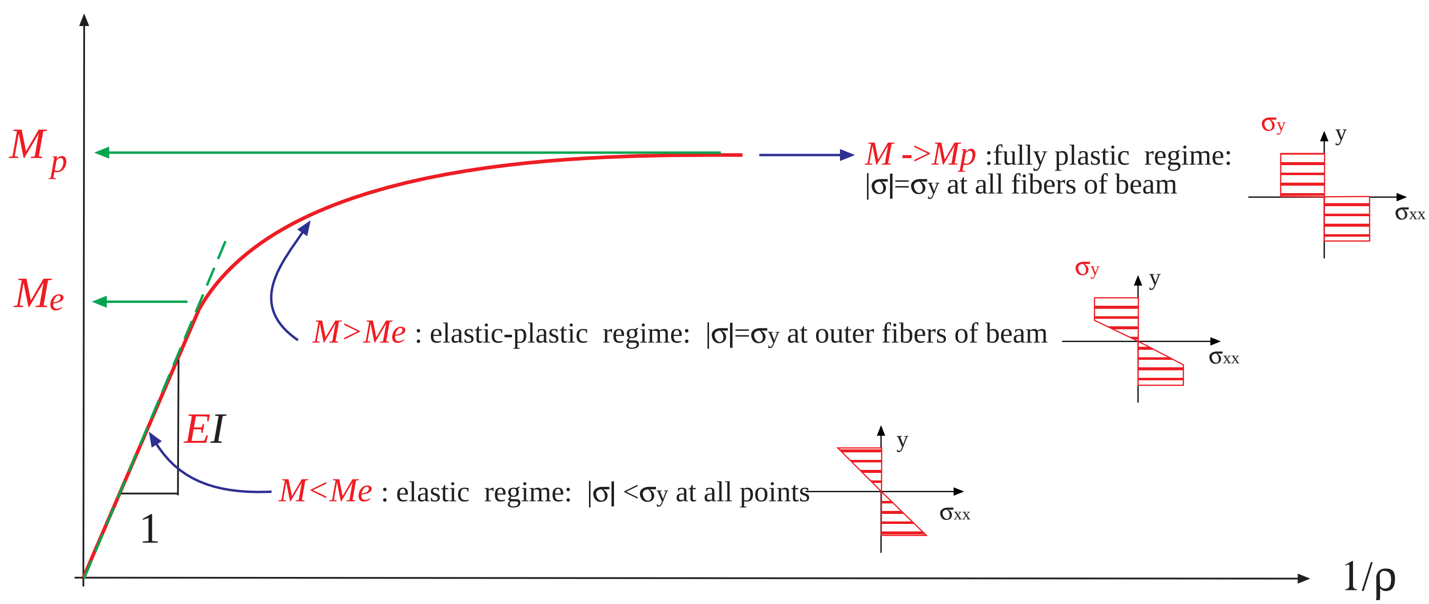
$I = (BH^3 - bh^3)/12$

Elastic -plastic beam bending

elastic-perfectly plastic material behavior



nonlinear Moment-curvature relationship



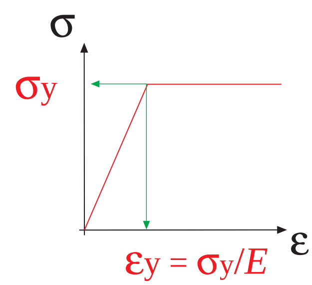
In the elastic regime $\sigma_{xx}=-M\,y/I$ -> $|\sigma_{max}| = M\;y_{max}/I$

Yielding initiates at the outer fibers of the beam when $|\sigma_{max}| = \sigma_y$. This corresponds to a bending moment Me :

$$Me = (\sigma_y\, I\,)/y_{max}$$

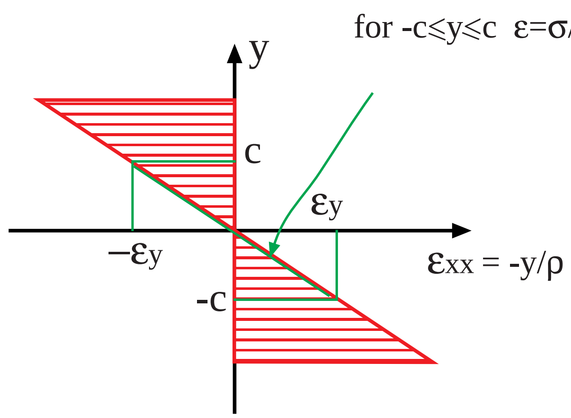
for rectangular beams : $Me = (\sigma_y\, BH^2)/6$

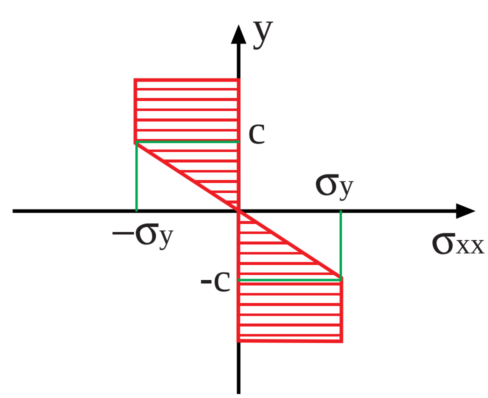
Elastic -plastic beam bending



elastic-perfectly plastic
material behavior

For $M>M_e$ the beam is in the elastic-plastic regime: the core of the beam (between y=- c and y=c) is in the elastic regime, while the outer fibers are in the plastic regime ($\sigma=\sigma_y$).





For $|y| = c$, $\varepsilon=-\sigma_y/E$ --> $-y/\rho = -c/\rho=-\sigma_y/E$--> the extension of the elastic region is given by:

$$c= \rho\,(\sigma_y/E)$$

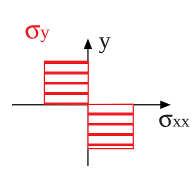
The moment-curvature relationship is then given by:

$$M = \int_A -\sigma_{xx}\, y\, dA \;=\; E/\rho \int_{-c}^{c} y^2\, dA + \int_{-y_{max}}^{-c} -\sigma_y\, y\, dA + \int_{c}^{y_{max}} -\sigma_y\, y\, dA$$

For a rectangular beam :

$$M = \sigma_y\,(3H^2-4c^2)\,B/12$$

For very large curvatures c-->0 and the moment approaches the limit moment M_p, where the entire section is in the plastic regime



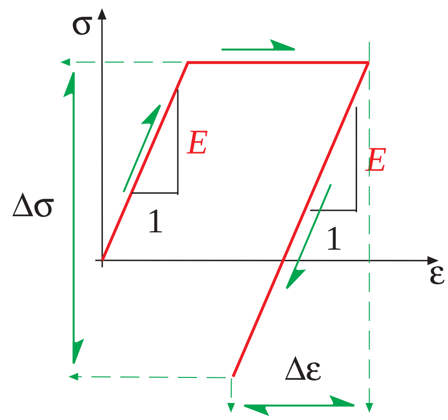
$$M_p = \int_A -\sigma_y\, y\, dA$$

for rectangular beams : $M_p = (\sigma_y B H^2)/4 = 1.5\, M_e$

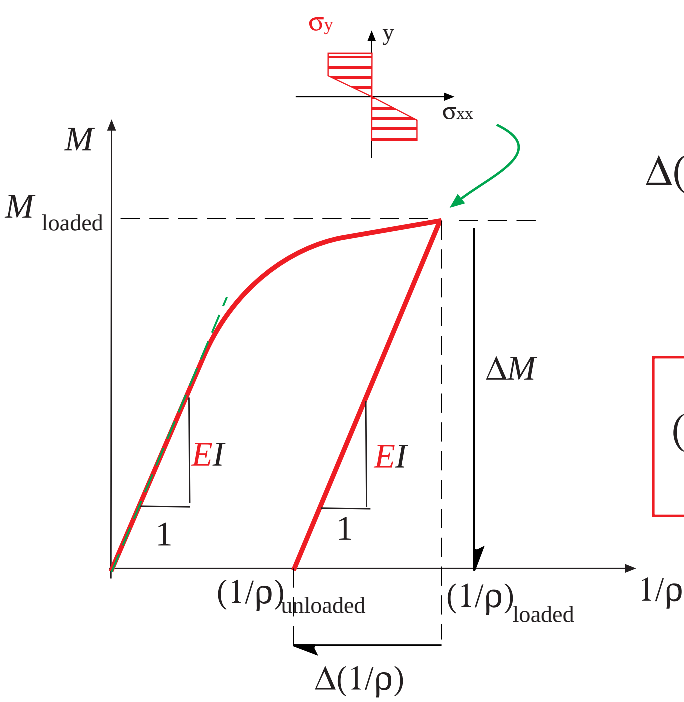
Elastic -plastic beam bending . Unloading /springback

material behavior: elastic unloading

$$\Delta\sigma = E\,\Delta\varepsilon$$



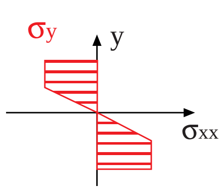
The beam unloads elastically



$$\Delta(1/\rho)=\Delta M / EI = -M_{loaded} / EI$$

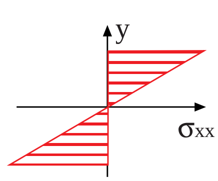
$$(1/\rho)_{unloaded} = (1/\rho)_{loaded} - M_{loaded} / EI$$

Stress distribution upon unloading



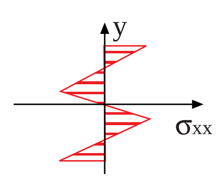
stress distribution in the loaded configuration

σ_{loaded}



Change in stress upon unloading

$\Delta\sigma = (M_{loaded}\, y)/I$



Residual stress distribution

$\sigma_{residual} \longrightarrow M = 0$