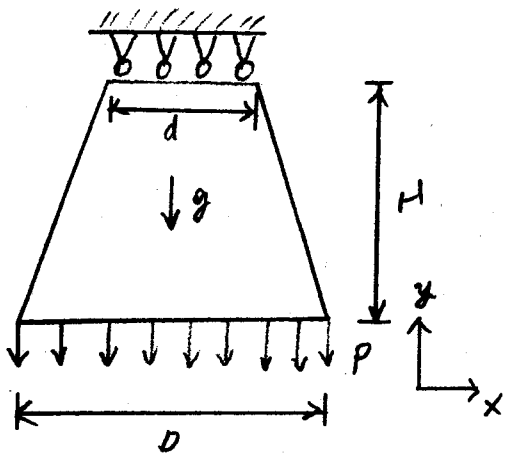
2.31 Assignment 4 (Due Oct 3) → Solutions



GIVEN:

$D = 10m$
$H = 10m$
$d = 2m$
$t = 1m$
$P = 0.1MPa$

Material Properties:

$E = 200\,GPA$
$\nu = 0.3$
$\rho = 7140\ kg/m^3$

Assume: 1-D model, consider $\sigma = \sigma_{yy}$

a) Find the total vertical reaction at the support

$R_T = W_{plate} + F_{(PRESSURE)}$

① $W_{PLATE} = \rho g \int dV \qquad dV = (D - \frac{4}{5}y)\, t \cdot dy$

$= \rho g \int_0^H (D - \frac{4}{5}y)\, t\, dy$

$= \rho g t \left[Dy - \frac{2}{5}y^2 \right]_0^H = \rho g t (DH - \frac{2}{5}H^2)$

Substituting Numbers

$\boxed{W_{plate} = 4.19 \times 10^6 N}$

② $F_{PRESSURE} = P \cdot A_{plate\ bottom}$

$= P(Dt)$

$= (0.1 \times 10^6 Pa)(10m)(1m)$

$= 10^6 N$

$\therefore R_T = (4.19 \times 10^6 N) + (1 \times 10^6 N) = \boxed{5.19 \times 10^6 N}$

1. b) Calculate & plot the axial stress profile.

$\sigma_y = \sigma_{weight} + \sigma_{pressure}$

$\sigma_{PRESSURE} = \frac{P_0 \cdot A_0}{A(y)} = \frac{P_0 \cdot (Dt)}{[5D-4y]t/5} = \frac{5P_0D}{5D-4y}$ ✓

$\sigma_{weight} = \frac{F_{weight}}{A(y)}$ $\quad F_{weight} = \frac{\rho g t [5Dy - 2y^2]}{5}$ (from (a))

$A(y) = [D - \frac{4}{5}y]t = \frac{[5D-4y]t}{5}$

$\therefore \sigma_{w(y)} = \frac{5\rho g x y [5D-2y]}{5t\,[5D-4y]} = \boxed{\frac{\rho g y [5D-2y]}{[5D-4y]}}$

So,

$$\boxed{\sigma_y = \frac{\rho g y [5D-2y]}{[5D-4y]} + \frac{5P_0D}{5D-4y}}$$

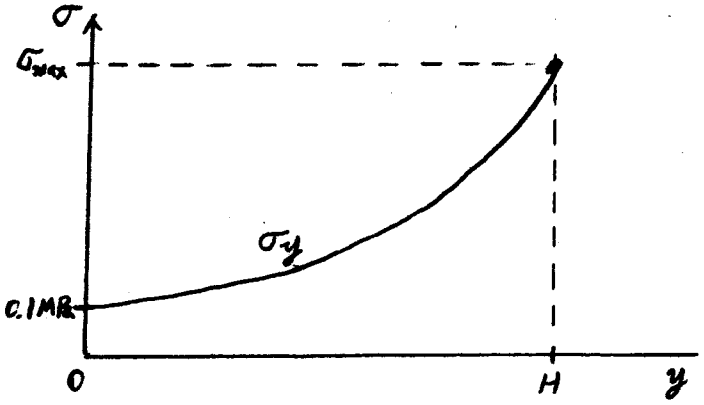
c) Max Stress level & location

Max Stress σ_{max} occurs at $y = 10m$ (at the support)

Substituting values into the formula

$$\boxed{\sigma_{(y=10)} = 2.59\ MPa}$$

* Stress plot.



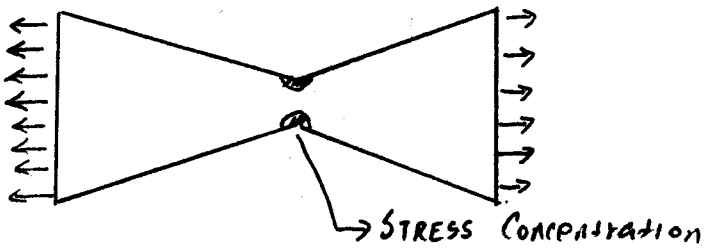
Part II. Analysis

a) Comparing σ_{yy} Results

$\sigma_{THEORY} = 2.59$ MPa (max)

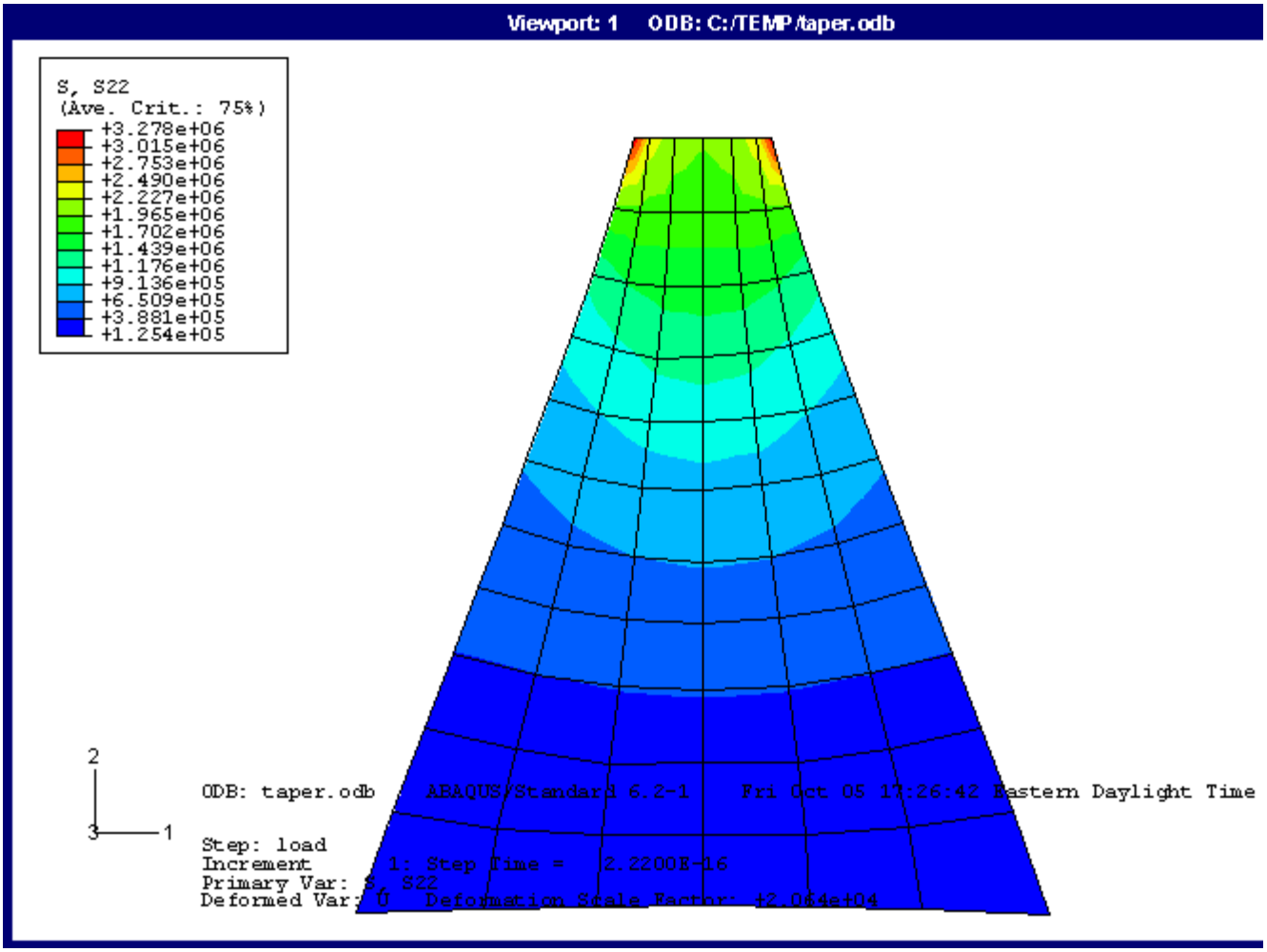
$\sigma_{computer} = 3.30$ MPa (max)

Computed stress, σ_{yy}, was higher than the theoretical calculation. On the computer analysis, the corner of the support was subjected to the highest level of stress. This is caused by the stress concentration on that region, due to the shape of the structure



This tells that by theorizing this shape as a linear beam, we have underestimated the structure stress level.

b) The reaction at the support agreed with our theoretical value. This will be true for all structures in the static equilibrium.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+1.965e+06
+1.702e+06
+1.439e+06
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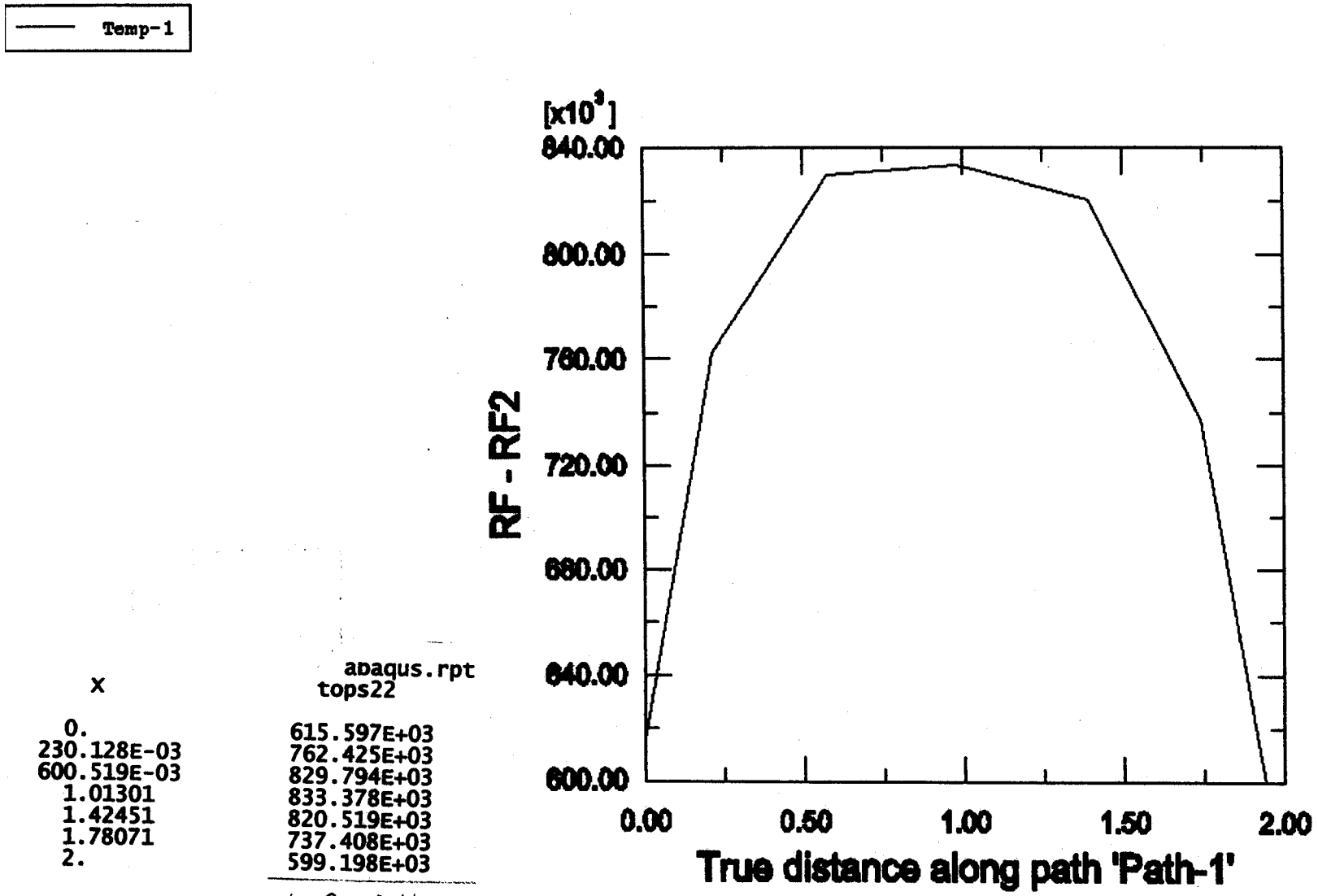
+9.136e+05
+6.509e+05
+3.881e+05
+1.254e+05
2
3
1
ODB: taper.odb ABAQUS/Standard 6.2-1 Fri Oct 05 17:26:42 Eastern Daylight Time
Step: load
Increment 1: Step Time = 2.2200E-16
Primary Var: S, S22
Deformed Var: U Deformation Scale Factor: +2.064e+04

Viewport 1 Graph XYPlot 1



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1.78071	737.408E+03
2.	599.198E+03

5.19 MN ✓