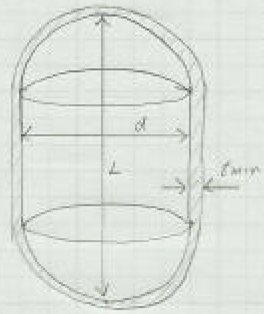
Problem Set #6 Solution



Given:

$d = 2m$

$L = 5m$

$P = 200\ MPa$

$E = 200\ GPa$

$\nu = 0.3$

$\sigma_y = 1.5\ GPa$

- Criteria

$\sigma_{\theta\theta} = 1.0\ GPa$

- Find t_{min}

I) Paper Calculation (No FE)

a) Using thin wall assumption (From Engineering Reference Manual)

In general, tank can be considered as a thin walled tank if,

$\frac{t}{2r} < 0.1$ t = thickness, r = inner radius

- For $\sigma_{\theta\theta}$

$\sigma_{\theta\theta} = \frac{Pr}{t}$ Known: $\sigma_{\theta\theta} = 1.0 \times 10^9\ Pa$, $r = 1m$, $P = 200 \times 10^6\ Pa$

$$t_1 = \frac{Pr}{\sigma_{\theta\theta}} = \frac{(200 \times 10^6\ Pa)}{(1.0 \times 10^9\ Pa)}(1m) = \boxed{0.2\ m}$$

- Using $t = 0.2$

$$\sigma_{zz} = \frac{Pr}{2t} = \frac{(200 \times 10^6\ Pa)(1m)}{2\ (0.2m)} = \boxed{500\ MPa}$$

$\sigma_{rr} = \boxed{200\ MPa}$ (Thin wall)

c) Using t_1, obtained from thin wall assumption, on the thick wall tank stress calculation

$$\sigma_{rr} = \frac{a^2 P}{b^2 - a^2}\left[1 - \frac{b^2}{r^2}\right] = \frac{(1m)^2(200MPa)}{(1.2m)^2 - (1m)^2}\left[1 - \frac{(1.2m)^2}{(1m)^2}\right]$$

$$= \boxed{200\,MPa}$$

$$\sigma_{\theta\theta} = \frac{a^2 P}{b^2 - a^2}\left[1 + \frac{b^2}{r^2}\right] = \frac{(1m)^2(200MPa)}{(1.2m)^2 - (1m)^2}\left[1 + \frac{(1.2)^2}{(1m)^2}\right]$$

$$= 1.11 \times 10^9\,Pa = \boxed{1.11\,GPa}$$

$$\sigma_{zz} = \frac{Pa^2}{(a+b)t} = \frac{(200MPa)(1m)^2}{(1m + 1.2m)(0.2m)} = \boxed{454\,MPa}$$

* It does not satisfy design requirement ($\sigma_{\theta\theta} > \sigma_{allowed}$)

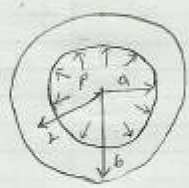
b) Using thick wall assumption

$$\frac{t}{2r} > 0.1$$

- Using formula from the handout

(Cylinder under Internal pressure Only)

$$\sigma_{rr} = \frac{a^2 p}{b^2 - a^2}\left[1 - \frac{b^2}{r^2}\right] \quad ; \quad \sigma_{\theta\theta} = \frac{a^2 p}{b^2 - a^2}\left[1 + \frac{b^2}{r^2}\right]$$



For maximum stress

Let $r = a$

$b = a + \text{thickness}$

Since we know every value except b, we can calculate the thickness (by hand or using spreadsheet)

$\therefore\ t_2 = 0.224\ m$

$$* \sigma_{zz} = \frac{pa^2}{(a+b)t}$$ (Engineer's Reference Manual)

$\sigma_{\theta\theta} = 1.0\ GPa$

$\sigma_{rr}\ (\text{at } r=1) = 200\ MPa$

$\sigma_{zz} = 401\ MPa$

- The initial estimate t_1 was less than the improved estimate t_2, which was obtained by thick wall assumption.
- If thin wall assumption is used for this design, it will underestimate the thickness of the tank, resulting in tank failure.

Comments

1. By assuming this tank as a thin walled tank, we obtained the thickness which was unsatisfactory in terms of thick walled estimate. As it was mentioned in the calculation above, the tank can be assumed as a thin walled tank when the Thickness to Diameter ratio is less than 0.1

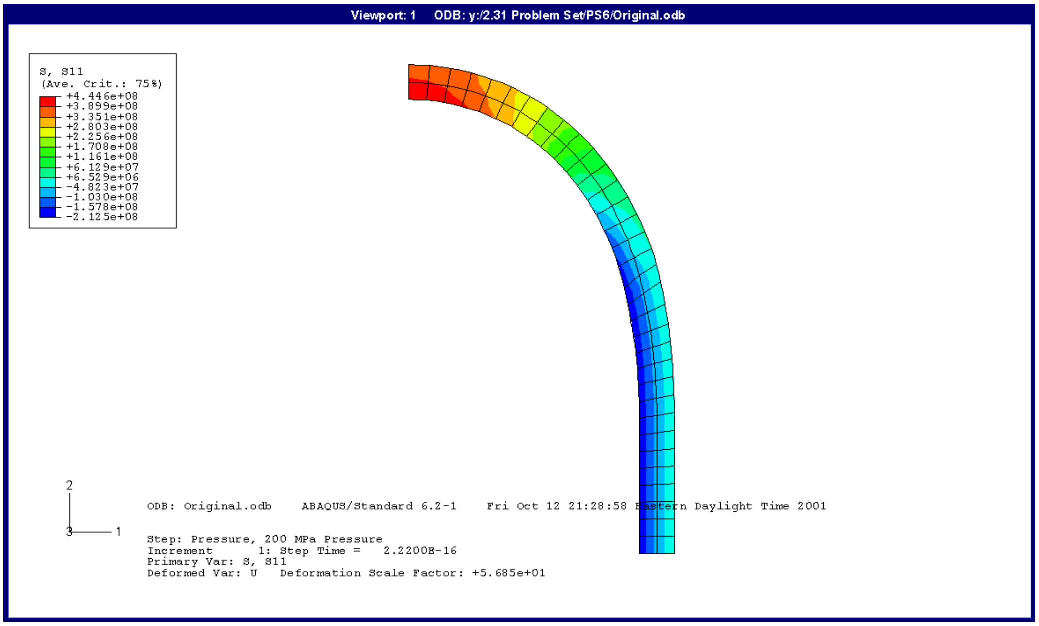
2. The FE analysis value for $\sigma_{\theta\theta}$ was higher than the result obtained by theoretical calculation for the given thickness, t2.
   This is an effect of the actual tank geometry which differs from the infinite cylinder for which the theoretical calculations where carried out. The emispherical cap creates an additional stress concentration at the inner surface that does not decay entirely in the relatively short axial distance corresponding to the length of the tank.

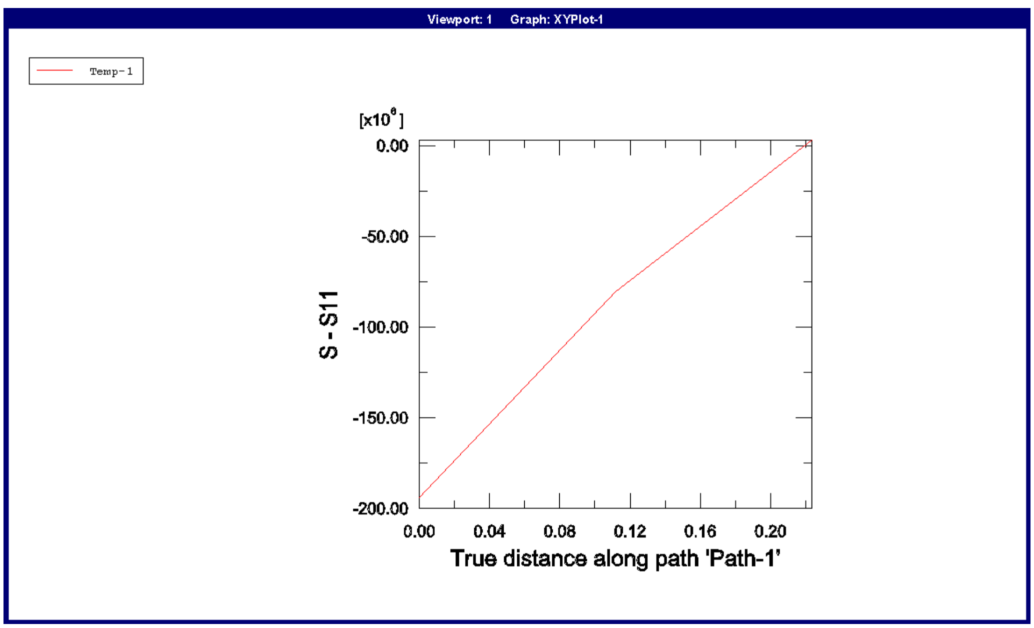
3. For σ_{rr}, the result was as expected, with 200 Mpa compressive stress inside of the cylinder and no stress on the outer surface. This was the same for both the coarse and the fine mesh.

4. With a finer mesh, the value for $\sigma_{\theta\theta}$ became slightly lower.
   This effect is related to the way the contour plot values are obtained. The nodal values are estrapolated from the actual values at the integration points using shape functions. Since these are quadratic elements, the shape functions used to extrapolate the values to the inner surface nodes are steeper than the actual stress distributions and they tend to overshoot the nodal values at the inner surface. The finer mesh has actual values at the integration points closer to the inner surface and therefore a shorter extrapolation distance and lower overshoot.

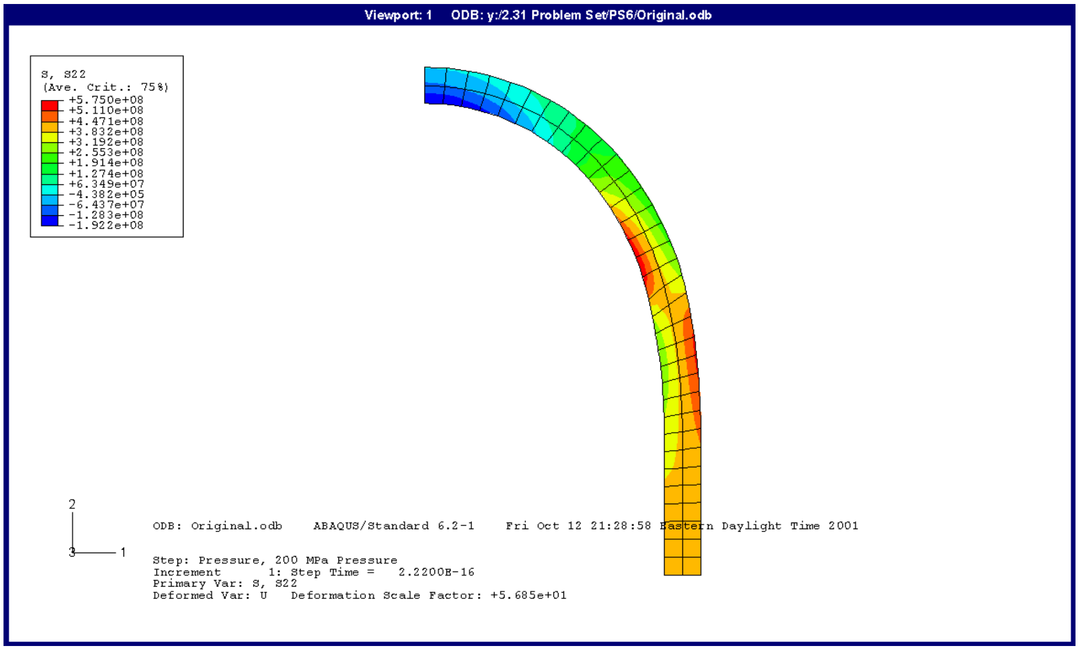
5 The tank thickness should be designed by relying on the fine mesh results. The hand calculations cannot account for the effects of the hemispherical caps on the stress at the inner surface.
The thickness and the inner stress do not scale linearly, but, based on the FE results with t=t2, we can try a tank with t=t3=0.23 m. For this thickness we get a max circumferential stress of 992 MPa, which has a reasonable safety margin to account for possible FE uncertainties.

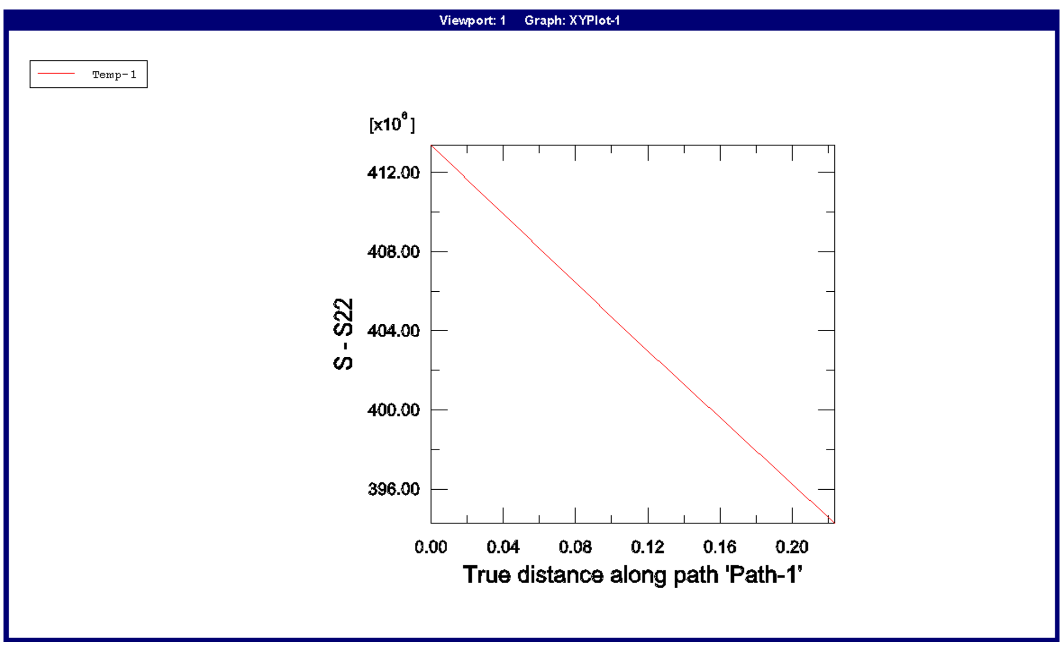
Coarse Mesh (σ rr) t=t2
Contour & Node Plot



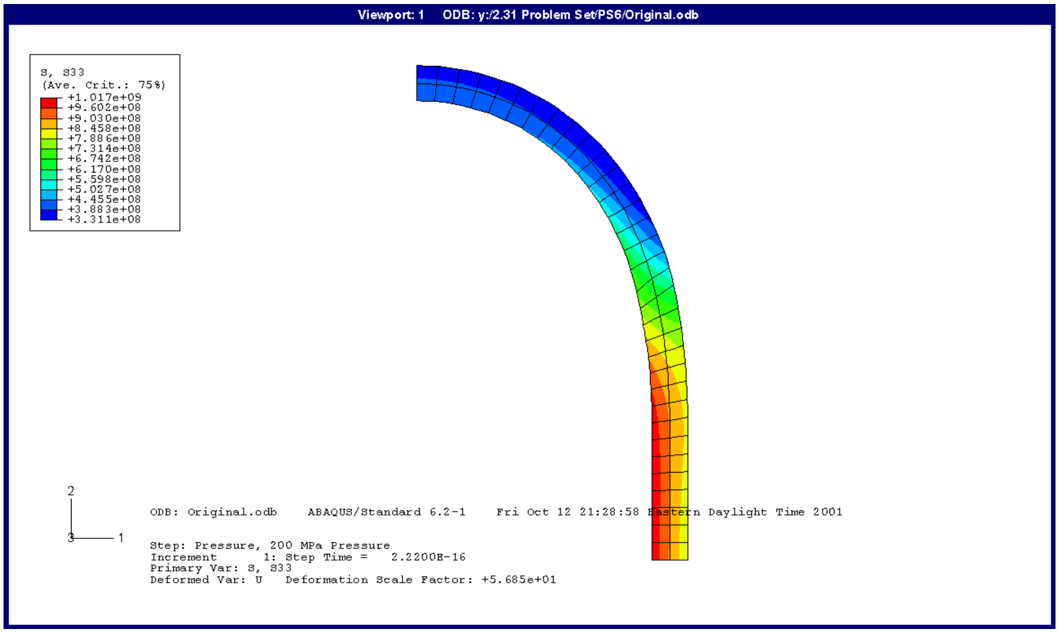


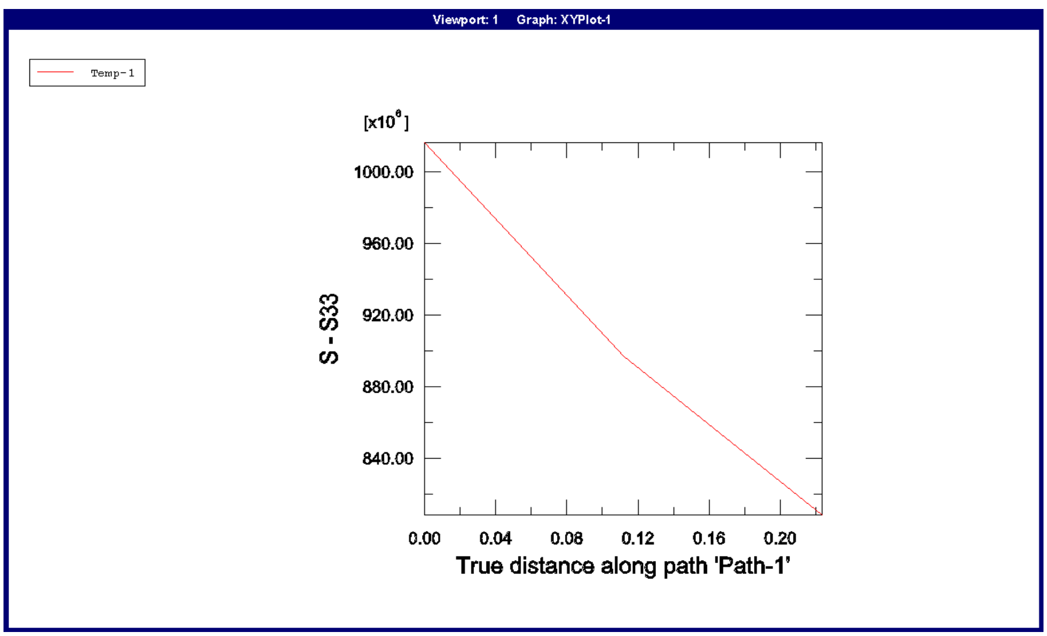
Coarse Mesh (σ zz) t=t2
Contour & Node Plot



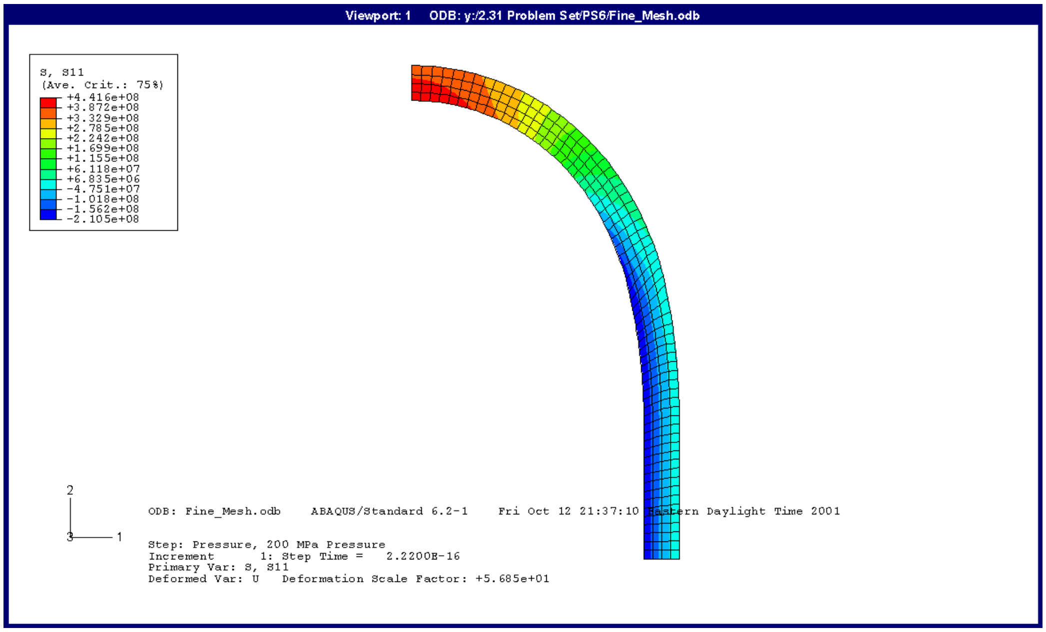


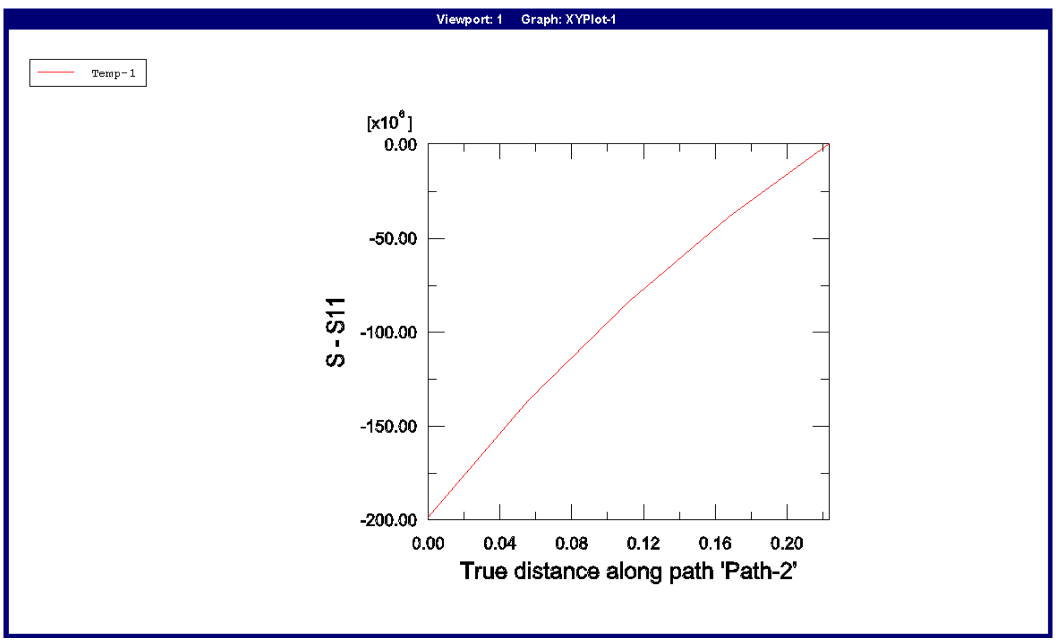
Coarse Mesh (s qq) t=t2
Contour & Node Plot



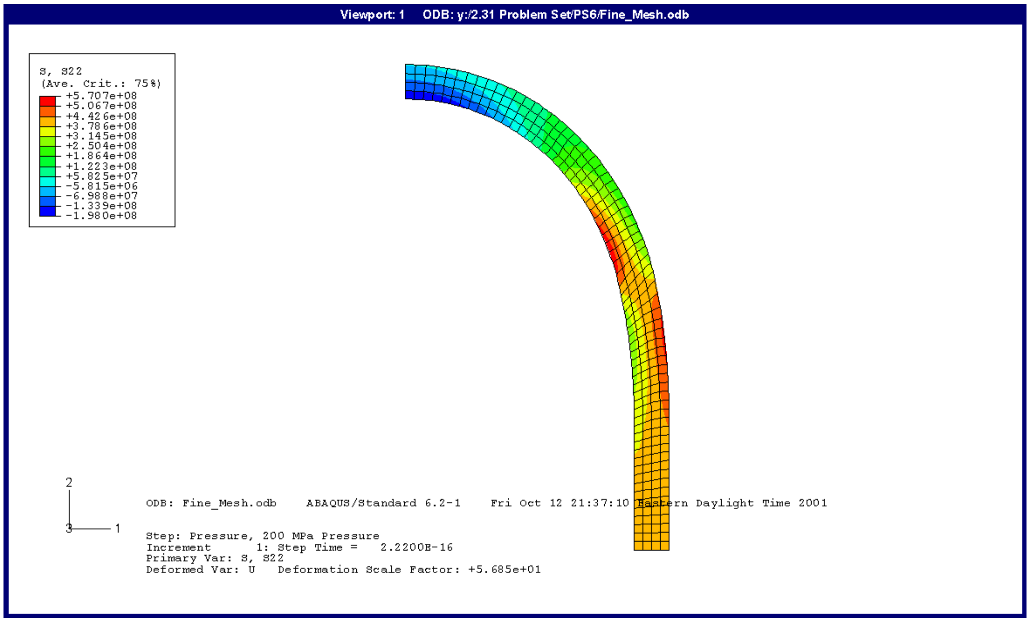


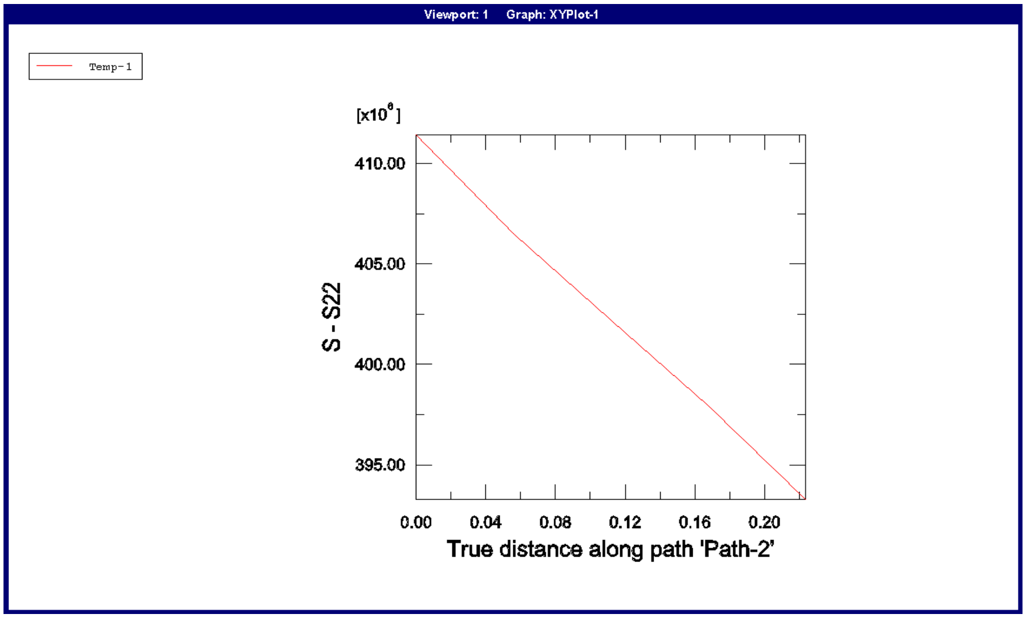
Fine Mesh (σ rr) t=t2
Contour & Node Plot



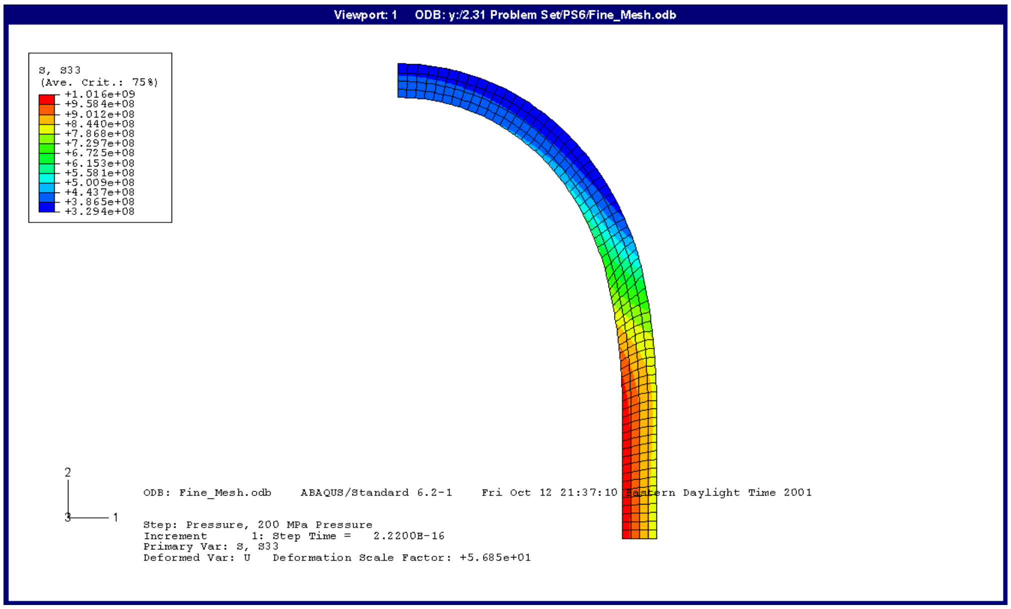


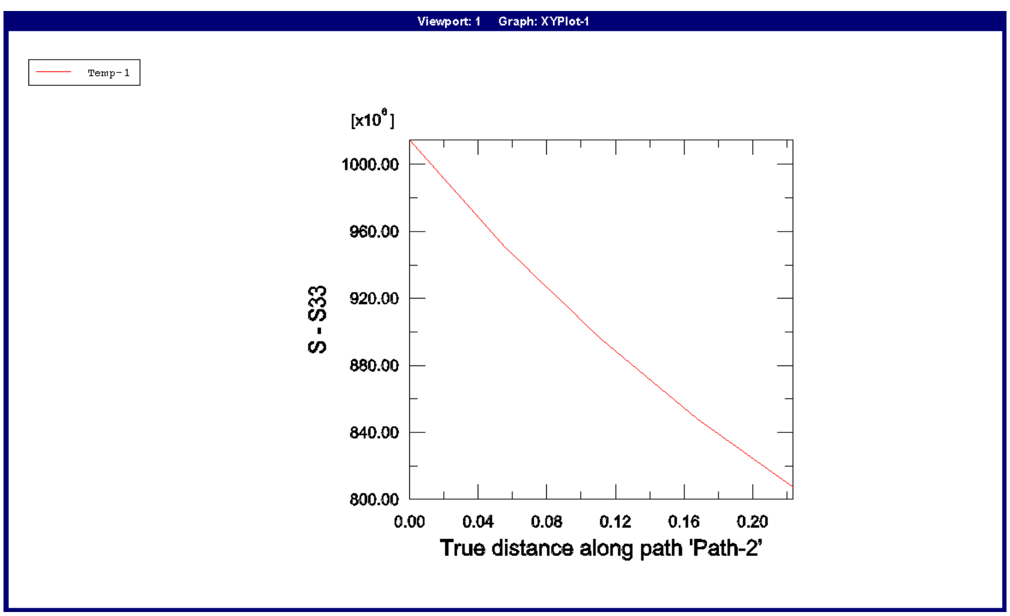
Fine Mesh (σ_{zz}) t=t2
Contour & Node Plot





Fine Mesh (s qq) t=t2
Contour & Node Plot





Fine Mesh (s qq) t=t3=0.23 m
Contour Plot