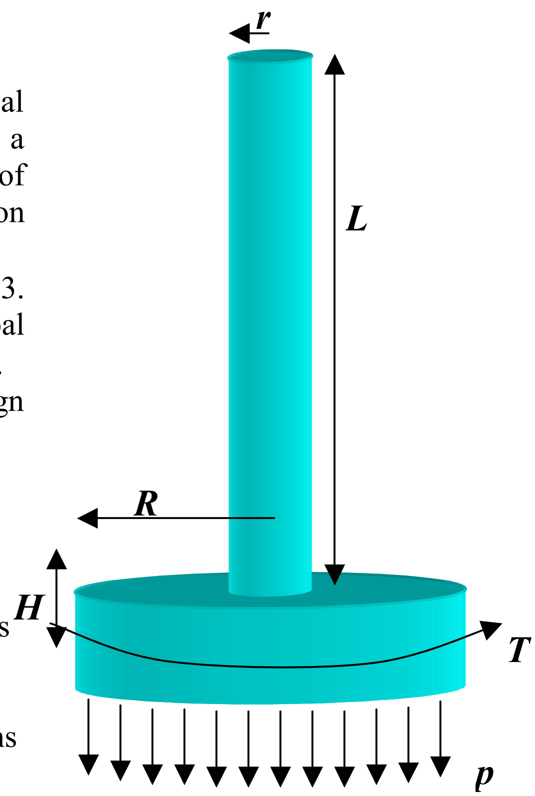
2.31 Assignment 7

Due Mon, Oct 22 at 9:30 am



A torsion pendulum consists of a solid rigid cylindrical disk of radius ***R***=3m and height ***H***=1m suspended by a shaft of length ***L***=10m with a circular cross section of radius ***r***=1m. The disk is subjected to an axial traction on its lower surface ***p***=10 MPa, and a torque ***T***=240MNxm. The material elastic properties are E=200 GPa, and ν=0.3. In order to avoid fatigue failure, the maximum principal stress (σ_1) in the component must be limited to 300 MPa. Our task is to verify if the component meets the design requirements.

1) Pen and paper work (no FE)

Calculate the maximum principal stress in the shaft subjected to the axial load resulting from ***p***, and to the applied torque, ***T***. A quick review of the formulas to calculate the principal stresses resulting from the superposition of shear and tension is attached at the end of this assignment. According to your calculations does the component meet the design requirement?

2) FE model

Create a FE model of the component using linear full integration brick (3D stress) elements.

Things to keep in mind as you set up the model:

PART: You want to create a 3D Deformable Solid Part using Extrusion. Set the part approximate size to 10. Create the circular cross section of the shaft in the sketch tool (radius 1) and then extrude it by a length of 10.

After you create the shaft, you add the disk by using the Create Solid: Extrude tool (also under Shape→ Solid→ Extrude), extruding from the face at the bottom of the shaft, sketching a circle of radius 3 in the sketch tool, and then extruding (Blind) by a depth of 1.

PROPERTY : In section property you want to create a *solid* *homogeneous* section of thickness 1. In material properties you have to input only the Mechanical props (E, ν).

STEP : Choose static, linear perturbation.

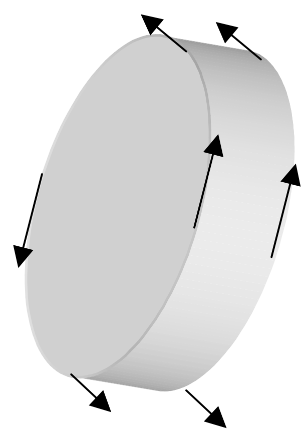
Do the mesh before you do the load!

MESH : When you enter the mesh module the part looks orange because it cannot be automatically meshed. You must partition the part to help the mesh generator. 1) Separate the disk from the shaft using the Tool→Partition→Cell→Extend Face tool, and clicking on the top face of the disk to extend it and partition the disk from the shaft. (Remember to click on create partition when the button appears!). 2)You also want to enforce symmetric meshes by creating partitions by splitting the parts in four quarters across reference (XZ, YZ) planes. The easiest way to do this is by creating two datum planes coinciding with the XZ, YZ reference planes, and then using them to split the part. The procedure is then: a) create the two datum planes Tools→datum→plane→Offset from principal plane→Apply→ choose YZ and Offset by 0.0 (if you centered the shaft at 0.0). Repeat to create an offset (by 0.0) of the XZ plane. Now you have two datum planes showing, and you can proceed with

b) partitioning the cells :

Tool→Partition→Cell→ Use datum plane→apply→select the whole part (click and drag rectangle) →select a datum plane(XZ) by clicking on the edge→create partition. Repeat the thing with the other plane (YZ). Now your part is split and you can mesh it: SEED with size 0.5, Choose linear, full integration elements, mesh the whole thing.

LOAD : Now you are ready to load the part. Create a pressure load (-10MPa) on the bottom plane of the disk (you will have to rotate the part to click on the correct plane. Remember to select all 4 quadrants of the bottom plane). Apply the torque as 8 concentrated forces acting counterclockwise, applied at the 4 +4 corners of the disk. (The corner points exist because you split the part with the daum planes).



Encastre the top plane of the shaft. (All four quadrants!)

Submit the job and look at the results in the VISUALIZATION module of ABAQUS/CAE:
Plot the contours of Mises stress. Notice that they do not look quite right at the corner between the shaft and the disk: you expect a stress concentration right at the corner, but it is not there! This is because of the nodal averaging at of stresses, as the corner nodes also belong to the bulky disk elements. To see the real stress profile, you have to modify the defaults for field contours: In the Result→Field Output window click on the Result option tag, and modify the nodal averaging threshold by moving the slider to 50%. →Apply.

Now you can actually see the stress concentration. Plot and print the Mises contours. Now plot and print the maximum principal stress(σ_1). Note: this is not S11: it is one of the invariant stresses!

Check the value of σ_1 along the shaft away from the disk. Is it in agreement with your pen and paper calculation? What is the maximum σ_1 in the component? Why is it so high?
Does the component meet the design requirement? What could you do to improve the design and meet the requirement?

7

Analysis of Plane Stress and Plane Strain

7.1 General Case of Plane Stress

In preceding discussions of beams bent by transverse loads, we have seen how an element of material in the beam can be subjected to both normal and shearing stresses on its edges as shown in Fig. 7.1a. A similar situation will occur in the case of an element of a shaft subjected to axial loads and twisting moments as shown in Fig. 7.1b. Such a state of stress

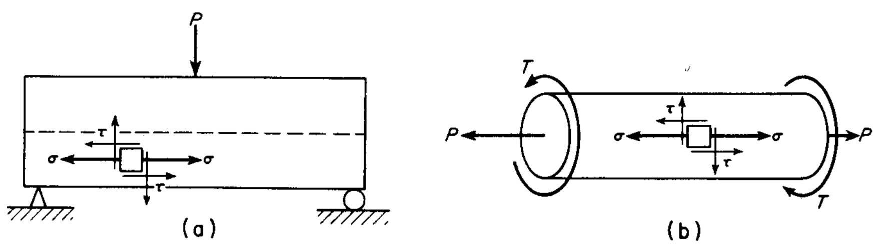


Fig. 7.1

on the edges of a rectangular element in which there are no stresses normal to its face is called *plane stress*. After such normal and shearing stresses as those shown in Fig. 7.1 have been found, it is frequently necessary to examine further the state of stress within the element to find the magnitudes and directions of maximum stresses.

Let us consider now the general case of an element under plane stress as shown in Fig. 7.2a. The normal stress in the x direction is denoted by σ_x, that in the y direction by σ_y, and tension is considered positive. The shear stresses on the edges of the element that are normal to the x-axis are denoted by τ_{xy} while those on the edges normal to the y-axis are denoted by τ_{yx}. The shear streseses τ_{xy}, having a *clockwise* sense of rotation about a point inside the element, are to be considered *positive* in accordance with our previous rule (see p. 28). The shear stresses τ_{yx}, having a *counter-*

clockwise sense of rotation, are *negative*. From the requirement of equality of complementary shear stresses (see p. 30), we have $\tau_{xy} = -\tau_{yx}$. Because of this equality of orthogonal shear stresses, it is customary to use only one notation τ_{xy} for these stresses without regard to order of subscripts, but in so doing, it is necessary to remember that shear stresses giving counterclockwise rotation are to be treated as negative.

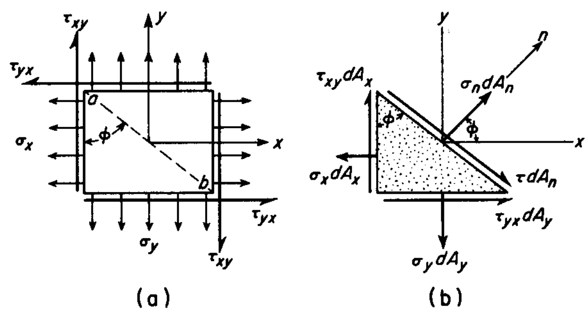


Fig. 7.2

Given the state of plane stress shown in Fig. 7.2a, the normal stress σ_n and the shear stress τ on any plane whose normal n makes the angle ϕ with the x-axis can easily be found from the equilibrium conditions of the triangular element shown in Fig. 7.2b. Let the area of the inclined face of this element be denoted by dA_n; then the areas of the other two faces are $dA_x = dA_n \cos\phi$ and $dA_y = dA_n \sin\phi$. Multiplying the various stresses by the areas of the faces on which they act, the total forces on the triangular element will be as shown in Fig. 7.2b. Then for equilibrium in the n direction, we must have

$$\sigma_n dA_n = \sigma_x dA_n \cos^2\phi + \sigma_y dA_n \sin^2\phi - 2\tau_{xy} dA_n \cos\phi \sin\phi. \tag{a}$$

Similarly, for equilibrium in the direction perpendicular to n, we must have

$$\tau dA_n = \sigma_x dA_n \cos\phi \sin\phi - \sigma_y dA_n \sin\phi \cos\phi + \tau_{xy} dA_n(\cos^2\phi - \sin^2\phi). \tag{b}$$

Equations (a) and (b) are readily reduced to

$$\left.\begin{aligned} \sigma_n &= \sigma_x \cos^2\phi + \sigma_y \sin^2\phi - 2\tau_{xy} \sin\phi \cos\phi \\ &= \tfrac{1}{2}(\sigma_x + \sigma_y) + \tfrac{1}{2}(\sigma_x - \sigma_y)\cos 2\phi - \tau_{xy} \sin 2\phi, \\ \tau &= (\sigma_x - \sigma_y)\sin\phi\cos\phi + \tau_{xy}(\cos^2\phi - \sin^2\phi) \\ &= \tfrac{1}{2}(\sigma_x - \sigma_y)\sin 2\phi + \tau_{xy}\cos 2\phi, \end{aligned}\right\} \tag{7.1}$$

which are analogous to eqs. (3.2) in Art. 3.2.

To find the location of the plane of maximum normal stress σ_n, we make the derivitive $d\sigma_n/d\phi = 0$ from the first of eqs. (7.1) and obtain

$$-(\sigma_x - \sigma_y)\sin 2\phi - 2\tau_{xy}\cos 2\phi = 0, \tag{c}$$

from which

$$\tan 2\phi = -\frac{2\tau_{xy}}{\sigma_x - \sigma_y}. \tag{7.2}$$

This condition defines two values of 2ϕ differing by 180° and hence two values of ϕ differing by 90°. For one of these values, σ_n is a maximum and for the other, a minimum.

Considering the second of eqs. (7.1) and setting the shear stress τ equal to zero, we again obtain eq. (c): From this, it may be concluded that on those planes where σ_n is a maximum or a minimum the shear stress τ vanishes. The corresponding normal stresses $(\sigma_n)_{\max}$ and $(\sigma_n)_{\min}$ are called *principal stresses*, and the planes on which they act are called *principal planes* of stress.

Referring to the second of eqs. (7.1) and setting $d\tau/d\phi = 0$, we obtain

$$(\sigma_x - \sigma_y)\cos 2\phi - 2\tau_{xy}\sin 2\phi = 0, \tag{d}$$

from which

$$\cot 2\phi = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}. \tag{7.3}$$

Comparing this with eq. (7.2), we see that the maximum shear stresses occur on orthogonal planes bisecting the angle between principal planes, i.e., at 45° to the planes of principal stress as already concluded in Art. 3.2.

To evaluate the maximum normal and shear stresses in the element, it is necessary to replace ϕ in eqs. (7.1) by the values defined by eqs. (7.2) and (7.3). Because of the transcendental character of the equations, this becomes somewhat involved and, for this purpose, it will be simpler to use Mohr's circle.

The general case of plane stress is shown again in Fig. 7.3a. To construct Mohr's circle for this case, one proceeds as follows: Lay out first the coordinate axes σ and τ with origin at O as shown in Fig. 7.3.b Then to locate the point D representing the stress conditions on the x-plane, i.e., the plane normal to the x-axis, lay off the value of σ_x as abscissa OF and the shear stress τ_{xy} as a positive ordinate FD. Next, locate the point D_1, representing the state of stress on the y-plane, by laying off the abscissa OF_1 to represent the normal stress σ_y and the negative ordinate F_1D_1 to represent the shear stress $-\tau_{xy}$. Since the x- and y-planes are orthogonal, the corresponding points D and D_1 on Mohr's circle are 180° apart and represent the ends of a diameter. Connecting these points with a straight line locates the center C on the σ-axis and the circle can be drawn as shown.

The maximum and minimum normal stresses are represented in Fig. 7.3b by OA and OB, respectively. These principal stresses are denoted by $(\sigma_n)_{\max} = \sigma_1$ and $(\sigma_n)_{\min} = \sigma_2$, as shown. To locate their directions in Fig. 7.3a, we start with point D on the circle, corresponding to the x-plane of the element, and label this point $\phi = 0$, as shown. Then to reach point A on the circle, corresponding to the plane of maximum prinicpal stress, it is necessary to pass through the *clockwise* angle $2\phi_a$. Hence, the direction of σ_1 in Fig. 7.3a is found by laying out, also clockwise, the angle ϕ_a from the x-axis, as shown. The direction of σ_2 is then at right angles to that of σ_1 and the principal planes are located as shown.

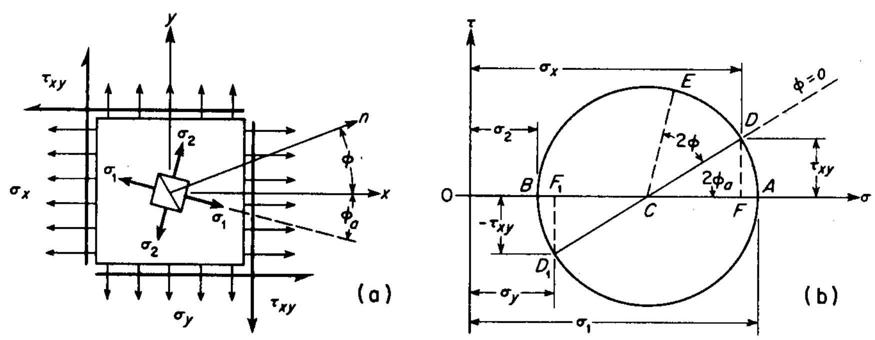


FIG. 7.3

In general, any plane through the element whose normal n makes the angle ϕ with the x-axis and the corresponding point E on the circle, representing the state of stress on this plane, are related in the same way, namely: the angle DCE in Fig. 7.3b is always double the angle ϕ in Fig. 7.3a and is to be measured in the same direction.

Expressions for the principal stresses σ_1 and σ_2 are easily found in terms of σ_x, σ_y, and τ_{xy}, from the geometry of Mohr's circle, Fig. 7.3b, as follows:

$$\left.\begin{aligned} \sigma_1 &= OA = OC + CD = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}, \\ \sigma_2 &= OB = OC - CD = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}. \end{aligned}\right\} \tag{7.4}$$

These are the same values which would be found by substituting the value of 2ϕ from eq. (7.2) into the first of eqs. (7.1) on p. 174.

EXAMPLE 1. A square element of a thin plate subjected to plane stress is shown in Fig. 7.4a. The given stresses on its mutually perpendicular faces are $\sigma_x = -500$