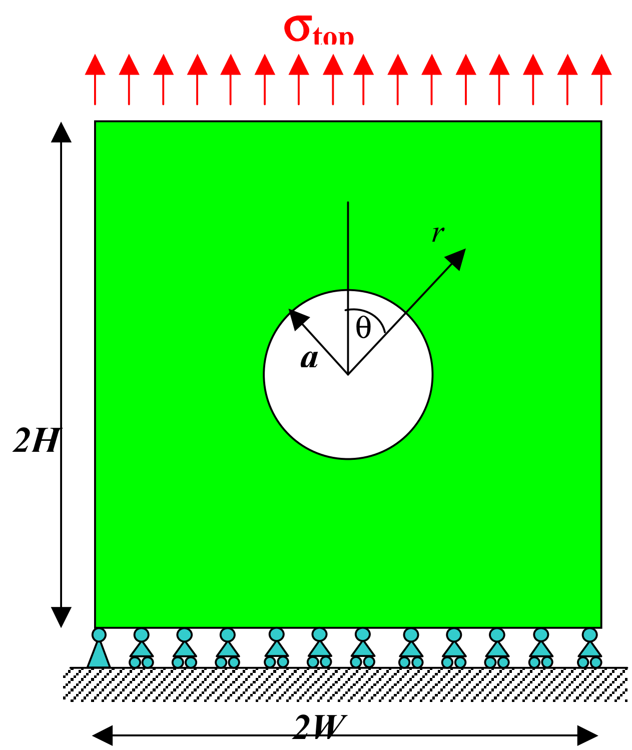
2.31 Assignment 8

Due Wed, Oct 31 at 9:30 am



Ok, I know all the MIT undergraduates thought that they had had enough of the hole in the plate story in the 2.002 lab to last the rest of their lives, and instead here we are again. We have a 6X6 square plate (*H*=3m *W*=3 m, unit thickness) with a hole of radius a =1m right in the middle. For the nostalgic, the graduate students, and the people that have burned their 2.002 handouts, I have attached the corresponding 2.002 document at the end of the assignment. We are going to compare FE and theoretical evaluation of stress concentrations for different meshes and element types. The plate is loaded with an axial stress acting on the top edge of the plate σ_{top}= 100 MPa. And yes you guessed it, the material is steel.

1) Pen and paper work (no FE) (aka: The Return of 2.002)

For the case of a circular hole in an infinitely wide plate, using equation *1b* in the 2.002 handout to derive an analytical expression for the distribution of axial stress along the equatorial ligament ($\theta=\pi/2$) as a function of distance (r) from the center of the hole. Plot the curve for $r = a \rightarrow W$

For the actual case of a circular hole in the finite width plate above, obtain the value of the stress concentration factor, Ktg , from the graph in Figure 3 of the 2.002 handout. Compare the max level of stress σ_{max} [=axial stress at the edge of the hole] in the finite plate, to that obtained above for the infinite width plate. Explain the reasons for the discrepancy between the two stress concentration values.

2) FE model

Create FE models of the component using plane stress elements. The objective is to investigate cost/benefits of mesh refinement, mesh structuring as well as the effects of various element choices.

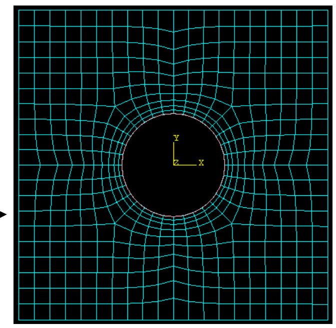
Please construct and run the following FE models. Make a table where for each model, you give the max s22 stress and the CPU time it took to run the model (it is at the bottom of the .dat file: USER TIME (SEC)). For all models partition the plate in 4 quadrants (top-bottom, left-right) before you seed, so as to have symmetric meshes.

1) Coarse quadrilateral meshes : seed=1.0. Try four types of quadrilateral elements: (**1a**) linear, reduced integration elements (CPS4R), (**1b**): linear, full integration (CPS4), (**1c**): quadratic, reduced integration (CPS8R), (**1d**): quadratic full integration(CPS8).

2) Refined quadrilateral meshes uniform seed, seed= 0.3. Try the same 4 types of elements: **2a**:CPS4R, **2b**:CPS4, **2c**:CPS8R, **2d**:CPS8.

3) Very refined quadrilateral meshes uniform seed, seed= 0.1. Try the same 4 types of elements: **3a**:CPS4R, **3b**:CPS4, **3c**:CPS8R, **3d**:CPS8.

4) Refined quadrilateral meshes with biased seeds. Give a general seed of 0.3, and then give local (edge) seeds around the hole (8 elements for each of the four quarters), and biased seeds on the 4 ligaments on the x,y axes. Bias with a factor 6 and put 10 elements along these edges. I get a mesh that looks like this →



   Try the same 4 types of elements: **4a**:CPS4R, **4b**:CPS4, **4c**:CPS8R, **4d**:CPS8.

5) One last set of models using triangular elements. Get rid of the edge seed, and of the global seeds. Reseed with a global seed of 0.3. Use the **Mesh→Control** tool : select the entire model and assign **Tri** element shape to the whole plate. Now go on as usual: assign element type (Tri) linear, and mesh the part. Run a job with linear elements (**5a**: CPS3). Change the elements to quadratic standard formulation (CPS6) and run your last job: **5b**:CPS6.

Comment on the results of this parametric study.
What did you learn in terms of cost/benefits of using different discretizations/formulations? The stress contours are symmetric about the 2-axis (left-right) but not about the 1-axis (top-bottom): why? Where do you think that the inconsistency with the theoretical $\boldsymbol{\sigma_{max}}$ comes from?

I will demonstrate the following in class: you do not need to do this

Compare the estimated stress fields for models **2a**, **2b**, **2c**, **2d**, **5a**, **5b**, by superposing the FE profiles of axial stress along the ligament with the theoretical estimate of the stress profile (for the hole in the infinite plate). Also on the same plot, mark the theoretical $\boldsymbol{\sigma_{max}}$ for the finite plate obtained using the stress concentration factor, K_{tg}. Comment on the consistencies/inconstistencies of the *stress field* for different FE models, as opposed to just looking at the max stress level.

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2.002 Mechanics and Materials II

Just when you thought you could forget about this....

The stress distribution around a hole in an <u>infinite</u> plate

The stress distributions around a central hole can be estimated for the simple case of an infinitely wide plate subjected to elastic tensile loading. The overall stress distributions in the plate are given by (Figure 1):

$$\sigma_{rr} = \frac{\sigma}{2}\left(1 - \frac{a^2}{r^2}\right) + \frac{\sigma}{2}\left(1 + 3\frac{a^4}{r^4} - 4\frac{a^2}{r^2}\right)\cos 2\theta \tag{1a}$$

$$\sigma_{\theta\theta} = \frac{\sigma}{2}\left(1 + \frac{a^2}{r^2}\right) - \frac{\sigma}{2}\left(1 + 3\frac{a^4}{r^4}\right)\cos 2\theta \tag{1b}$$

$$\tau_{r\theta} = -\frac{\sigma}{2}\left(1 - 3\frac{a^4}{r^4} + 2\frac{a^2}{r^2}\right)\sin 2\theta \tag{1c}$$

where σ is the magnitude of the applied uniaxial far-field stress.

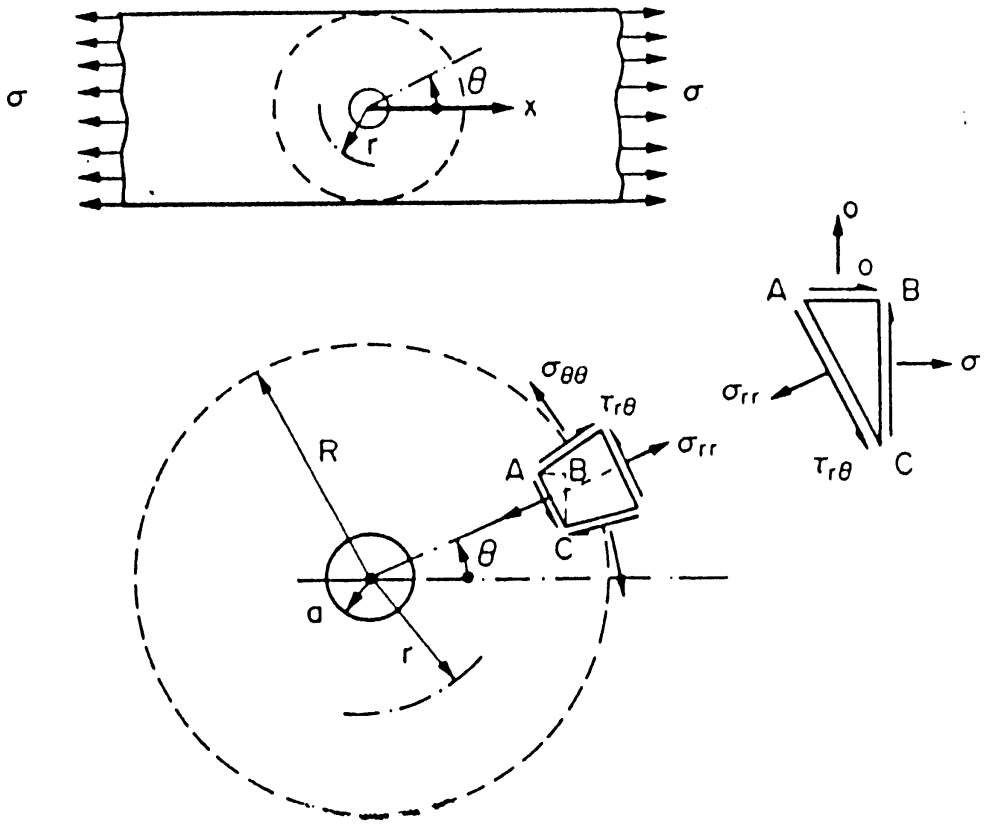

Figure 1 Stress distribution around a circular hole.

For r = a, the stress distribution around the hole is given by:

$$\sigma_{rr} = 0 \tag{2a}$$

$$\sigma_{\theta\theta} = \sigma\,(1 - 2\cos 2\theta) \tag{2b}$$

$$\tau_{r\theta} = 0 \tag{2c}$$

For $\theta = \pi/2$, $\sigma_{\theta\theta} = \sigma_{max} = 3\sigma$. This corresponds to the peak stress of the stress distribution (Figure 2). Hence, we may state that the stress concentration factor (the ratio of the maximum stress to the far field stress) for this notch geometry is equal to 3. The concept of a stress concentration factor will be further discussed in the following section.

However, it is important to note that the stresses in the immediate vicinity of the hole are much higher than the far field stresses. Failure may therefore initiate prematurely from the edge of hole.

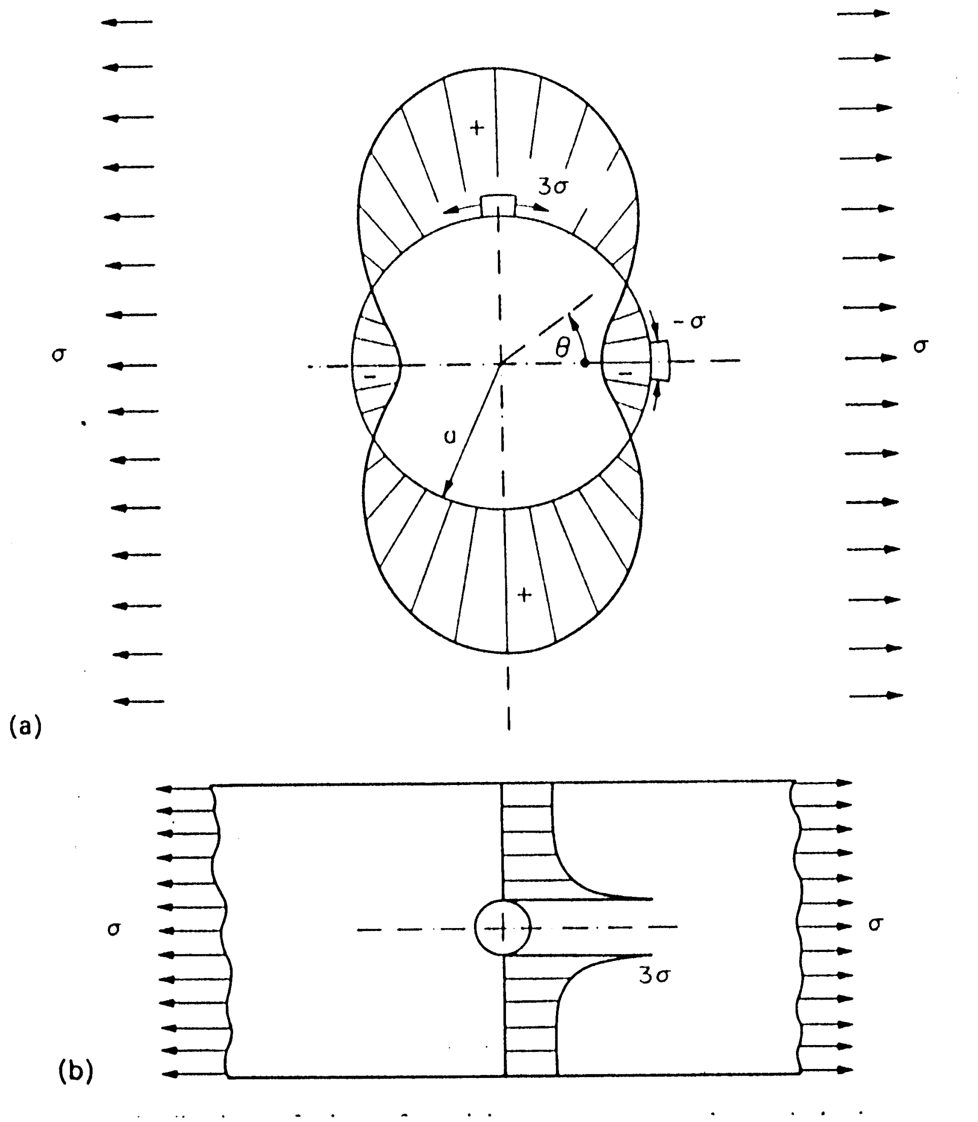

Figure 2 Distribution of $\sigma_{\theta\theta}$: (a) around the hole, and (b) along the ligament ($\theta = \pi/2$).

The stress along the ligament, where $\theta = \pi/2$ (Figure 2), decreases rapidly with increasing distance from the notch. This is a clear example of the St. Venant principle which states that the perturbations in the stress field due to a small geometrical discontinuity (with size d) are localized within regions with ~ 3d from the discontinuity. The stress levels outside this region are therefore close to the nominal applied stress levels.

Stress Concentration Factors For Different Geometries

Stress concentration factors have been obtained for several geometries of engineering significance. These are usually tabulated in mechanical engineering handbooks. Two types of stress concentration factors are generally found in the literature. The first is the stress concentration factor based on gross stress, K_{tg}. It is given by

$$K_{tg} = \frac{\sigma_{max}}{\sigma} \tag{4}$$

where σ_{max} is the maximum stress at the edge of the hole and σ is the applied far field stress remote from the hole. Similarly, we may also define a stress concentration based on the nominal applied stress, K_{tn}. This is given by:

$$K_{tn} = \frac{\sigma_{max}}{\sigma_{nom}} \tag{5}$$

where σ_{nom} is the nominal (average) stress along the ligament. [For a plate of width 2w with a circular hole of radius a , σ_{nom} is given by: $\sigma_{nom}=\sigma/(1-a/w)$].

Values for the stress concentration factors, K_{tg} and K_{tn} for circular holes in plates of finite width are given in the graph in Figure 3.

For the case of elliptical holes in an infinite plate subjected to tensile loading, the stress concentration factor, K_{tg}, can be obtained analytically, and is given by:

$$K_{tg} = 1+2(b/a) \tag{6}$$

where 2a is the length of the minor diameter (in the stress direction), and 2b is the length of the major diameter. Equation 6 is plotted as a solid line in Figure 4.

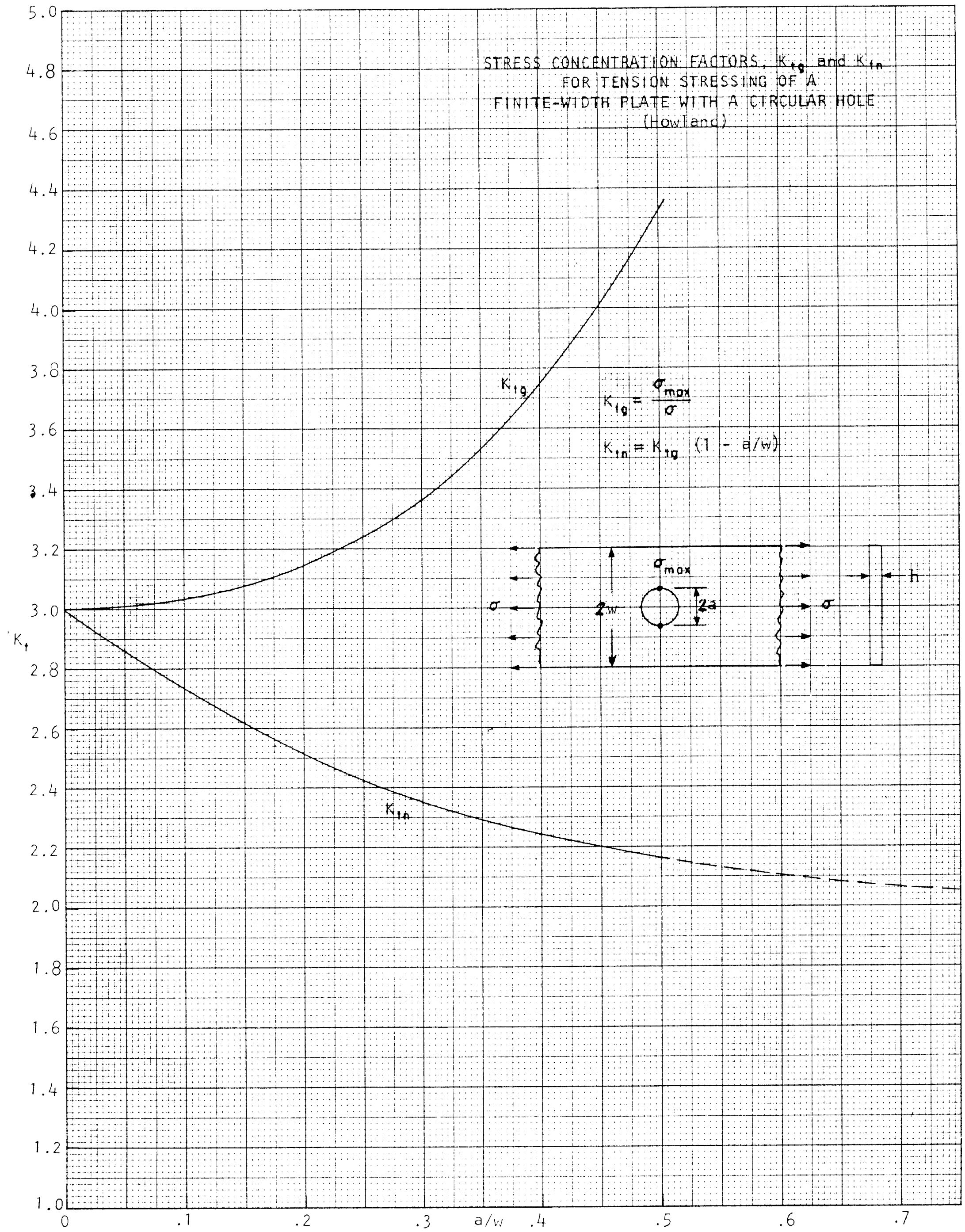

Figure 3 Stress concentration factor for a circular hole in a plate of <u>finite</u> width.

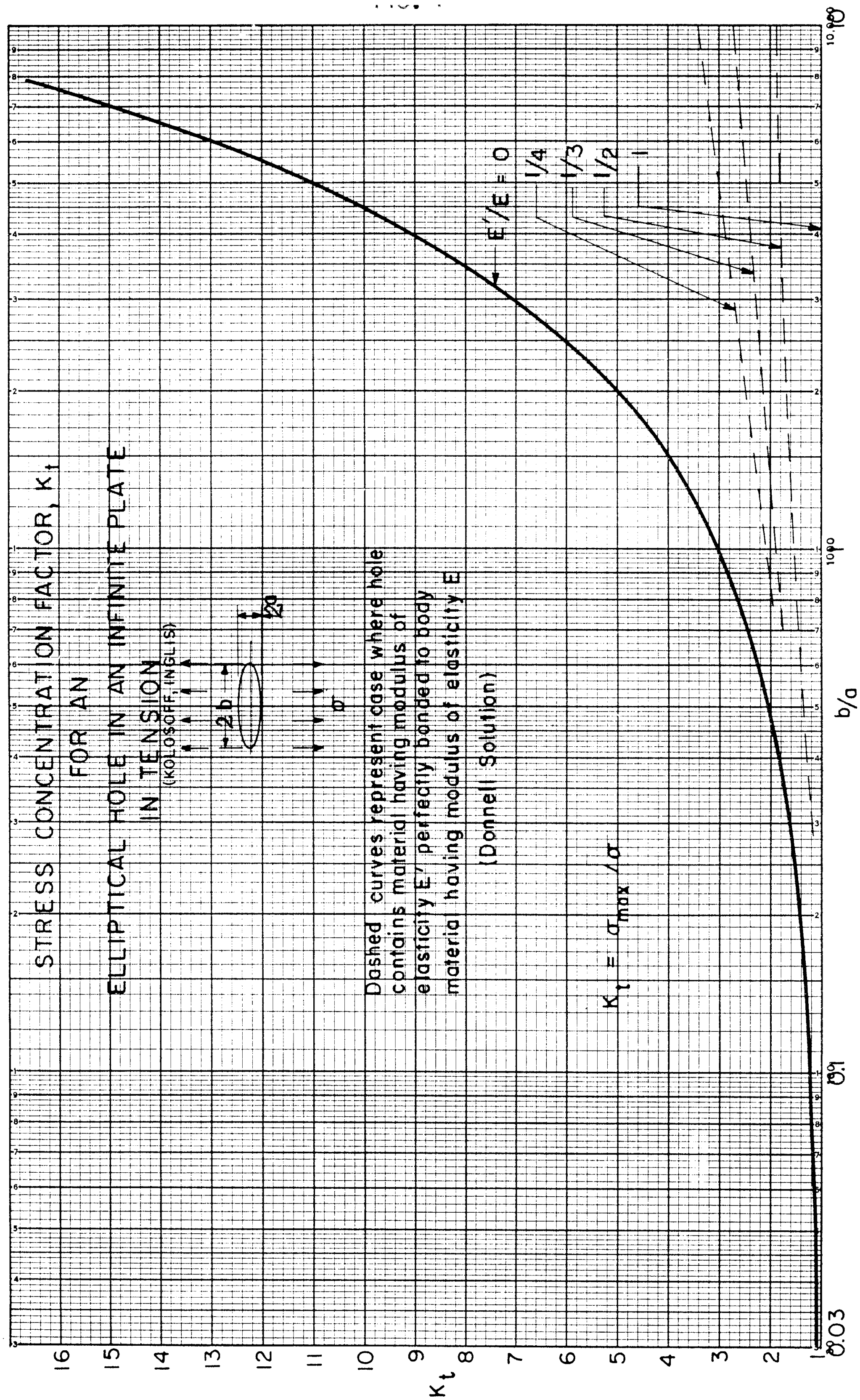

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Figure 4 Stress concentration factor for an elliptical hole in an <u>infinite</u> plate.

Equation 6 may also be expressed as:

$$K_{tg} = 1 + 2\sqrt{\frac{b}{r}} \quad , \tag{7}$$

where r is the radius of curvature at the edge of the ellipse along the axis perpendicular to the applied stress ($r = a^2/b$). Equation 7 may also be used to estimate the stress concentration factors for other hole geometries by introducing the idea of an equivalent ellipse (Figure 5).

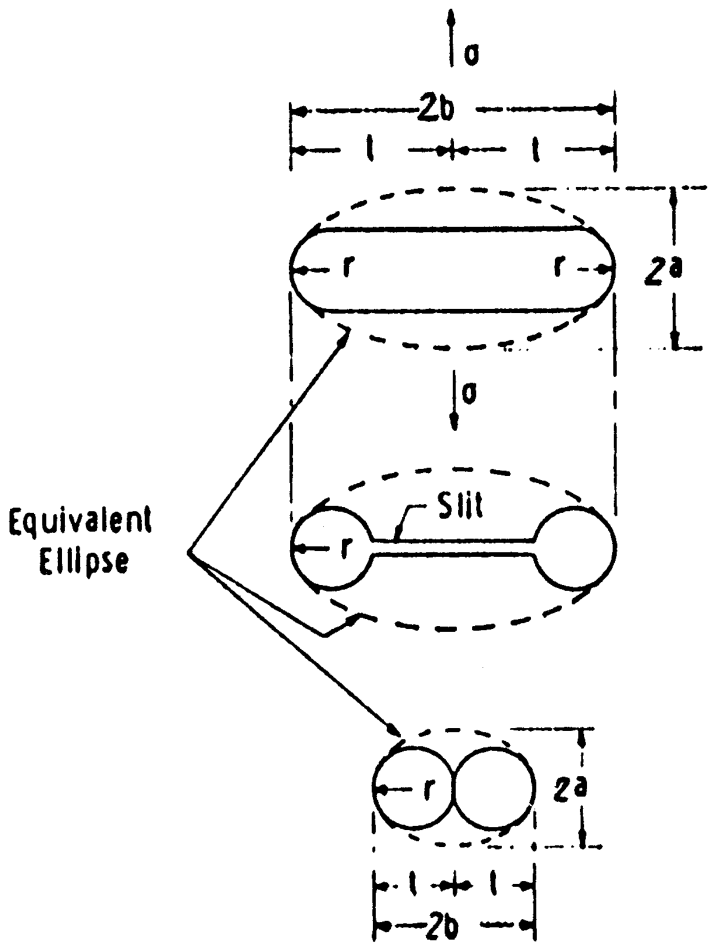

Figure 5 Schematic illustration of equivalent ellipses for various notch geometries. [Note that the top geometry (slot) is the geometry of the hole in the PC plate tested in this Laboratory session (see Figure 8b)].