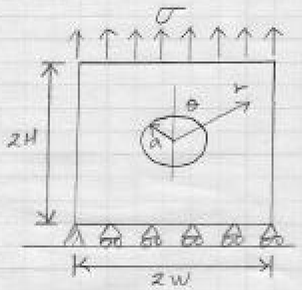
Problem Set #8 Solution



Known:

$H = 3m$; $W = 3m$

$\sigma = 100$ MPa

Material = Steel

$E = 200$ GPa

$\nu = 0.3$

Part I)

1. Axial Stress along $\theta = \pi/2$

From the handout, use eqn. 1b)

$$\sigma_{\theta\theta} = \frac{\sigma}{2}\left[1+\frac{a^2}{r^2}\right] - \frac{\sigma}{2}\left[1+3\frac{a^4}{r^4}\right]\cos 2\left(\frac{\pi}{2}\right)$$

$$\sigma_{\theta\theta}(r) = \sigma\left[1+\frac{a^2}{r^2}+\frac{3a^4}{2r^4}\right]$$

Refer to the attached Plot for σ distribution.

σ_{max} (Theoretical) = **300 MPa**

2. Actual Stress using Stress Concentration factor Chart

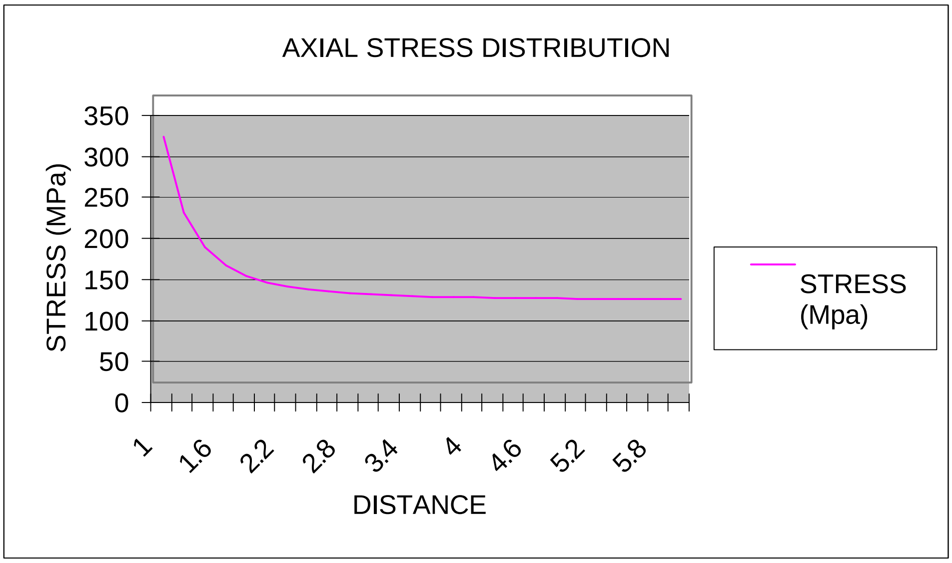
From graph in Fig. 3 of handout

K_{tg} (a/W) ≈ 3.5

$K_{tg} = \sigma_{max}/\sigma \rightarrow \sigma_{max} = \sigma K_{tg}$

∴ σ_{max} (Finite Plate) = **350 MPa**

AXIAL STRESS CURVE AT p/2 (THEORETICAL)



AXIAL STRESS & TIME CHART

	NO.	ELEMENT TYPE	S22 (Mpa)	USER TIME
ELEMENT SEED = 1.0	1A	CPS4R	177	0.2
	1B	CPS4	286	0.2
	1C	CPS8R	372	0.2
	1D	CPS8	353	0.2
ELEMENT SEED = 0.3	2A	CPS4R	304	0.4
	2B	CPS4	371	0.5
	2C	CPS8R	375	0.7
	2D	CPS8	370	0.9
ELEMENT SEED = 0.1	3A	CPS4R	337	2.0
	3B	CPS4	374	2.9
	3C	CPS8R	367	5.5
	3D	CPS8	369	7.3
BIASED MESHING	4A	CPS4R	336	0.4
	4B	CPS4	371	0.5
	4C	CPS8R	372	0.8
	4D	CPS8	368	1.1
TRIANGLE ELEM	5A	CPS3	342	0.5
	5B	CPS6	379	1.0

Part I Comments :

1) The first value obtained by equation 1b) assumes that the plate in tension is an infinite width plate with local stress concentration on the edge of the circular hole.

The second value obtained from the Chart III is for a plate of finite width, and infinite axial length. Because the plate is finite, the axial force in the plate must be redistributed over a smaller area at the equatorial cross section, thus increasing the average stress at that section. The stress concentration due to the geometry of the hole is superposed to this increase. For an infinite width plate, there is no actual area reduction due to the hole. In the graph, as the a/w ratio approaches 0, meaning the width goes to infinity, Kt approaches 3, which is the infinite width value.

Part II Comments :

1) If the mesh is coarse, it doesn't make much of a difference as far as the user time is concerned, whether it is linear or quadric, or it is reduced or fully integrated.

2) As the mesh becomes finer, there are noticeable differences in the user time, but the maximum stress values generally converge to the same value. If it is the maximum stress level that we are looking for, it would be more efficient to use CPS4 elements with 0.3 seed size rather than CPS8R elements with 0.1 seed size. The results were both accurate, with CPS4 elements taking up less user time. (0.5 vs. 5.5)

3) In general, quadratic elements took longer to mesh (more nodes and integration points), but yielded better stress results. Linear reduced integration elements did poorly in general.

4) A biased mesh represents the best solution, where accurate results are obtained with lower computational times. This suggests an approach where a part is first analyzed with a uniform coarse mesh. Then, based on the coarse mesh results, the areas of steep stress gradients are identified, and the mesh is selectively refined in the critical regions.

5) The discrepancy of stress contours in the vertical direction is due to the asymmetric boundary conditions of the geometry. We have imposed uniform stress on the top boundary, and uniform displacement (0) on the bottom boundary. These boundary conditions, enforced at a relatively small distance from the hole, are also responsible for the enhanced stress concentration at the hole (~370 MPa) which exceeds the value obtained from the chart for an infinitely long plate, where the uniform axial stress is imposed far away from the hole.

6)The profiles of axial stress along the ligament for models 2a, 2b, 2c, 2d, 5a, 5b, are superposed in the figure below. We see that even for this fairly coarse mesh all the quadratic elements are in good agreement and give a satisfactory solution. The linear reduced integration elements grossly underestimate the stress concentration, and the stress distribution is not accurate in the vicinity of the hole. The linear triangular element also underestimates the stress. Linear triangles are constant strain elements and can only be used in very refined meshes, or as (few) filler elements for a mostly quadrilateral mesh for oddly shaped domains.

S22 along the equatorial ligament (MPa)