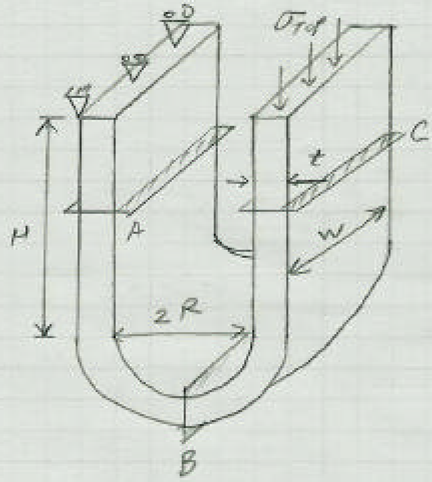
Problem Set #9 Solution



Known

$\sigma_{Top} = 100\ MPa$

$t = 0.5\ m$

$W = 2.0\ m$

$R = 2.0\ m$

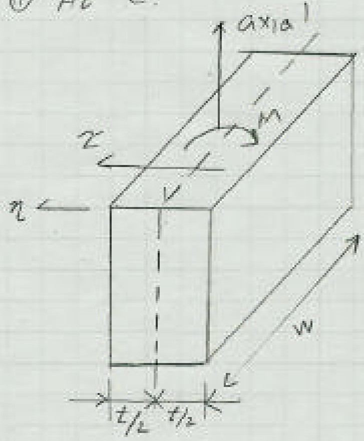
$H = 4.0\ m$

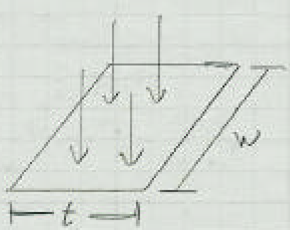
$E = 0.1\ GPa$

$\nu = 0.3$

Part I.

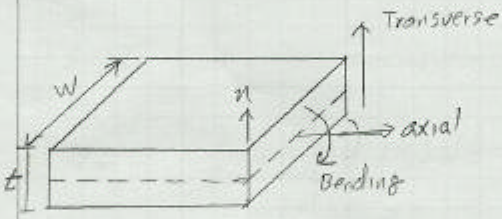
① At C.





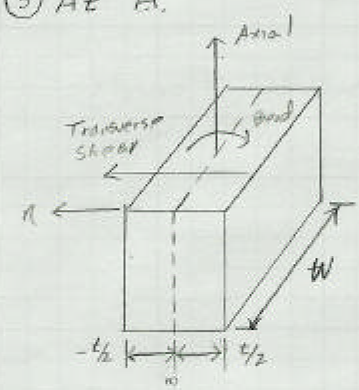
1. $F_{axial} = \frac{-\sigma_{Top}}{bw} = -100\ MN$ (compression)
2. $N_{axial} = \frac{F_{axial}}{W} = -50\ MN$
3. $\sigma_{m\_axial} = -\sigma_{Top} = -100\ MPa$
4. $F_{shear} = 0$
5. $N_{shear} = 0$
6. $\tau_{shear} = 0$
7. $MT = 0$
8. $M = 0$
9. $\sigma_{b\_axial} = 0$
10. $\sigma_{Top\_axial} = -100\ MPa$
11. $\sigma_{Bot\_axial} = -100\ MPa$.

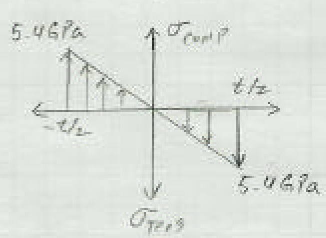
② At B.



1. $F_{axial} = 0$
2. $N_{axial} = 0$
3. $\sigma_{m\_axial} = 0$
4. $F_{shear} = 100 MN$
5. $N_{shear} = F/W = 50 MN/m$
6. $\tau_{m\_shear} = \frac{F_{shear}}{wt} = 100 MPa$
7. $MT = F(R + \frac{1}{2}t) = (100 MN)(2.25 m) = 225 MN \cdot m$
8. $M_{Bend} = MT/W = 112.5 MN \cdot m$ per unit depth

③ At A.





1. $F_{axial} = 100 MN$
2. $N_{axial} = F_{axial}/W = 50 MN/m$
3. $\sigma_{m\_axial} = \frac{F_{axial}}{tW} = 100 MPa$ (Tension)
4. $F_{shear} = 0$
5. $N_{shear} = 0$ 6. $\tau_{shear} = 0$
7. $MT = F_{axial} * (2R + t) = 450 MN \cdot m$
8. $M = MT/W = 225 MN \cdot m$ per unit
9. $\sigma_{b\_axial} = \frac{MC}{I} = \frac{MC}{wt^3/12} = \left[2.16 \times 10^{11} N/m^3\right] C$
10. $\sigma_{top}(-t/2) = -5.4 GPa + 0.1 GPa = -5.3 GPa$
11. $\sigma_{Bot}(t/2) = 5.4 GPa + 0.1 GPa = 5.5 GPa$

Additional Calculation

I. To Find σ_m, Average Stress

$$\sigma_m = \frac{1}{t}\int_{-t/2}^{t/2} \sigma(z)\,dz \quad \text{(For Section A)}$$

$$\sigma(z) = \sigma_0 + \sigma_{BEND}\left[\frac{z}{t/2}\right] \quad (-t/2 \le z \le t/2)$$

$$= \sigma_0 + \frac{2z}{t}\sigma_{BEND}$$

$$\sigma_m = \left[\left[\sigma_0 z + \frac{z^2}{t}\sigma_{BEND}\right]_{-t/2}^{t/2}\right]\frac{1}{t} = \sigma_0 = 100\ MPa\ \text{(Tension)}$$

For Section C, it is just **100 MPa** Compression

II. To Find Bending Moment M_{BEND}

$$M_{BEND} = \int_{-t/2}^{t/2} \sigma(z)\,z\,dz$$

$t = 0.5\ m$
$\sigma_0 = 100\ MPa$
$\sigma_B = 5.4\ GPa$

$$= \left.\frac{\sigma_0 z^2}{2} + \frac{2}{3}\frac{z^3}{t}\sigma_{BEND}\right]_{-t/2}^{t/2}$$

$\therefore M_{BEND} = 225\ MN\cdot m$ (Section A)

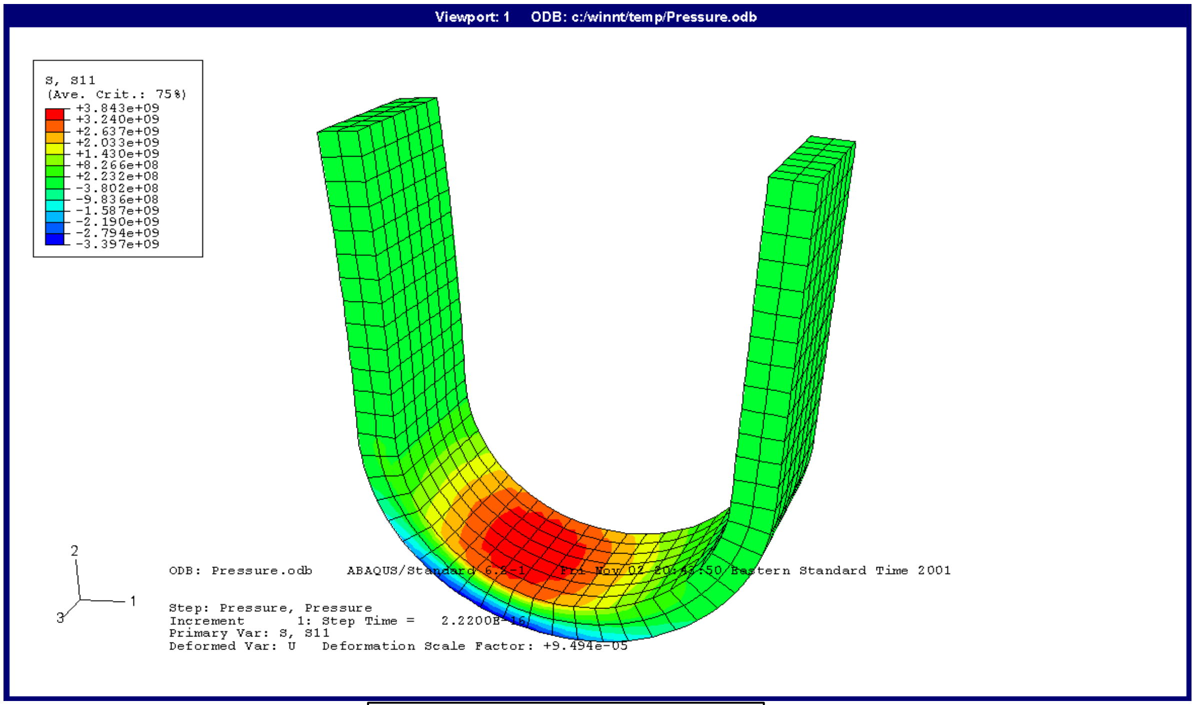
Comments

For sections A and C, S21 corresponds to the transverse shear stress on the sections, and S22 corresponds to the axial stress. The distributions of axial stress on both sections are consistent with the theoretical predictions.

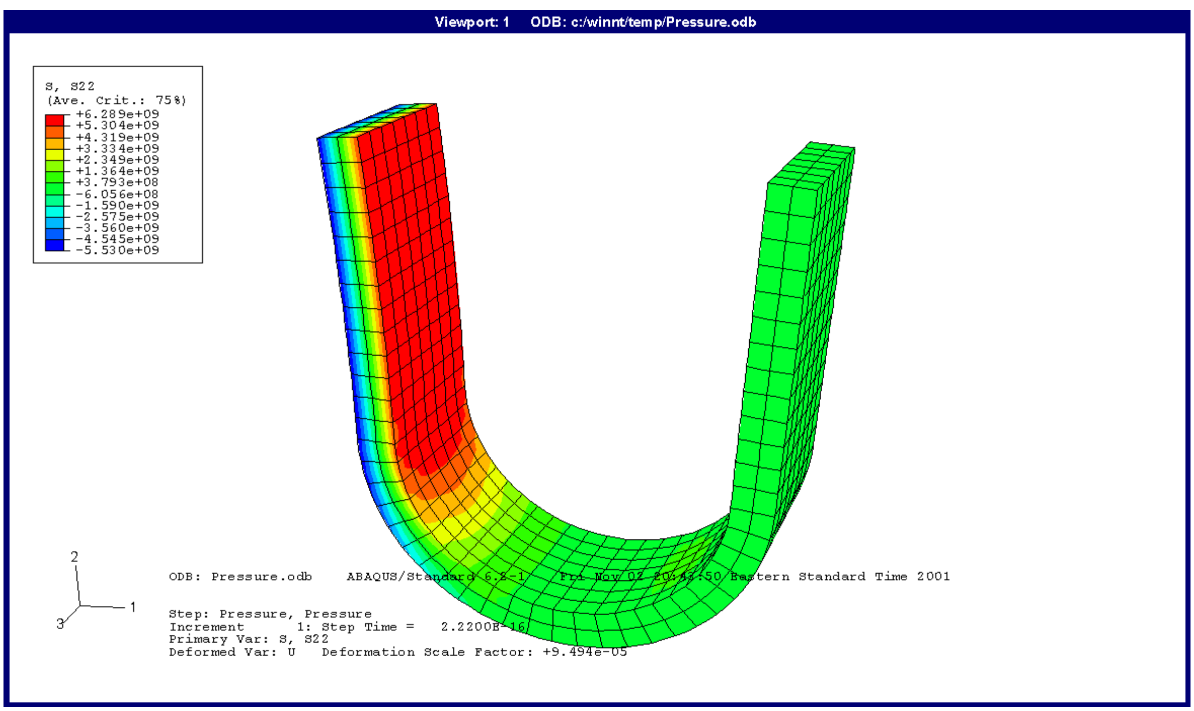
The FE model gives a distribution of transverse shear stress on both sections as well, but the magnitude of the transverse shear stress is orders of magnitude lower than the axial stress.
The shear stress distribution for section A is shown in the countour plot below.
This shear stress distribution originates from "plane strain" constraints due to the high width/height ratio of the cross section, coupled with the geometry of the curved section of the U-channel. It cannot be captured by elemental beam theory.

For Section B, the shear stress component is still S21, but the axial stress component is now S11. Note that the max S11 is not at B, because the moment increses with the distance from the load.

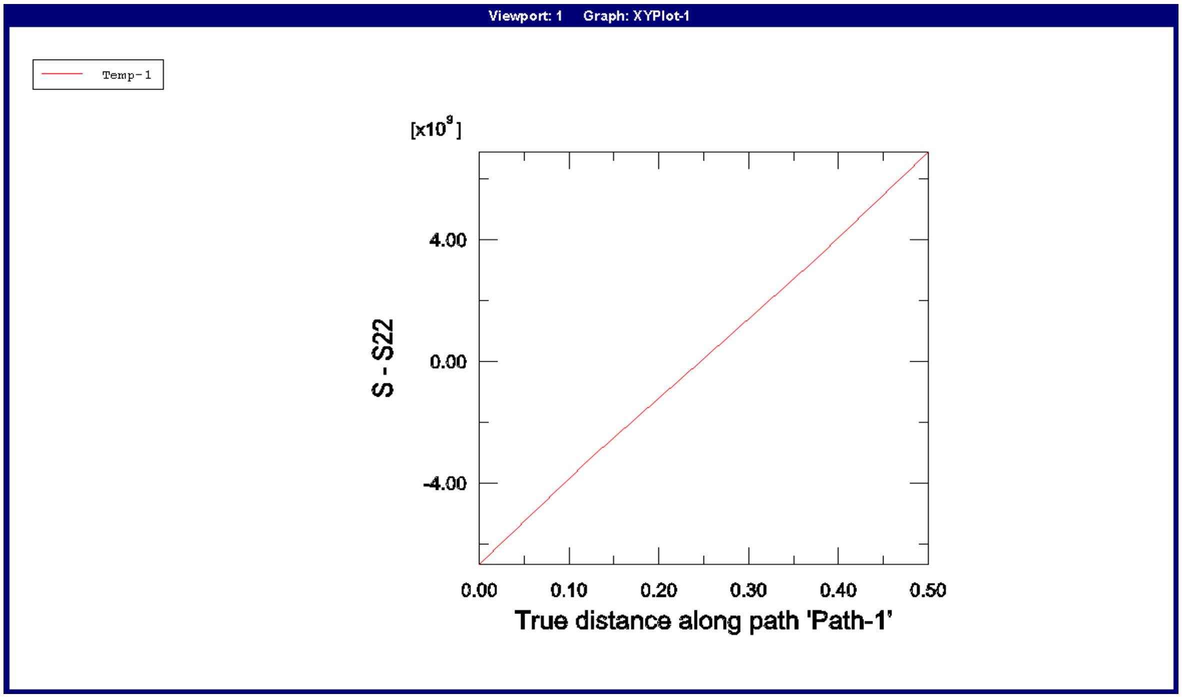
PLOT OF S11



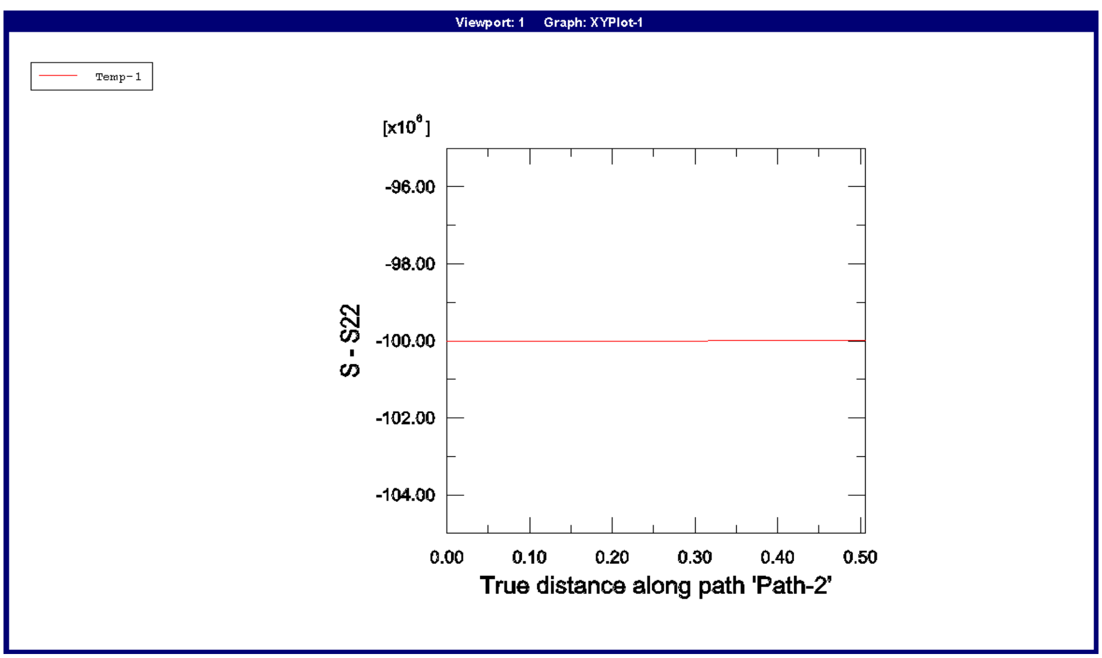
PLOT OF S22



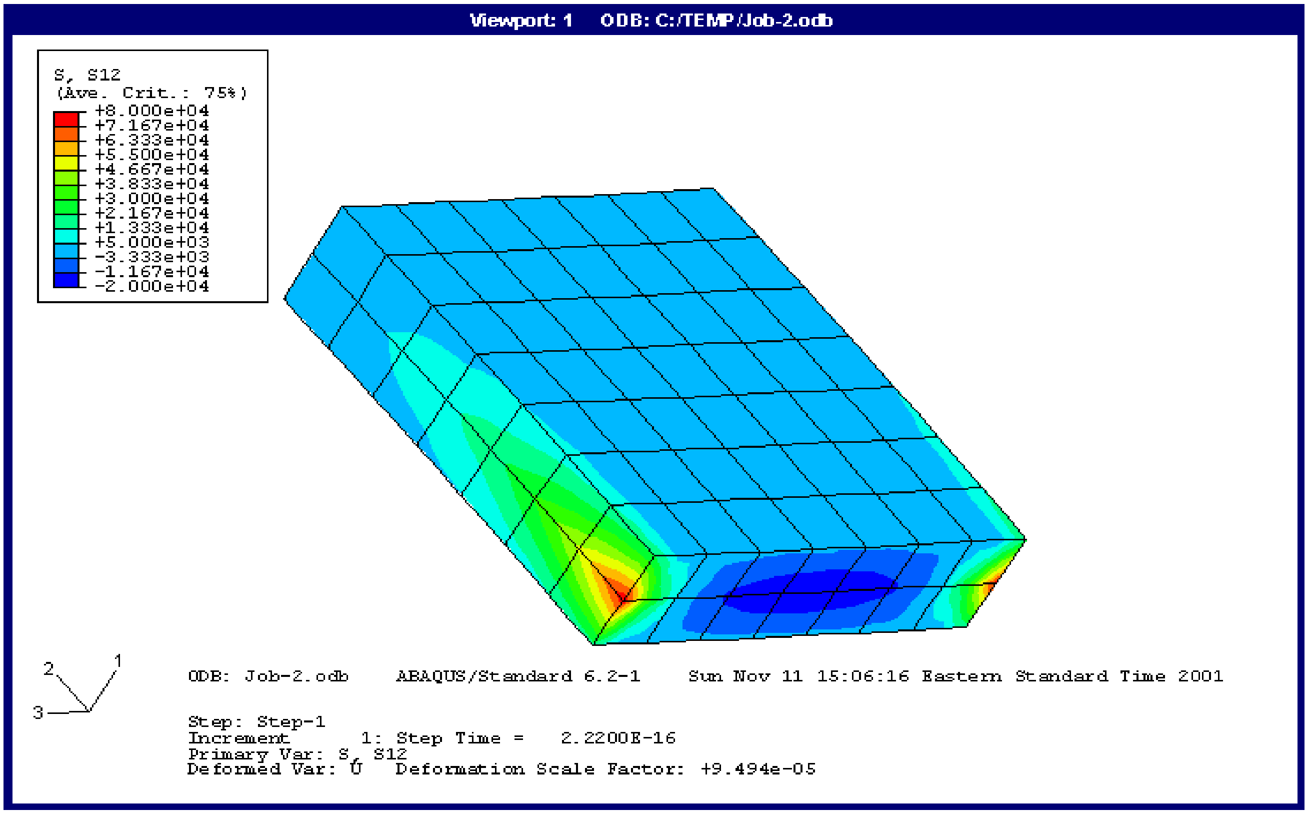
SECTION A-A axial s DISTRIBUTION s top,bottom



SECTION C-C axial s DISTRIBUTION



SECTION A-A transverse shear DISTRIBUTION



SECTION C-C s DISTRIBUTION
tshear