Precision Machine Design

Topic 10

Vibration control step 1:
Modal analysis[1]

Purpose:

The manner in which a machine behaves dynamically has a direct effect on the quality of the process. It is vital to be able to measure machine performance.

Outline:

- **Introduction**
- **Measurement process outline**
- **Practical issues**
- **Vibration fundamentals**
- **Experimental results**
- **Data collection: Instrumentation summary**
- **Case study: A wafer cassette handling robot**
- **Case study: A precision surface grinder**

"There is nothing so powerful as truth"

Daniel Webster

[1] This section was written by Prof. Eric Marsh, Dept. of Mechanical Engineering, Penn State University, 322 Reber Bldg., University Park, State College, 16802; erm7@psu.edu

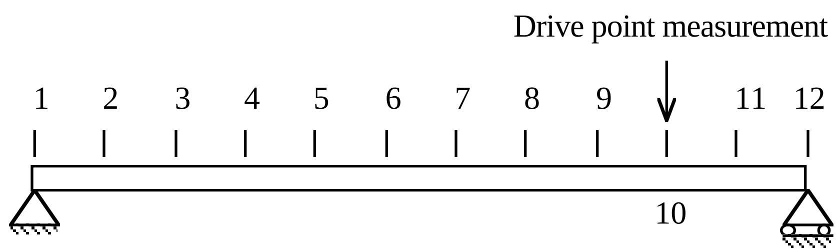
Introduction

- **Experimental Modal Analysis allows the study of vibration modes in a machine tool structure.**
- **An understanding of data acquisition, signal processing, and vibration theory is necessary to obtain meaningful results.**
- **The results of a modal analysis are:**
  - **Modal natural frequencies**
  - **Modal damping factors**
  - **Vibration mode shapes**
- **This information may be used to:**
  - **Locate sources of compliance in a structure**
  - **Characterize machine performance**
  - **Optimize design parameters**
  - **Identify the weak links in a structure for design optimization**
  - **Identify modes which are being excited by the process (e.g., an end mill) so the structure can be modified accordingly.**
  - **Identify modes (parts of the structure) which limit the speed of operation (e.g., in a Coordinate Measuring Machine).**
- **Use modal analysis to measure an older machine that achieves high surface finish, but is to be replaced with a more accurate machine.**
  - **The new machine can be specified to have a dynamic stiffness at least as high as the old machine.**

Measurement process outline

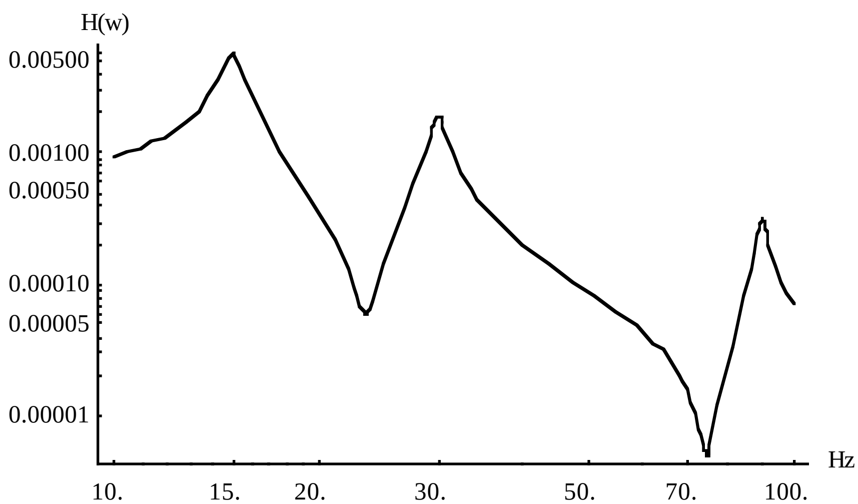
1. **Measure input and output of system using the appropriate transducers and analog to digital converters.**

   - **Input is usually a force excitation.**
   - **Output may be measured with an interferometer, a capacitance probe, an accelerometer, or another response transducer.**
   - **Many machine tool structures may be conveniently analyzed with inexpensive piezoelectric force and acceleration sensors.**
   - **16-bit A/D with analog anti-aliasing filters is required to obtain good quality time histories.**



2. **Fast Fourier transform discrete time data to obtain the frequency response function between input and output.**

- **Calculation of input and output FFT's allows the computation of the transfer function.**
    - **It is evaluated along the jω-axis and therefore called the frequency response function (frf).**
- **The coherence may also be calculated which gives an indication of the quality of the data.**
    - **0 indicates poor quality, 1 indicates high quality.**
- **Frf is stored on disk.**



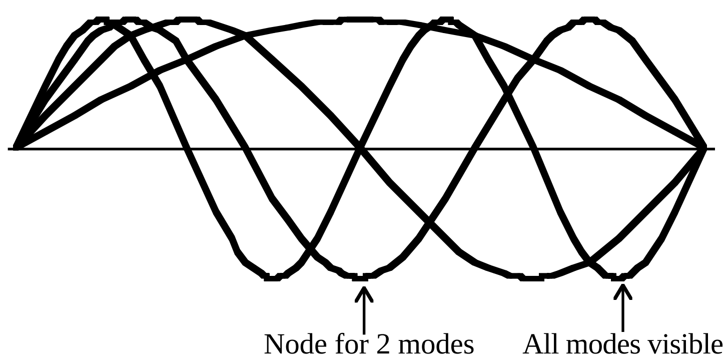
3. Repeat process over many points on the structure.

- **Either the location of the input or the output measurement point is changed and the process is repeated, including the calculation and storage of the new frf.**
- **Either channel, but not both, may be moved as a result of reciprocity in linear systems.**
- **An entire data set is collected by repeating the measurement process over many locations on the test article.**

4. Use collection of frequency response functions to locate natural frequencies and modal damping factors.

- **All the collected frf's will show the same modes of vibration.**
- **Each frf will have peaks at the same frequencies with the same amount of damping. The difference will be in the magnitude of each peak.**
- **The drive point frf is typically a good frf to use for locating the modal frequencies and damping factors.**

5. **For each mode, measure fluctuation of response amplitude over all the collected frf's.**
   - **Each frf is now used to estimate the mode shapes of vibration.**
     - **The magnitude of each vibration mode is recorded for each of the collected frf's.**
     - **A large magnitude for a given mode in a given frf indicates that the structure has a large amplitude at that location and frequency (anti-node).**
     - **Small magnitudes indicate that the structure is barely moving at the indicated location and frequency (node).**
6. **Animate mode shapes to visualize results.**
   - **The magnitudes of the modes can be used to animate a wireframe mesh on a computer.**
   - **This helps visualize each vibration mode and identify sources of compliance in the test article.**



Practical issues

- **In practice, several factors make the modal measurement and identification process difficult.**

1. **Non-linearities in the test article.**

    - **Modal analysis is built upon the assumption of linear, time-invariant system analysis.**

        - **Any non-linearities in a structure distort the results.**

    - **The solution is to either remove the non-linear portion of the system or ignore it.**

        - **Mild non-linearities will not overly distort results (which is good because every system has at least some non-linearity).**

    - **In some cases, removing the non-linear components will be necessary.**

        - **A correction must be developed that will account for the dynamics of the removed components.**

2. Noise in measurement.

- **Measurement noise may result from background excitation such as floor vibration, 60 Hz noise, and other sources.**
- **When using transient excitation techniques, such as impulse hammers:**
  - **The noise may be greater than the true signal after the transient vibration has decayed.**
- **The noise can significantly alter the results, so the effects of noise should be minimized in one of two ways:**
  - **Reduce sample time or use time windowing.**
- **By reducing the sample time, less of the noise will be present to corrupt the transient decay.**
- **Time windowing can also be used to filter out the noise in a record:**
  - **But this causes irreversible distortion of the final data (with care, the distortion can be minimized).**

3. High modal density/high damping.

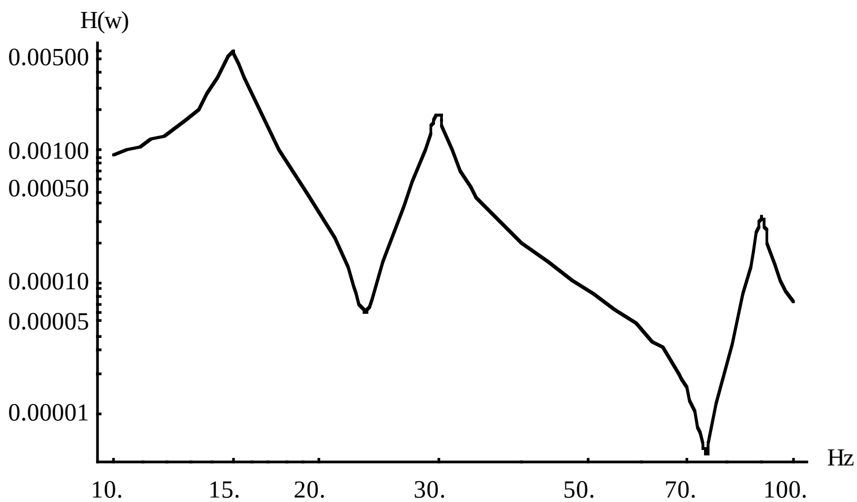
- **If the modes are closely spaced, the amplitude of a given mode will be effected by neighboring modes.**
- **A closely coupled system requires more sophisticated methods of extracting the modal parameters from the frf's.**
- **There are a wide variety of time and frequency-based modal parameter extraction procedures in the public domain.**
    - **While too complicated to discuss in this introduction:**
        - **Many have been included in commercially-available modal analysis software.**
- **The method of peak picking mentioned in the introduction is often used only as an approximation of the true mode shapes.**
    - **More sophisticated algorithms are very frequently used for higher accuracy results.**

4. Multiple modes at a single frequency.

- **If two or more modes are very closely spaced, they may not be resolved by even a sophisticated extraction algorithm.**
  - **In this case, two or more input sources must be used to identify the proper modal parameters.**
- **Detailed modal analyses of complicated structures use multiple channel instrumentation with multiple IO capability.**
  - **Some lab facilities can measure 400 or more channels of data simultaneously.**
- **Field testing is more likely to be carried out with a 2 or 4 channel analyzer.**

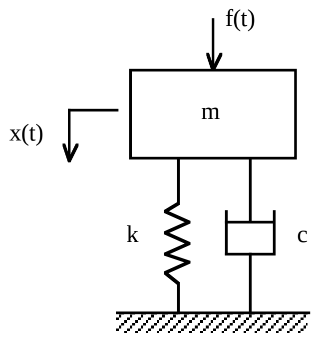
Vibration fundamentals

- **Modal analysis is based on an understanding of lumped-parameter systems.**
- **Although any real structure has an infinite number of modes:**
  - **Modal analysis always fits the data to a finite-order model of discrete masses, springs, and velocity-proportional dampers.**
- **Experimental modal analysis reduces the measurements taken on a real-world test article (continuous system) to:**
  - **A lumped parameter model of the vibration modes of interest (lumped-parameter system).**
- **A sample frequency response function with three modes each with its own natural frequency and damping:**



Dynamics of a Single Degree of Freedom System

- A single degree of freedom system is a mathematical idealization of a single mode of vibration.
- In many structures, the vibration modes are spaced far enough apart in frequency that each mode may be measured independently of the others.
- For this reason, a study of the dynamics of a single degree of freedom system is important.
    - A SDOF model has a mass, a spring, and a dashpot:



- The spring stores potential energy and the mass stores kinetic energy as the system vibrates.
- The dashpot dissipates energy at a rate typically assumed to be proportional to velocity.
    - This idealized damping model is called viscous damping.
        - Although there are other models of damping such as hysteretic and friction damping:
        - Viscous damping is often assumed because it is most conveniently cast into a workable analysis problem.
        - Assuming viscous damping does not usually introduce large errors into an experimental analysis because damping forces are usually small.

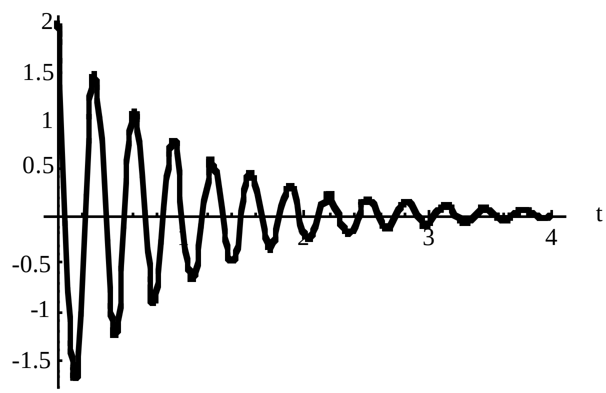
- **The equation of motion of this SDOF system may be obtained by a force balance acting on the mass.**

$$m\ddot{x} + c\dot{x} + kx = f(t)$$

- **The undamped natural frequency ω_n and damping factor ζ are given by:**

$$\omega_n = \sqrt{k/m}, \quad 2\zeta\omega_n = c/m$$

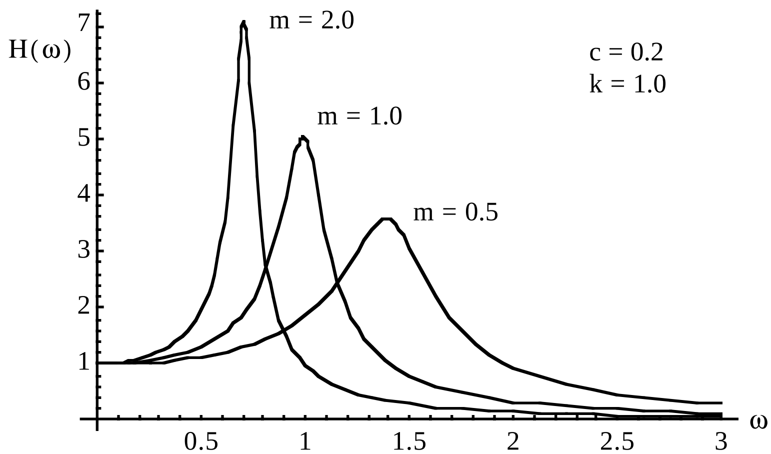
- **The form of the solution can take one of three forms depending on the value of the damping factor ζ.**
    - **For $\zeta > 1$, the system is considered over-damped and the time response to an impulse force is an exponential decay in position *x(t)*.**
    - **For $\zeta = 1$, the response is critically damped and the impulse response is a well-damped sinusoid with no overshoot in position *x(t)*.**
    - **For $\zeta < 1$, the impulse response is an under-damped sinusoid. The smaller ζ is, the longer the settling time of the sinusoid.**



- **$\zeta = 0.44$, and amplification at resonance Q = 11.5.**
- **Most mechanical systems have damping factors less than unity.**
- **A welded structure may have a damping factor of $\zeta = 0.001$.**
- **A bolted structure may have a damping factor closer to $\zeta = 0.01$.**

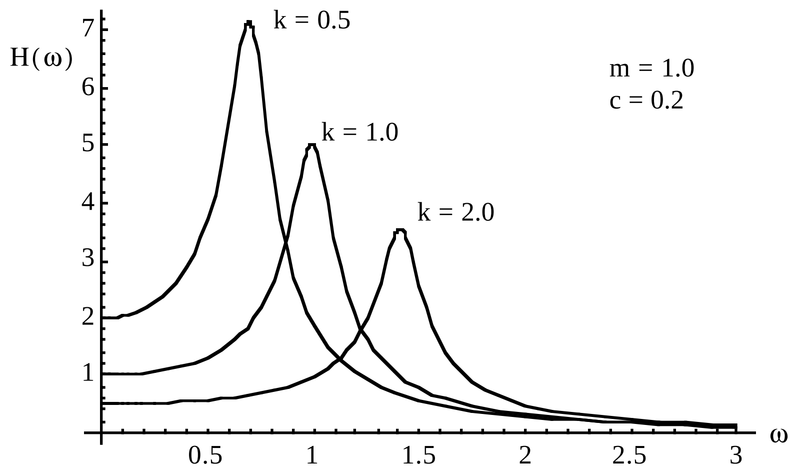
Effects of Removing Mass from the System

- ***Lower*** **mass results in higher natural frequency and increased damping** ***with*** **loss of high frequency noise attenuation.**



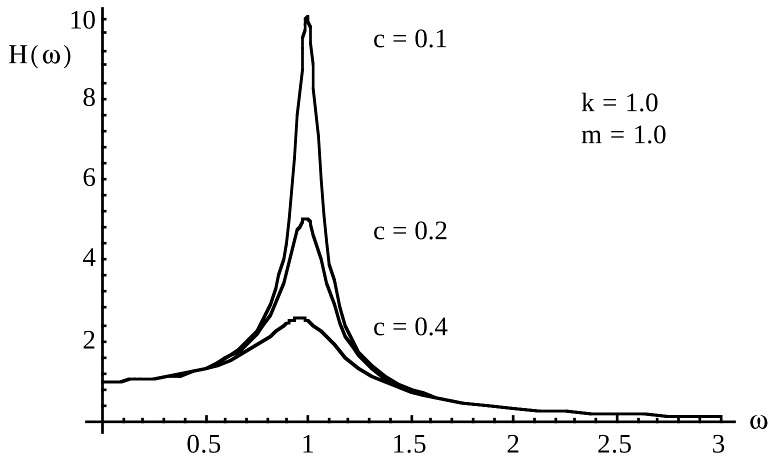
Effects of Adding Stiffness to the System

- ***Higher*** **stiffness results in higher natural frequency and increased damping** ***without*** **loss of high frequency noise attenuation.**



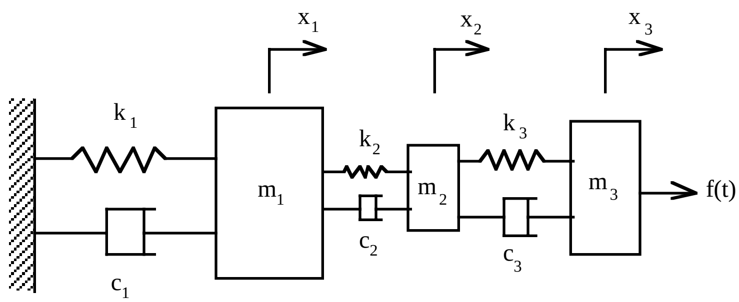
Effects of Adding Damping to the System

- **Higher damping helps reduce the vibration amplitude near the natural frequency of the system.**

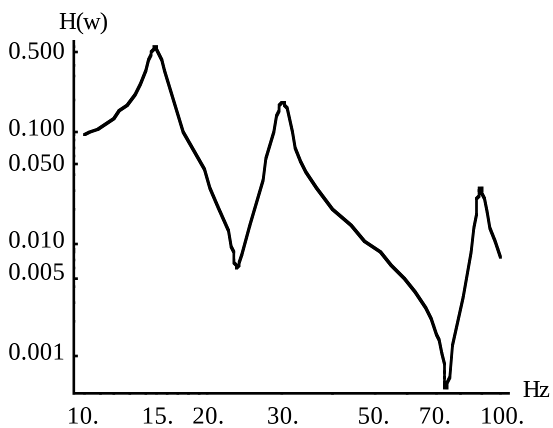


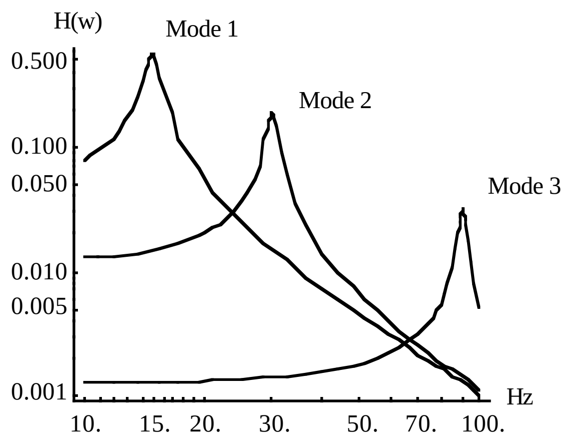
Dynamics of a Multiple Degree of Freedom System

- **A sample MDOF system:**



- **A typical frequency response plot (individual contributions shown to illustrate mode superposition):**





- **The equation of motion of a MDOF system is now a matrix problem:**

$$[M]\{\ddot{x}\}+[C]\{\dot{x}\}+[K]\{x\}=\{f(t)\}$$

- **The Fourier transformed equations are:**

$$\left(-\omega^2[M]+j\omega[C]+[K]\right)\{X\}=\{F\}$$

- **The eigenvalues and eigenvectors may now be calculated. This is done by finding the roots of the determinate of**

$$-\omega^2[M]+j\omega[C]+[K].$$

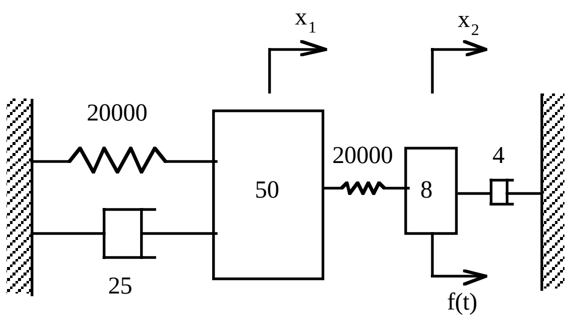
- **In the general case, the eigensolution will be complex.**

- **The eigenvectors [*Φ*] of the system give the mode shapes of the different vibratory modes.**

- **The eigenvalues** $[\omega]=\left[-\omega_n\zeta\pm\omega_n\sqrt{1-\zeta^2}\right]$ **give the natural frequency and damping factor of each mode.**

Example - Two DOF system - Vibration Analysis

- **Consider the two degree of freedom system:**



- **The equation of motion of this system is:**

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}\begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix}+\begin{bmatrix} c_1+c_2 & c_2 \\ c_2 & c_2 \end{bmatrix}\begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{Bmatrix}+\begin{bmatrix} k_1+k_2 & k_2 \\ k_2 & k_2 \end{bmatrix}\begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix}=\begin{Bmatrix} f \\ 0 \end{Bmatrix}$$

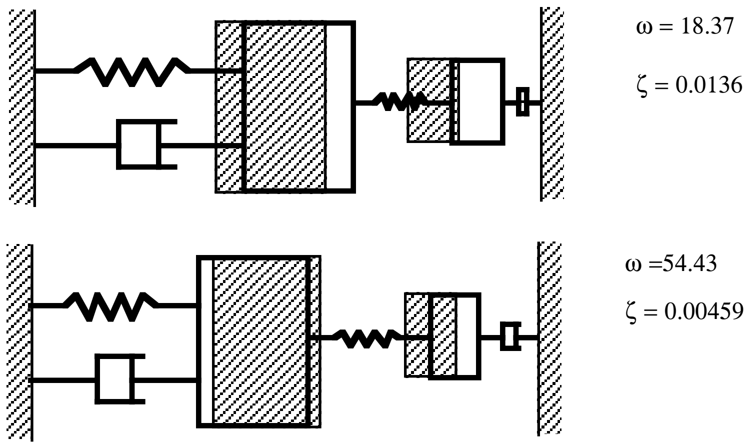
- **The Fourier transform of this equation may be written:**

$$\begin{bmatrix} -\omega^2 m_1 + j\omega(c_1+c_2)+k_1+k_2 & j\omega c_2 + k_2 \\ j\omega c_2 + k_2 & -\omega^2 m_2 + j\omega c_2 + k_2 \end{bmatrix}\begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix}=\begin{Bmatrix} F \\ 0 \end{Bmatrix}$$

- **The solution may be found (on a computer).**

$$[\omega_d]=\begin{bmatrix} 18.37 & \\ & 54.43 \end{bmatrix}, \quad [\zeta\omega_n]=\begin{bmatrix} .25 & \\ & .25 \end{bmatrix}, \text{ and } \quad [\Phi]=\begin{bmatrix} 0.908 & -0.168 \\ 1.049 & 0.908 \end{bmatrix}$$

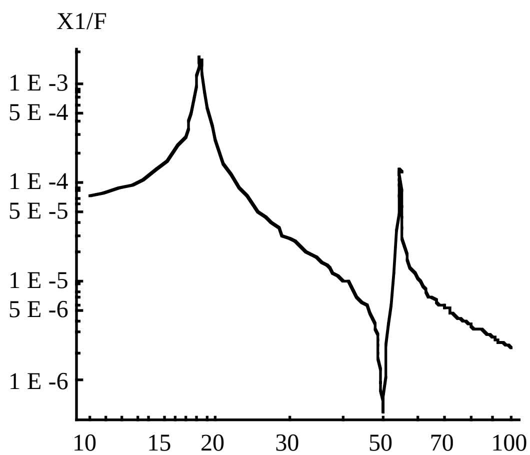
- **The deformed mode shapes (in bold) may be plotted over the undeformed masses (shaded):**

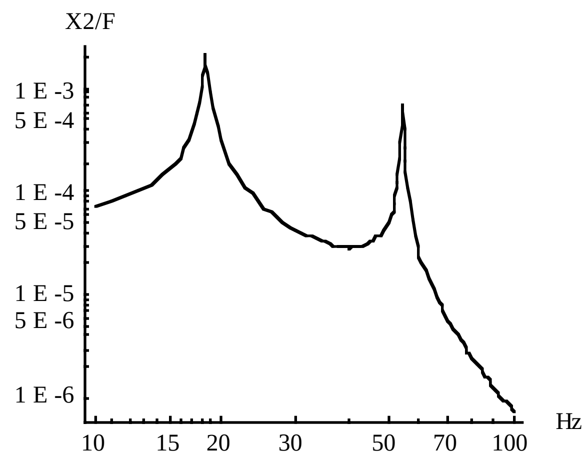


- **Note in one case, the masses move in phase, and in the other they move out of phase.**

Example - Two degree of freedom system - Experimental Modal Analysis

- **The modal parameters (natural frequency, damping, and mode shape) may also be determined experimentally:**
  - **Given the frequency response of the two masses $X(\omega)_1/F(\omega)$ and $X(\omega)_2/F(\omega)$:**





- **The damping factor may be estimated roughly by using the half power bandwidth of the frequency response.**
  - **The half power bandwidth relates the damping factor to the width of a modal peak at** $2^{-1/2}$ **the amplitude of each peak (using magnitude frf).**
  - **The formula for the half power bandwidth calculation for a force excited system is given by:**

$$\frac{\Delta\omega}{\omega_n} = 2\zeta$$

  - **The damping factors for the two modes are thus 0.016 and 0.0055.**

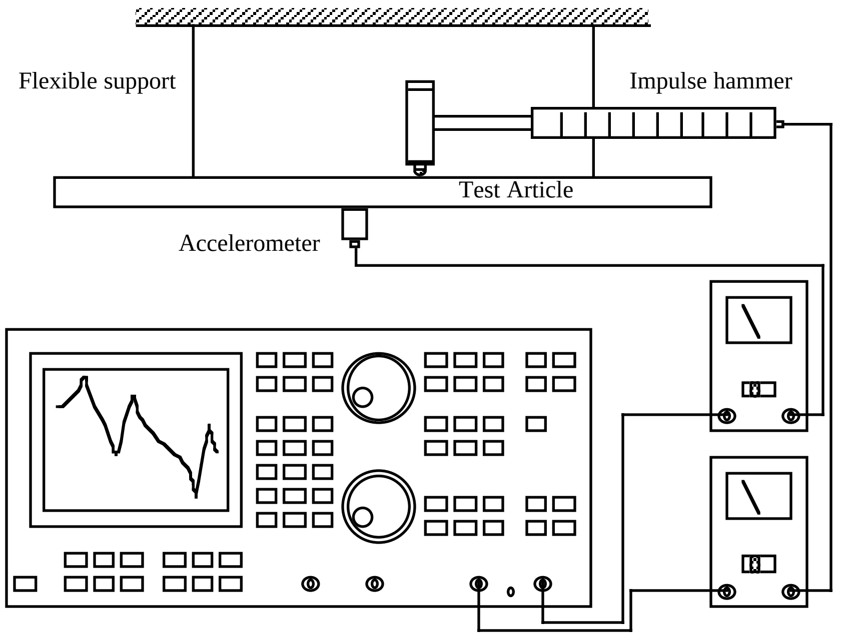
- **Comparison of analytical and experimental modal analysis results:**

	Experimental	Analytical
Natural frequency - first mode	18 rad/sec	18.4 rad/sec
Natural frequency - second mode	54 rad/sec	54.4 rad/sec
Damping factor - first mode	0.016	0.0136
Damping factor - second mode	0.0055	0.00460
Mode shape - first mode	{.875,1.00}	{.865,1.00}
Mode shape - second mode	{-.179,1.00}	{-0.185,1.00}

- **Close agreement!**
- **In practice, a closed form analysis of a structure is usually impractical because of unknowns such as bolted joint stiffness.**
- **A modal analysis allows you to check a machine's dynamic properties.**
  - **It gives you fitted equations of the machine's response.**
  - **It shows you where dampers can be attached.**
- **With the performance modeled, you can design and try dampers on the computer before you ever have to build one.**
- **Once you build the damper, you have a greater confidence level that it will actually work.**

Data collection: Instrumentation summary

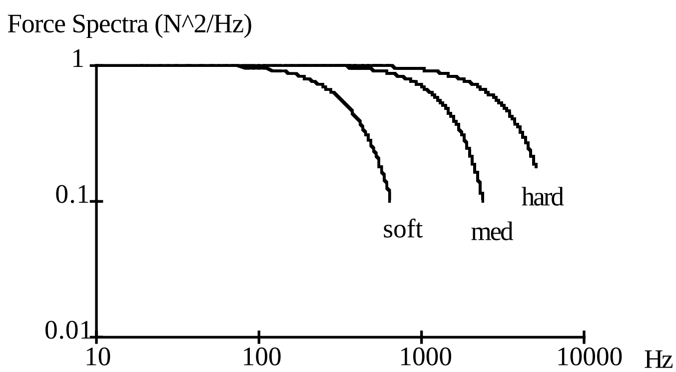
- **Impulse hammers and accelerometers are commonly used in modal analyses of machine tool structures.**
- **A digital signal analyzer and signal conditioning hardware is also needed to complete the necessary equipment.**



FFT-based signal analyzer (includes anti-aliasing filters) DC signal conditioners

Instrumentation - Sensors

- **Hammer testing requires the proper selection of an impact tip.**
    - **Soft tips have longer impact giving better time domain resolution.**
    - **Soft tips do not inject as much high frequency energy. Some modes may not be properly excited as a result.**
    - **The best compromise is to use the softest hammer tip that still excites the modes of interest.**



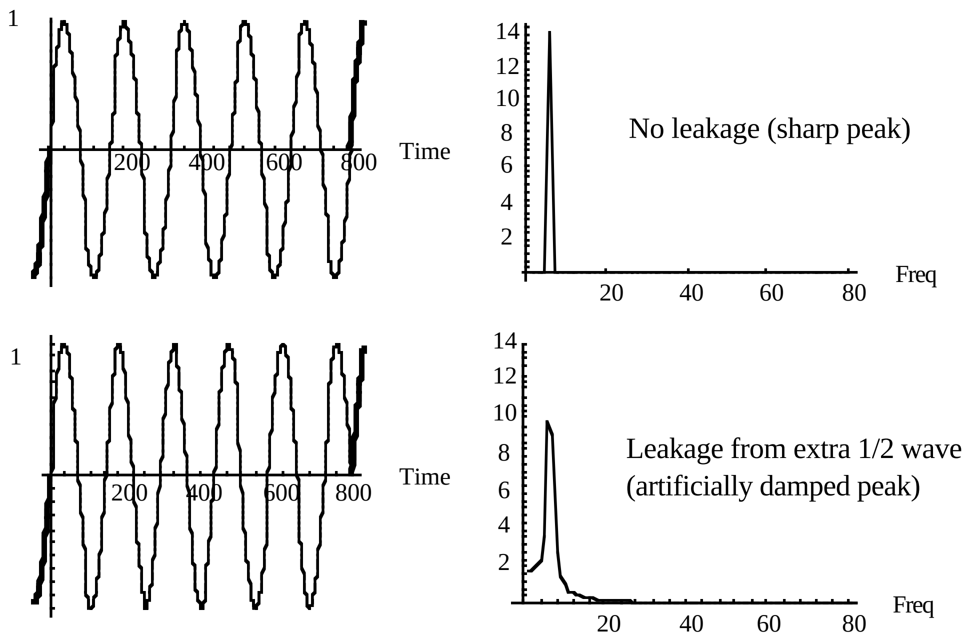
- **Some of the trade-offs with force transducers are also found in accelerometers.**
- **Accelerometers are chosen as a compromise between weight and resolution.**
  - **Heavier accelerometers have higher resolution.**
  - **Heavier accelerometers also mass load a structure and can noticeably alter the dynamics of the measured system.**
  - **Heavier accelerometers typically have a lower maximum range.**

- **Shakers may also be used to excite a structure if care is taken to avoid leakage in the measurement.**
- **There are a variety of excitation waveforms that may be used with a shaker:**

	Steady sine	Swept sine	Burst sine	True random	Periodic random	Burst random	Impact
Leakage	poor	poor	good	poor	good	good	good
Signal to noise ratio	good	good	good	fair	fair	fair	poor
Characterizes non-linearity	yes	yes	yes	no	no	no	no

Leakage

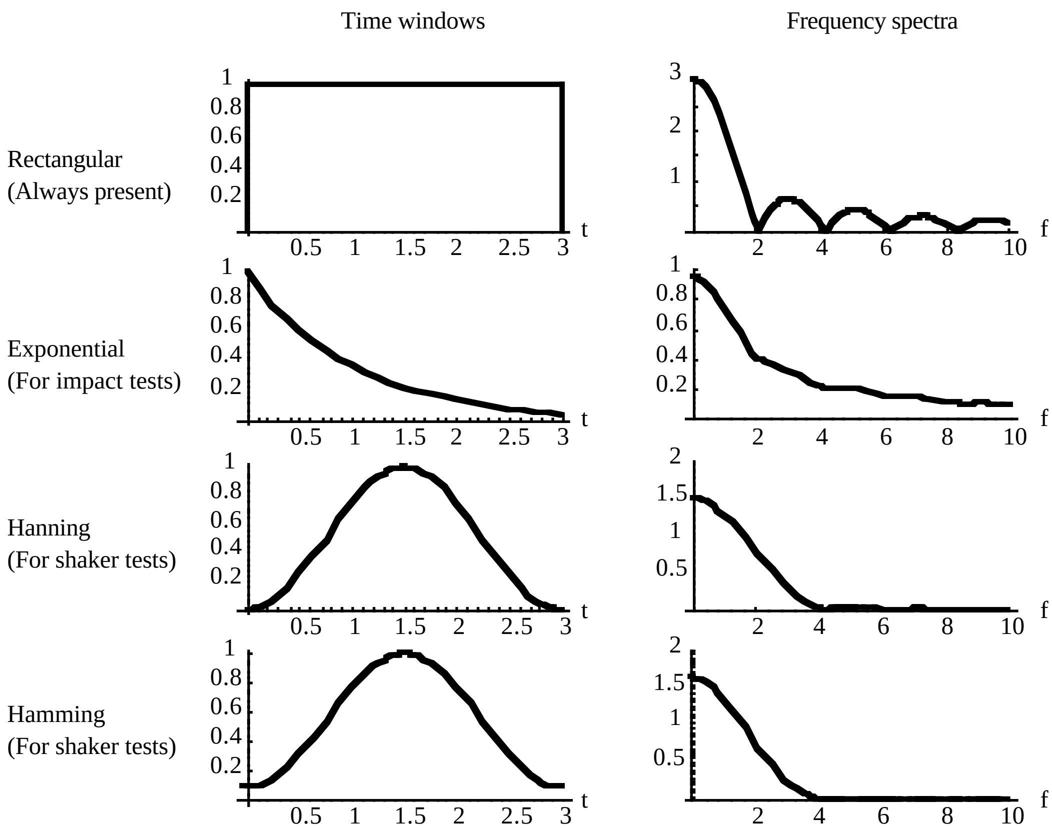
- **Leakage results from violating the Fourier assumption that the sampled series represents the infinite series:**



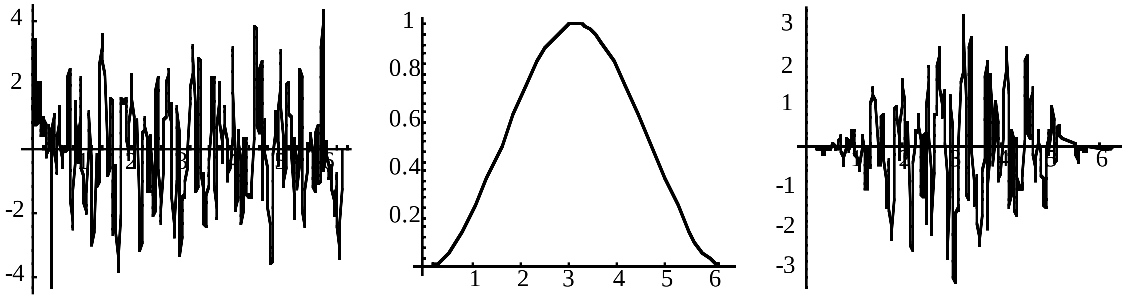
- **Leakage may be avoided by:**
  - **Carefully constructing an excitation waveform.**
    - **It should only contain components that will be sampled an integer number of times during the time record (pseudo-random excitation).**
  - **Making sure that the excitation and response is zero at the beginning and end of the data record.**
    - **This is automatic in hammer testing with sufficient sampling time.**
  - **Time windowing.**

Time windowing

- **The effect of a window is always to smooth the data in the frequency domain.**



- **Filtered data is guaranteed to minimize leakage because waveform will be periodic in time:**



Actual time history * Hanning window = Filtered time window

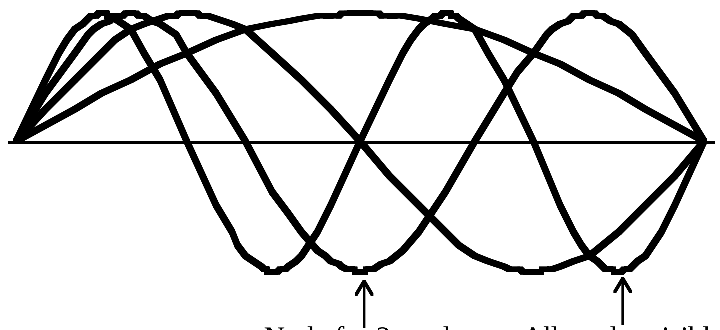
- **Averaging can also be used to improve data quality (by virtue of smoothing).**

- **The improvement in quality varies approximately with the square root of the number of averages:**

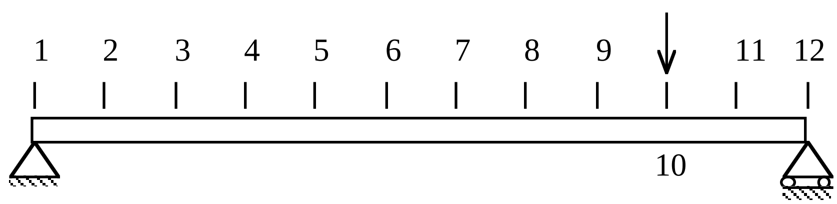
$$\sigma = \sqrt{\frac{\sum deviations^2}{N-1}}$$

Data reduction

- **The location of the fixed sensor must not be on a node of a mode of interest.**



- **A grid can be set out marking the locations on the structure where data will be taken.**



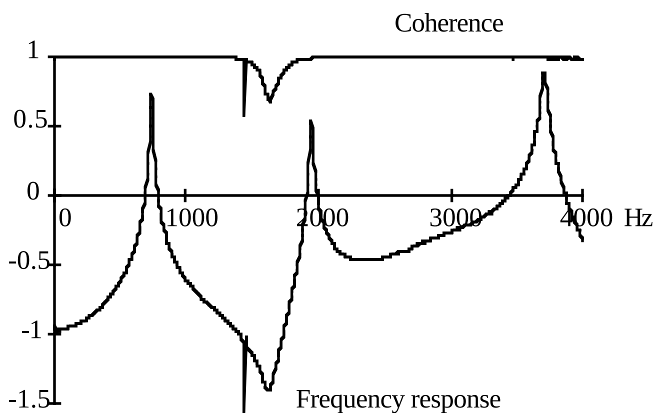
- **Because of reciprocity, the accelerometer can be fixed at a point, and the impact point location can be moved along the beam.**

- **Alternatively, the *drive point* can remain fixed, and the measurement can be made at each of many locations.**

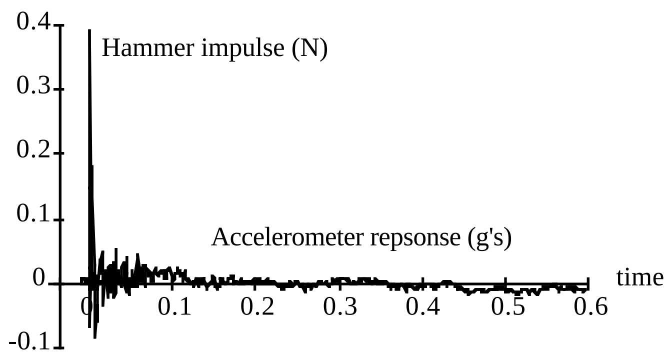
- **The coherence must be checked to make sure that the output is properly related to the input (and not some other noise source).**

$$\eta_{yx}^{2} = \left| \frac{H_{yx}(\omega) H_{xy}(\omega)}{H_{xx}(\omega) H_{yy}(\omega)} \right|$$

  - **The coherence function should be as close as possible to unity.**
  - **In practice, the coherence should be greater than 0.85 for a measurement to be considered usable.**
  - **In many test cases, the coherence can be consistently 0.99 or better, indicating that the data is probably very good.**

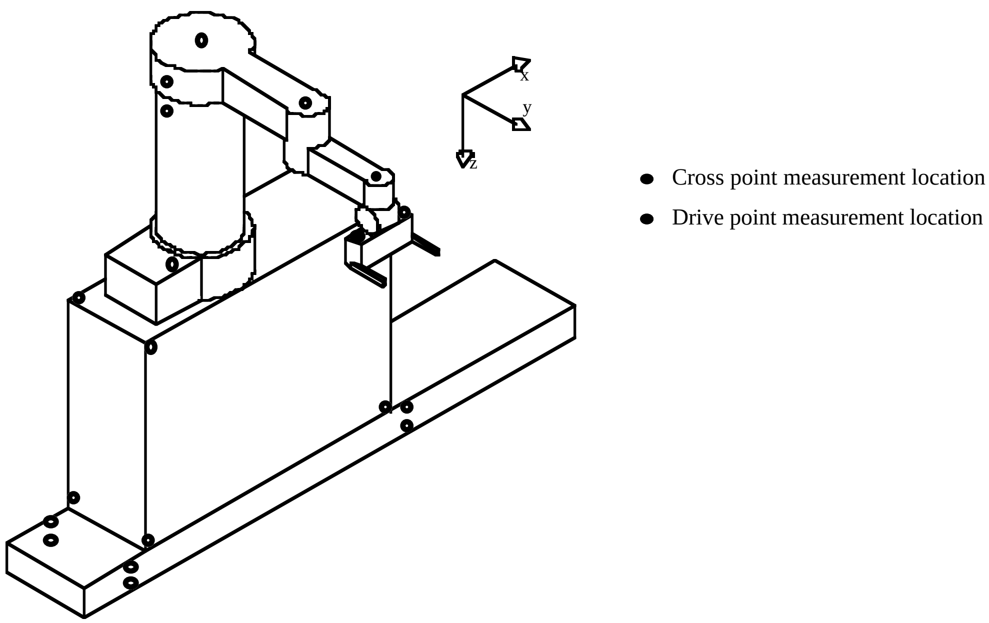


- **Poor coherence can indicate several problems**
  - **If the coherence is low at modal peaks:**
    - **Leakage is probably effecting the measurement (increase sample time or change excitation waveform).**
  - **A sensor is on a node (change measurement point location).**
  - **Low coherence at low frequencies (common in piezoelectric sensors - switch to laser interferometer for response measurement).**
  - **Non-linearities present in system (identify and remove non-linearity)**
  - **Noise in measurement (check time histories and make sure data acquisition board is auto-ranged to correct voltage level).**
- **Here are the input and output time histories of an impact test showing severe noise.**



Case study: A wafer cassette handling robot

- **A complete modal survey was performed on a linear track-mounted robot system.**
- **Survey began with some preliminary measurements being taken to optimize the instrumentation setup and data filtering parameters.**
    - **The location of the drive point measurement was also selected using the pre-test measurements.**
    - **The location of the other test points was made.**
    - **The coherence of the drive point frf was checked.**

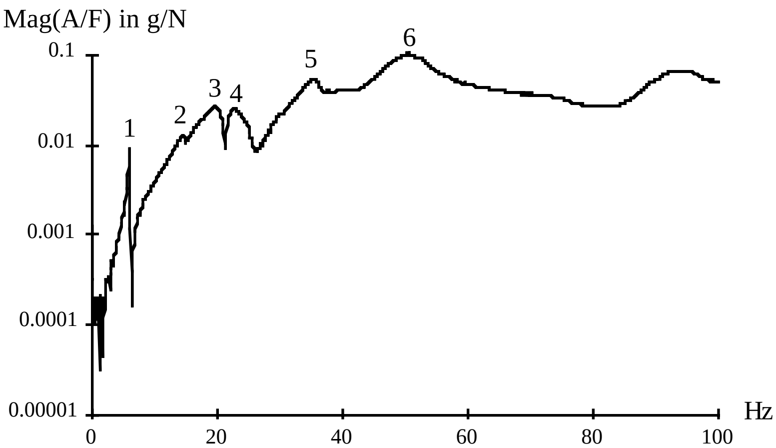


Test equipment and configuration:

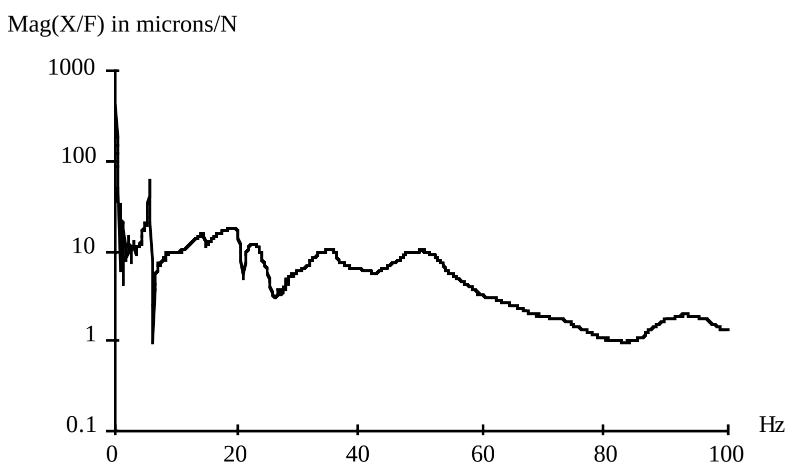
Frequency range	100 Hz
Sample time	8 seconds
Pre-triggering	0.25 seconds
Excitation	roving PCB 3 lb impulse hammer
Response (accelerometer)	PCB low frequency accelerometer - fixed on end effector
Excitation window	uniform
Response window	uniform
Number of averages	8

Drive point frequency response functions

- **The drive point measurement in acceleration per unit force is taken at the most sensitive error motion point (e.g., the spindle or gripper):**



- **The drive point measurement in displacement per unit force helps to identify the dominant error motion mode:**



Modal results

- **Frequency and damping of the first six modes of vibration:**

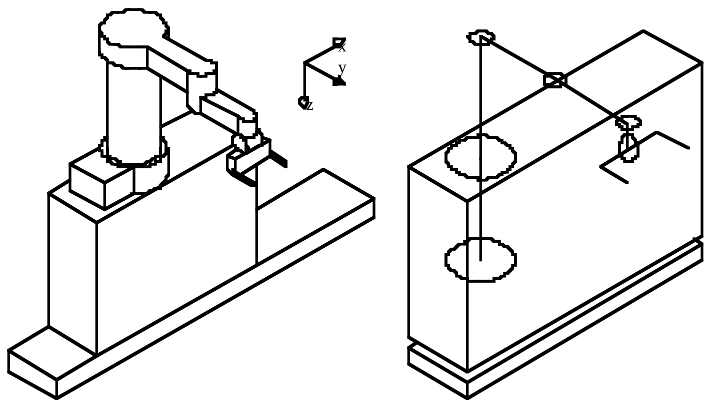
	Nat. Freq. (Hz)	Damping (%)
Mode 1	6.00	1.36
Mode 2	15.60	2.33
Mode 3	20.46	6.14
Mode 4	22.70	5.23
Mode 5	35.93	4.31
Mode 6	50.70	6.75

- **The MAC matrix shows the orthogonality of the identified experimental mode shapes.**
  - **Ideally, all modes should be mutually orthogonal to each other so the off diagonal terms should be 0's.**
    - **A "good" MAC matrix shows that the data is good, and the modes measured are "clean" and real.**
  - **The main diagonal of the MAC matrix should be unity because each mode coincides with itself.**

	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5	Mode 6
Mode 1	1.00	0.57	0.05	0.17	0.01	0.01
Mode 2	0.57	1.00	0.07	0.08	0.00	0.00
Mode 3	0.05	0.07	1.00	0.23	0.02	0.02
Mode 4	0.17	0.08	0.23	1.00	0.11	0.21
Mode 5	0.01	0.00	0.02	0.11	1.00	0.44
Mode 6	0.01	0.00	0.02	0.21	0.44	1.00

Test structure and wireframe mesh for computer animation:[2]

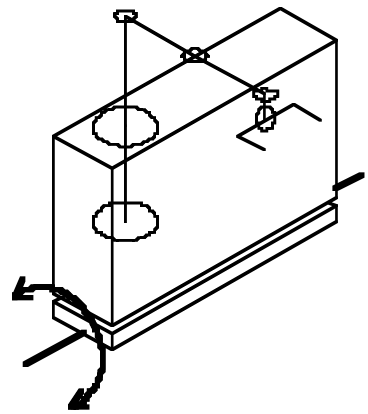
- **A wire mesh drawing is created.**
- **The measurement points are defined.**
- **The test data taken at the points is loaded.**
- **At each point, one defines a frequency band.**
- **The software moves each point according to the amplitude in the selected frequency band.**



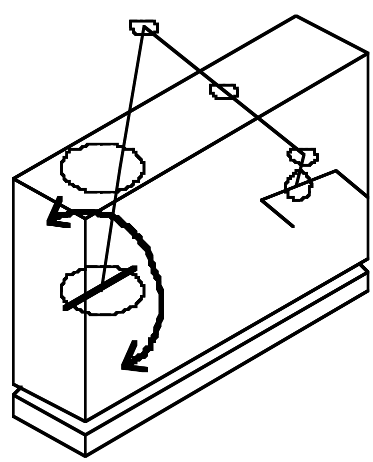
- **The robot was mounted by a contractor on pedestals that went from the concrete floor to just under the tiles of a cleanroom raised floor.**

[2] These animations were done using software from *Structural Measurement Systems*, 510 Cottonwood Drive, Milpitas, CA 95035.

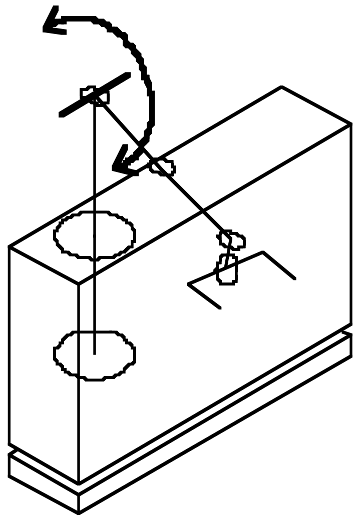
- **Mode 1:**



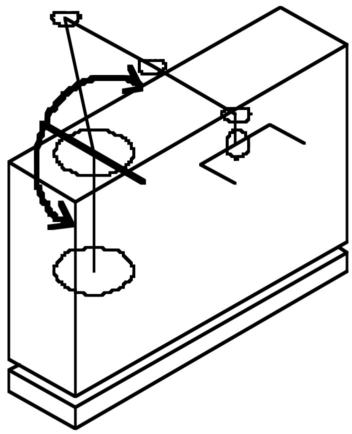
- **Mode 1 shows the entire machine rocking as a rigid body, and the effective dynamic stiffness was very low.**
    - **The contractor never installed the steel pedestals! The contractor just bolted the robot to the floor tiles!**
- **Mode 2:**



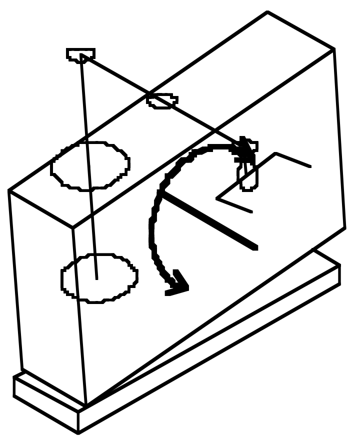
- **Modes 3 and 4:**



- **Mode 5:**



- **Mode 6:**



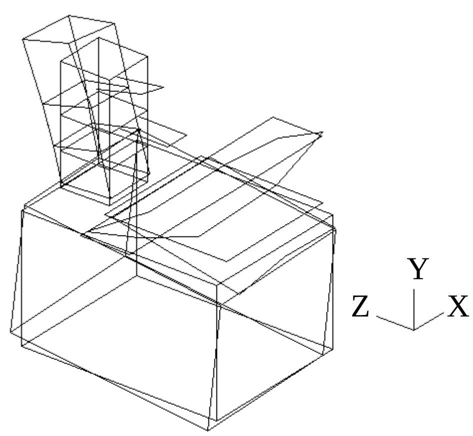
- **The rest of the modes showed the robot to be well-designed.**

Case study: A precision surface grinder[3]

- **A good quality surface grinder was tested to see if it could be made stiffer so it would grind ceramics better.**

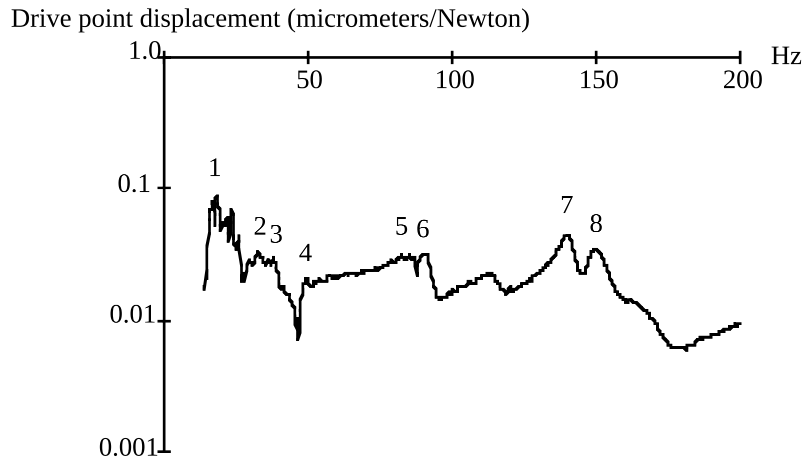
- **Test equipment and analyzer configuration:**

Frequency range	**10 -210 Hz**
Sample time	**4 seconds (random)**
Pre-triggering	**none**
Excitation	**50 pound shaker (fixed location)**
Response	**tri-axial accelerometer (roving)**
Excitation window	**Hanning**
Response window	**Hanning**
Number of averages	**20**



[3] These animations were done using software from *Structural Measurement Systems*, 510 Cottonwood Drive, Milpitas, CA 95035. Contact Dan Sylvester for further information: (408) 435-5559.

Displacement response at the drive point of the grinder:



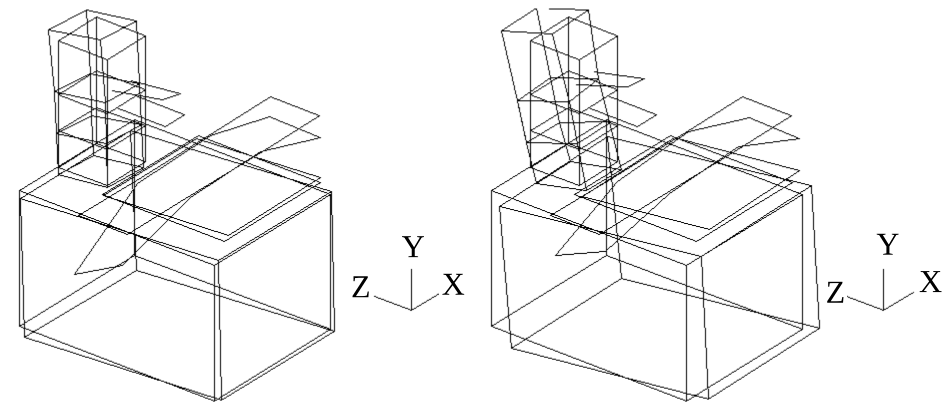
- **Most of the vibration modes yield a compliance of about 0.05 to 0.10 μm/N.**
- **This corresponds to a stiffness of around 10 to 20 N/μm.**
- **Note that the first cluster of modes around 20 to 30 Hz will be found to be rigid body modes of the structure vibrating on its ground supports.**

Summary of first eight modes:

	Nat. freq. (Hz)	Z-direction drive point stiffness (N/μm)
Mode 1	26	24.1
Mode 2	37	45.6
Mode 3	48	115.5
Mode 4	53	118.4
Mode 5	87	44.4
Mode 6	92	35.2
Mode 7	141	25.0
Mode 8	151	28.6

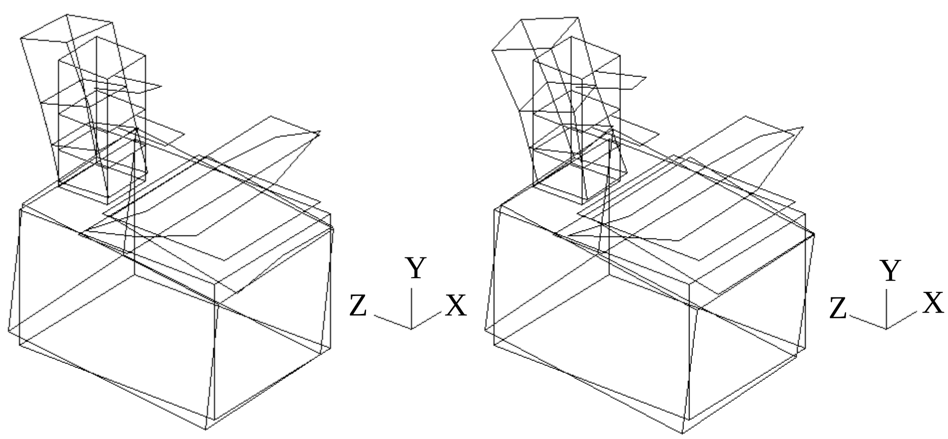
- **The first four modes are rigid body modes of the entire machine rocking on its mounts.**
- **Overall, for general purpose shop use, the grinder is well designed and damped.**
- **The primary areas for improvement identified include:**
    - **The spindle overhanging structure (Y axis).**
    - **The column structure and bearings (Z axis)**
    - **The table at the ends of travel (X axis)**
- **Ideally, all the dynamic stiffnesses should be similar.**

- **Modes 5 and 6 are table bending modes:**



  - **This effect is due to the overhang of the table.**

- **Modes 7 and 8 are column bending/Z axis bearing deflection modes:**



  - **This effect is due to the cantilever nature of the design.**
  - **Since no mode clearly stood out as being a problem:**
    - **The machine cannot easily be modified to increase performance for ceramics grinding.**

Conclusion:

- **Much of the black art of precision machine design is due to misunderstanding of machine dynamics.**

- **Modal analysis and the resulting animations showing the mode shapes can be a very powerful identification tool.**

- **If you have an existing machine that machines well, use it to set a minimum dynamic stiffness specification.**

- **All machine tool companies and most buyers of machine tools should have at least one person and the equipment to do modal analysis.**