Precision Machine Design

Topic 11

Vibration control step 2:
Damping

Purpose:

Damping is one of the most important, yet misunderstood, factors in machine design. Without proper damping, a machine or process can shake itself into ineffectiveness.

Outline:

- **The importance of damping**
- **Damping, stiffness and mass effects on system servo bandwidth**
- **Material damping**
- **Tuned mass dampers**
- **Constrained layer dampers**
- **Replicated internal viscous dampers**

"Be always sure you're right - then go ahead"

David Crockett

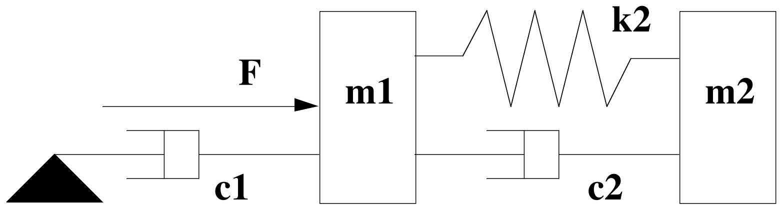
The importance of damping

- **Damping is needed to absord energy from the process:**
  - **To prevent chatter and damage to the surface.**
  - **To absorb energy from structural modes excited by the servos.**
- **Damping can be obtained by internal means:**
  - **Material damping.**
  - **Damping by microslip in joints.**
- **Damping can be obtained by external means:**
  - **Tuned mass dampers.**
  - **Constrained layer dampers.**
  - **Active dampers.**
    - **Velocity feedback in servos**
    - **Actively controlled damped masses attached to the structure.**

Damping, stiffness and mass effects on system servo bandwidth

- **There are three primary sources of excitation in a system that require the servo to have a minimum bandwidth:**
  - **Self-excited structural vibrations**
    - **Step response**
    - **Contouring speed requirements**
    - **A simple model can help the designer ensure that sufficient damping is made available.**
  - **External disturbance force requirements**
    - **Difficult to determine the effects of system parameters without a complete dynamic simulation including the controller.**

- **For determining system parameters to prevent self-excited structural vibrations:**
  - **Model the system with a motor driving a carriage in the following manner:**



  - **m_1 is the mass of a linear motor forcer or:**
  - **m_1 is the reflected inertia of the motor rotor and leadscrew (or just a linear motor's moving part):**

$$M_{reflected} = \frac{4\pi^2 J}{l^2}$$

  - **M_2 is the mass of the carriage.**
  - **C_1 is the damping in the linear and rotary bearings.**
  - **C_2 is the damping in the actuator-carriage coupling and the carriage structure.**
  - **K_2 is the stiffness of the actuator and actuator-carriage-tool structural loop.**

- **The equations of motion are:**

$$\left|\begin{matrix} m_1 & 0 \\ 0 & m_2 \end{matrix}\right| \left|\begin{matrix} \ddot{x}_1 \\ \ddot{x}_2 \end{matrix}\right| + \left|\begin{matrix} c_1 + c_2 & -c_2 \\ -c_2 & c_2 \end{matrix}\right| \left|\begin{matrix} \dot{x}_1 \\ \dot{x}_2 \end{matrix}\right| + \left|\begin{matrix} k_2 & -k_2 \\ -k_2 & k_2 \end{matrix}\right| \left|\begin{matrix} x_1 \\ x_2 \end{matrix}\right| = \left|\begin{matrix} F(t) \\ 0 \end{matrix}\right|$$

- **The transfer function x2/F (dynamic response of the carriage) is:**

$$\frac{x_2}{F} = \frac{k_2 + c_2 s}{c_1 s\left(k_2 + c_2 s + m_2 s^2\right) + (m_1 + m_2) s^2 (k_2 + c_2 s) + m_1 m_2 s^4}$$

- **Note the product of the masses term which tends to dominate the system.**

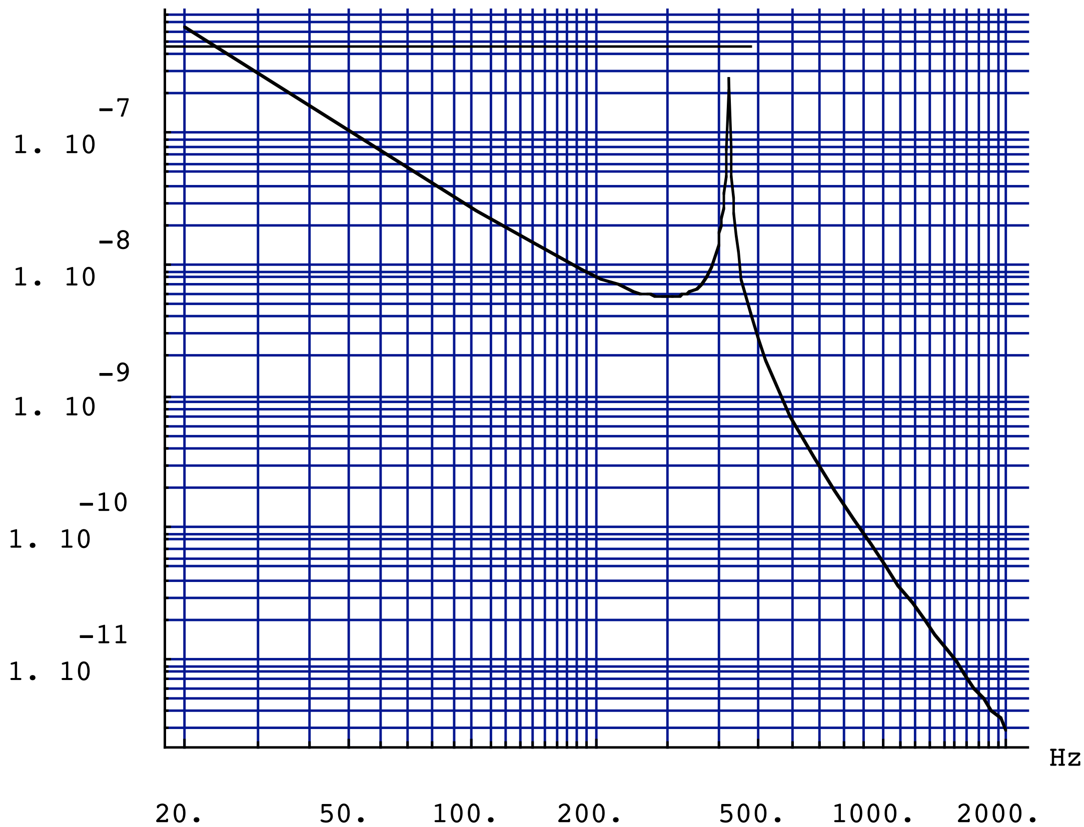
- **Calculated parameters of four possible systems are:**

Actuator	**ballscrew**	**lin. motor**	**lin. motor**	**lin. motor**
Bearings	**linear ball**	**linear ball**	**air**	**air**
Structural damping	**no**	**no**	**no**	**yes**
material damping zeta	**0.005**	**0.005**	**0.005**	**0.1**
actuator to ground zeta	**0.05**	**0.03**	**0**	**0**
m1 (actuator) (kg)	**50**	**5**	**5**	**5**
m2 (carriage), kg	**50**	**50**	**50**	**50**
c1 (N/m/s)	**355**	**187**	**0**	**0**
c2 (N/m/s)	**19**	**19**	**19**	**374**
k1 (N/m)	**1.75E+08**	**1.75E+08**	**1.75E+08**	**1.75E+08**
Bandwidth (Hz)	**25**	**100**	**30**	**100**

- **As a guideline, the servo bandwidth of the system is:**
  - **Generally limited by the frequency the servo can drive the system at without exciting structural modes.**
  - **Without special control techniques can be no higher than the frequency:**
    - **Found by drawing a horizontal line 3 dB above the resonant peak to intersect the response curve.**
- **This method is used only to initially size components.**
- **A detailed controls simulation must be done to verify performance, and guide further system optimization.**

- **The response of the ballscrew driven carriage supported by rolling element linear bearings will be:**

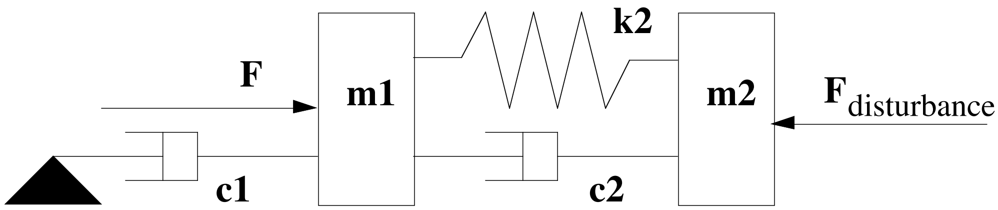
Transfer Function x2/f



- **In this case, since preloaded linear guides and a ballnut are used, damping to ground will be high.**
- **The inertia of the screw will lower the system frequency considerably (note the $m_1m_2s^4$ term in the TF)**
- **The system bandwidth will be limited to about 25 Hz.**
- **The Ballscrew was inertia matched to the carriage:**

Diameter	**0.025**
Lead	**0.003**
Length	**0.5**
Axial K (N/μm)	**203**
J	**9.6E-06**
Reflected inertia = $4\pi^2J/l^2$	**42**

- **External disturbance force effects on required servo bandwidth**



  - **Difficult to determine the effects of system parameters without a complete dynamic simulation including the controller.**
  - **A high degree of structural damping is required as before.**
    - **Use the previous analysis method to determine the degree of structural damping required.**
  - **Either a very high static and dynamic stiffness actuator is required (e.g., a ballscrew) or:**
  - **A modest stiffness actuator (linear motor) with a high degree of damping to ground is required.**
  - **This topic is discussed in greater detail in the context of linear power transmission system requirements.**

Material damping

- **Hysteresis losses from the motion of dislocations in a material under stress are typically 2-3%/cycle.**
- **Quantifiers of the amount of damping include:**

□	**Loss factor of material (%)**
$□_s$	**Loss factor of material (geometry and load dependent)**
Q	**Amplification at resonance factor (A_r)**
□	**Phase angle □ between stress and strain (hysteresis factor)**
□	**Logarithmic decrement ($□_{Ld}$)** [1]
□U	**The energy dissipated during one cycle**
□	**The damping factor associated with second order systems**

$$\frac{k_{dynamic}}{k_{static}} = \frac{1}{Q} = □ = \frac{1}{A_r} = \frac{□}{□} = □ = \frac{□U}{2□U}$$

[1] Most texts on vibration refer to the log decrement as □; however, to avoid confusion with discussions on displacement termed □, the log decrement will be referred here to as $□_{Ld}$.

- **The *logarithmic decrement* δ_{Ld} is used to relate impulse response data to system parameters:**
  - **δ_{Ld} is a measure of the relative amplitude between N successive oscillations of a freely vibrating system:**

$$\delta_{Ld} = \frac{-1}{N}\log_e\left(\frac{a_N}{a_1}\right)$$

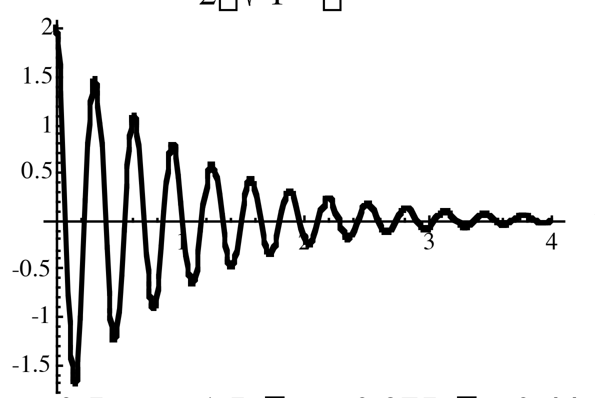
- **The logarithmic decrement can also be related to:**
  - **The damping factor ζ.**
  - **Velocity damping factor b.**
  - **Mass m.**
  - **Natural frequency ω_n of a second order system model.**

$$\zeta = \frac{\delta_{Ld}}{\sqrt{4\pi^2 + \delta_{Ld}^2}}$$

$$b = 2m\zeta\omega_n$$

- **The peak amplification at resonance of a second order system is given by**

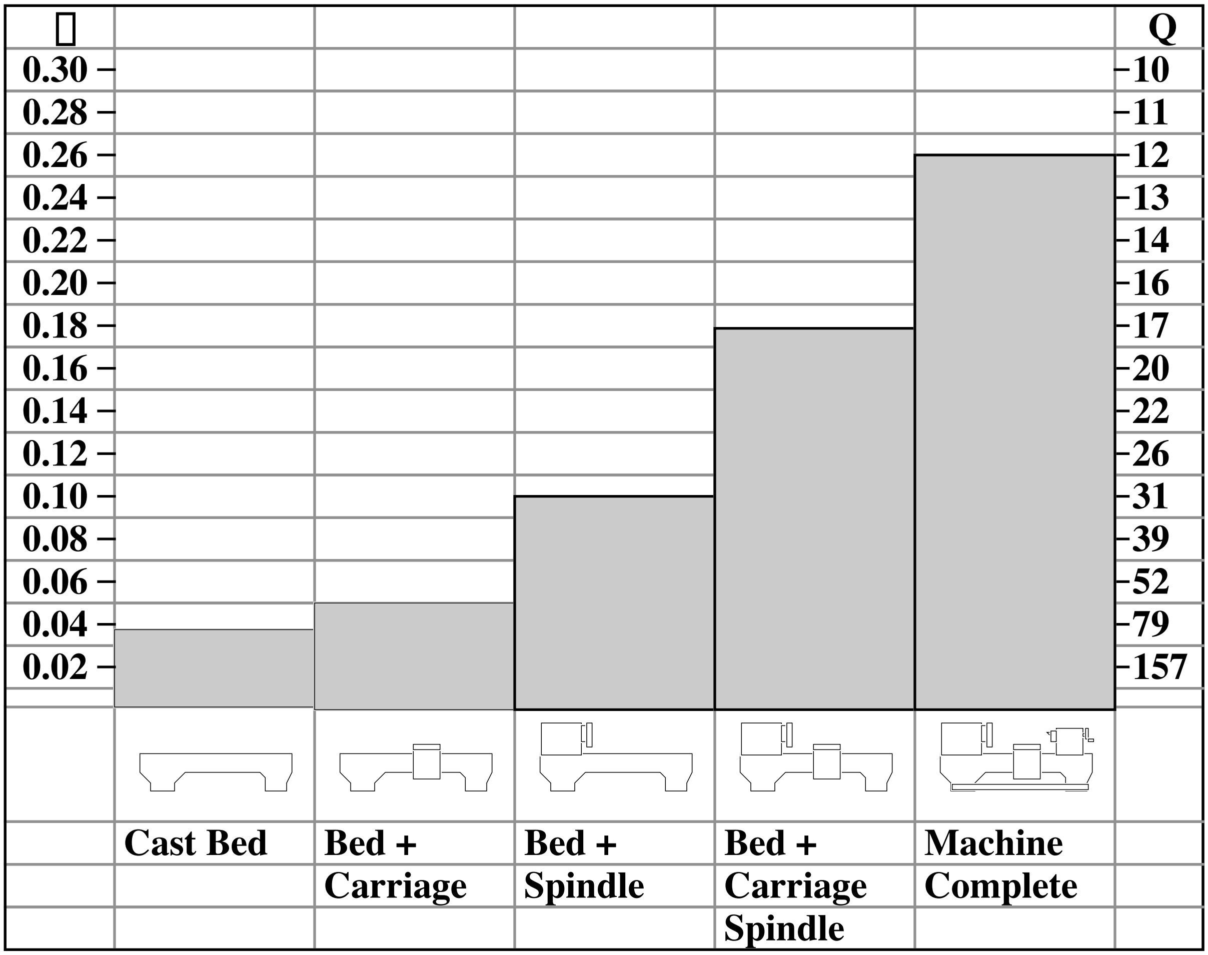
$$Q = A_r = \frac{1}{2\zeta\sqrt{1-\zeta^2}} \quad (\zeta \le 0.707)$$



- **Here, n = 4, a_5 = 0.5, a_1 = 1.5, δ_{ld} = 0.275, ζ = 0.44, and Q = 11.5**

Damping from joints

- **Microslip in the joints dissipates energy by friction.**
- **Contact between the machine bed and the ground, and joints between components creates the total damping[2]:**



2 This historical figure (from data from the 1800's) was provided by Dr. Richard Kegg of Cincinnati Milacron.

Case Study

- **An old sliding contact bearing machine was rebuilt with linear motion ball bearings and more range of motion.**
    - **Rolling element bearings have 1/10th the friction (damping) of sliding contact bearings.**
    - **Rolling element bearings allow for 10x better positioning accuracy than sliding contact bearings.**
- **The structural loop static stiffness was maintained.**
- **The structural loop length (distance from the bearings to the tool) was greatly increased.**

Cutting tests

- **The new machine made very accurate parts.**
- **When making heavy cuts, the surface finish was very poor, even though the same tooling was used as with the old machine!**
- **The bearing vendor suggested using a bigger stiffer (more expensive) linear motion guide.**
- **The Nerd consultant noticed, that when cutting, there was a high pitch squeal.**
    - **Squealing sounds are in the kHz range.**
    - **Most machine tool structural vibration s are in the 100's Hz range.**
    - **kHz frequencies are caused by short stiff structures (the tooling).**
- **The Nerd wrapped the tool shank in electrical tape, and a mirror surface finish was obtained!**
- **The tool was unwrapped, and tape was place between the tooling block and the crosslide, and good surface finish was also obtained.**
    - **Dowel pins can keep establish the planar position of the tooling block.**
    - **The damping tape (e.g., a 1/4 mm layer of ScotchDamp™ from 3M) provides enhanced joint damping.**

What was the difference between the new and old machines?

- The old machine had strong dampers (sliding bearings) closer to the tool.
  - When the tool was vibrating, it coupled with a mode of the crosslide which was damped by the bearings.
- The new machine structure was larger and although it was statically stiff, its frequency was far lower than the tool.
  - The tool acted independently and chattered.
- Morals of the story:
  - All modes (machine elements) in the structural loop should be damped.
  - Any element left undamped can become the resonance source.
  - Keep a supply of viscoelastic damping tape handy!
  - Be prepared to do more dynamic analysis before and after the machine is built!

Material damping:

- **Typical data shows wide variations because damping is so sensitive to boundary conditions:**[3]

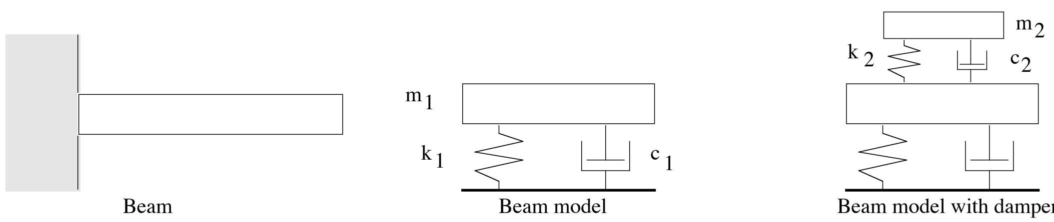
Material	Load	T_1 (°K)	T_2 (°K)	□$_1$ (ksi)	□$_2$ (ksi)	f_1 (Hz)	f_2 (Hz)	□$_1$	□$_2$	Q1	Q2
Alumina								5.00E-06	1.50E-05	100000	33300
Aluminum (6063-T6)	bending			1	6			2.50E-04	2.50E-03	2000	200
Aluminum (pure annealed)	axial	50	300					3.50E-06	1.00E-05	143000	50000
Beryillium (18.6%Be)	unspec.			2	50			7.50E-03	4.10E-01	66.7	1.3
Copper (brass)	bending					50	600	1.50E-03	3.00E-03	333	167
Copper (pure annealed)	bending					20	550	3.50E-03	1.00E-03	143	500
Glass	bending					10	100	1.00E-03	3.00E-03	500	167
Granite (Quincy)	bending					140	1600	2.50E-03	5.00E-03	200	100
Iron (cast, annealed)	bending					100	2000	6.00E-04	1.50E-03	833	333
Iron (mild steel)	bending			2.5	5.5			4.50E-04	7.00E-04	1110	714
Lead	bending					20	160	4.00E-03	7.00E-03	125	71.4
Polymer concrete	bending							3.50E0-03		143	
Portland cement concrete	bending							1.20E-02		41.7	
Quartz (ground, piezo)	unspec.					65k		5.00E-06		100000	
Sand (loose on an Al beam)											
beam alone	bending					1000	4000	1.00E-03		500	
50% wt. layer of sand	bending					1000	4000	4.00E-02	9.95E-02	12.5	5.1
100% wt. layer of sand	bending					1000	4000	9.95E-02	4.10E-01	5.1	1.3
Silica (fused, annealed)	axial	73	1073					5.00E-07	5.00E-05	1000000	10000
Silicon nitride (n)	unspec.							1.25E-05		40000	
Soil (misc.)	unspec.					6	30	4.99E-02		10.0	

- **Note that the greatest degree of damping is obtained when sand is loaded on top of the beam.**
- **Material damping is very low compared to the damping from a damping mechanism.**
- **"Old timers" knew that a machine could be damped by filling it with fine sand or lead shot.**
  - **One had to be sure the added weight did not deform the machine.**
- **Viscous dampers (discussed in detail later) can be designed to yield an 10x more damping than that of structural materials.**

3 Data from sources in B. J. Lazan, Damping of Materials and Members in Structural Mechanics, Pergamon Press, London, 1968.

Tuned mass dampers

- **In a machine with a rotating component (e.g., a grinding wheel):**
  - **There is often enough energy at multiples of the rotational frequency (harmonics) to cause resonant vibrations.**
  - **Some of the machine's components are usually affected.**
- **A tuned mass damper is simply a mass, spring, and damper attached to a structure at the point where vibration motion is to be decreased.**
- **The size of the mass, spring, and damper are chosen so they oscillate out of phase with the structure.**
  - **They help to reduce the structure's vibration amplitude:**



- **The equations of motion of the system are**

$$m\ddot{x}_1(t) + (c_1 + c_2)\,\dot{x}_1(t) - c_2\dot{x}_2(t) + (k_1 + k_2)\,x_1(t) - k_2x_2(t) = F(t)$$

$$m_2\ddot{x}_2(t) - c_2\dot{x}_1(t) + c_2\dot{x}_2(t) - k_2x_1(t) - k_2x_2(t) = 0$$

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}\ddot{x}(t) + \begin{bmatrix} c_1 + c_2 & -c_2 \\ -c_2 & c_2 \end{bmatrix}\dot{x}(t) + \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix}x(t)$$

- **In the frequency domain, in order to present a solution for the motion of the system, the following notation is introduced:**

$$Z_{ij}(\omega) = -\omega^2 m_{ij} + i\omega c_{ij} + k_{ij} \qquad i, j = 1, 2$$

- **The amplitudes of the motions of the component and the damper as a function of frequency are given by**

$$X_1(\omega) = \frac{Z_{22}(\omega)\,F_1}{Z_{11}(\omega)\,Z_{22}(\omega) - Z_{12}^2(\omega)}$$

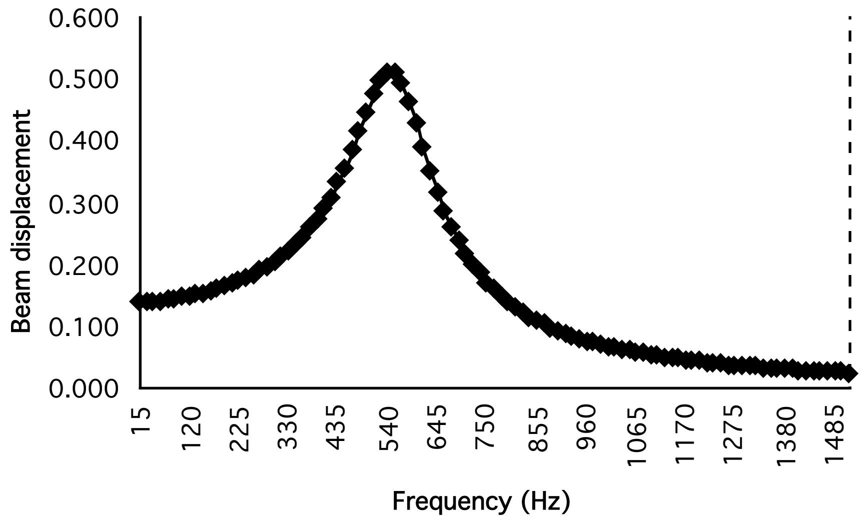
$$X_2(\omega) = \frac{-Z_{12}(w)\,F_1}{Z_{11}(\omega)\,Z_{22}(\omega) - Z_{12}^2(\omega)}$$

- **The design of a tuned mass damper system for a machine component may involve the following steps:**
    - **Determine the space available for the damper and calculate the mass (m_2) that can fit into this space.**
    - **Determine the spring size (k_2) that makes the natural frequency of the damper equal to the natural frequency of the component.**
    - **Use a spreadsheet to generate plots of component amplitude as a function of frequency and damper damping magnitude (c_2).**
- **Note that tuned mass dampers work well at specific frequencies, but the structural loop changes with cutting loads.**
- **Example: Portion of a spreadsheet for the design of a tuned mass damper design:**

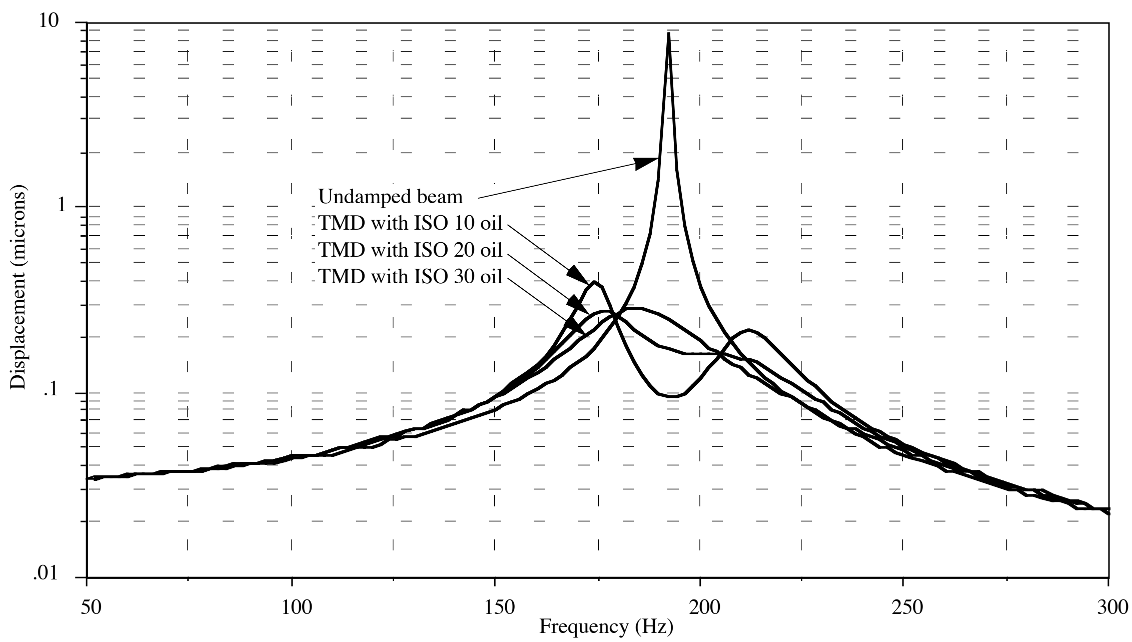
TMDdes.xls
Tuned mass damper design for a cantilever beam
Written by Alex Slocum. Last modified 9/3/95 by Alex Slocum
Enter numbers in bold

Cantilever beam characteristics (input)		
Modulus (N/m^2)	**2.07E+11**	
Density (Kg/m^3)	**7800**	
Length (m)	**0.25**	
The beam to be damped:		
For a circular beam	**TRUE**	
Outside Diameter (m)	**0.1**	
Inside Diameter (m)	**0.05**	
For a rectangular beam		
Height (m)	**0.2**	
Width (m)	**0.5**	
Max. static deflection (microns)	**0.25**	
Applied dynamic force (N)	**25**	
Beam mass	11.49	
Added mass on the end	**5**	
Calculated beam properties		
Area A (m^2)	5.89E-03	
Inertia I	4.60E-06	
Est. first natural frequency (rad/s, Hz)	3606	574
Stiffness (N/m, N/micron)	182899597	182.90
Equivelant mass (kg)	14	
Static deflection (microns)	0.14	
Damping coefficient (for steel) (N/(m/s))	161.47	
Cylindrical damper characteristics (input)		
Cylinder diameter (m)	**0.035**	
Cylinder length (m)	**0.06**	
Outer core density	**7800**	
Inner core density	**7800**	
% size of core (IDcore = %OD)	**0.75**	
Damper fluid viscosity (N-sec/m^2)	**10**	
Bore radial clearance (microns)	**10**	
Number of damping cylinders	**1**	
maximum dynamic displacement (μm)	0.511	
Calculated damper properties		
Damper mass (kg)	0.62	
Damper damping (N/(m/s))	6597	
Damper stiffness (N/m)	8048991	
Unit spring stiffness (N/m, lbf/in)	4024496	22997
Maximum damper displacement (microns)	0.54	

- **Simulated response of the above-modeled damper:**



- **Simulated response of an 80-mm diameter, 400-mm-long steel cantilever beam equipped with a tuned mass damper.**



- **Cross section of a tuned mass damper design for an 80-mm diameter, 400-mm-long steel cantilever beam.**

Threaded plu 16 Fill with ISO 20

1 micron radial clearan

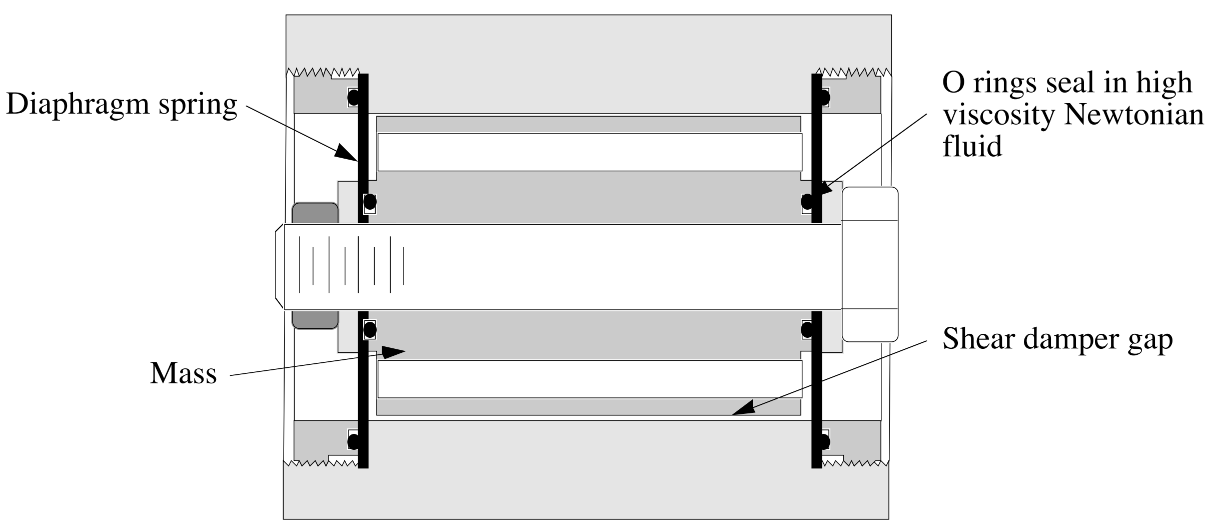
190,000 N/m stiffness disk washer. Washers must be preloaded.

49 31

Hous

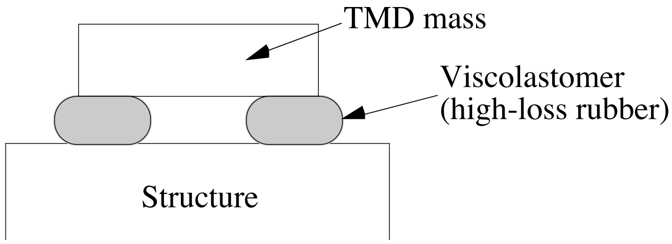
30

- **There are many different tuned mass damper designs:**

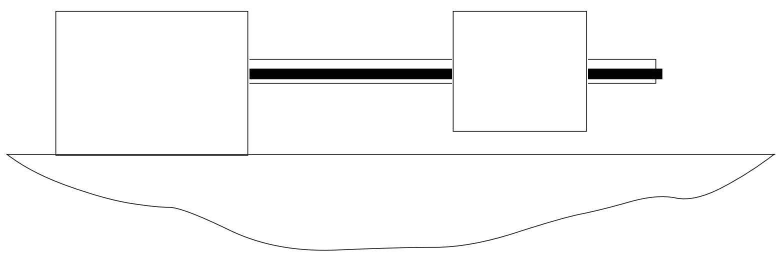


Low cost tuned mass damper

- **The spring and the damper can be combined in a high-loss elastomer:**

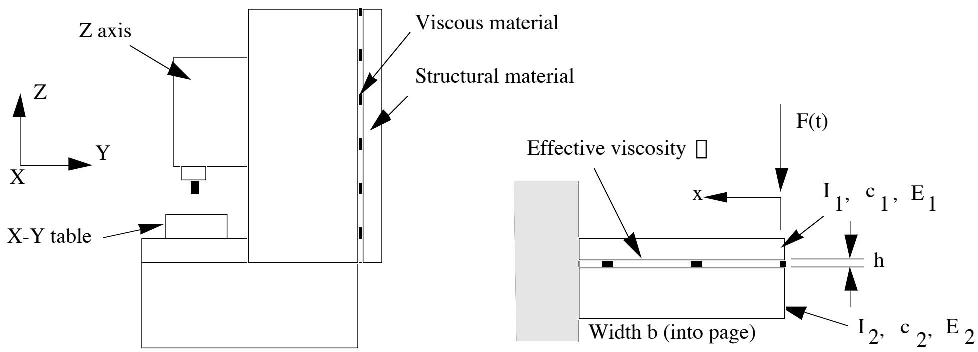


- **One can also use a mass on a beam damped with a constrained layer damper, and even make it adjustable:**



Constrained Layer Dampers

- **Damping is achieved by viscoelastic shear.**
- **Viscoelastic shear dampers work well at all frequencies and can be analytically modeled.**



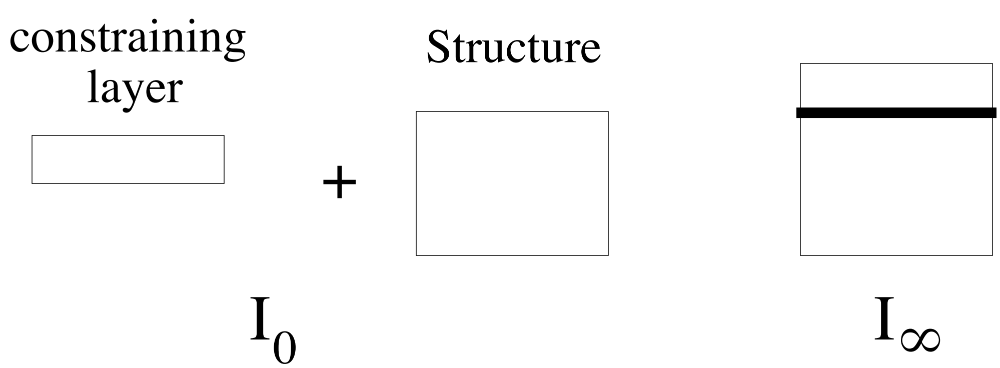
- **Delamination can occur with time unless surfaces and adhesives are very carefully prepared.**
    - **Exterior surfaces subject to impact etc. are not viable candidates.**
    - **Replication can be used to obtain smooth flat surfaces for the damping mechanism.**
- **Machine tool structures often have other components mounted to their surfaces.**
- **Cast iron (and rough surface) structures require the layers to be epoxied together.**

Design theory[4]

- **Motion of a structure is greatest far from the neutral axis.**
- **Consider dynamic stiffness of other parts of the structure. A balanced design must be obtained!**
- **Determine the moments of inertia of the system:**
- **Structure by itself: (EI_s) about its own respective neutral axis.**
- **Constraining layers (EI_{cli}) about their own respective neutral axes.**
- **The system as if the constrained layer had an infinite modulus: (I_∞) about the system nuetral axis.**
- **The system as if the constrained layer had a zero modulus:** $\left(\mathbf{I_o = I_{structure} + \sum_{i=1}^{N} I_{cli}}\right)$ **about the system nuetral axis.**
- **The maximum damping that can be obtained with this system (given the ideal damping material) is:**

$$Q_{max} = \frac{1}{\eta_{\text{effective for the system}}}$$

$$\eta_{max} = \frac{\left(\frac{I_\infty}{I_0} - 1\right)}{4}$$



4 This theory was developed by Layton Hale. It builds upon work done by Eric Marsh as part of his Ph.D. thesis.

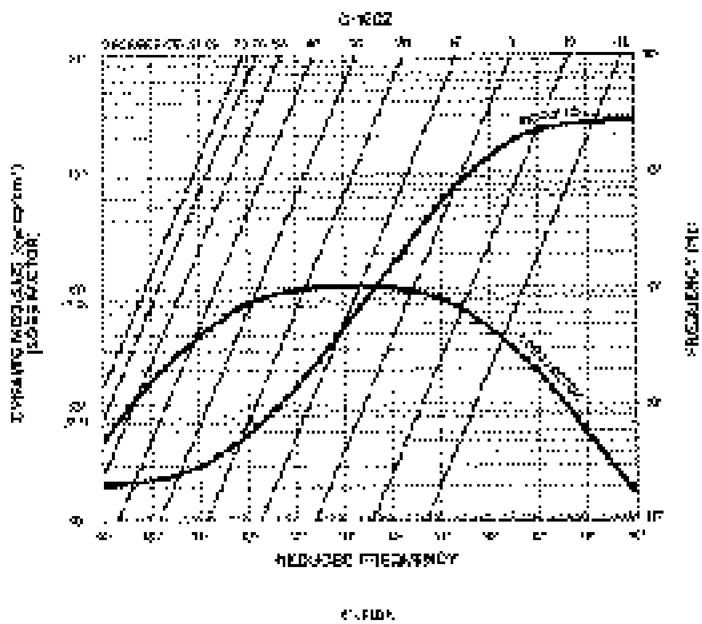
- **Calculate the stiffness ratio:**

$$\mathbf{r} = \frac{\mathbf{EI}_{\infty} - \mathbf{EI}_0}{\mathbf{EI}_0}$$

- **Calculate the optimal damping parameter to maximize dynamic stiffness:**

$$\square_{\mathbf{optimal}} = \frac{1}{\sqrt{\left(1 + \square^2\right)}}$$

- **Properties of a typical viscoelastic damping material:**



- **Calculate optimal damping layer thickness**

$$\mathbf{h_{optimal}} = \frac{\mathbf{G_{damping\ material}} \sum_{w=1}^{\mathbf{N}} \mathbf{w_i c_i^2}}{\mathbf{\eta_{optimal} E_{structure}}\left(\mathbf{I_\infty} - \mathbf{I_0}\right)} \mathbf{L_{eff}}^2$$

- **The constraining layer should be attached at the point of zero shear in the beam.**
- **The effective length is thus dependent on the beam mounting:**

End Condition	Zero Shear location	L_{eff}
Fixed-free	Fixed end	0.613L
	Free end	0.314L
	0.4L	0.229L
Pinned-Pinned	Center	0.318L
Fixed-Fixed	Center	0.160L
Free-Free	Center	0.314L

- **Calculate the dynamic compliance ratio:**

$$Q = \frac{1 + (2+r)\square + (1 + r)\square^2(1 + \square^2)}{\square r \square}$$

- **Often, the damping material is only available in incremental thicknesses, so calculate □ and then calculate Q:**

$$\square = \frac{G_{\text{damping material}} \sum_{i=1}^{N} \frac{w_i c_i^2}{h_i}}{E_{\text{structure}}\left(I_\infty - I_0\right)} L_{\text{eff}}^{\;2}$$

- **The stiffness and damping factor of the constrained layer (damping material) as a function of frequency.**

- **This theory is a starting point only. The final design should be checked using FEA.**

Spreadsheet CLDdes.XLS for the design of constrained layer dampers

- **The primary issue is that the damping material data (modulus in particular) is frequency dependent.**

CLDdes.XLS
To design constrained layer dampers for a rectangular beam with plate CLDs
Written by Alex Slocum. Theory by Layton Hale. Last modified 12/19/95 by AS
Only change cells with boldface numbers.

Structural beam

Outside height (m)	**0.01**
Outside width (m)	**0.025**
Inside height (m)	**0**
Inside width (m)	**0**
Length (m) [L]	**0.25**
Modulus of elasticity (Pa) E	**2.07E+11**
Moment of inertia (m^4)	2.08E-09
Cross section area (m^2)	2.50E-04
Distance: structure nuetral axis and I∞ nuetral axis (m)	0.0051

Beam constraints

Cantilever [cant]	FALSE	
Simply supported [simple]	TRUE	
Free-free [free]	FALSE	
Cantilever-simple [other]	FALSE	
Fundemental mode shape [mode]		0.1013

Viscoelastic damping layer properties

Re(Gv) (Elastic storage shear modulus) Gv	**9.00E+05**
Loss factor n [eta]	**1**
Optimal thickness (calculated below) (mm)	0.02
Desired thickness to use (available damping tape thickness) (mm) [htape]	**0.125**

Top surface constraining layer (may be 0)	
Height (m)	**0**
Width (m)	**0.025**
Width constraining layer covers (m) [wt]	**0.025**
Moment of inertia (m^4)	0.00E+00
Cross section area (m^2)	0.00E+00
Distance: constraining layer and structure's nuetral axes (m) [ct]	0.0051
Distance: constraining layer nuetral axis and I∞ nuetral axis (m)	0.0102
Bottom surface constraining layer (must exist)	
Height (m)	**0.01**
Width (m)	**0.025**
Width constraining layer covers (m) [wb]	**0.025**
Moment of inertia (m^4)	2.08E-09
Cross section area (m^2)	2.50E-04
Distance: constraining layer and structure's nuetral axes (m) [cb]	0.0101
Distance: constraining layer nuetral axis and I∞ nuetral axis (m)	0.0051
System cross section properties	
Location of I∞ nuetral axis from bottom of bottom constraining layer	0.0101
I∞ (m^4) [Iinfinity]	1.70E-08
Io (m^4) [Io]	4.17E-09
Stiffness factor r = (I∞/Io-1) [rr]	3.075
Damping calculations	
Minimum Q with the viscoelastic material selected & hopt	***3.5***
Theoretical minimum possible Q	***1.3***
Q obtained wih the damping tape thickness [htape] available	***7.7***
Kviscolayer/kconstraining layer = alpha optimal [alphaopt]	0.350
Optimal damping layer thickness (mm) [hopt]	***0.02***
alpha for given damping tape thickness htape [alpha]	0.055

How much damping and how much static stiffness?

Qinitial 20

Structure h	Damper h	Q	Kstatic	Kdynamic
1.0	0.0	20.00	1.000	0.050
0.9	0.1	7.02	0.730	0.104
0.8	0.2	3.56	0.520	0.146
0.7	0.3	2.10	0.370	0.176
0.6	0.4	1.44	0.280	0.194
0.5	**0.5**	**1.25**	**0.250**	**0.200**
0.4	0.6	1.44	0.280	0.194
0.3	0.7	2.10	0.370	0.176
0.2	0.8	3.56	0.520	0.146
0.1	0.9	7.02	0.730	0.104

- **The error budget should tell you how much static stiffness you need, then match the static and dynamic stidffness.**

Predicted responses of damped free-free[5] beams

For a 25 mm wide, 250 mm long, cold rolled steel beam:

	Plain steel beam	3M IsoDamp 112	EAR C1002	Soundcoat GP3
Beam height (mm)	20	10	10	10
Constrained layer height (mm)	NA	.125	1.5	1.5
Constraining layer height (mm)	NA	10	10	10
Beam or damping material modulus (Pa)	$E=2.0 \times 10^{11}$	$G=9.0 \times 10^{5}$	$G=1.0 \times 10^{7}$	$G=2.2 \times 10^{7}$
FEA ω_1 (Hz)	1678	902	908	971
Measured ω_1 (Hz)[6]	1638	902	910	970
FEA Q (includes material damping)	56	5.9	5.2	3.3
Measured Q	65	6.3	5.1	3.1
CLDdes.XLS Spreadsheet predicted Q (does not include material damping)	NA	7.8	6.4	4.0

5 In order to obtain the Q, the free-free case is modeled as a beam supported at its minimum deflection points (0.225L from the ends) by very soft springs. This results in less than a 0.1% difference in natural frequency calculations (1678 Hz vs 1676.6 Hz).

6 The predicted first natural frequency using fundemental theory is 1666 Hz

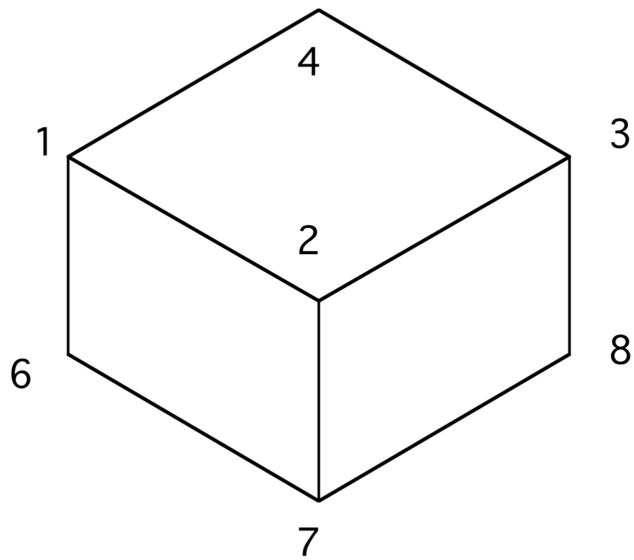
Predicted responses of damped simply supported beams

For a 25 mm wide, 250 mm long, cold rolled steel beam:

	Plain steel beam	3M IsoDamp 112	EAR C1002	Soundcoat GP3
Beam height (mm)	20	10	10	10
Constrained layer height (mm)	0	.125	1.5	1.5
Constraining layer height (mm)	0	10	10	10
Beam or damping material modulus (Pa)	E=2.0×10^{11}	G=9.0×10^{5}	G=1.0×10^{7}	G=2.2×10^{7}
Material □	0.015	1	1	1
FEA □$_1$ (Hz)	730	331	375	421
ANSYS FEA SOLID45's BEATAD = □/2□□ steel/damping	3.27×10^{-6}	4.26×10^{-4} 6.38×10^{-6}	4.24×10^{-4} 6.37×10^{-6}	3.87×10^{-4} 5.81×10^{-6}
FEA □$_{static}$ (m)	9.8×10^{-8}	3.7×10^{-7}	3.6×10^{-7}	3.0×10^{-7}
FEA □$_{dynamic}$ (m)	6.4×10^{-6}	2.1×10^{-6}	1.8×10^{-6}	1.1×10^{-6}
FEA Q (includes material damping)	65	5.7	5.0	3.7
Measured Q		6.3	4.8	3.4
CLDdes.XLS Spreadsheet predicted Q (does not include material damping)	NA	7.7	6.3	3.9

Using finite element models to design damped systems

- **Now consider the finite element method, which will be required for analysis of more complex structures.**
- **A designer should be able to digitally design different structures with different damping treatments to compare performance.**
    - **Using FEA, you can model a beam as an 8 node solid element (e.g., SOLID45 element in Ansys™) with 3 DoF per node.**



    - **Most machine tool structures can be modeled with these elements.**
- **For the beam tested above, going from one 8 node element for the cross-section to 4 elements, to 16 elements changes the results by about 3%**
    - **The computed first frequency is 1672 Hz.**
- **The theory, experiment, and FEA results match very well!**

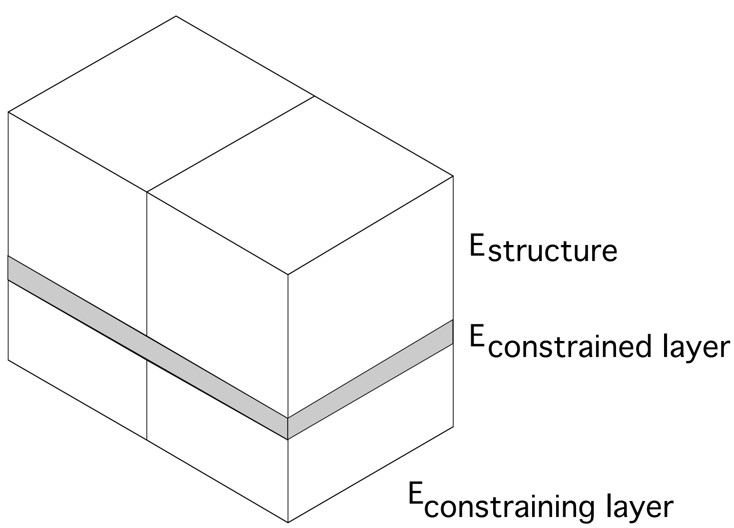
How can FEA be used to compute the damped vibration response of a structure?

- **Bolted joints can only be modeled as “solid” if the bolt stress cones overlap!**
- **“Bulk” machine damping factor, η, can be applied to all structural elements.**
- **ANSYS’ SOLIDS45 element, for example, allows you to input a value for the damping (DAMP = η)**
    - **This is based on a certain amount of damping occurring in each mode, so it is linked to the stiffness matrix.**
    - **The damping is the Imaginary part of the response, so it is given by η in the response of a classic second order system:**

$$m\ddot{x}(t) + k(1 + i\eta)x(t) = Ake^{i\omega t}$$

    - **In terms of the modal damping coefficient ζ (e.g., from the modal analyses or material property data, remember, f is in Hz!):**

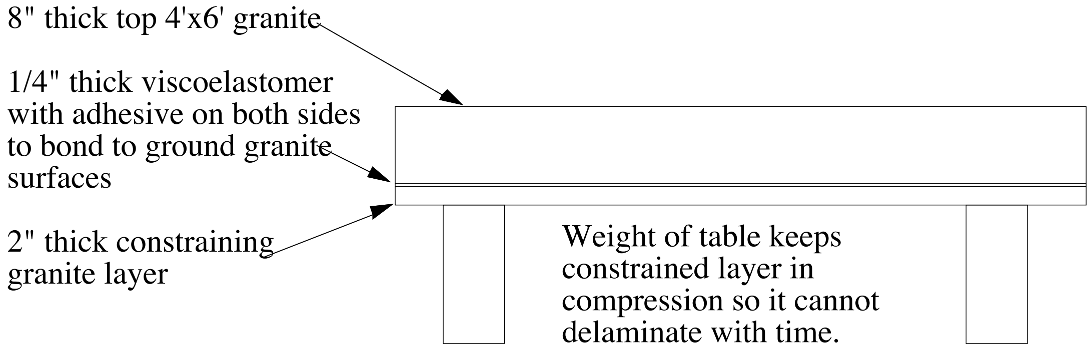
$$\eta = \frac{\zeta}{2\pi f}$$



General procedure

- **Design the structure for the desired static stiffness**
    - **Deflections due to gravity and axis force loads should be within budgeted values.**
- **Use the spreadsheet to design a constrained layer damper.**
- **Use FEA to determine the dynamic performance of the machine:**
    - **DAMP of the structure's material set to that of a typical similar machine (obtained for example from the Log Decrement).**
- **Use FEA to determine the dynamic performance of the machine with the constrained layer damper.**
- **Use FEA to determine the dynamic performance of the machine were the constrained layer damper material is removed, and the constraining layer is added directly to the structure.**
- **This will determine the most efficient use of added material to the system.**

- **Example: Bonding a bottom damping plate to a structure (e.g., a surface plate) allows the weight of the structure to make sure it will not delaminate:**

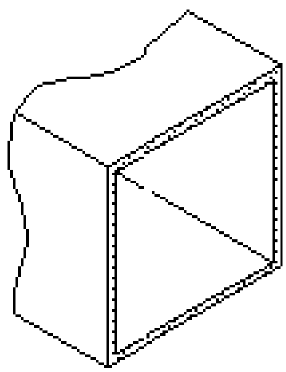


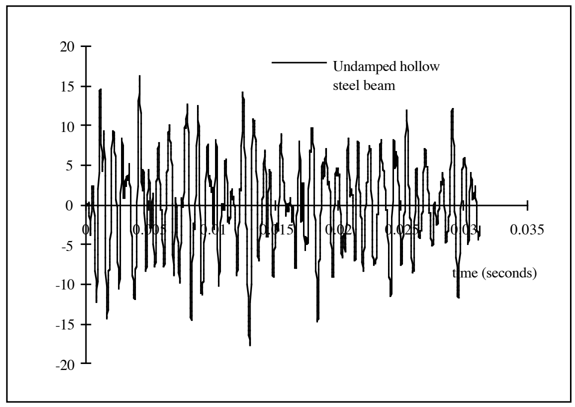
- **Example: A damper plate on the bottom of a machine tool base:**
  - **EI_o = 1.7 x 10^9 N·m^2**
  - **I = 2.5 x 10^9 N·m^2**
  - **η_b = 0.17 Q = 6**
  - **ω = 380 Hz**
  - **At 20 ^{o}C, properties of the material are G = 12 MPa and η = 1.**
  - **70% of the ribbed cast iron machine base is in contact with the damper plate.**
  - **Solve for optimum damper material thickness h to be 2.2 mm.**

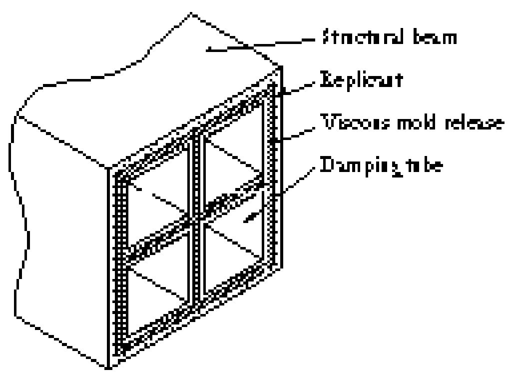
- **If the machine structure is ribbed, you may decrease a bending more, but miss a local plate mode that may be the dominant error source.**

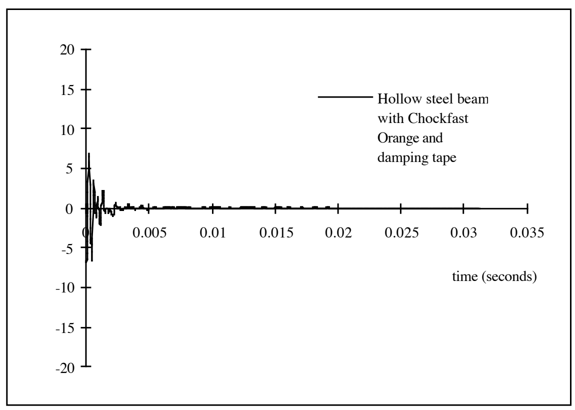
Replicated Internal ShearTube™ ShearDampers™[7]

- **An internal constrained layer damper, that will also damp local surface modes in a structure.**
- **A ShearTube damper can highly damp a structure without imposing strict limits on the structure's geometry or materials.**
- **The ShearTube damper can decrease the amplification at resonance of a metal beam from 500 to 10.**









7 Patents pending. For more information, contact Richard Slocum, Aesop Inc., 200 Forest Trail, Nicholasville, KY 40356-9150 Phone (606) 224 4140, fax (606) 224-8080, email slocuminky@aol.com

- **A ShearTube™ damper is incorporated into a structure in the following manner:**

  1. **The structure has rough holes formed in it (casting, drilling, or welding in a pipe).**

     - **The holes may be any shape, but should maximize the cross-section perimeter (e.g., a square).**

     - **The neutral axes of the holes must be as far away as possible from the neutral axis of the structure.**

     - **Ideally, the holes almost fill the structure (e.g., four squares inside of a large square beam.**

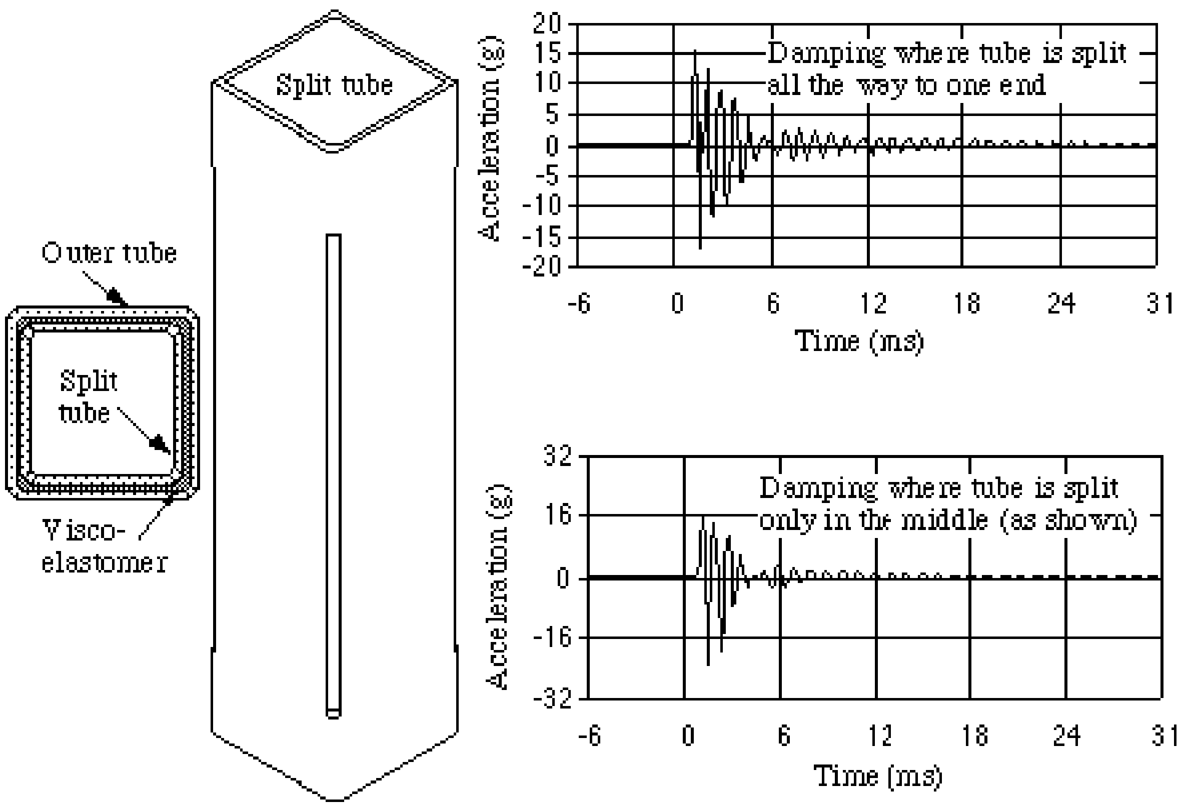
  2. **Modestly smooth-surfaced tubes (0.5 mm R_a) that are 3-5 mm smaller than the hole are covered with a high loss damping material (e.g., 3M Scotchdamp™, Soundcoat GP3, or EAR C1002)**

  3. **The tubes are suspended into the hole, and an epoxy replicant (e.g., Vibradamp™ from Philadelphia Resins) or grout is injected (poured) around the tubes.**

  4. **After the epoxy hardens, the component is ready to be used.**

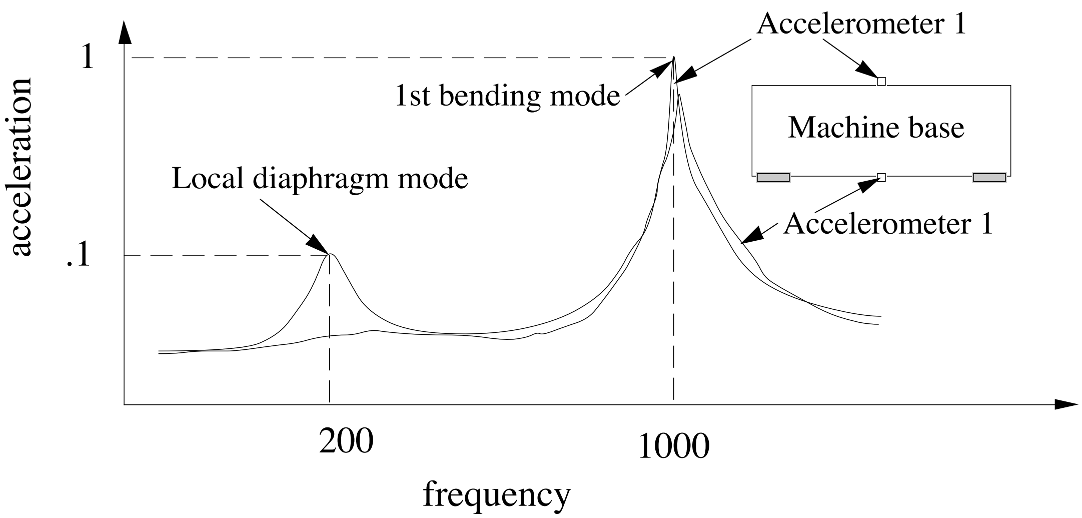
  5. **To achieve precise temperature control of the machine, the ShearTubes™ are used like heat exchanger tubes to channel temperature controlled fluid inside the machine.**

- **Instead of using multiple shear tubes (not practical for smaller structures such as CMM beams and tooling :**
  - **A single inner concentric tube can be used, which has been slit through its neutral axes to within one diameter of each end.**
  - **This can be referred to as a *Split Sheartube* ™[8] which typically has 50% greater damping, and 50% greater static stiffness.**
  - **The minimum dynamic stiffness of the system where the inner damping tube is slit all the way to one end is 20.3 N/μm.**
  - **On the other hand, the minimum stiffness of the system where the damping tube is slit only in its center portion is 38.3 N/μm!**



8 ibid.

- **Frequency Response Functions may show that a machine base's first mode could be damped using a constrained layer damper on the bottom of the machine.**
    - **Local diaphragm modes could still cause dominant error motions near the surface where the part is:**
    - **displacement is proportional to acceleration/frequency^2**



- **A ShearTube damper or filling the base with concrete will also damp these local diaphragm modes.**
- **The same effect can occur in columns, which are generally weight sensitive, so ShearTube dampers are more effective.**

- **ShearDampers and Hydroguide water hydrostatic bearings can be used to greatly increase part finish and accuracy.**