Purpose:

From the flip-top on a shampoo bottle to the piezo-beams on a scanning tunneling microscope, flexural bearings are one of the most widely used types of bearings. However, they are one of the least well-understood bearings by most engineers. This lecture introduces the concept of flexural bearings and their characteristics.

Outline:

- Introduction
- General characteristics
- Types of flexures

"Genius can only breathe in an atmosphere of freedom"

John Stuart Mill
Introduction

- Monolithic designs:
  
  ![Diagram of monolithic designs]

  - Most accurate.
  - Sometimes difficult to machine:
    - Abrasive water jet machining is an economical way to cut non-critical areas (links).
    - Wire EDM can be used for precision cuts in critical hinge areas.
    - May require localized heat treating of the material in the flexure.
    - Applications include flexural couplings, mirror mounts, STM's, and many others.
  
  - Size is about 20 times the range of motion
• Clamped designs:

![Diagram of clamped design]

• Nanometer accuracy and better can be obtained if the bolted joint is properly designed.
  • Rounded edges
  • bolt cones-of-action overlap
  • Easy to assemble from annealed parts and hardened spring steel.
  • Careful when tightening bolts, as the torque can twist the flexure!
  • Lubricated thrust washers can help.
  • Care is required to make sure residual stresses don't create asymmetries that create parasitic error motions.
  • Cut spring metal with EDM or waterjet.
  • Use dual lubricated washers under bolt heads and guides for clamp plates.
  • Applications include wafer steppers, mirror mounts, and many others.
  • Size is about 10 times the range of motion
Speed and acceleration limits
- Limited only by yield strength and design.

Range of motion
- Typically used for motions less than a few millimeters.
- Monolithic designs: flexure length/motion = 20 or more.
- Clamped designs: flexure length/motion = 5-10

Applied loads
- Design goal is to obtain load capacity with minimum spring constant.

Repeatability
- Axial: limited only by the drive system.
- Lateral: less than nanometers for monolithic designs.

Resolution
- Axial: limited only by the drive system.
Preload

- Inherently preloaded.

Stiffness

- The greater the motion and the lower the spring rate, the less the stiffness.
- Theory of elasticity or finite element analysis yields very accurate predictions of performance.
- Flexures often have low transverse stiffness, so they are more susceptible to parasitic forces!

Vibration and shock resistance

- Very good.

Damping capability

- Material damping only (2-5%).
- Damping mechanisms (constrained layer dampers) can be used to obtain high damping.
Accuracy

- Axial: limited only by the drive system.
- Lateral: can be less than nanometers for monolithic designs.
- Depends on how well the bearing was assembled or machined.
- Even if there is a small off-axis error motion associated with the primary motion:
  - The error motion is usually very predictable and highly repeatable.
- Flexural bearings cannot attain perfect motion because of:
  - Variation in spring strength
    - Elastic modulus varies with rolling direction in steels.
  - Variation in spring geometry.
  - Overall inaccuracies of manufacture.
  - Bending of the bearing in an unintended manner.
  - Bending of structure.
  - External applied loads (e.g., gravity and the manner in which the actuation force is applied)
• The most common errors are the pitch angle and vertical motion.
  • They accompany linear motion in a four bar linkage flexure.
  • For small displacements, the errors are a function of:

\[
\begin{align*}
\theta_{\text{pitch}} &= \left( \frac{6(\ell - 2a) t^2}{3b^2 \ell - 2t^2 \ell + 6at^2} \right) \frac{x}{\ell} \\
\delta_{\text{vertical}} &= \frac{x^2}{2\ell}
\end{align*}
\]

• Note that there is no pitch error when \( a = \ell/2 \)
If the force is applied at a point other than halfway between the platforms:

- A bending moment is generated which causes a pitch error to occur.

The pitch errors caused by the difference in spring length and difference in platform length are respectively:

\[ \theta_{\text{spring}} = \frac{\delta_{\text{spring}} x^2}{2 \ell^2 b} \]

\[ \theta_{\text{platform}} = \frac{\delta_{\text{platform}} x}{\ell b} \]

Proven by Jones who gives typical tolerances on the spring and platform length to be 25 \( \mu \)m and 1-3 \( \mu \)m respectively.
Types of flexures

- Four bar linkage flexure:

  ![Diagram of four bar linkage flexure]

  - In the Y direction there is a small error motion $\delta_Y$ that is a function of the X motion $\delta_X$ and flexure length $\ell$:

    $$\delta_Y \approx \frac{\delta_X^2}{2\ell}$$

  - Consider other parasitic motions mentioned earlier.

  - X direction stiffness is equal to two fixed-fixed beams acting together in a side by side mode.

  - Y direction stiffness is equal to that of two columns.

  - Buckling effect is negligible for small motions and should be prevented by the X direction servo.

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1 Modular flexural bearing components can be purchased: Alson E. Hatheway Inc., Suite 400, 595 East Colorado Blvd., Pasadena, California 91101.
• To minimize parasitic motions:
  • Symmetrical dual four bar linkage eliminates $\delta_Y$ error.
  • Strains along flexure's length resisted by the frame:
    • Stiffness is increased: range of motion is decreased:

\[
\delta = \frac{FL^3}{192EI}
\]

• Use a two stage (stacked) four bar linkage:

• The parasitic motions of one 4-bar cancel the parasitic motions of the second 4-bar.

• Lateral and yaw stiffness will not be high, however, due to buckling effect.

• Overcome by use of monolithic hourglass-type members.
• A differential actuation system can be used to increase resolution.

• A wobble pin will minimize parasitic force actuation errors:

![Diagram of differential actuation system]

- Diameter D1 and lead L1
- Diameter D2 and lead L2
- Wobble pin (ideally rests in three ball clusters)

• Alternate differential design:

![Diagram of alternate differential design]

- Differential moving platform
- Fixed platform
- Wobble pin

• Left and right platforms would be connected to a common rigid top-plate.
• When the flexure is built in to "walls", it needs extensional capability: (after F. Zhu at Delft):
Many thin blades can be used to increase lateral stiffness and load capacity while keeping axial stiffness and stress low:

\[
F_{\text{load capacity}} \approx Ntw
\]

\[
K_{\text{vertical}} = \frac{NtwE}{L}
\]

\[
K_{\text{axial multiblade}} \approx \frac{L^3}{ENt^3w}
\]

\[
K_{\text{axial monolithic}} \approx \frac{L^3}{E(Nt)^3w}
\]

- Be careful of alignment and slip problems between the blades.
- EDM process can be used to prevent these problems.
- Rubber placed between the blades can increase damping.
- Careful when tightening bolts, as the torque can twist the flexure!
• Lever and fulcrum transmission:

![Diagram of lever and fulcrum transmission]

• A bowed flexure used as a high reduction transmission:

![Diagram of bowed flexure transmission]

• A downward motion \( \delta \) causes a lateral motion \( \Delta \):

\[
\Delta = \frac{4\delta \ell_h}{\ell_w}
\]

• It is possible to chain together a series of these bowed flexures at right angles to each other.

• This can yield a very high transmission ratio.

• Assume that \( \ell_h = 2 \text{ mm} \) and \( \ell_w = 20 \text{ mm} \), then the transmission ratio is 5.

• In series, the ratios become 25, 125, 625... for 2, 3, 4... units respectively.

• A variation to this bowed beam approach is to use two beams laid on top of each other and tied together at one end.

• When the beams bend there will be differential motion along the interface between them.
Flexures that provide angular motion

- **Device for adjusting pitch and yaw** (Courtesy of Polaroid Corp.):

- Note the transmission effect, where the screws push up on the upper beam, using the lower beam as a base.

- If the top beam were 1/2” thick, and the bottom beam were 1/8” thick, then the transmission ration would be \( (((1/2)/(1/8))^3 = 64! \)

- The slope at the end of the beam causes an Abbe error in the beam that holds the optic which cancels the beam deflection \( \delta = \alpha(2/3L) : \)

\[
\delta = \frac{FL^3}{3EI} \\
\alpha = \frac{FL^2}{2EI}
\]
• Cross-strip flexure for angular motion:

- Lucas flexural pivots (Courtesy of Lucas Aerospace Power Transmission Corporation):

  - For limited motion applications use in place of rolling element bearings.
  - Flexural pivots are not subject to fretting problems.
Flexures to guide and lock components into place

- Flexures can be used as one-shot hinges to guide components into alignment and then lock them into place:

![Diagram of anvil and flexural locking member](image)

- X direction stiffness of the clamping system is equal to:
  - The sum of the X direction stiffness of the anvil and locking member due to the preload effect.

- Small taper keeps component preloaded in both the X and Y directions.

- To minimize distortion caused by bending, line of force must pass through the support ledge.

- A lens could be kinematically held by two anvils and a single locking member.

- The geometry is tolerant of manufacturing errors.
• For precise X position location, utilize a grooved structure:

![Diagram showing X and Y axes, anvil, flexural locking member, and preload forces.]

• Numerous permutations are possible.

• The moral of the story is don't do this:
- Flexures allow for snap assembly of components which make the design of systems more efficient:

(From US Patent 5,102,235 by Peter Mugglestone of Thomson Industries Inc.)

- End caps click together.
- Seal is a single molded piece.
- No screws required.
• Flexures for fixturing: Opposed L’s support a platform so it can move in six degrees of freedom\(^2\).

• This allows chips held in the platform to be operated on (tested in this case).

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\(^2\) Patent Pending, Kinetrix, Inc. 33 Constitution Drive, Bedford, MH 03110-6000. Contact Dr. Alexander Slocum.