

Precision Machine Design

Topic 17

Flexures

Purpose:

From the flip-top on a shampoo bottle to the piezo-beams on a scanning tunneling microscope, flexural bearings are one of the most widely used types of bearings. However, they are one of the least well-understood bearings by most engineers. This lecture introduces the concept of flexural bearings and their characteristics.

Outline:

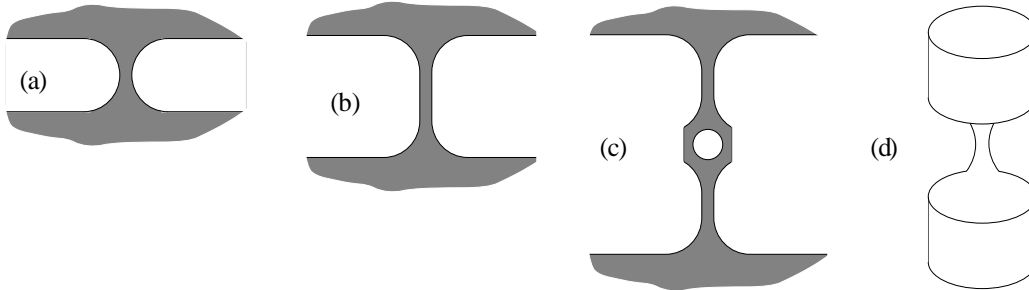
- Introduction
- General characteristics
- Types of flexures

"Genius can only breathe in an atmosphere of freedom"

John Stuart Mill

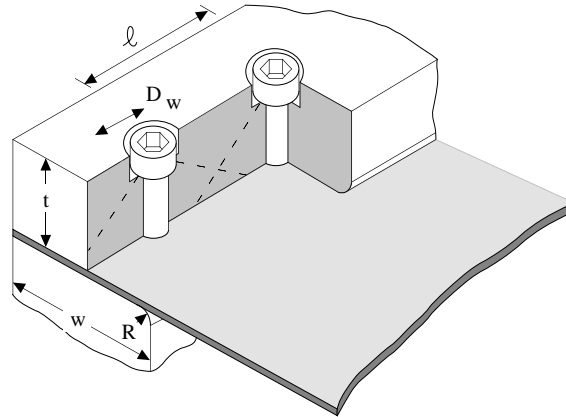
Introduction

- **Monolithic designs:**



- **Most accurate.**
- **Sometimes difficult to machine:**
 - **Abrasive water jet machining is an economical way to cut non-critical areas (links).**
 - **Wire EDM can be used for precision cuts in critical hinge areas.**
- **May require localized heat treating of the material in the flexure.**
- **Applications include flexural couplings, mirror mounts, STM's, and many others.**
- **Size is about 20 times the range of motion**

- **Clamped designs:**



- **Nanometer accuracy and better can be obtained if the bolted joint is properly designed.**
 - **Rounded edges**
 - **bolt cones-of-action overlap**
- **Easy to assemble from annealed parts and hardened spring steel.**
 - **Careful when tightening bolts, as the torque can twist the flexure!**
 - **Lubricated thrust washers can help.**
- **Care is required to make sure residual stresses don't create asymmetries that create parasitic error motions.**
 - **Cut spring metal with EDM or waterjet.**
 - **Use dual lubricated washers under bolt heads and guides for clamp plates.**
- **Applications include wafer steppers, mirror mounts, and many others.**
- **Size is about 10 times the range of motion**

Speed and acceleration limits

- **Limited only by yield strength and design.**

Range of motion

- **Typically used for motions less than a few millimeters.**
- **Monolithic designs: flexure length/motion = 20 or more.**
- **Clamped designs: flexure length/motion = 5-10**

Applied loads

- **Design goal is to obtain load capacity with minimum spring constant.**

Repeatability

- **Axial: limited only by the drive system.**
- **Lateral: less than nanometers for monolithic designs.**

Resolution

- **Axial: limited only by the drive system.**

Preload

- **Inherently preloaded.**

Stiffness

- **The greater the motion and the lower the spring rate, the less the stiffness.**
- **Theory of elasticity or finite element analysis yields very accurate predictions of performance.**
- **Flexures often have low transverse stiffness, so they are more susceptible to parasitic forces!**

Vibration and shock resistance

- **Very good.**

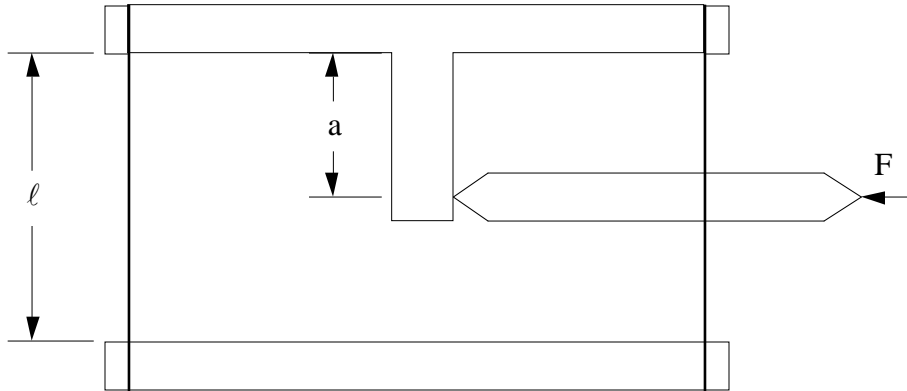
Damping capability

- **Material damping only (2-5%).**
- **Damping mechanisms (constrained layer dampers) can be used to obtain high damping.**

Accuracy

- **Axial: limited only by the drive system.**
- **Lateral: can be less than nanometers for monolithic designs.**
- **Depends on how well the bearing was assembled or machined.**
- **Even if there is a small off-axis error motion associated with the primary motion:**
 - **The error motion is usually very predictable and highly repeatable.**
- **Flexural bearings cannot attain perfect motion because of:**
 - **Variation in spring strength**
 - **Elastic modulus varies with rolling direction in steels.**
 - **Variation in spring geometry.**
 - **Overall inaccuracies of manufacture.**
 - **Bending of the bearing in an unintended manner.**
 - **Bending of structure.**
 - **External applied loads (e.g., gravity and the manner in which the actuation force is applied)**

- The most common errors are the pitch angle and vertical motion.
 - They accompany linear motion in a four bar linkage flexure.
- For small displacements, the errors are a function of:



- The distance moved x .
- The length of the springs ℓ .
- The spring thickness t .
- The platform length b .
- The distance of force application a above the fixed end of the springs

$$\theta_{\text{pitch}} = \left(\frac{6(\ell - 2a) t^2}{3b^2\ell - 2t^2\ell + 6at^2} \right) \left(\frac{x}{\ell} \right)$$

$$\delta_{\text{vertical}} \approx \frac{x^2}{2\ell}$$

- Note that there is no pitch error when $a = \ell/2$

- **If the force is applied at a point other than halfway between the platforms:**
 - **A bending moment is generated which causes a pitch error to occur.**
- **The pitch errors caused by the difference in spring length and difference in platform length are respectively:**

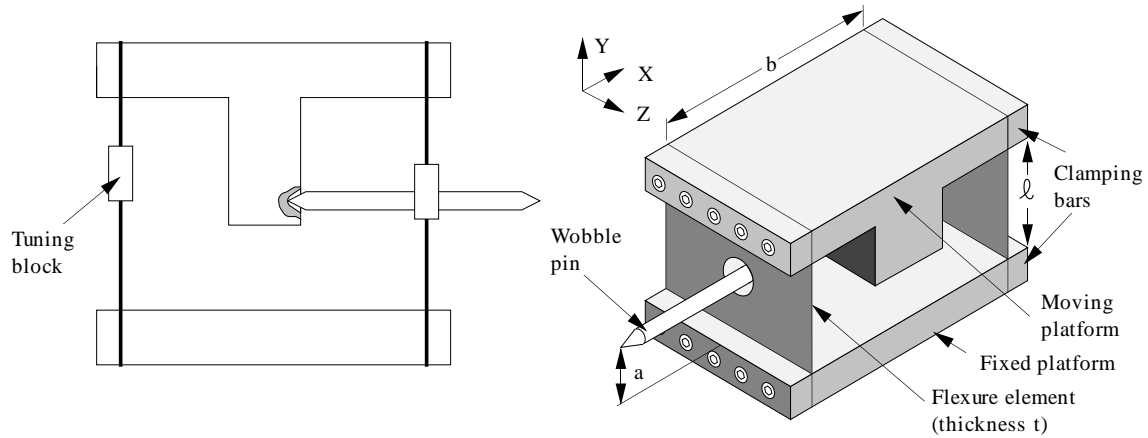
$$\theta_{\text{spring}} = \frac{\delta_{\text{spring}} x^2}{2\ell^2 b}$$

$$\theta_{\text{platform}} = \frac{\delta_{\text{platform}} x}{\ell b}$$

- **Proven by Jones who gives typical tolerances on the spring and platform length to be 25 μm and 1-3 μm respectively.**

Types of flexures¹

- **Four bar linkage flexure:**



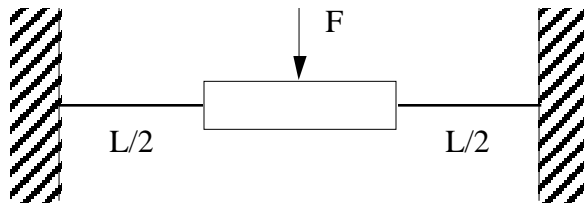
- **In the Y direction there is a small error motion δY that is a function of the X motion δX and flexure length l :**

$$\delta_y \approx \frac{\delta_x^2}{2l}$$

- **Consider other parasitic motions mentioned earlier.**
- **X direction stiffness is equal to two fixed-fixed beams acting together in a side by side mode.**
- **Y direction stiffness is equal to that of two columns.**
- **Buckling effect is negligible for small motions and should be prevented by the X direction servo.**

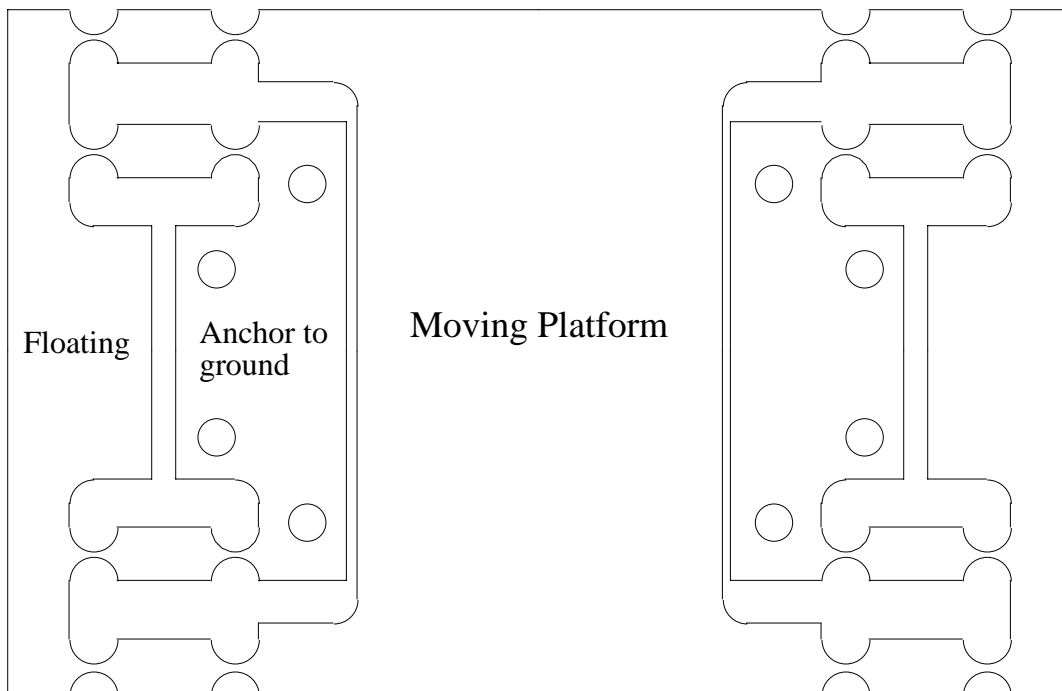
¹ Modular flexural bearing components can be purchased: Alson E. Hatheway Inc., Suite 400, 595 East Colorado Blvd., Pasadena, California 91101.

- **To minimize parasitic motions:**
 - **Symmetrical dual four bar linkage eliminates δY error.**
 - **Strains along flexure's length resisted by the frame:**
 - **Stiffness is increased: range of motion is decreased:**



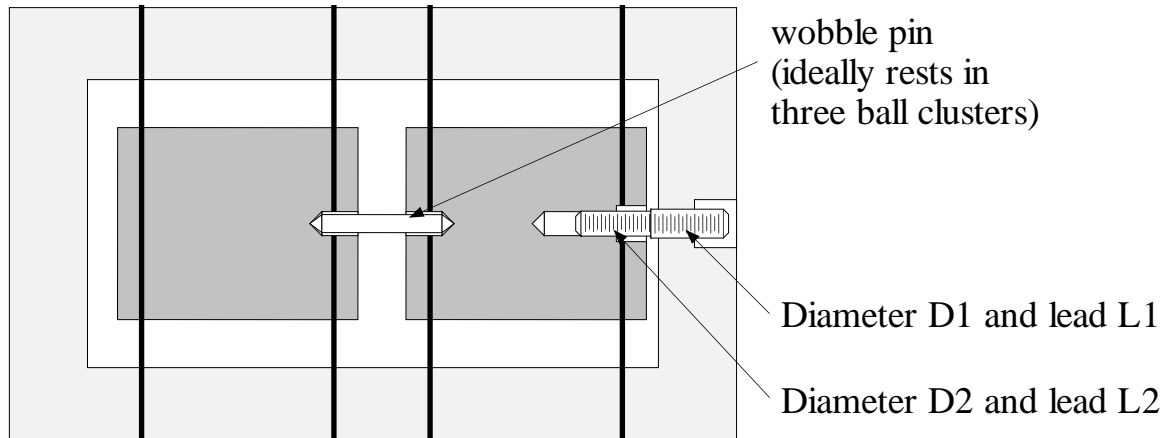
$$\delta = \frac{FL^3}{192EI}$$

- **Use a two stage (stacked) four bar linkage:**

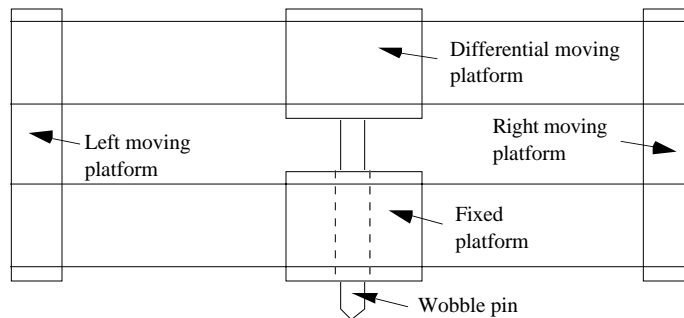


- **The parasitic motions of one 4-bar cancel the parasitic motions of the second 4-bar.**
- **Lateral and yaw stiffness will not be high, however, due to buckling effect.**
- **Overcome by use of monolithic hourglass-type members.**

- **A differential actuation system can be used to increase resolution.**
- **A wobble pin will minimize parasitic force actuation errors:**



- **Alternate differential design:**



- **Left and right platforms would be connected to a common rigid top-plate.**

- **When the flexure is built in to "walls", it needs extensional capability: (after F. Zhu at Delft):**



- **Many thin blades can be used to increase lateral stiffness and load capacity while keeping axial stiffness and stress low:**



$$F_{\text{load capacity}} \approx Ntw$$

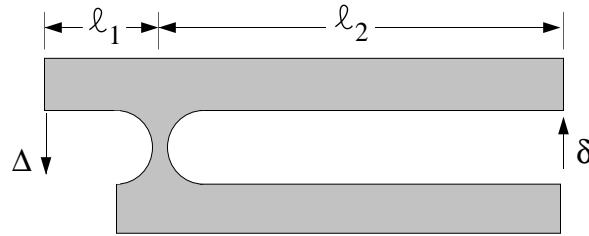
$$K_{\text{vertical}} = \frac{NtwE}{L}$$

$$K_{\text{axial multiblade}} \approx \frac{L^3}{ENt^3w}$$

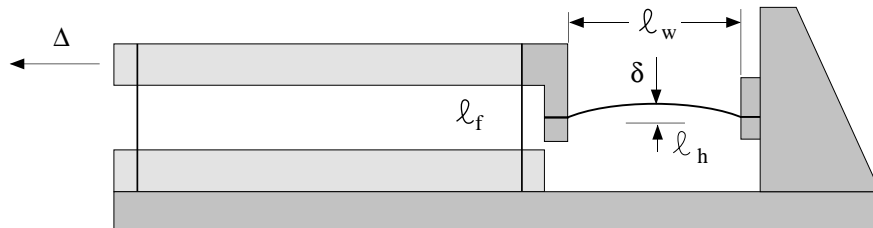
$$K_{\text{axial monolithic}} \approx \frac{L^3}{E(Nt)^3w}$$

- **Be careful of alignment and slip problems between the blades.**
- **EDM process can be used to prevent these problems.**
- **Rubber placed between the blades can increase damping.**
- **Careful when tightening bolts, as the torque can twist the flexure!**

- **Lever and fulcrum transmission:**



- **A bowed flexure used as a high reduction transmission:**



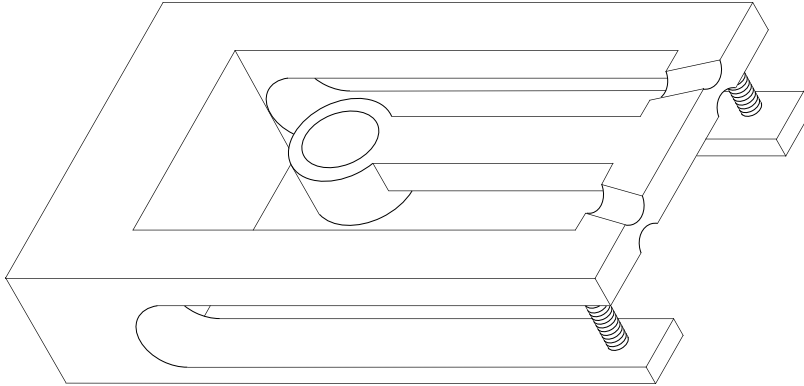
- **A downward motion δ causes a lateral motion Δ :**

$$\Delta \approx \frac{4\delta l_h}{l_w}$$

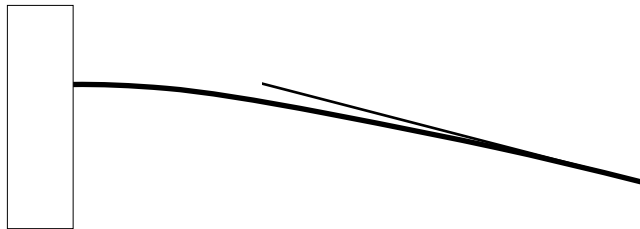
- **It is possible to chain together a series of these bowed flexures at right angles to each other.**
 - **This can yield a very high transmission ratio.**
 - **Assume that $l_h = 2$ mm and $l_w = 20$ mm, then the transmission ratio is 5.**
 - **In series, the ratios become 25, 125, 625... for 2, 3, 4... units respectively.**
- **A variation to this bowed beam approach is to use two beams laid on top of each other and tied together at one end.**
- **When the beams bend there will be differential motion along the interface between them.**

Flexures that provide angular motion

- **Device for adjusting pitch and yaw** (Courtesy of Polaroid Corp.):



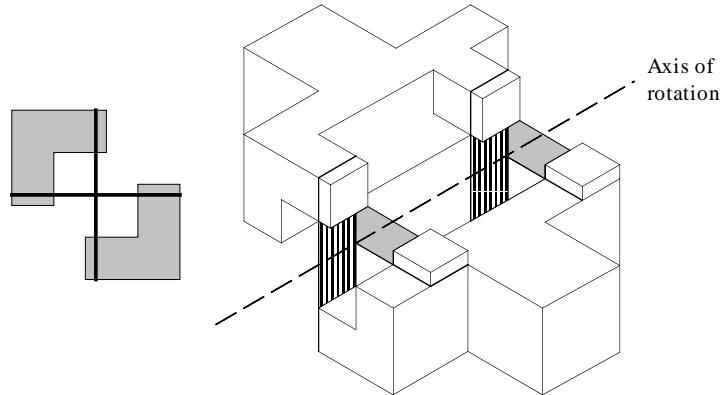
- **Note the transmission effect, where the screws push up on the upper beam, using the lower beam as a base.**
- **If the top beam were 1/2" thick, and the bottom beam were 1/8" thick, then the transmission ration would be $((1/2)/(1/8))^3 = 64!$**
- **The slope at the end of the beam causes an Abbe error in the beam that holds the optic which cancels the beam deflection ($\delta = \alpha(2/3L)$):**



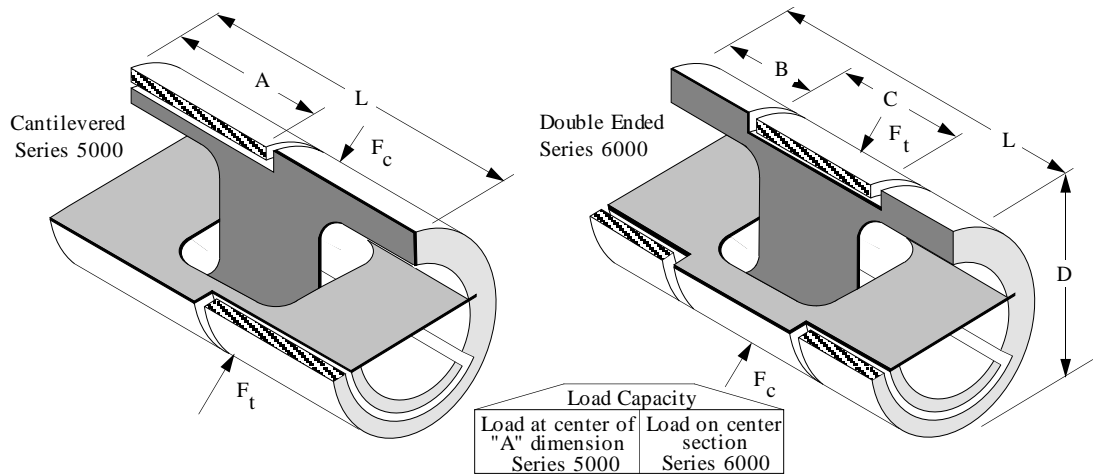
$$\delta = \frac{F L^3}{3EI}$$

$$\alpha = \frac{F L^2}{2EI}$$

- **Cross-strip flexure for angular motion:**



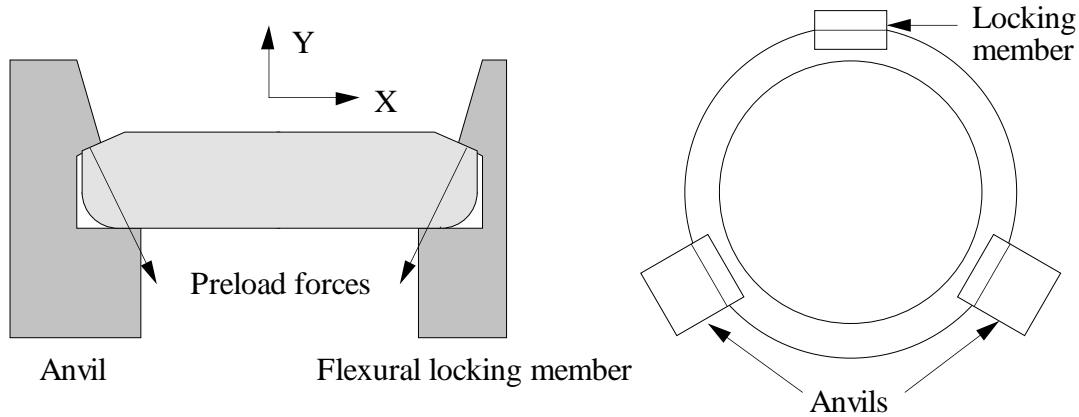
- **Lucas flexural pivots** (Courtesy of Lucas Aerospace Power Transmission Corporation.):



- **For limited motion applications use in place of rolling element bearings.**
- **Flexural pivots are not subject to fretting problems.**

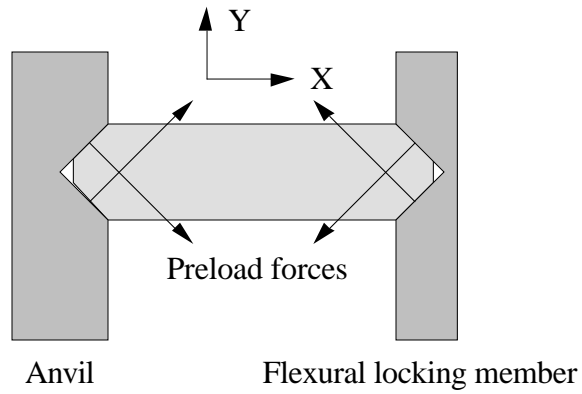
Flexures to guide and lock components into place

- Flexures can be used as one-shot hinges to guide components into alignment and then lock them into place:

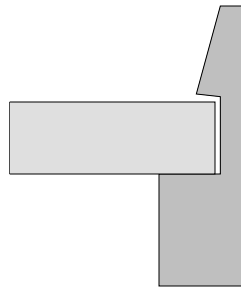


- **X direction stiffness of the clamping system is equal to:**
 - The sum of the X direction stiffness of the anvil and locking member due to the preload effect.
- Small taper keeps component preloaded in both the X and Y directions.
- To minimize distortion caused by bending, line of force must pass through the support ledge.
- A lens could be kinematically held by two anvils and a single locking member.
- The geometry is tolerant of manufacturing errors.

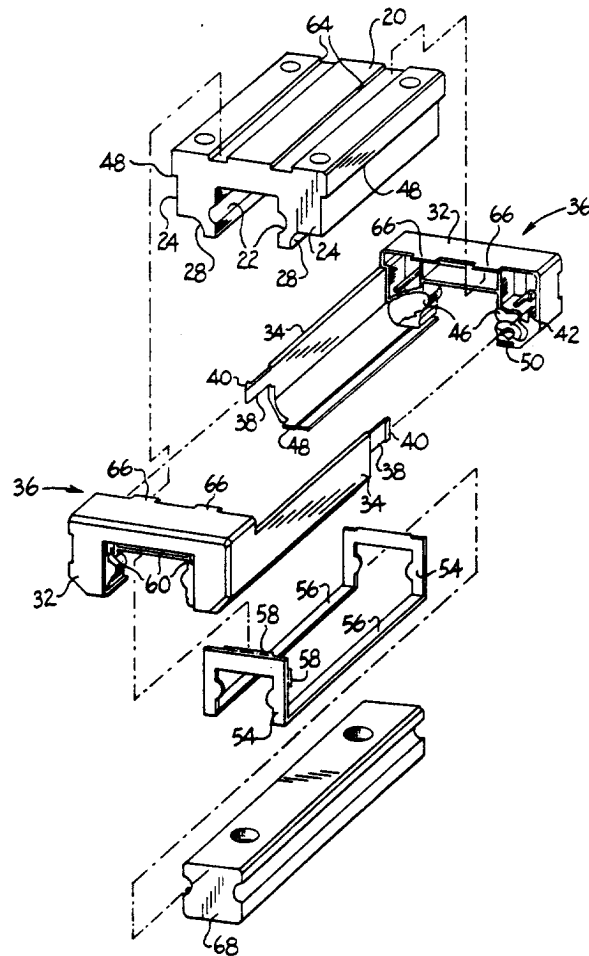
- **For precise X position location, utilize a grooved structure:**



- **Numerous permutations are possible.**
- **The moral of the story is don't do this:**



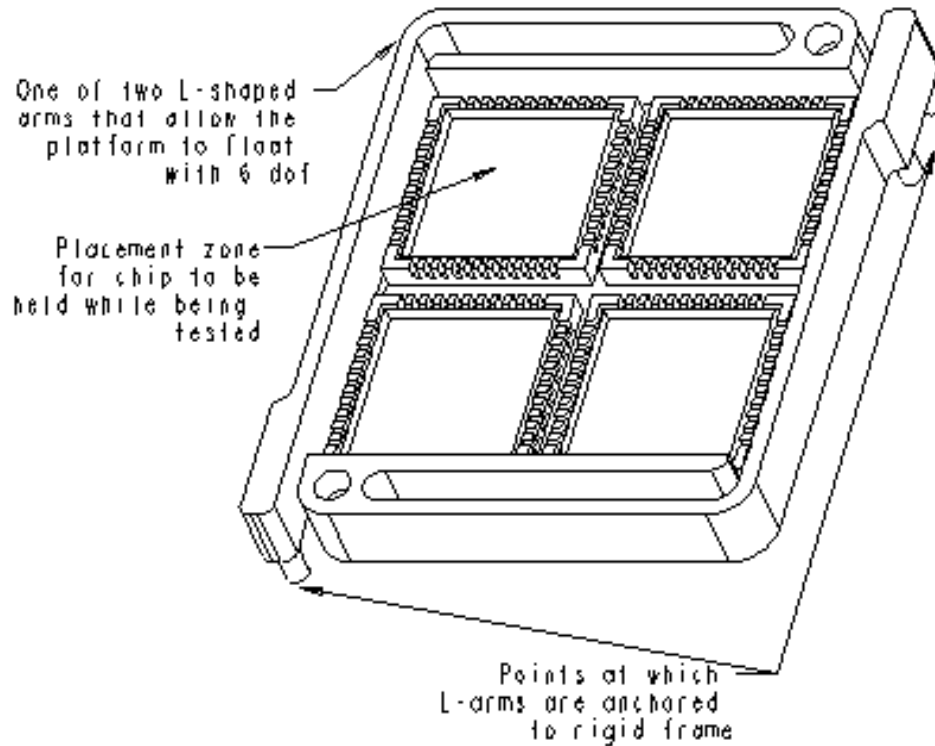
- **Flexures allow for snap assembly of components which make the design of systems more efficient:**



(From US Patent 5,102,235 by Peter Muggleston of Thomson Industries Inc.)

- **End caps click together.**
- **Seal is a single molded piece.**
- **No screws required.**

- **Flexures for fixturing: Opposed L's support a platform so it can move in six degrees of freedom².**



- **This allows chips held in the platform to be operated on (tested in this case).**

² Patent Pending, Kinetrix, Inc. 33 Constitution Drive, Bedford, MA 03110-6000. Contact Dr. Alexander Slocum.