Precision Machine Design

Topic 13

Design of joints and bearing rails

Purpose:

The characteristics of joints between a machine's components are strong functions of the components themselves, and the means by which they are joined. The latter is often thought of as being non-deterministic, but as this lecture illustrates, design and manufacturing methods can make joint design more deterministic.

Outline:

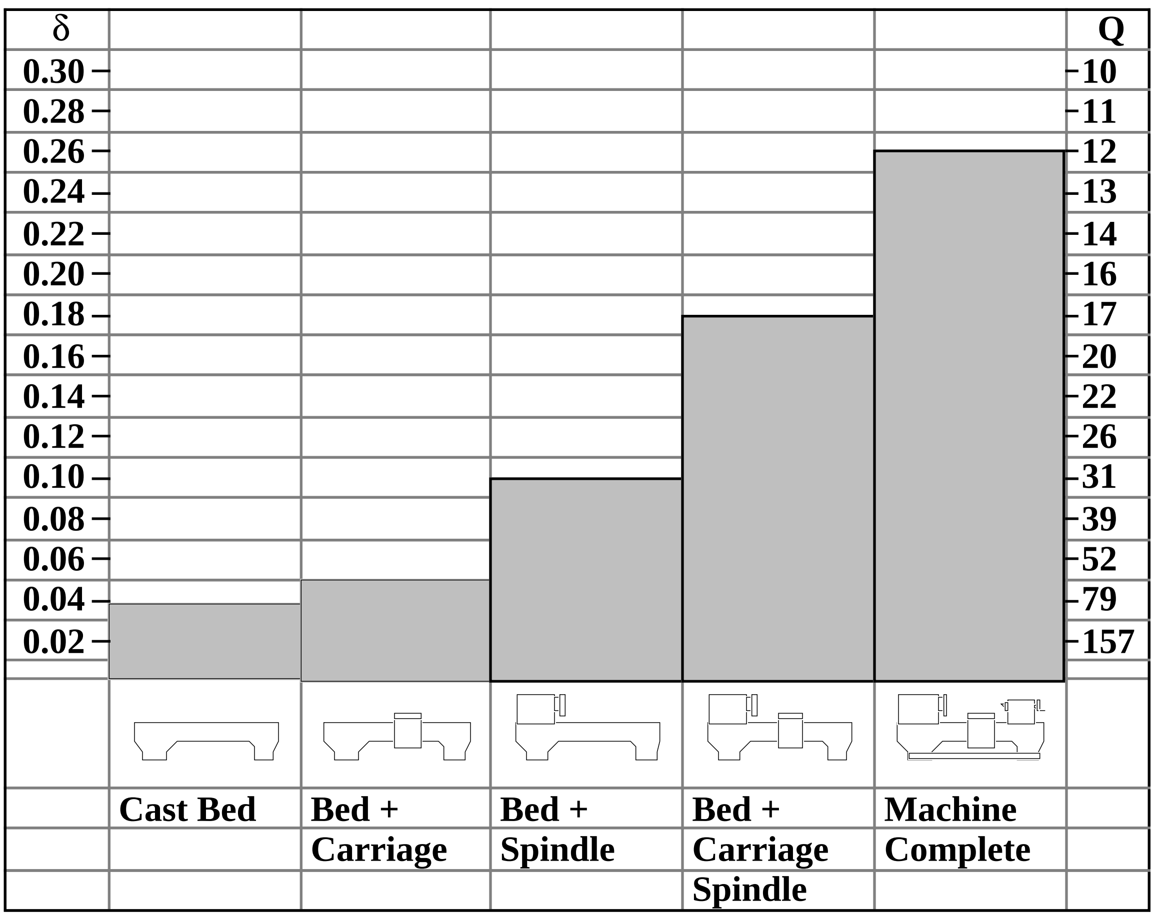
- **Bolted joints**
- **Bearing rail design**
- **Adhesive joints**

"Tis not a lip, or eye, we beauty call,
But the joint force and full result of all"

Alexander Pope

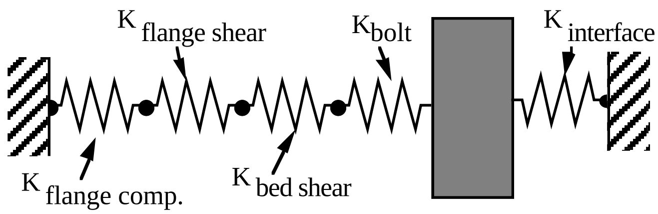
Bolted joints

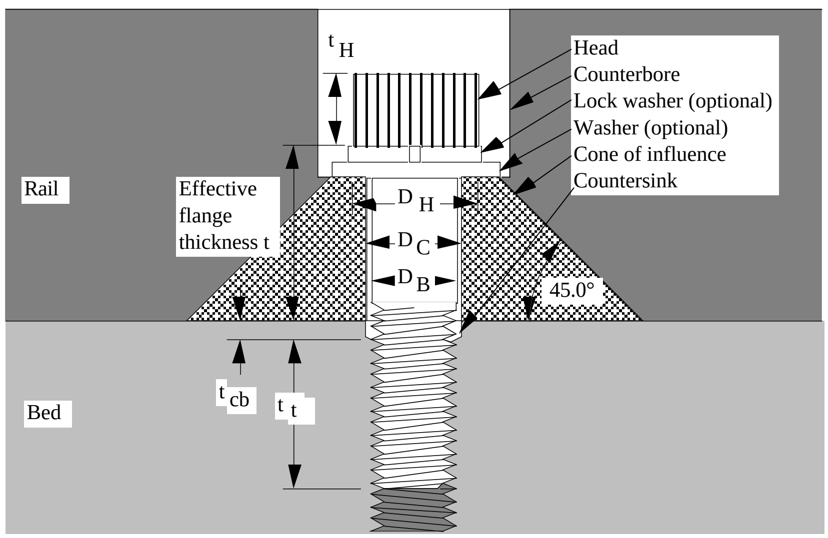
- A necessary evil in most machines.
    - Bolts act as point forces that create localized deformations in the material.
        - The goal in precision bolted joint design is to minimize variation in deformation between the bolts.
            - This is accomplished by carefully designing the geometry of bolt placement and the tightening method used.
- Remember, most of the damping in a machine tool comes from the bolted joints (and sliding bearings):



Mechanics can be used to model bolted joints

- **Many elements in a bolted joint act in series, which as a set acts in parallel with the stiffness of the parts' interface:**





- **As long as the applied load does not exceed the preload of the joint, the effective stiffness of the bolted joint will be:**

$$K = K_{\text{interface}} + \frac{1}{\frac{1}{K_{\text{flange comp.}}} + \frac{1}{K_{\text{flange shear}}} + \frac{1}{K_{\text{bed shear}}} + \frac{1}{K_{\text{bolt}}}}$$

- **Including the effects of part stiffness (e.g., a bearing rail), the total stiffness of the system is:**

$$K_{\text{system}} = \frac{1}{\frac{1}{K_{\text{part}}} + \frac{N_{\text{bolts}}}{K_{\text{Interface/bolt}} + \frac{1}{\frac{1}{K_{\text{Flange comp}}} + \frac{1}{K_{\text{Flange shear}}} + \frac{1}{K_{\text{Bed Shear}}} + \frac{1}{K_{\text{Bolt}}}}}}$$

- **The stiffness of a 45º cone of material under the bolt head is approximately:**

$$K_{\text{flange comp.}} = \frac{1}{\displaystyle\int_0^t \frac{dy}{\pi E_f \left\{\left(\frac{D_H}{2} + y\right)^2 - \frac{D_C^2}{4}\right\}}} = \frac{\pi\, E_f\, D_C}{\log_e \dfrac{(D_C - D_H - 2t)\,(D_C + D_H)}{(D_C + D_H + 2t)\,(D_C - D_H)}}$$

- **The shear deflection of the flange of thickness t due to the bolt being in tension is found using energy methods:**

$$\tau = \frac{F}{2\pi Rt}$$

$$\delta = \frac{\partial U}{\partial F} = \frac{F}{2\pi tG}\int_{R_B}^{2R_B} \frac{dR}{R} = \frac{F \log_e 2}{2\pi tG}$$

$$K_{\text{flange shear}} = \frac{\pi t E_f}{(1 + \eta)\log_e 2}$$

- **From geometric compatibility with the bolt head, this includes the effect of shear strain in the bolt head.**

- **Assume that the effective length of thread engagement is equal to D_B.**

  * **The shear stiffness of the threaded region in the bed (neglecting the countersunk region) is:**

$$K_{\text{bed shear}} = \frac{\pi D_B E_t}{(1 + \eta) \log_e 2}$$

- **Assume the bolt is threaded in one bolt diameter.**

  - **The threads start a distance t_{cb} below the surface to avoid forming a crater lip upon tightening.**

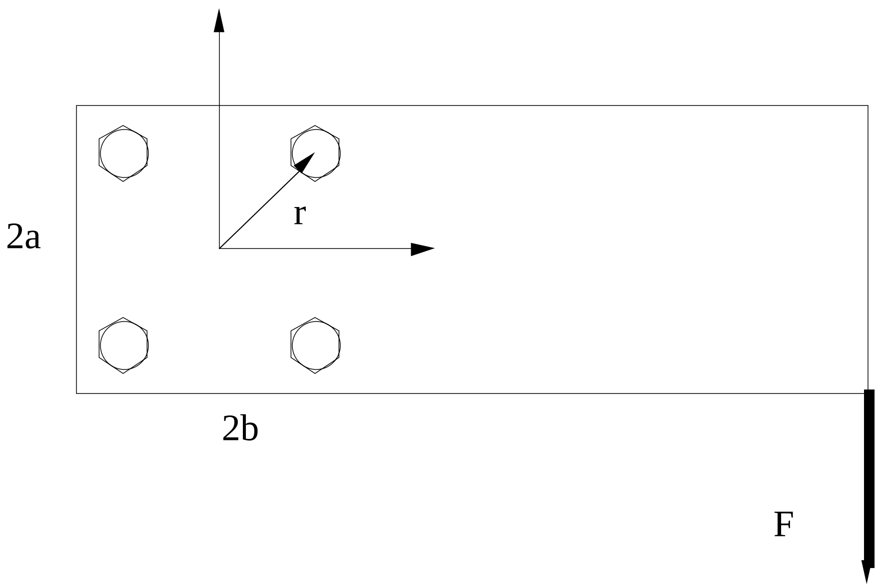
  - **Then the bolt stiffness is approximately (effective length = $t + D_B/2 + t_{cb}$):**

$$K_{\text{Bolt}} = \frac{\pi E_B D_B^2}{4\,(D_B / 2 + t + t_{cb})}$$

- **An example of the application of these formulas is given in conjunction with the discussion of bearing rail design.**

Shear Resistance of Bolted Joints

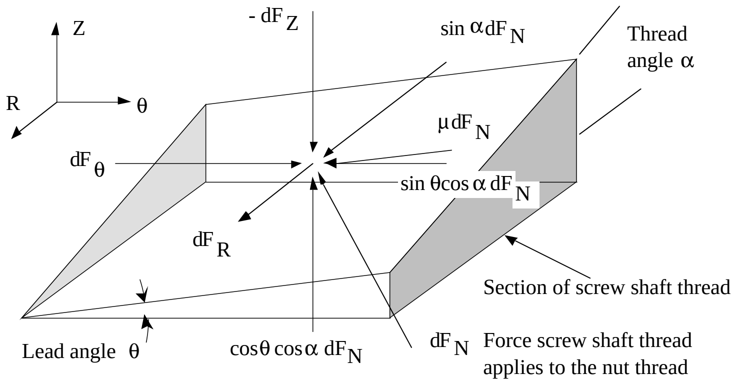
- **Bolted joints resist shear ONLY by clamping action and friction!**
- **NEVER rely on a bolt to take a shear load**
- **UNLESS the bolt is a shoulder bolt!**



- **Find the center of the bolt pattern, and compute the moments about it, using the products of bolt force, friction, and distance from the center point:**

$$F_{\max} = \frac{\sum_{i=1}^{numberofbolts} F_{bolt\_i}\, r\mu}{L}$$

- **The amount of force a bolt can generate, given friction in the threads, is determinable from basic physics of the force generated by a wedge[1]:**



Where:

- **Γ is the applied torque**
- **μ is the coefficient of friction**
- **r is the pitch radius of the bolt**
- **p is the lead of the thread (e.g., inches/rev)**
- **R is the bolt head radius**
- **α is the thread angle**
- **β is 2R/p**

- **To raise a load (tighten a joint), the torque required is:**

$$\Gamma_{required} = F_{desired}\left(\left(\frac{2\pi\mu r + p\cos\alpha}{2\pi r\cos\alpha - \mu p}\right)r + R\mu\right)$$

[1] For a REALLY thorough examination of this subject, including off-axis moments created by the applied torque, see A. Slocum Precision Machine Design, published by SME, Dearborn, MI.

- **To lower a load F, the torque required is:**

$$\Gamma_{required} = F_{desired}\left(\left(\frac{2\pi\mu r - p\cos\alpha}{2\pi r\cos\alpha + \mu p}\right)r + R\mu\right)$$

- **The screw will not back-drive when:**

$$p < \frac{2\pi r\mu}{\cos\alpha}$$

- **The efficiency η is:**

$$\eta = \frac{\cos\alpha(\pi\beta\cos\alpha - \mu)}{\pi\beta\cos\alpha(\cos\alpha + \pi\beta\mu)}$$

The torque applied to the bolt creates stresses

- **The force that is generated creates tensile and torsional shear stresses.**
    - **The thread root is a stress concentration area (a factor of 2-3), which is somewhat mitigated when the threads are rolled (as opposed to cut).**
- **Assuming a thread root diameter r_{tr} , the stresses are:**

$$\sigma_{tensile} = \frac{F_{Tensile}}{\pi r_{tr}^2}$$

$$\tau_{shear} = \frac{2\Gamma r_{tr}}{\pi r_{tr}^4}$$

The equivalent (Von Mises) stress is:

$$\sigma_{tensileequivelent} = \sqrt{\sigma_{tensile}^2 + 3\tau_{shear}^2}$$

Some heuristic design rules for bolted joints:

- **Never use a bolt diameter smaller than 6 mm on a structure: Small bolts are too easily twisted off!**
    - **Washers are rarely used with socket-head cap screws because stiffness would be reduced by the addition of an interface.**
        - **The purpose of a washer is to distribute the load on thin sections, or to keep a hex head bolt from chewing the surface as it is tightened!**
- **Ideally, the bedded portion of the rail should be at least as wide as the cantilevered portion.**
- **The flange should be at least one to two bolt diameters thick.**
    - **Make the stress zone cones beneath the bolt heads overlap.**
- **Make sure that the surfaces have been stoned with precision diamond ground stones just prior to assembly to remove raised lips around holes.**

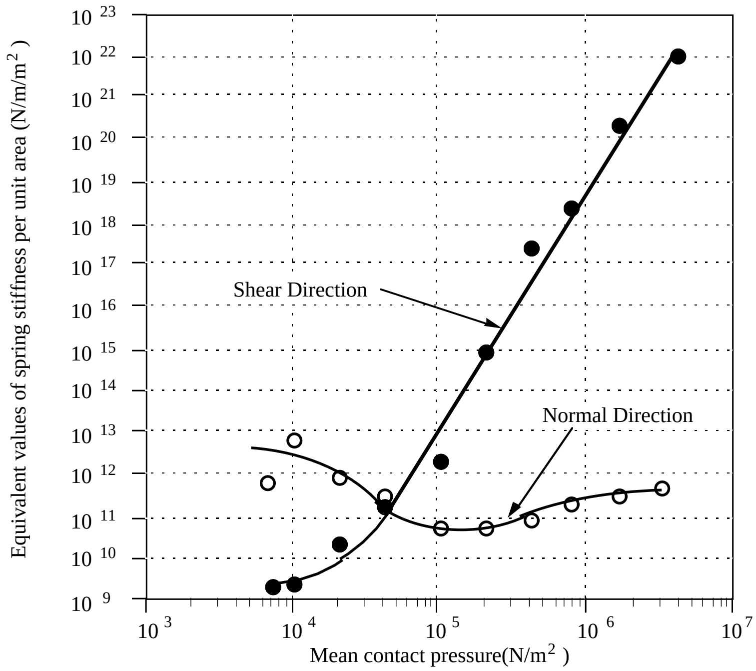
Bolt installation

- **Make sure the threads are cleaned and lubricated.**
- **Incrementally tighten the bolts.**
- **Consider using vibration to stress relieve the joint.**
- **To lock a bolt in place, use one or more of the following:**
  - **Use a thread lubricant-locking agent.**
  - **A dab of epoxy on the bolt head.**
  - **A lock washer.**
  - **A "Nylock™" bolt.**
  - **Yield tighten the bolt (tighten to the yield point).**

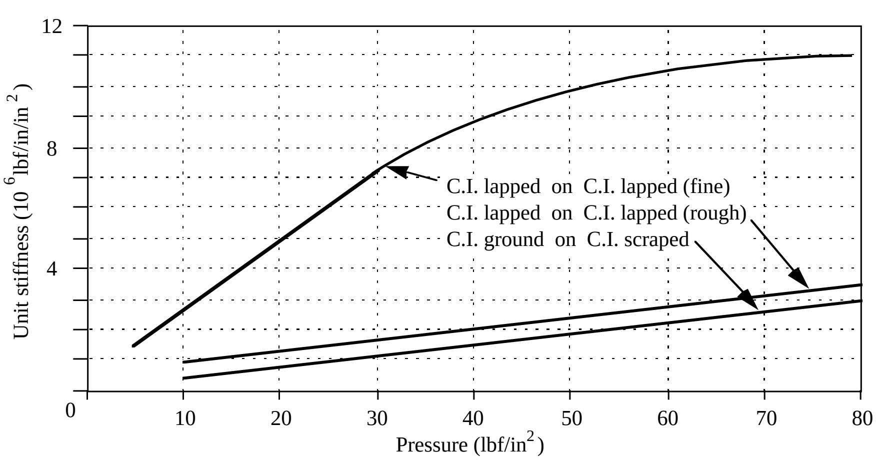
Joint interface stiffness:

- **Attempts to analytically model the contact between hundreds of have never been successful.**
    - **There are just too many unknowns.**
- **The stiffness of a bare joint interface can only be accurately determined by experimental means.**
- **However, typically, the interface stiffness will be 5 times the bolt system stiffness.**
- **If more determinism is sought in joint interface stiffness, then an adhesive can be used.**
    - **Parting can be obtained by having one surface plated and Teflon impregnated.**
        - **Most mold releases use a wax, and the wax layer may be too compliant.**
    - **Using an adhesive can lesson damping from a bolted joint!**

- **Empirical data must often be used to obtain joint stiffness values: joint stiffness of 0.55%C ground steel parts.** (After Yoshimura.)



- **Compressive stiffness at lower contact pressures** (after Dolby and Bell)**.**



Heuristic rules for increasing bolted joint stiffness

- **Bolts should not be spaced too far apart.**
  - **The joint pressure should be estimated as the total force provided by all bolts divided by the total joint area.**
- **The joint stiffness is the product of the stiffness per unit area value obtained from the empirical data and the bolted area.**
  - **Assume that the bolt force acts over an area of four to five bolt diameters (the area of the Cone of Influence).**
- **The finer the surface finish and the tighter the preload:**
  - **The fewer and smaller the gaps between surface asperities on the surfaces, and the higher the stiffness.**
- **Where compressive stiffness is critical, surfaces should be ground, scraped, or lapped:**
  - **Surfaces should always be stoned with a precision ground flat stone just prior to assembly. This is critical when mounting linear guides!**
  - **Bolt holes should also be countersunk to minimize the forming of "crater lips" during hole drilling.**
- **Vibration annealing can help to seat the asperities.**
- **Bolting through machined bosses, and then grouting between the bolted regions with an adhesive can save machining costs.**
  - **This greatly increases joint stiffness, but can decrease the damping.**

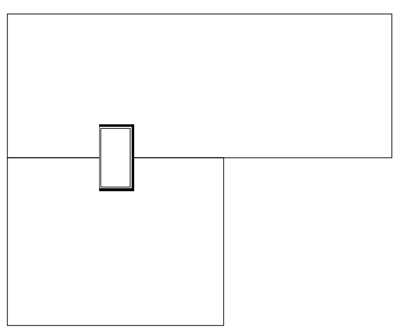
Design procedure to determine the required preload and bolt size for a joint:

- **The stiffness of the joint should be greater than the stiffness of the parts that make up the joint.**
- **Determine the required joint interface pressure to achieve the required stiffness.**
  - **Assume that the bolt force acts over an area of four to five bolt diameters (the area of the Cone of Influence).**
- **The minimum total preload force should be the larger of:**
  - **The sum of the maximum tensile force on the joint and the required joint preload force or:**
  - **Four times the maximum tensile force.**
- **The product of the minimum force and the coefficient of friction for the joint should be:**
  - **5-10 times greater than the maximum expected shear force on the joint.**
- **Space the bolts equal to the part thickness or a portion of the width of the bearing rail.**
- **On one extreme, size the bolts so the stresses in the bolts are below 25% of yield.**
  - **Compare this value to the bolt size found from a minimum stiffness criteria (as shown in the example).**
- **On another extreme, yield tighten the bolts.**
- **Check the bending, compression, and Poisson expansion of the rail to make sure they are within acceptable limits.**
  - **Do a wavelength FFT of the straightness to check for errors at a wavelength of the bolt spacing.**

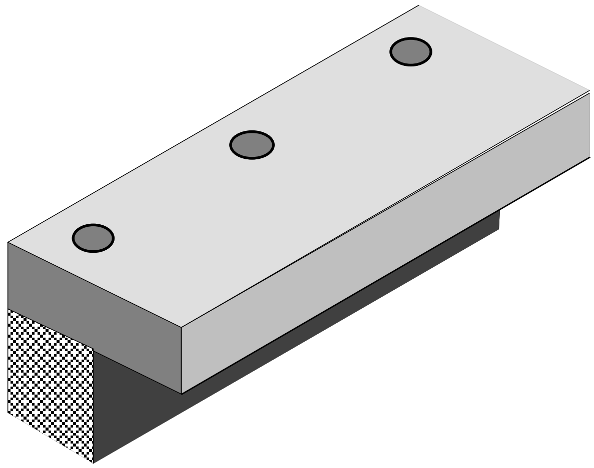
Stability of bolted joints

- **Common bolted joints may suffer a loss of initial preload with applied cyclic stress.**
  - **Loss of preload seems to be due to the applied stress causing microslip between the surfaces.**
- **If the joint cannot be pinned with dowels, one may wish to specify vibration stress relief and re-tightening of bolts.**
  - **The final tightening can be done while the vibration stress relief process is being done.**
- **If the joint is to be subject to vibration loads or extreme stability is required:**
  - **A hardening thread lubricant should be used.**
  - **A film adhesive can be spread on the joint.**
  - **After tightening, a large glob of epoxy should be placed on the bolt head and seating surface.**

- **Ideally, to ensure stability in a machine that may have a high degree of vibration, one could:**
    - **The bearing rail should be aligned and the bolts incrementally tightened according to a specific pattern:**
        - **For example, from the inside out to prevent clamping in a bow.**
    - **The alignment should be checked after each tightening increment.**
    - **The assembly should be vibration stress relieved.**
    - **The alignment should be checked and the bolts re-tightened.**
    - **The alignment should be checked, and then the assembly vibration stress relieved while the bolts are re-tightened.**
    - **A final checking of the alignment should be performed.**
    - **Ideally, the rail would be pinned to the structure with dowels.**
    - **It can also be potted in place with an epoxy which helps increase joint stability and damping.**
    - **A loose fitting key that runs the length of the rail can be grouted or epoxied in place.**



Bearing rail design

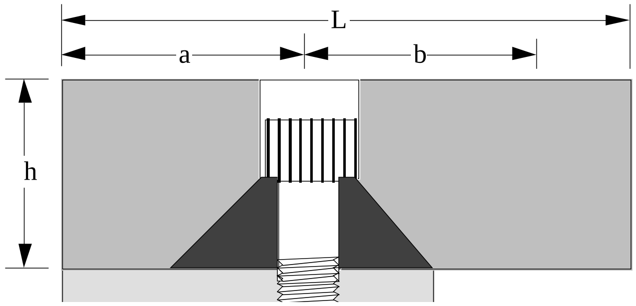


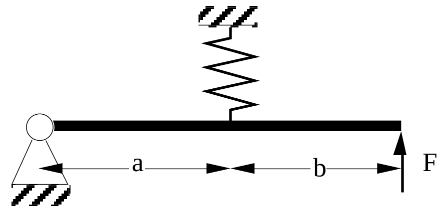
- Deterministic design is possible if the following are considered:
    - Bolted joint stiffness.
    - Bearing rail stiffness:

Bearing rail stiffness

- The rail is not as stiff as if it were ideally cantilevered, but it is stiffer than a simply supported beam.
    - A simply supported model should be conservative.
- The bolts do not provide knife edge support, thus the stiffness will be less than modeled.
- When a bearing pad exerts a force on the bearing rail, neighboring sections provide support also.
- The surface to which the rail is bolted only supports the rail at the rear point.
    - It actually provides some support due to the bowing of the rail.
    - Assume the rear support point is infinitely stiff and the surface beneath the rail doesn't resist bowing of the rail.
- Bending and shear deformations must be considered.
- For sliding contact bearings, force is applied at the center of the pad.
- For hydrostatic bearings, the deflection must be computed based on the pressure distribution.

Sliding contact bearing rail model:





- **The system behaves like a beam attached to a pivot point so the equivalent stiffness at the point of force application is:**

$$K_{eq\ bolt\ system} = \frac{Ka^2}{(a+b)^2}$$

$$K_{eq.\ bolt\ system} = CD_B$$

- **The bending deflection is:**

$$\delta_{bend} = F\,\frac{(a+b)\,b^2}{3EI}$$

- **The shear deflection is:**

$$\delta_{shear} = \frac{Fbh^2\,(a+b)\,(1+\eta)}{5aEI}$$

- **Assume that the bolt spacing ℓ is a function of the diameter:**

$$\ell = D_B(M - 1)$$

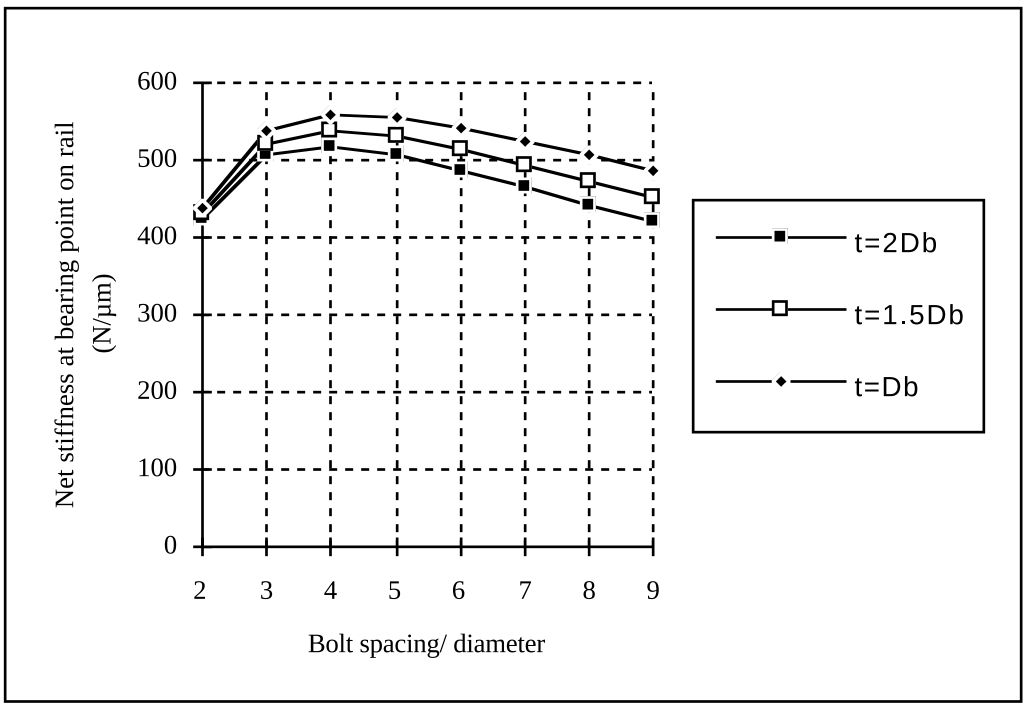
  - **This represents M bolt diameters minus the region occupied by the bolt.**

- **The moment of inertia for the rail section is then $I = \ell h^3/12$.**

- **The total stiffness for N segments of the rail (e.g., N segments under a bearing pad) is thus:**

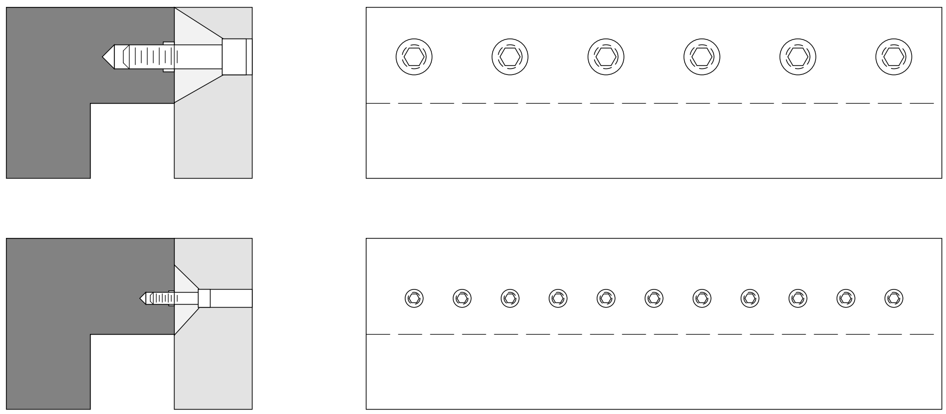
$$K_{Rail} = \frac{N_{Segments}}{\dfrac{1}{K_{Rail\ bend\ \&\ shear}} + \dfrac{1}{K_{Joint} + \dfrac{1}{\dfrac{1}{K_{Flange\ comp}} + \dfrac{1}{K_{Flange\ shear}} + \dfrac{1}{K_{Bed\ Shear}} + \dfrac{1}{K_{Bolt}}}}}$$

- **Bolt diameter is not a sensitive parameter for stiffness when bolt spacing is made a function of the bolt diameter**
    - **The length of bolt and the cone are expressed as bolt lengths:**
        - **Bolt joint stiffness becomes linearly dependent on the bolt diameter.**
    - **As a result, bolt diameter cancels out.**

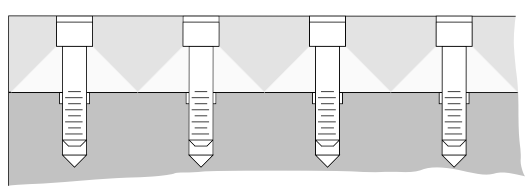


- **The interesting parameters are:**
  - **The ratio of joint interface stiffness to joint stiffness.**
  - **The total length of rail over which one wants to know the stiffness.**
- **Ideally, for a balanced design, the interface stiffness should be equal to the bolt and rail stiffness.**
- **If the interface stiffness is too high:**
  - **The displacement caused by the preload will be very small.**
  - **Manufacturing defects may cause loss of preload around a bolt.**
- **Realistically, the interface stiffness may be set to as high as five times the bolt system stiffness.**

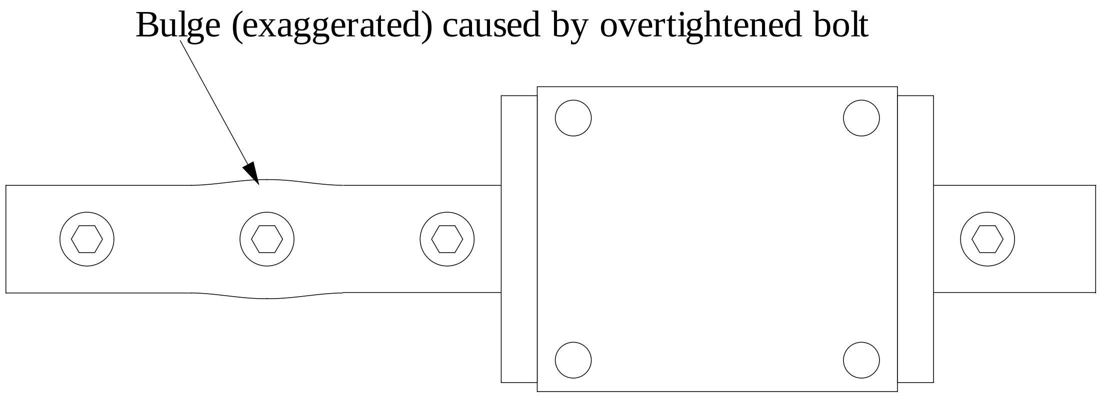
- For example, these two bolted systems have the same stiffness:



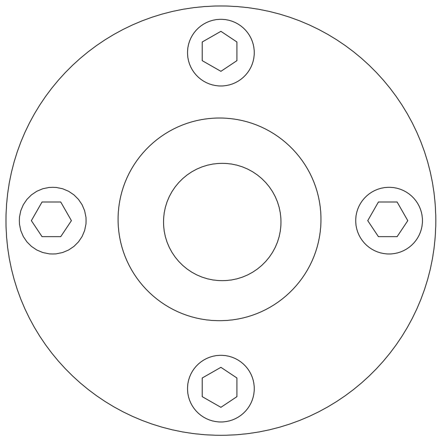
- The design goal is to achieve high stiffness and minimize manufacturing cost, while maintaining accuracy.
- For maximum accuracy, it is desirable for the cones of influence under the bolt heads to overlap.
    - This minimizes bending of the rail on an elastic foundation as described below.
- Thus even though a thicker flange has slightly lower stiffness, it allows for the use of less bolts.
    - For example, with a flange thickness of $2D_B$, the bolt spacing should not be more than $4D_B$



- **Lateral deformations due to the Poisson effect should also be prevented, particularly if the rail is also a guide rail:**



- **These effects are minimized by:**
  - **Maximizing the ratio of rail width to bolt diameter.**
  - **Minimizing the bolt spacing.**
  - **Minimizing the bolt torque.**
- **Poisson expansion is a leading cause of rotary bearing failure, when bolted flanges are used to preload bearing assemblies.**



- **Use more small bolts to minimize deformation.**
- **Remember St. Venant's principle: 3 characteristic dimensions (bolt head diameters) away, effects are no longer felt!**

- **In some cases, where joint loosening would be disastrous (e.g., in spindle components), yield tightening may be employed.**
    - **Yield tightened joints can use a torque limit or a displacement limit.**
    - **Torque limits (with a wrench) can vary due to friction changes.**
    - **Displacement criteria can be more deterministic (tighten until a fixed torque, and then apply 1/Nth of a turn).**
- **Be careful to measure the straightness of the system:**
    - **Do an FFT (straightness error vs. wavelength) and see if there is a peak at the bolt stiffness.**
        - **If there is a peak, then the bolts are too tight.**

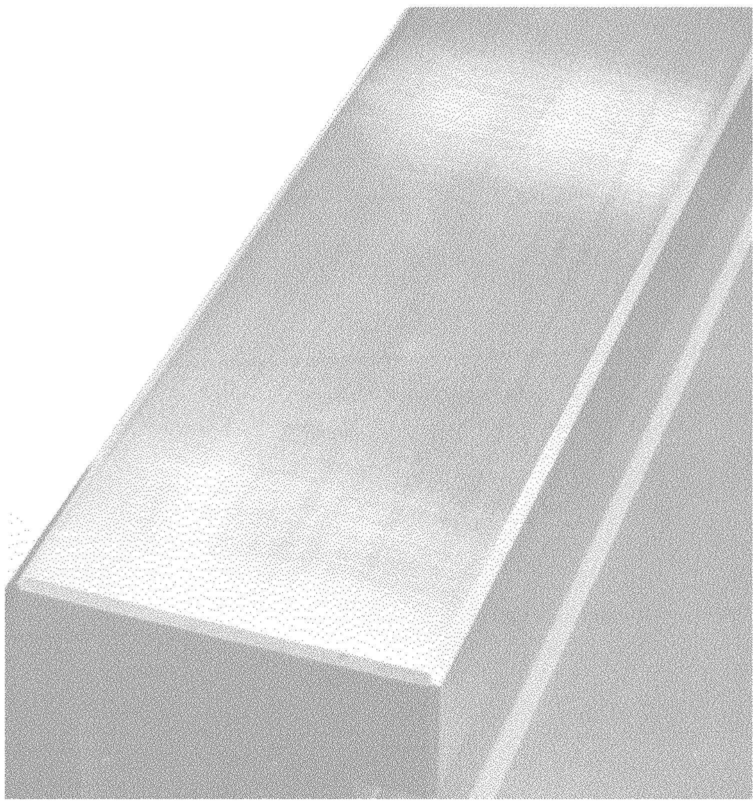
Example: Spreadsheet for bolt joint torque

Bolted rail design
Enter numbers in bold

	Meters and Newtons	Inches and pounds		
DB	**0.016**	0.016	0.62992	0.62992
Flange thickness (bolt diameters)	**2**	2	2	2
Bolt area	0.000201	0.000201	0.311645	0.311645
45 degree cone area	0.003016	0.003016	4.674681	4.674681
lead	**0.002**	0.002	0.08	0.08
Thread angle (deg)	**14.5**	14.5	14.5	14.5
mu	**0.1**	0.3	0.1	0.3
efficiency (Equation 10.8.18)	0.28	0.11	0.29	0.12
Kdesired	**5.26E+08**	5.26E+08	3.00E+06	3.00E+06
Kdesired/area	1.74E+11	1.74E+11	6.42E+05	6.42E+05
Joint pressure (use cell B13 & Fig. 7.5.7)	**4.00E+04**	4.00E+04	6	6
Joint pressure (Use cell D13 & Fig. 7.5.9)	1.38E+05	1.38E+05	**20**	20
Force required (Ground on scraped cast iron)	416	416	93	93
4*Alternating force on carriage	**20000**	20000	4494	4494
Number of bolts under carriage	**8**	8	8	8
Total force per bolt	2916	2916	655	655
Torque required	6.9	18.8	61.1	166.5
Polar inetria	6.43E-09	6.43E-09	1.55E-02	1.55E-02
Shear stress	8.52E+06	2.33E+07	1245	3392
Tensile stress	1.45E+07	1.45E+07	2103	2103
Mises equivelant stress	2.07E+07	4.29E+07	3012	6241

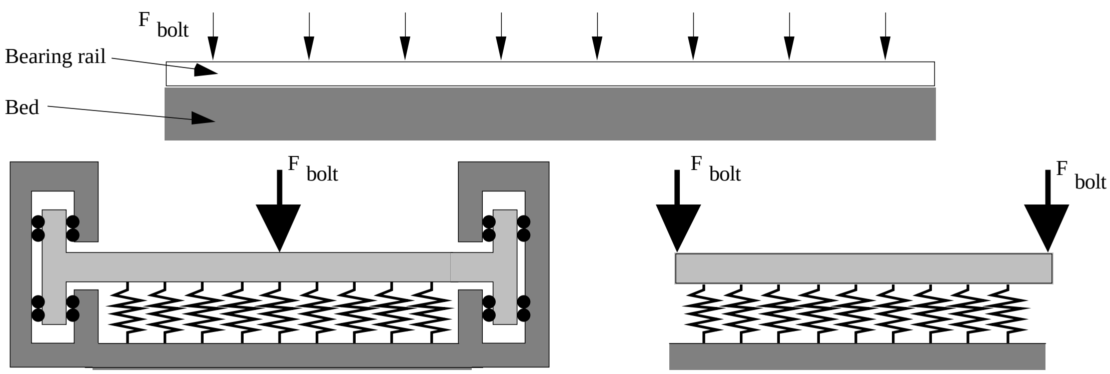
Rail deformation due to bolt preload

- **Even when a "perfect" rail is bolted to a "perfect" bed:**
  - **Significant (micron to submicron depending on bolt and rail size and preload) deformations can result.**
- **Bolt preload causes various types of deformation:**
  - **Bending.**
  - **Compression.**
  - **Poisson expansion.**



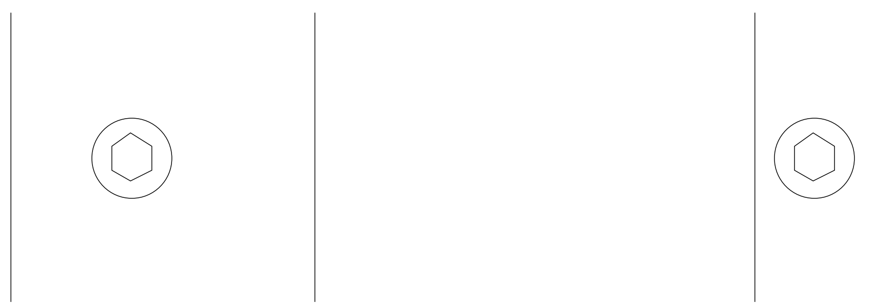
- **These errors can be corrected by lapping after bolting.**
- **The use of a large number of bolts can be used to deform a machine into a desired shape (e.g., to straighten an axis).**
- **If the rails were finished while bolted and then transferred to another machine:**
  - **Desirable to be able to determine the deformation that could result from a variation in the bolt force.**

- **Model of rail deformation due to bolt preload:**
  - **When the counterbore diameter is much less than the rail width:**
    - **The model of a guided beam on an elastic foundation of modulus k is used.**
  - **When the counterbore diameter is a significant portion of rail width:**
    - **The simply supported beam model on the right is used.**



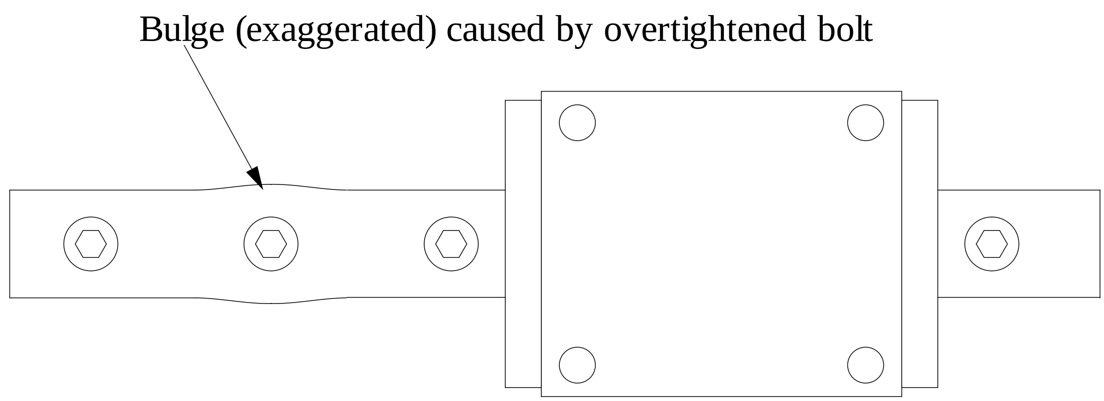
Rail width larger (> 4x) than bolt diameter

Rail width not much larger (< 4x) than bolt diameter



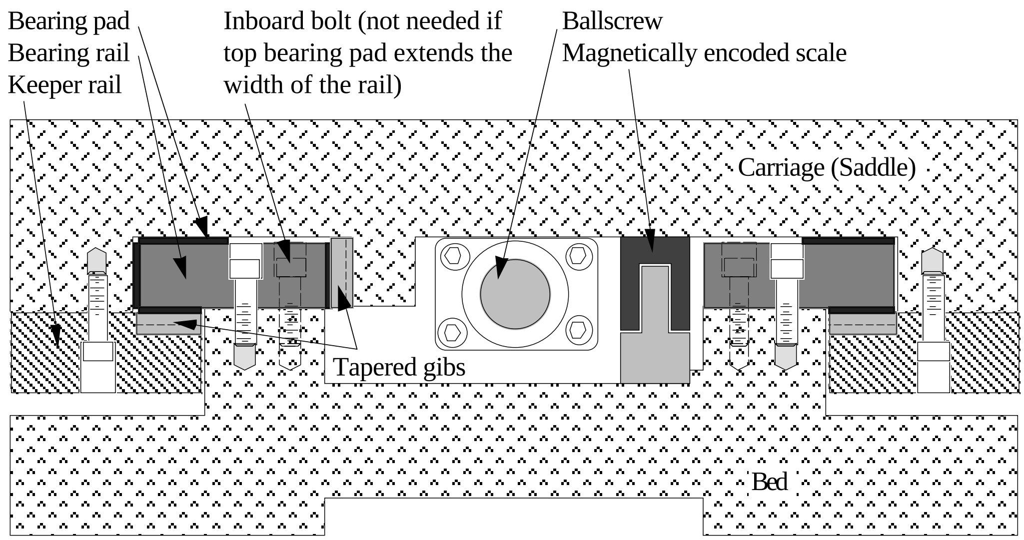
- **Analysis methods exist (see text) for estimating the amount of deformation.**

- **For the previous bolted rail spreadsheet example, the rail deformation is about 0.15 μm (6 μin.).**
- **Recall the Poisson effect:**
    - **Compressive deformation is accompanied by a lateral expansion:**
        - **On the order of 0.3 times the compressive deformation (for most metals), depending on the rail geometry.**
- **For large rails, the Poisson expansion is usually negligible because it is diffused into the surrounding metal.**
- **For thin rails, such as those used by many types of modular linear bearings, the effect can be more prominent.**
    - **Manufacturers' bolt-tightening recommendations should be carefully followed.**
    - **Beware of mounting to surfaces made from materials with a low modulus of elasticity.**
    - **Remember, a Fourier transform of the straightness as a function of wavelength can help spot bolting problems!**



- **Bolt torques and thread efficiencies are never exactly the same.**
  - **Varying vertical and horizontal straightness errors in the rail will often be present.**
    - **The period will often be equal to that of the bolt spacing.**
    - **Always do a Fourier transform of the straightness error.**
    - **Plot the results as error amplitude verses wavelength.**
- **When designing and manufacturing a precision machine:**
  - **Bolt preload, thread friction, and tightening methods should all be chosen with care.**

Modular "T" slide bearing configuration:

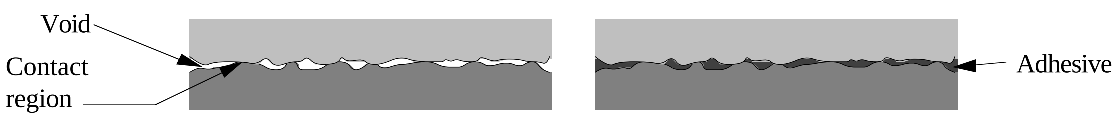


Bolt torque summary:

- **The theory will often yield a recommended torque level which is far below the allowable torque for a particular bolt:**
    - **The theory represents the minimum required torque.**
    - **The higher the torque, the better as far as machine stability is concerned.**
    - **Too high torques can deform components (e.g., cause rail straightness errors).**
- **It is best to experiment:**
    - **See what the component deformations are at different torque levels.**
    - **Use as high a torque level as possible.**
    - **Do not forget to stone surfaces before they are bolted together, in order to ensure that there are no burrs.**

Bonded joints:

- **Bonded joints can be strong and fatigue resistant.**
- **Adhesives fill small voids between mating parts and thus increase the following characteristics of the joint** (Courtesy of Loctite Corp.)**:**
  - **Strength**
  - **Stiffness**
  - **Damping (>welded, <bolted)**
  - **Heat transfer**



- **Even when two ground surfaces are to be bolted together:**
  - **It can be wise to specify the use of a few drops of low viscosity adhesive that will flow and fill the small voids.**
  - **Sometimes used on very high precision instruments and tools**
  - **Not used on machine tools**
- **A sliding fit joint held by an adhesive can make a joint more robust than a press fit joint:**
  - **More dimensionally stable.**
  - **More accurate.**
  - **Longer lasting.**
- **Bearing races can also be fixed to bores in this manner to avoid altering the preload by press fitting.**

- **A potted (grouted) joint uses adhesive (low shrinkage epoxy[2]) to lock in place a joint.**
- **Mechanical means hold parts in alignment and support the static weight of the structure.**
- **In some cases:**
  - **The mechanical means are often there just for initial alignment.**
    - **Kinematic couplings can be used.**
  - **The grouting carries large applied loads.**
  - **Allows for kinematic assembly for easy adjustment and then potting for added rigidity.**
- **In other cases:**
  - **The mechanical joint supports the static load.**
  - **The potting compound resists microslip of the mechanical joint and greatly increases joint stiffness and damping.**

[2] Low shrinkage potting epoxies are available from 1) Andrew Devitt, Devitt Machinery Co., Twin Oaks Center, Suite G, 4009 Market Street, Aston, PA 19014, (610) 494-2900; 2) Loren Power, ITW Philadelphia Resins, 10132 Spiritknoll Ln., Cincinnati, OH 45252-1937, (513) 385-4577

- **Examples include:**
    - **Grouting machine tool beds to concrete floors.**
    - **Potting major machine tool components to the machine tool structure.**
    - **Potting sensors in sensor mounts.**
- **Replication: a polymer is poured or injected around a master shape that has been coated with mold release.**
- **When the polymer cures, it has the shape of the master which can then be removed and used again.**
- **Polymer shrinkage is very low (typically 0.25%) shrinkage, BUT it can be significant: Example:**
    - **1 meter tall column 0.5 m wide at the base.**
    - **Wedge of polymer 2 mm thick at one side and 3 mm thick at the other side, and 0.25% shrinkage.**
    - **Differential shrinkage across the wedge will be 0.001m x 0.0025 = 2.5 microns.**
    - **Abbe error at the top of the column will be 5 microns!**

- **Hardening of many polymer resins is an exothermic reaction.**
  - **Minimize the amount of polymer used.**
  - **Maximize the stiffness and thermal diffusivity of the master and the part.**
- **This is required to prevent:**
  - **The heat of polymerization from heating the structure and deforming it.**
  - **Then the polymer hardening to the thermally deformed shape.**
- **Unlike bolted assemblies, once the polymer cures to form the desired shape realignment is no longer possible.**
- **Replicated parts and assemblies can be as dimensionally stable as the rest of the machine.**
- **In machine tools, mounting surfaces for linear bearing rails can be replicated.**
- **In some cases for sliding or hydrostatic bearings the replicated surface functions as the way surface.**