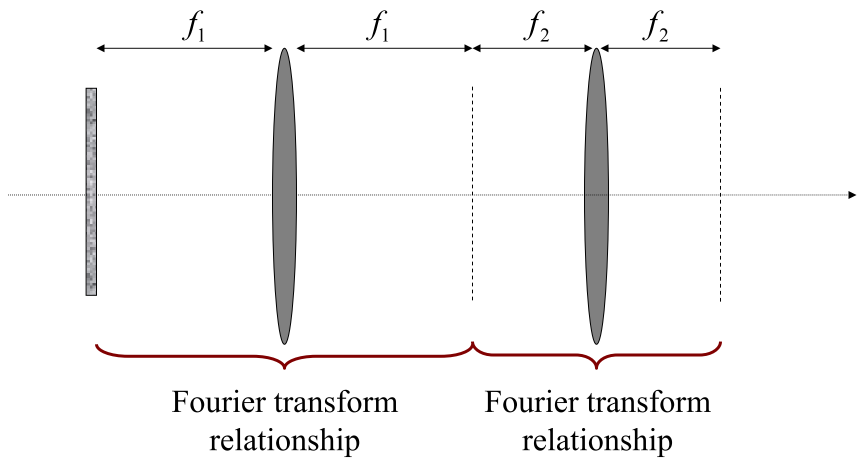
Today

Imaging with coherent light

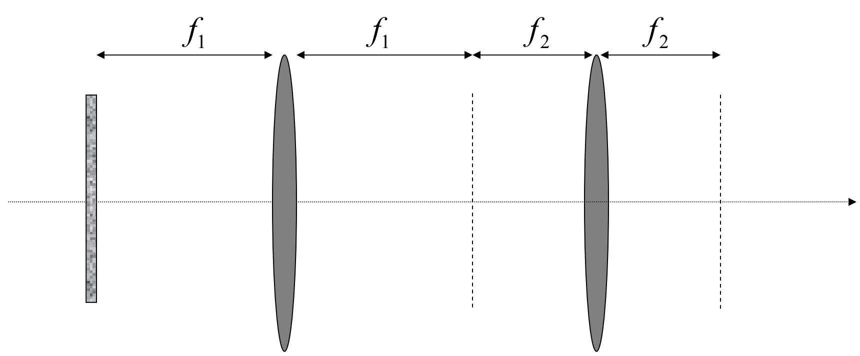
- Coherent image formation
  - space domain description: impulse response
  - spatial frequency domain description: coherent transfer function

Coherent imaging with a telescope

The 4F system



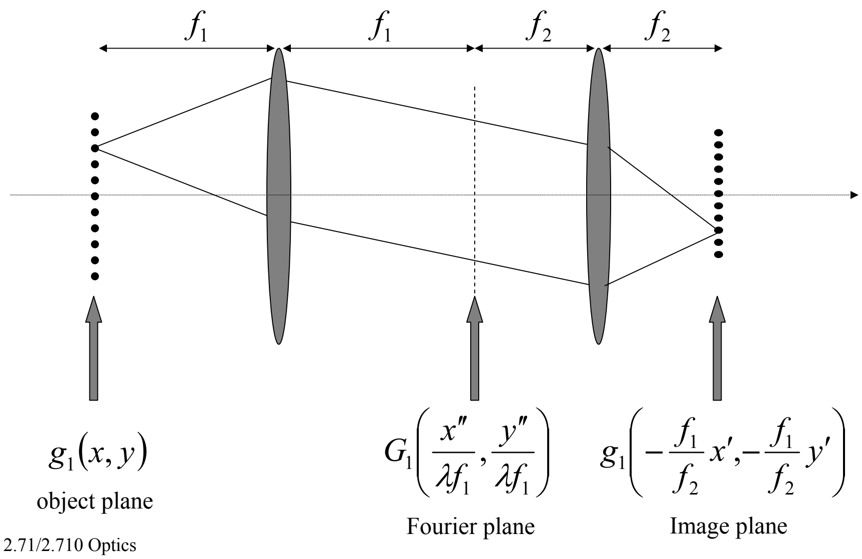
The 4F system



Theorem:

$$\Im\{\Im\{g(x,y)\}\} = g(-x,-y)$$

The 4F system



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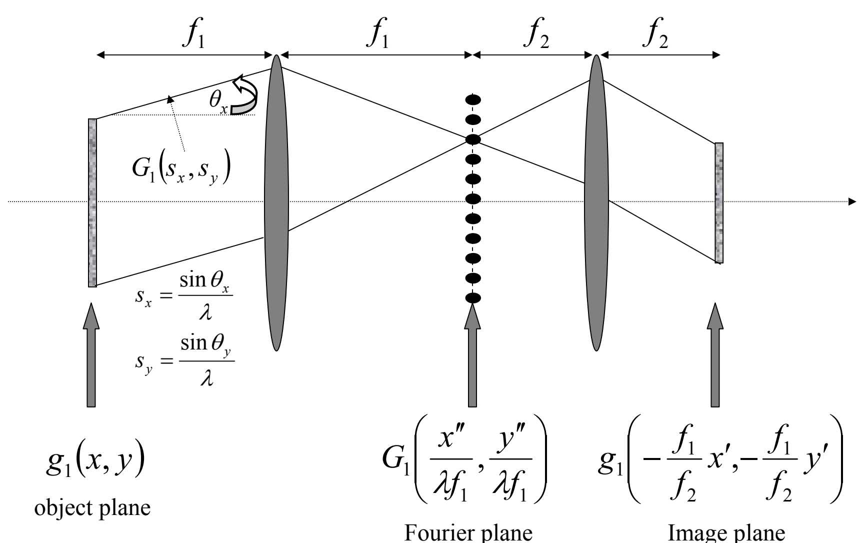
The 4F system



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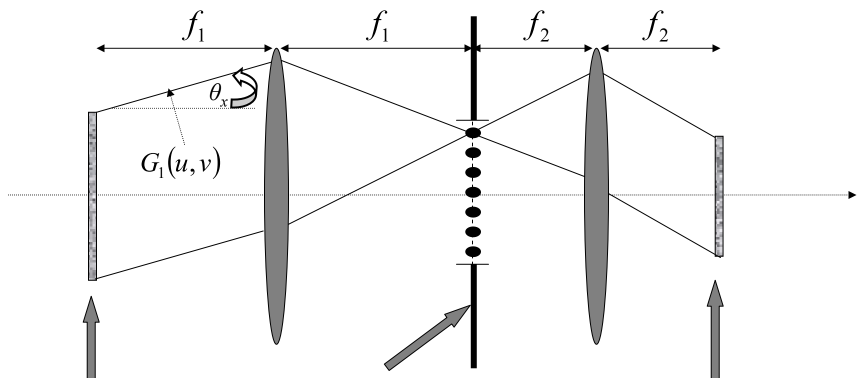
The 4F system with FP aperture



$g_1(x,y)$

object plane

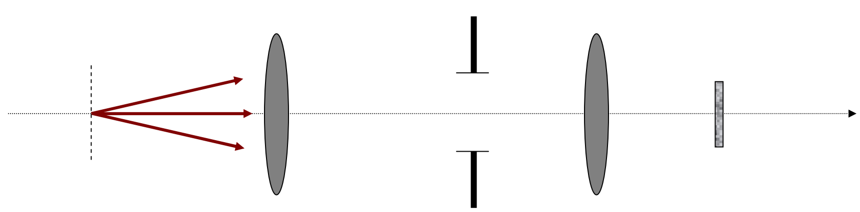
$$G_1\left(\frac{x''}{\lambda f_1},\frac{y''}{\lambda f_1}\right)\times\text{circ}\left(\frac{r''}{R}\right)$$

Fourier plane: aperture-limited

$$(g_1 * h)\left(-\frac{f_1}{f_2}x',-\frac{f_1}{f_2}y'\right)$$

Image plane: blurred
(i.e. low-pass filtered)

Impulse response & transfer function



A point source at the input plane ...

... results not in a point image but in a diffraction pattern $h(x',y')$

Point source at the origin ↔ delta function $\delta(x,y)$
$h(x',y')$ is the inpulse response of the system

More commonly, $h(x',y')$ is called the
Coherent Point Spread Function (Coherent PSF)

Coherent imaging
as a linear, shift-invariant system

Thin transparency $t(x,y)$

$g_1(x,y)$ illumination

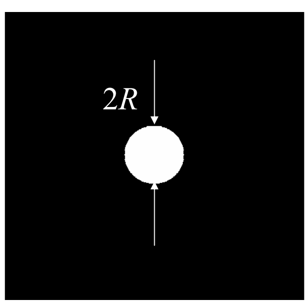
$g_2(x,y) = g_1(x,y)\,t(x,y)$ $\xrightarrow{\text{impulse response}}$ **convolution** output amplitude $g_3(x',y') = g_2(x,y) * h(x,y)$

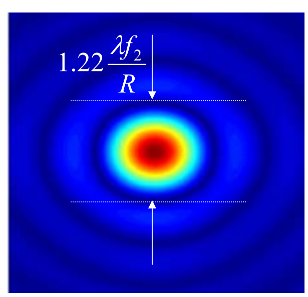
Fourier transform $\Updownarrow$ Fourier transform

(≡plane wave spectrum) $G_2(u,v)$ $\xrightarrow{\text{transfer function}}$ **multiplication** $G_3(u,v) = G_2(u,v)\,H(u,v)$

transfer function $H(u,v)$: aka pupil function

Transfer function & impulse response of circular aperture





Transfer function:
circular aperture

$$\operatorname{circ}\left(\frac{r''}{R}\right)$$

Impulse response:
Airy function

$$\operatorname{jinc}\left(\frac{r'R}{\lambda f_2}\right)$$

Coherent imaging
as a linear, shift-invariant system

Example: 4F system with ***circular*** aperture @ Fourier plane

Thin transparency $t(x,y)$

$g_1(x,y)$ ⇒ illumi nation

$$g_2(x,y) = = g_1(x,y)\,t(x,y)$$

$$h(x,y) = \mathrm{jinc}\left(\frac{r'R}{\lambda f_2}\right)$$

impulse response ⇒ **convolution**

output amplitude

$$g_3(x',y') = = g_2(x,y) * h(x,y)$$

⇕ Fourier transform

⇕ Fourier transform

(≡plane wave spectrum)

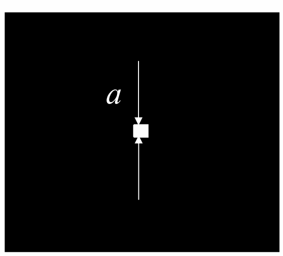
$$G_2(u,v)$$

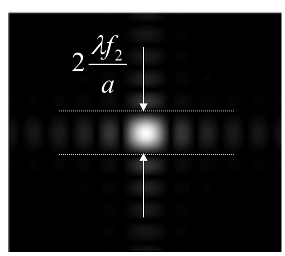
transfer function ⇒ **multiplication**

$$G_3(u,v) = = G_2(u,v)H(u,v)$$

$$H(u,v) = \mathrm{circ}\left(\frac{\lambda f\sqrt{u^2+v^2}}{R}\right)$$

Transfer function & impulse response of rectangular aperture





$$\mathrm{rect}\left(\frac{x''}{a}\right)\mathrm{rect}\left(\frac{y''}{b}\right) \qquad \mathrm{sinc}\left(\frac{x'a}{\lambda f_2}\right)\mathrm{sinc}\left(\frac{y'b}{\lambda f_2}\right)$$

Coherent imaging

as a linear, shift-invariant system

Example: 4F system with *rectangular* aperture @ Fourier plane

Thin transparency $t(x,y)$

$g_1(x,y)$ ⇒ illumi nation ⇒ $g_2(x,y) = g_1(x,y)\,t(x,y)$

$$h(x,y)=\mathrm{sinc}\left(\frac{x'a}{\lambda f_2}\right)\mathrm{sinc}\left(\frac{y'b}{\lambda f_2}\right)$$

impulse response ⇒ **convolution**

output amplitude $g_3(x',y') = g_2(x,y) * h(x,y)$

⇕ Fourier transform ⇕ Fourier transform

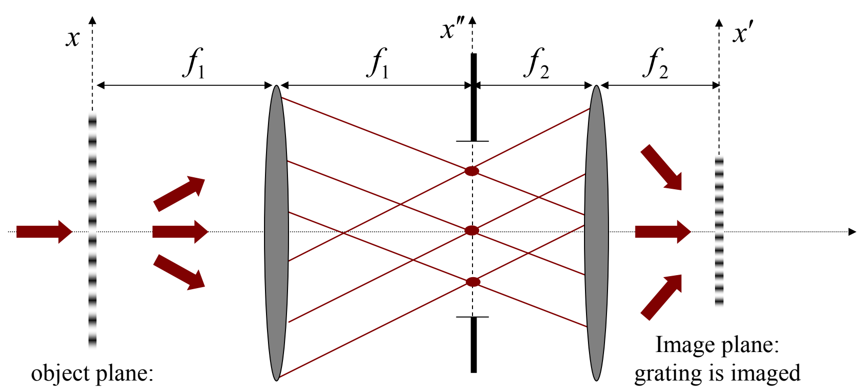
(≡plane wave spectrum) $G_2(u,v)$

transfer function ⇒ **multiplication**

$G_3(u,v) = G_2(u,v)\,H(u,v)$

$$H(u,v)=\mathrm{rect}\left(\frac{\lambda f u}{a}\right)\mathrm{rect}\left(\frac{\lambda f v}{b}\right)$$

Aperture–limited spatial filtering



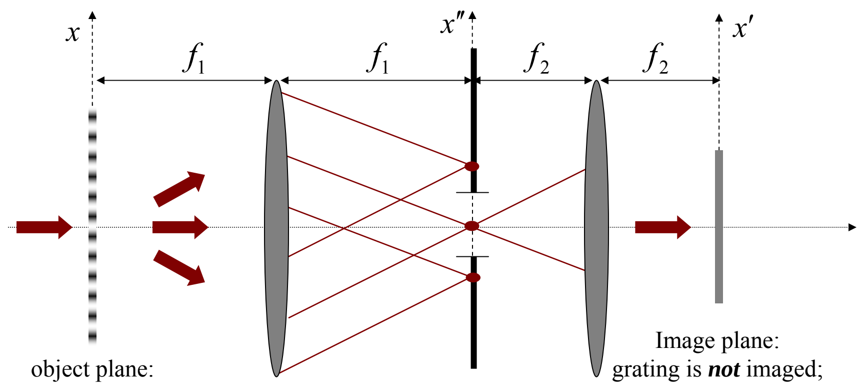
object plane: grating generates one spatial frequency

Fourier plane: aperture unlimited

(all orders pass)

Image plane: grating is imaged with lateral de-magnification

Aperture–limited spatial filtering



Spatial frequency clipping

field after input transparency

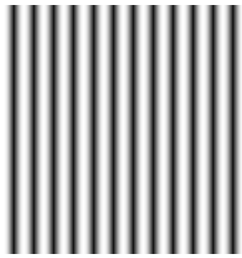
$$g_{in}(x) = \frac{1}{2}\left[1+\cos(2\pi u_0 x)\right] \Rightarrow G_{in}(u) = \frac{1}{2}\left[\delta(u) + \frac{1}{2}\delta(u-u_0) + \frac{1}{2}\delta(u+u_0)\right]$$

field before filter $\quad g_{f-}(x'') = \frac{1}{2}\left[\delta(x'') + \cancel{\frac{1}{2}\delta(x''-\lambda f_1 u_0)} + \cancel{\frac{1}{2}\delta(x''+\lambda f_1 v_0)}\right]$

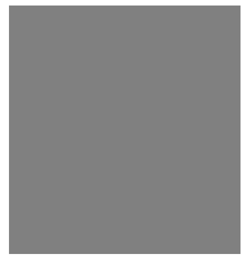
field after filter $\quad g_{f+}(x'') = \frac{1}{2}\delta(x'')$

field at output
(image plane) $\quad G_{out}(u) = \frac{1}{2}\delta(u) \Rightarrow g_{out}(x') = \frac{1}{2}$

Effect of spatial filtering



Fourier plane filter
with circ-aperture

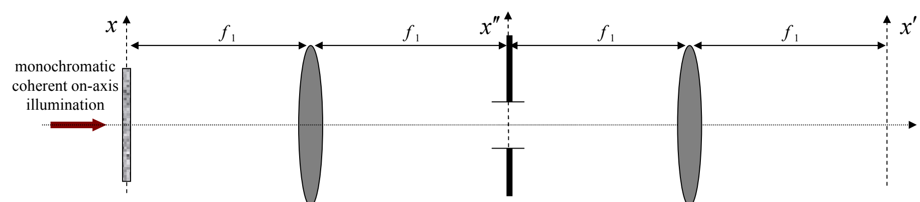


Original object
(sinusoidal spatial variation,
i.e. grating)

Frequency-filtered image
(spatial variation blurred out,
only average survives)

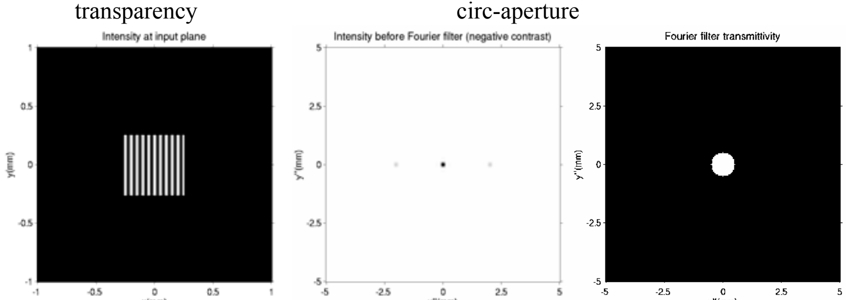
Spatial frequency clipping

f_1=20cm
λ=0.5μm

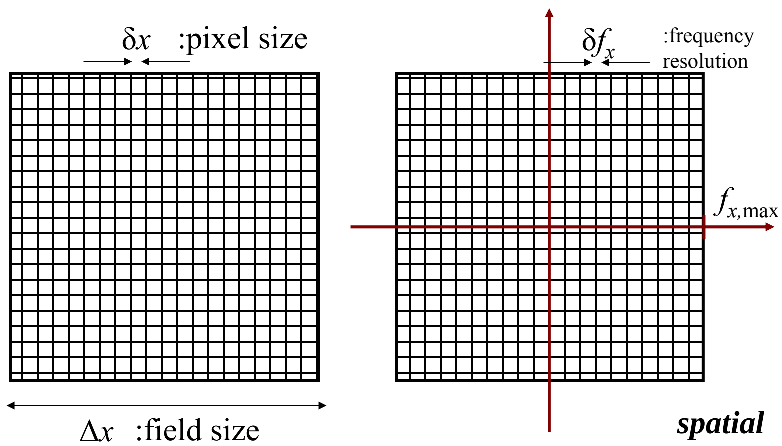


object plane
transparency

Fourier plane
circ-aperture



Space-Fourier coordinate transformations

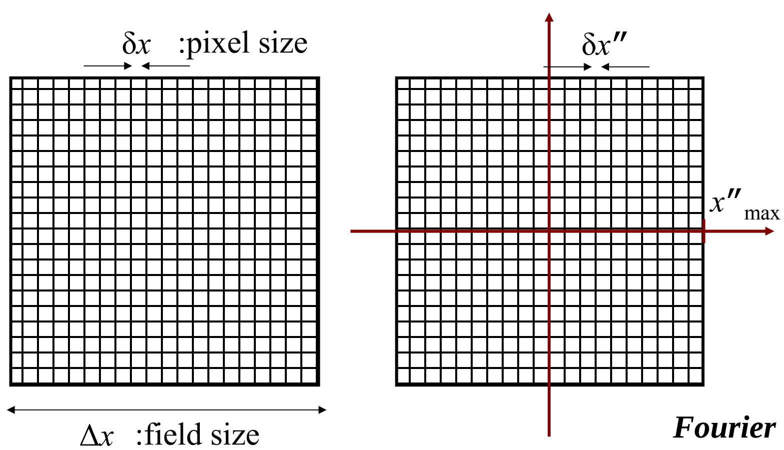


space domain

spatial frequency domain

Nyquist relationships: $f_{x,\max} = \dfrac{1}{2\delta x}$ $\quad \delta f_x = \dfrac{1}{2\Delta x}$

4F coordinate transformations



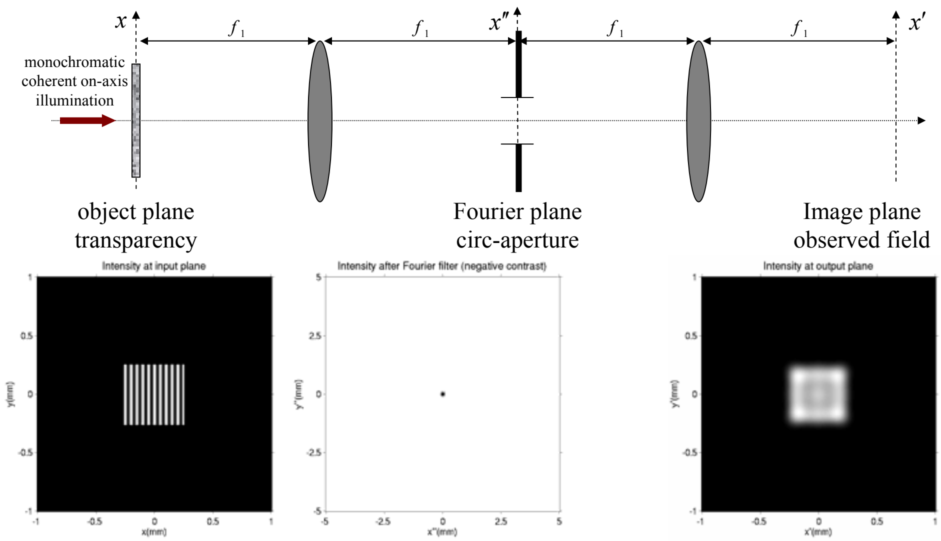
space domain

Fourier plane

Nyquist relationships: $x''_{\max} = \dfrac{\lambda f}{2\delta x}$ $\quad \delta x'' = \dfrac{\lambda f}{2\Delta x}$

Spatial frequency clipping

f_1=20cm
λ=0.5μm



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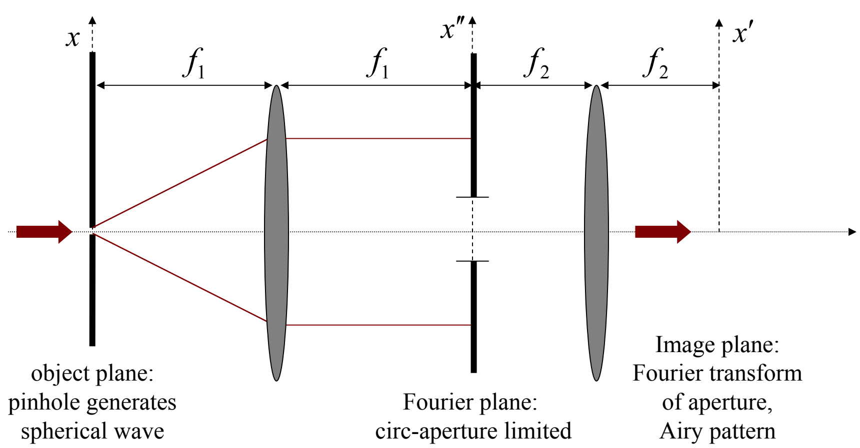
Formation of the impulse response



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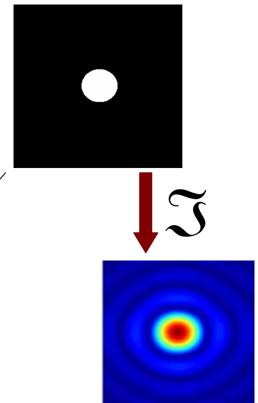
Low–pass filtering

field after input transparency

$$g_{\text{in}}(x,y)=\delta(x)\delta(y)\Rightarrow G_{\text{in}}(u,v)=1$$



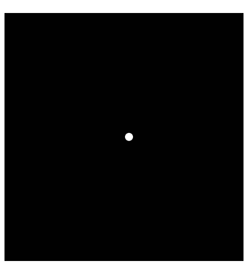
field before filter $\quad g_{\text{f}-}(x'',y'')=1$

field after filter $\quad g_{\text{f}+}(x'',y'')=\text{circ}\left(\dfrac{\sqrt{x''^2+y''^2}}{R}\right)$

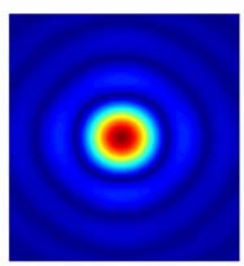
field at output (image plane) $\quad G_{\text{out}}(u,v)=\text{circ}\left(\dfrac{\lambda f\sqrt{u^2+v^2}}{R}\right)\Rightarrow$

$$g_{\text{out}}(x',y')\propto \text{jinc}\left(\frac{R\sqrt{x'^2+y'^2}}{\lambda f}\right)$$ (Airy pattern)

Effect of spatial filtering



Fourier plane filter with circ-aperture

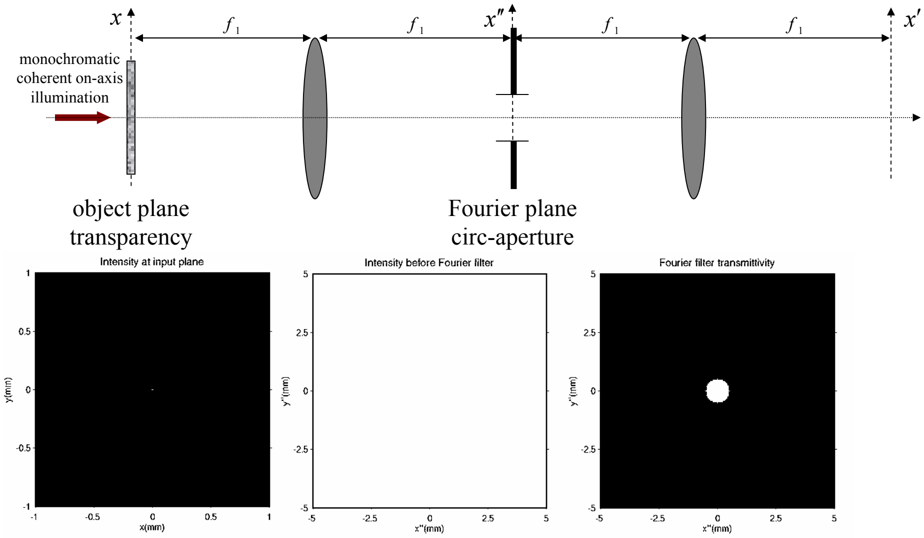


Original object
(small pinhole ⇔ impulse, generating spherical wave past the transparency)

Impulse reponse
(aka point-spread function, original point has blurred to an Airy pattern, or jinc)

Low–pass filtering the impulse

f_1=20cm
λ=0.5μm



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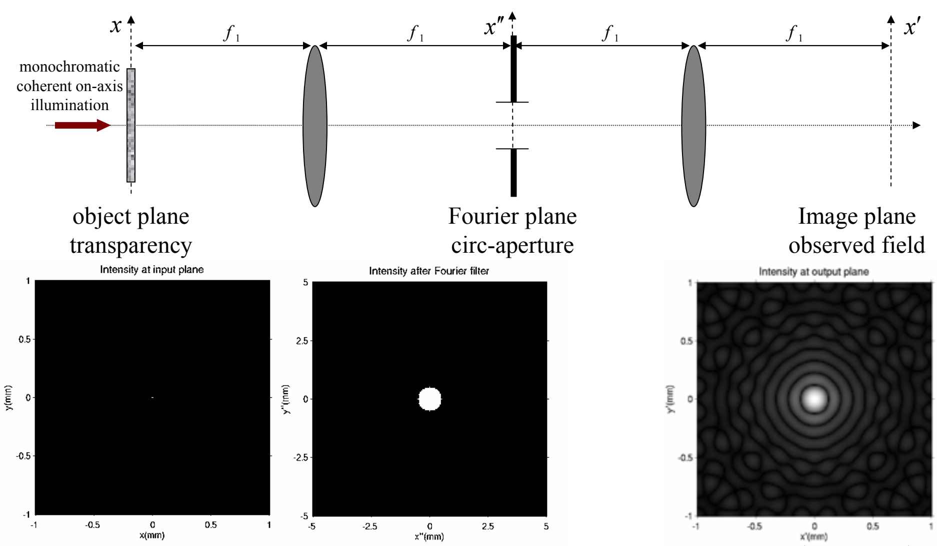
Spatial frequency clipping

f_1=20cm
λ=0.5μm



note: pseudo-accentuated sidelobes

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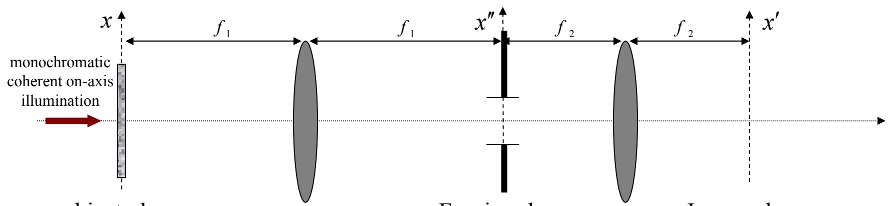
Low-pass filtering with the 4F system



object plane
transparency
$g_{\text{in}}(x,y)$

Fourier plane
circ-aperture

$$H(x'',y'')=\operatorname{circ}\left(\frac{\sqrt{x''^2+y''^2}}{R}\right)$$

Image plane
observed field

$$(g_{\text{in}} * \operatorname{jinc})\left(-\frac{f_1}{f_2}x',-\frac{f_1}{f_2}y'\right)$$

$\Im$ Fourier transform

field arriving at Fourier plane $G_{\text{in}}\left(\frac{x''}{\lambda f_1},\frac{y''}{\lambda f_1}\right)$

field departing from Fourier plane $G_{\text{in}}\left(\frac{x''}{\lambda f_1},\frac{y''}{\lambda f_1}\right)\times\operatorname{circ}\left(\frac{\sqrt{x''^2+y''^2}}{R}\right)$

$\Im$ Fourier transform

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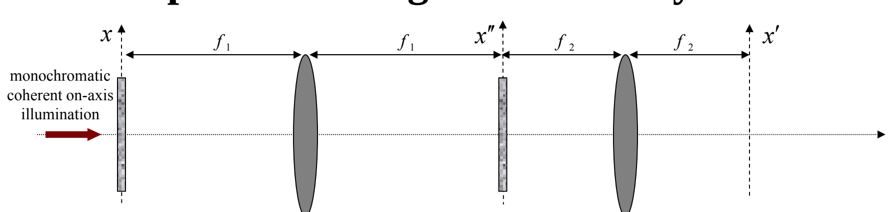
Spatial filtering with the 4F system



object plane
transparency
$g_{\text{in}}(x,y)$

Fourier plane
transparency
$H(x'',y'')$

Image plane
observed field

$$(g_{\text{in}} * h)\left(-\frac{f_1}{f_2}x',-\frac{f_1}{f_2}y'\right)$$

$\Im$ Fourier transform

field arriving at Fourier plane $G_{\text{in}}\left(\frac{x''}{\lambda f_1},\frac{y''}{\lambda f_1}\right)$

field departing from Fourier plane $G_{\text{in}}\left(\frac{x''}{\lambda f_1},\frac{y''}{\lambda f_1}\right)\times H(x'',y'')$

$\Im$ Fourier transform

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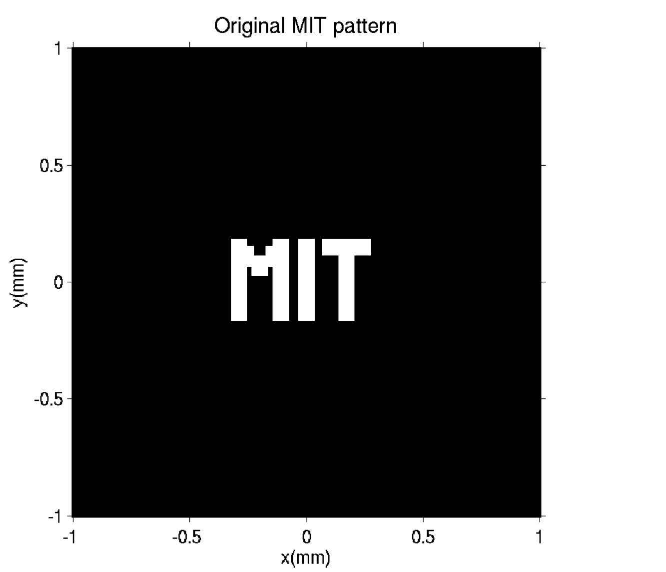
Examples: the amplitude MIT pattern



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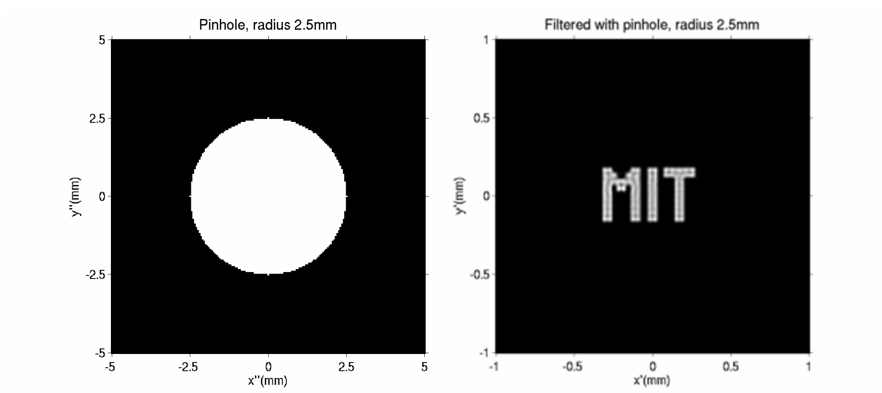
Weak low–pass filtering



f_1=20cm
λ=0.5μm

Fourier filter

Intensity @ image plane

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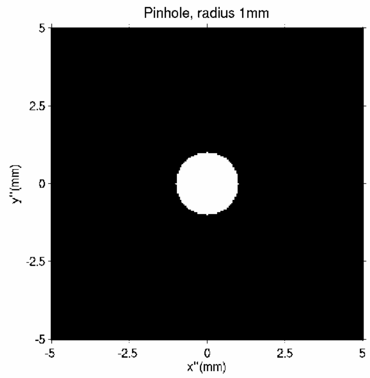
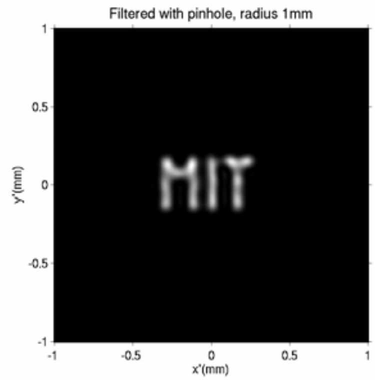
Moderate low–pass filtering

(*aka* blurring)





f_1=20cm
λ=0.5µm

Fourier filter

Intensity @ image plane

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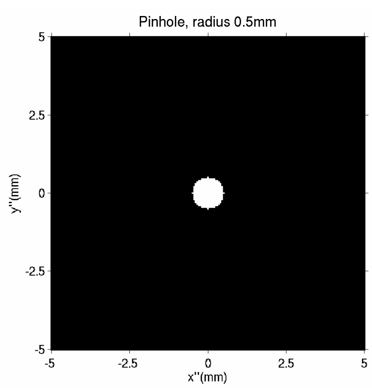
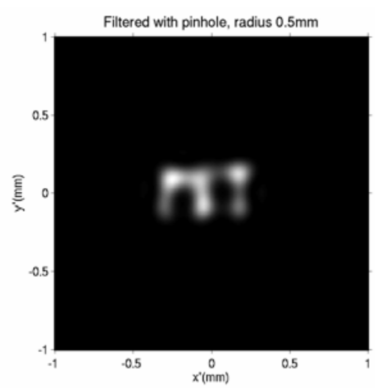
Strong low–pass filtering





f_1=20cm
λ=0.5µm

Fourier filter

Intensity @ image plane

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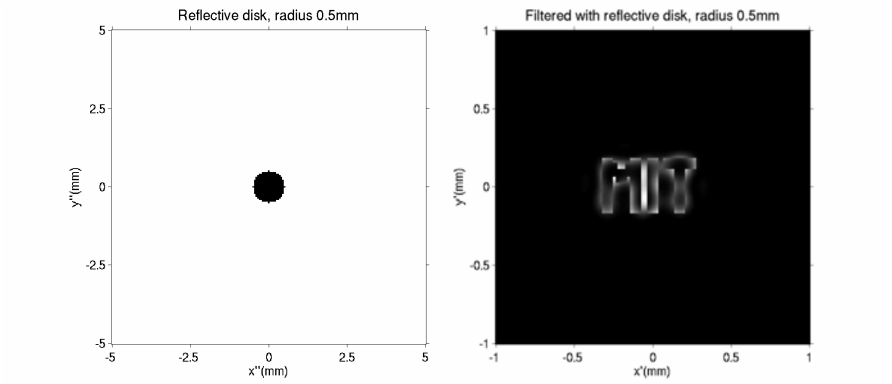
Moderate high–pass filtering



f_1=20cm
λ=0.5μm

Fourier filter

Intensity @ image plane

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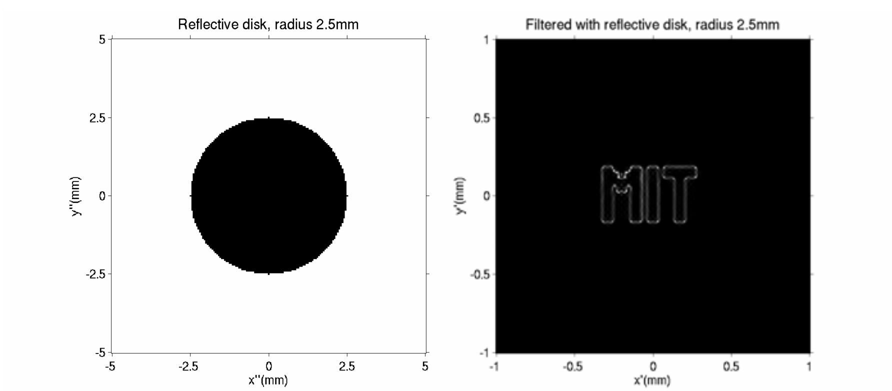
Strong high–pass filtering

(*aka* edge enhancement)



f_1=20cm
λ=0.5μm

Fourier filter

Intensity @ image plane

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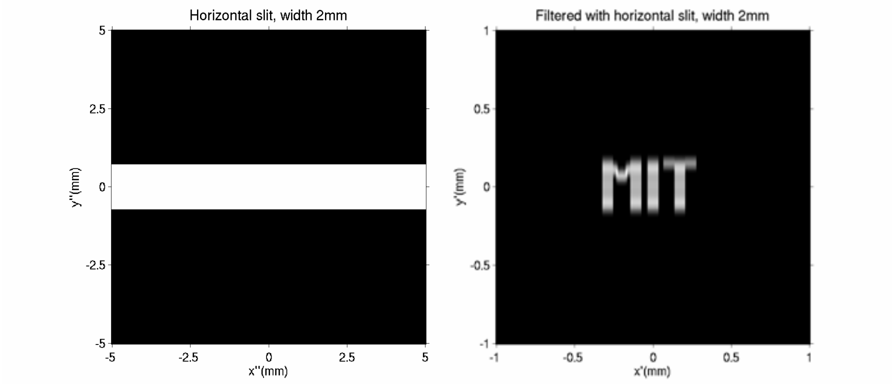
1-dimensional blurring



f_1=20cm
λ=0.5µm

Fourier filter

Intensity @ image plane

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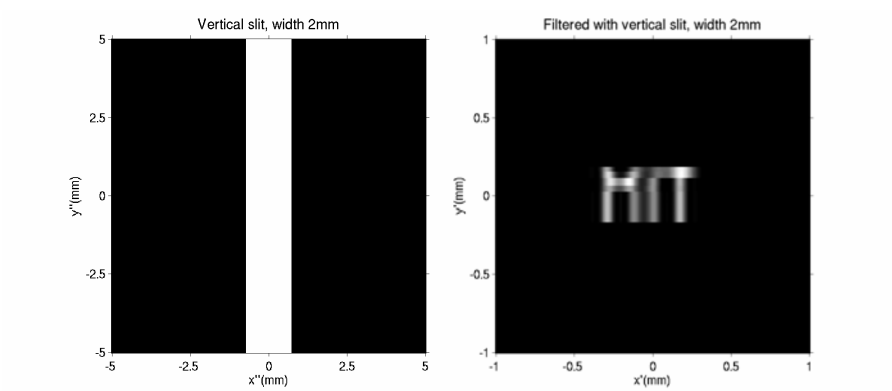
1-dimensional blurring



f_1=20cm
λ=0.5µm

Fourier filter

Intensity @ image plane

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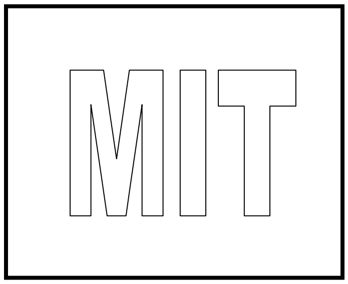
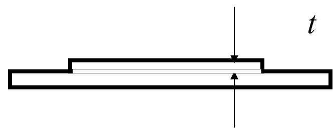
Phase objects



glass plate
(transparent)



protruding part
phase-shifts
coherent illumination
by amount $\varphi=2\pi(n-1)t/\lambda$

Often useful in imaging biological objects (cells, etc.)

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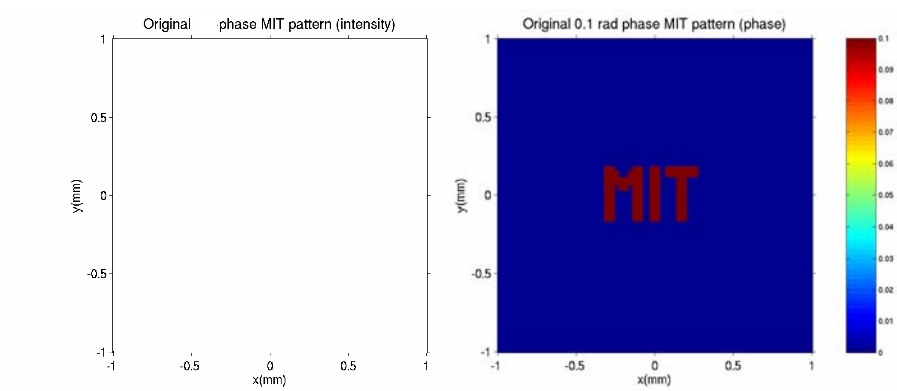
Viewing phase objects



Intensity
(object is invisible)

Amplitude
(need interferometer)

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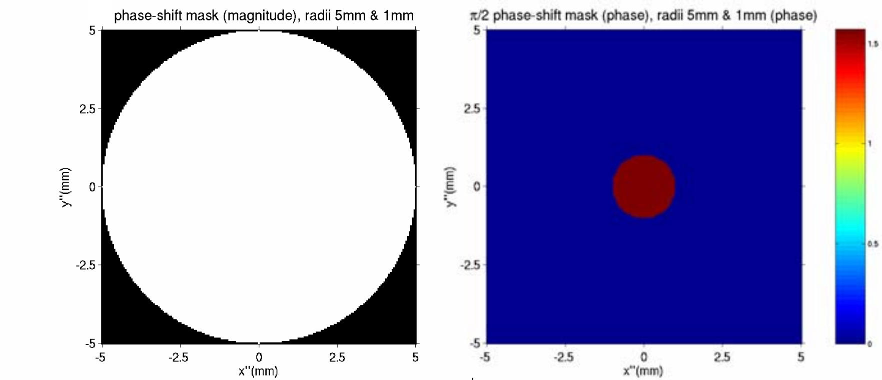
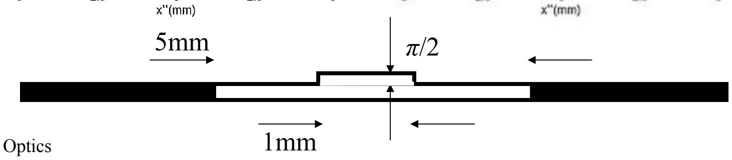
Zernicke phase-shift mask





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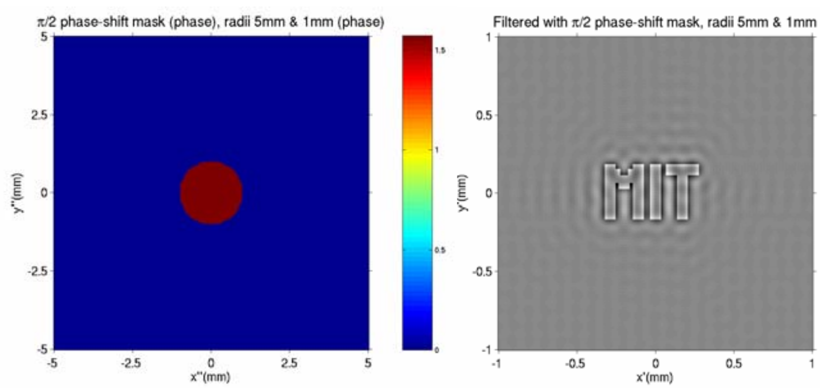
Imaging with Zernicke mask



f_1=20cm
λ=0.5μm

Fourier filter

Intensity @ image plane

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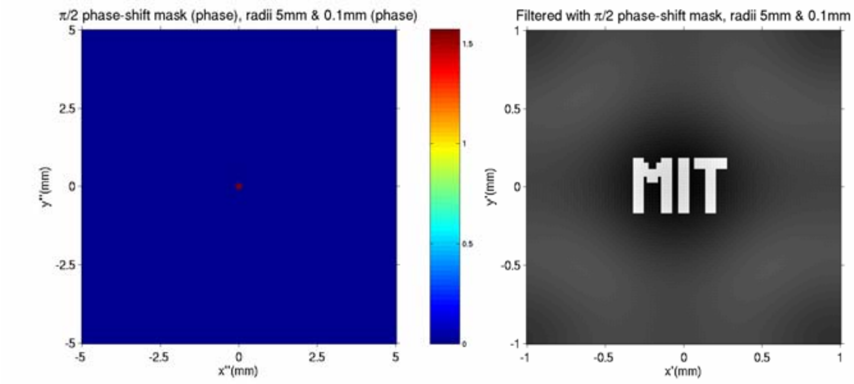
Imaging with Zernicke mask



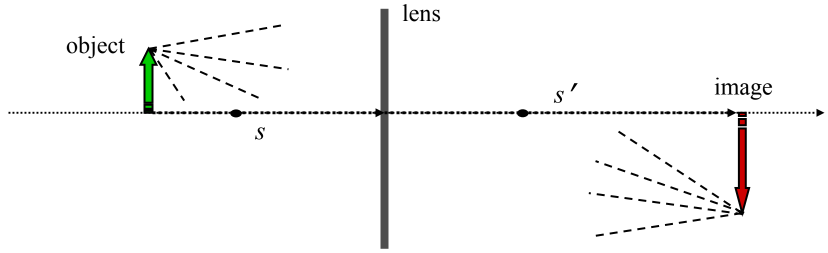
f_1=20cm
λ=0.5μm

Fourier filter

Intensity @ image plane

Coherent imaging with a single lens

Single-lens imaging condition



$$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$$

Imaging condition (*aka* Lens Law)

$$m_{\text{lateral}}=-\frac{s'}{s}$$

Magnification

Derivation using wave optics ?!?

MIT 2.71/2.710 Optics
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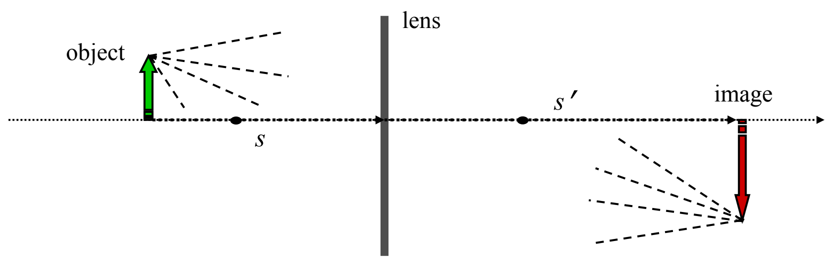
Single-lens imaging system



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Single-lens imaging system

Impulse response (PSF)

Ideal PSF: $$h(x,y;x',y') = \delta(x'-mx)\delta(y'-my)$$

Diffraction-limited PSF: $$h(x,y;x',y') = \text{jinc}\left(\frac{R}{\lambda}\sqrt{\left(\frac{x'}{s'}-\frac{x}{s}\right)^2+\left(\frac{y'}{s'}-\frac{y}{s}\right)^2}\right)$$