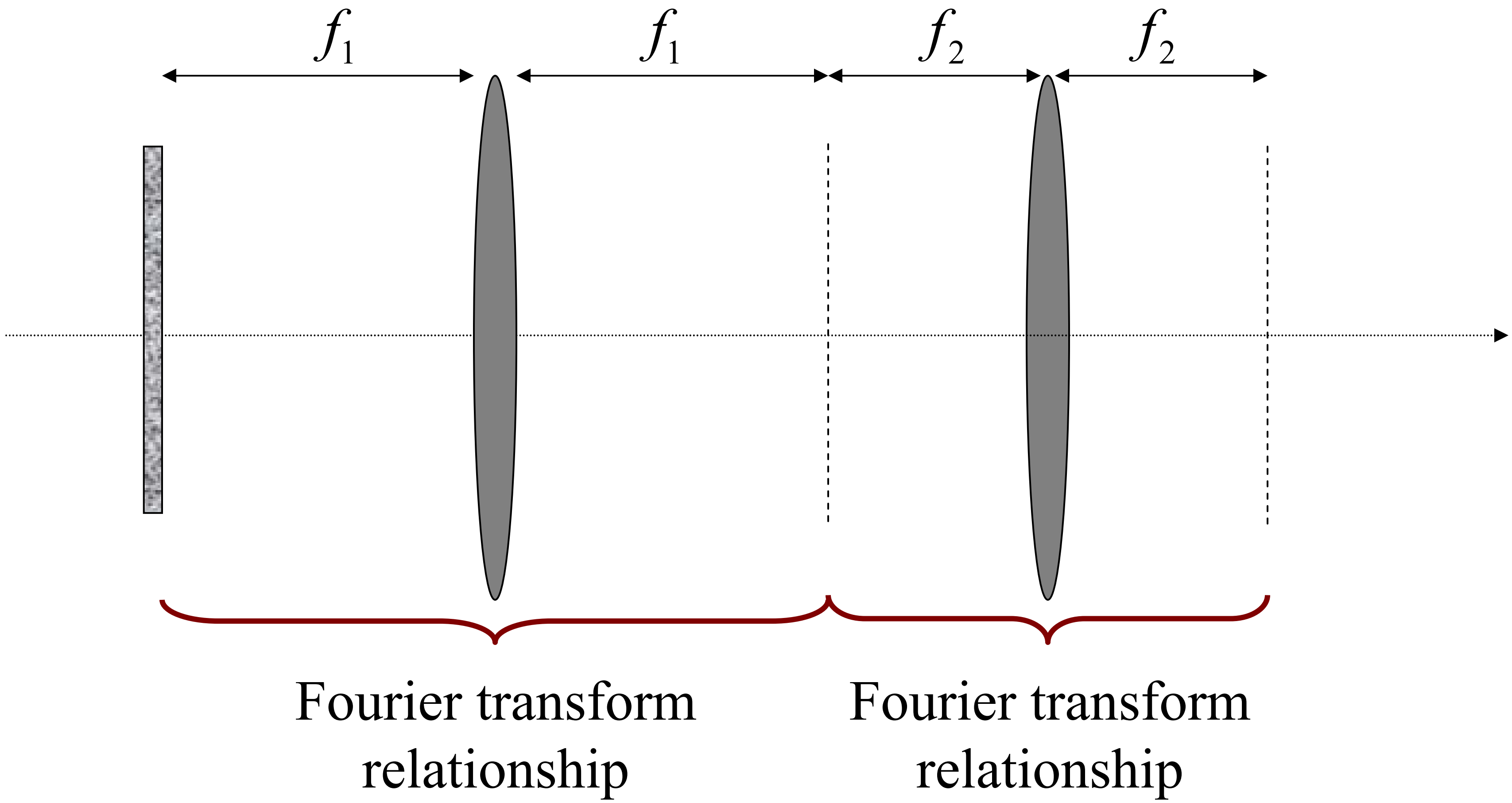
Today

Imaging with coherent light

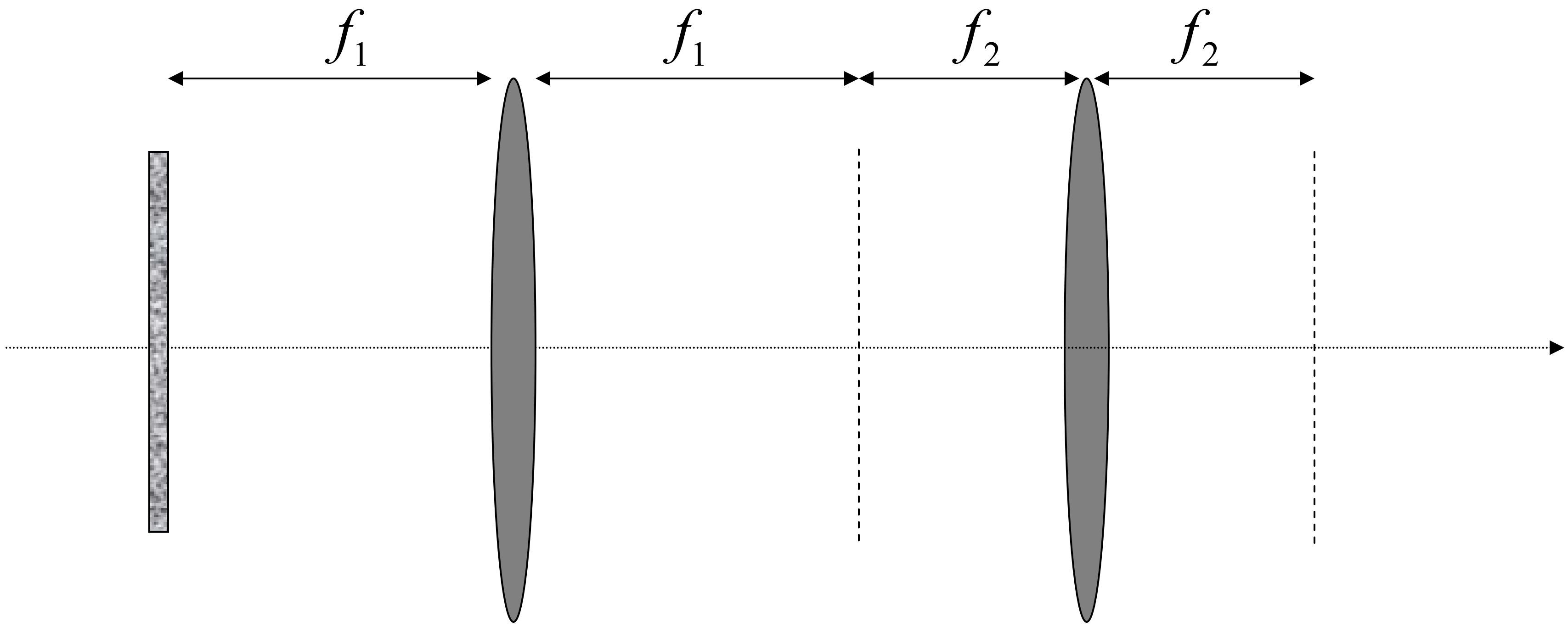
- Coherent image formation
  - space domain description: impulse response
  - spatial frequency domain description: coherent transfer function

Coherent imaging with a telescope

The 4F system



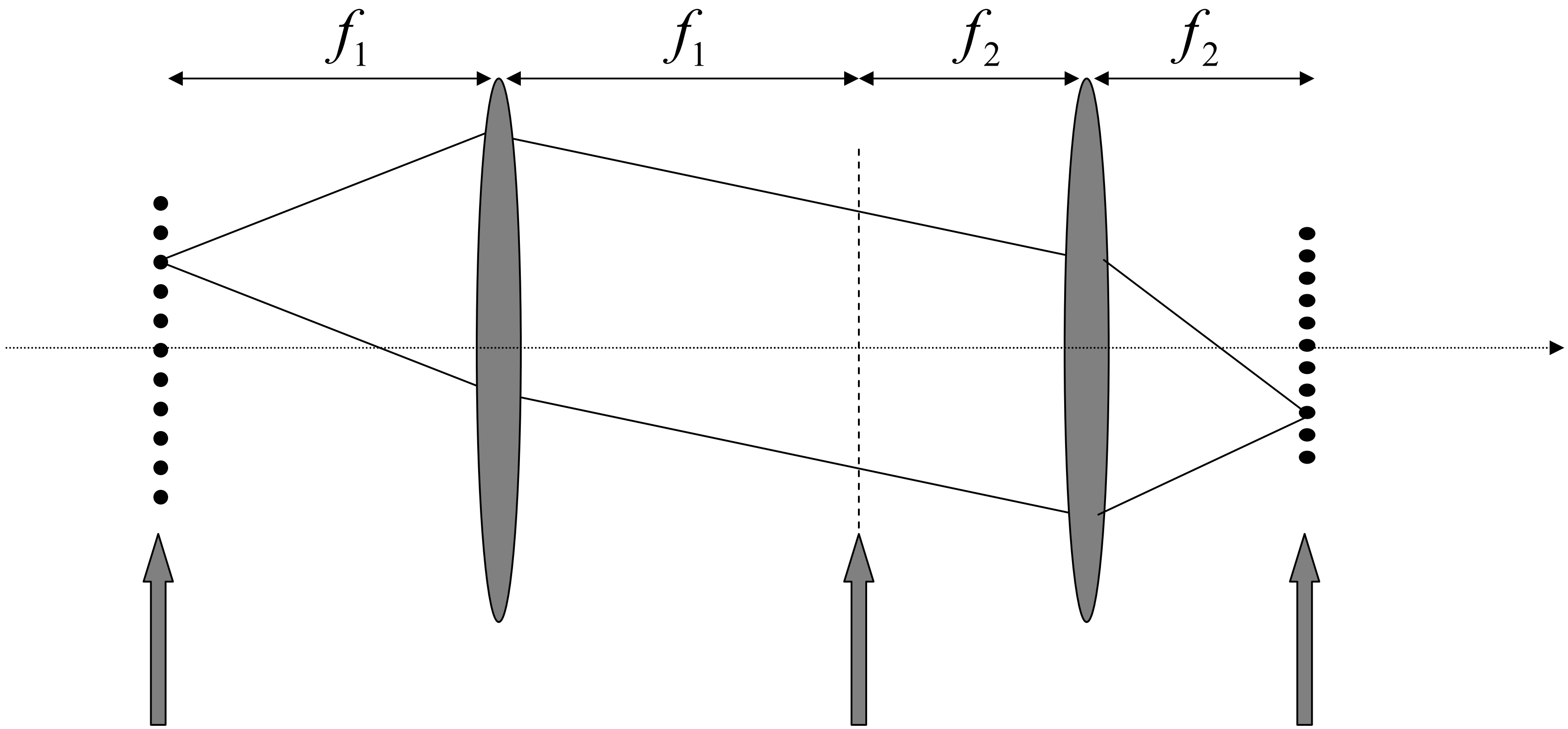
The 4F system



Theorem:

$$\Im\{\Im\{g(x,y)\}\} = g(-x,-y)$$

The 4F system



$g_1(x, y)$

object plane

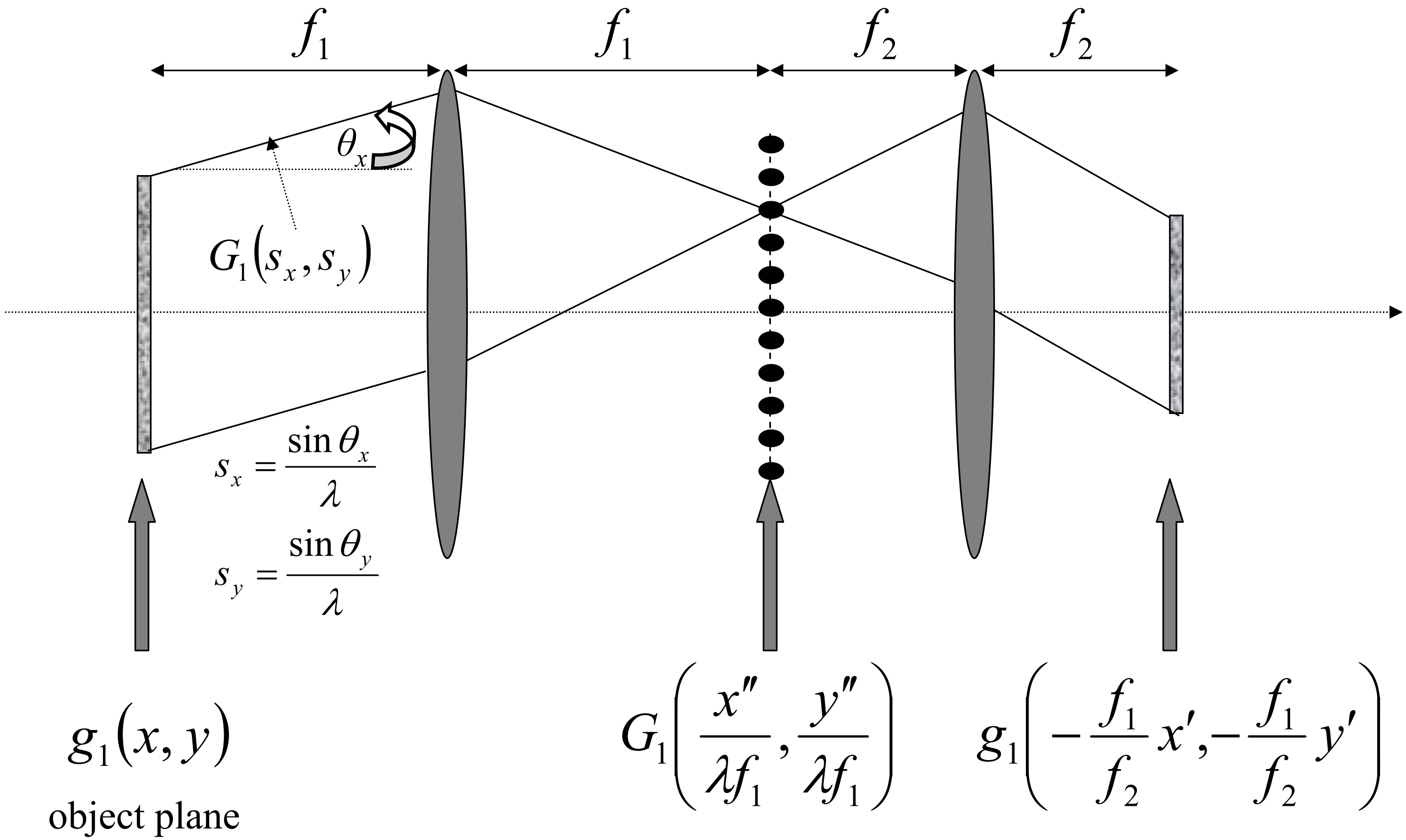
$$G_1\left(\frac{x''}{\lambda f_1}, \frac{y''}{\lambda f_1}\right)$$

Fourier plane

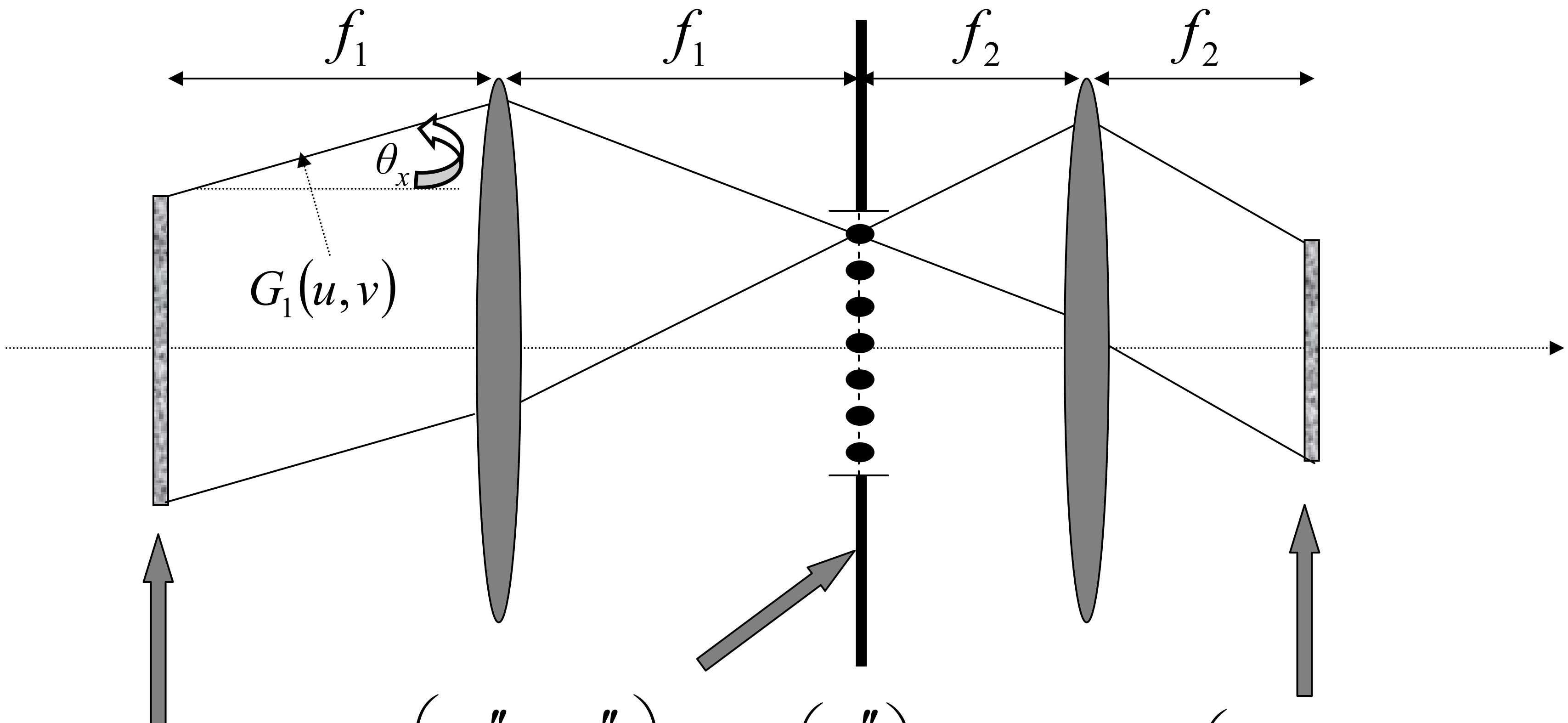
$$g_1\left(-\frac{f_1}{f_2}x', -\frac{f_1}{f_2}y'\right)$$

Image plane

The 4F system



The 4F system with FP aperture



$$g_1(x,y)$$

object plane

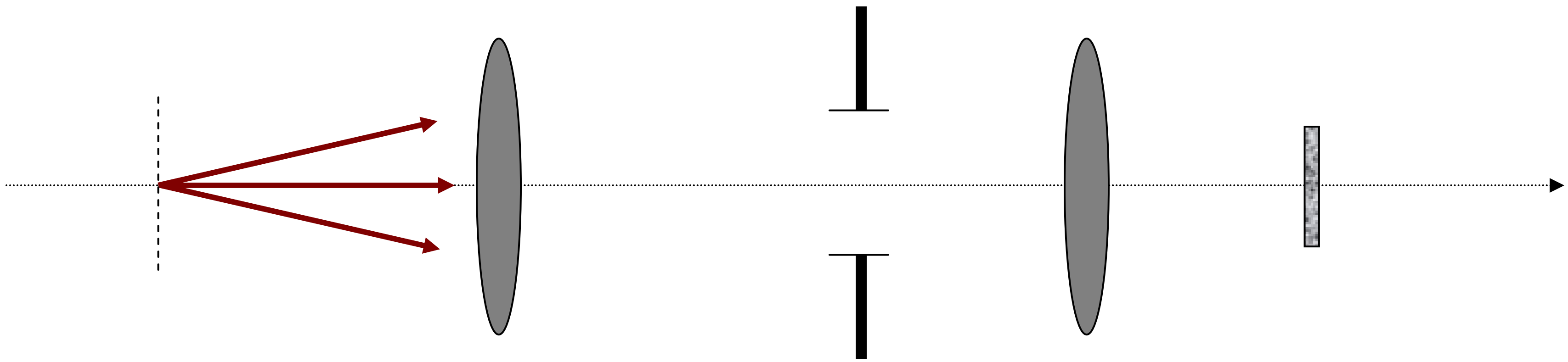
$$G_1\left(\frac{x''}{\lambda f_1},\frac{y''}{\lambda f_1}\right)\times \mathrm{circ}\left(\frac{r''}{R}\right)$$

Fourier plane: aperture-limited

$$(g_1 * h)\left(-\frac{f_1}{f_2}x',-\frac{f_1}{f_2}y'\right)$$

Image plane: blurred
(i.e. low-pass filtered)

Impulse response & transfer function



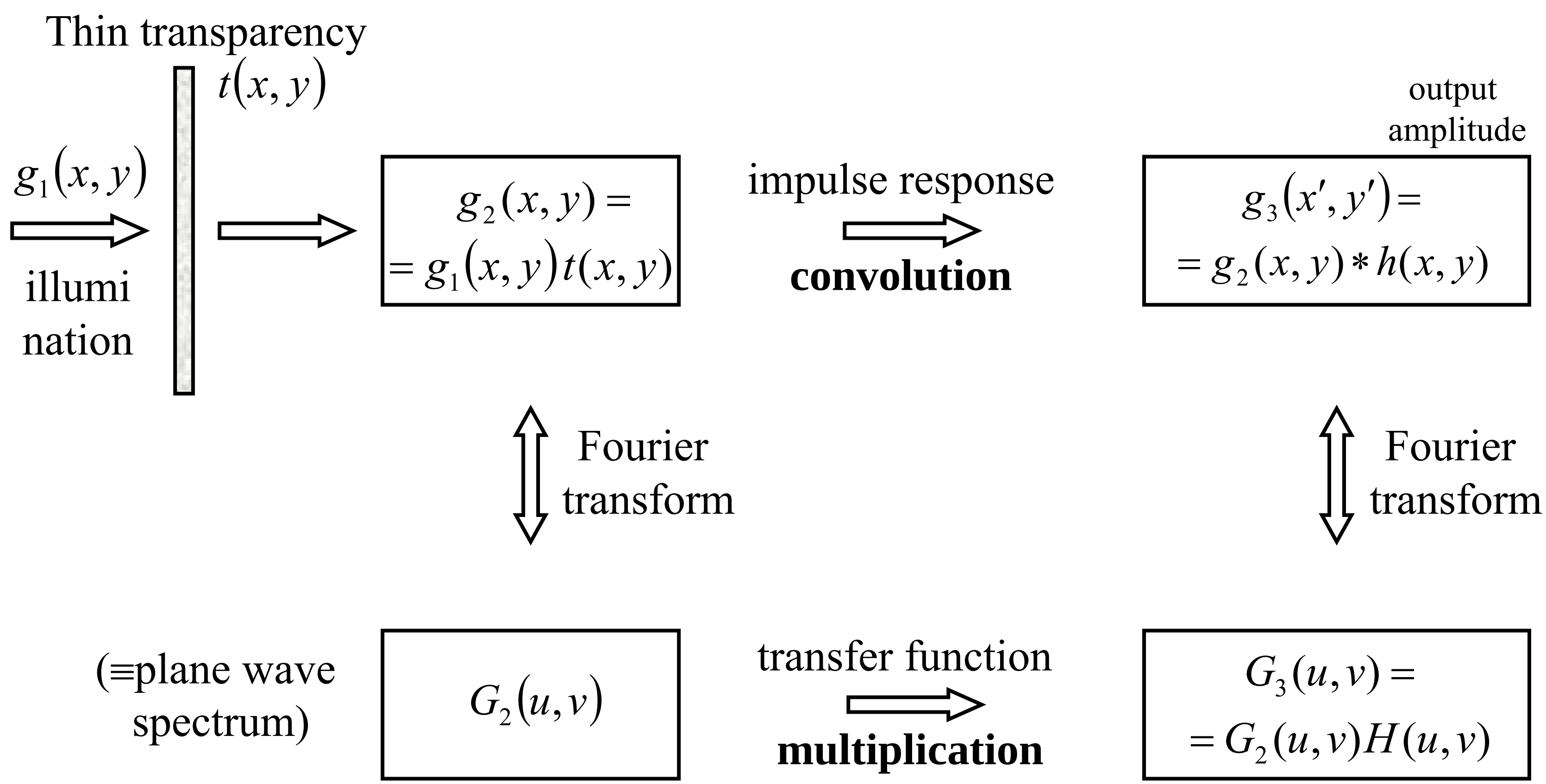
A point source at the input plane ...

... results not in a point image but in a diffraction pattern $h(x',y')$

Point source at the origin $\leftrightarrow$ delta function $\delta(x,y)$
$h(x',y')$ is the inpulse response of the system

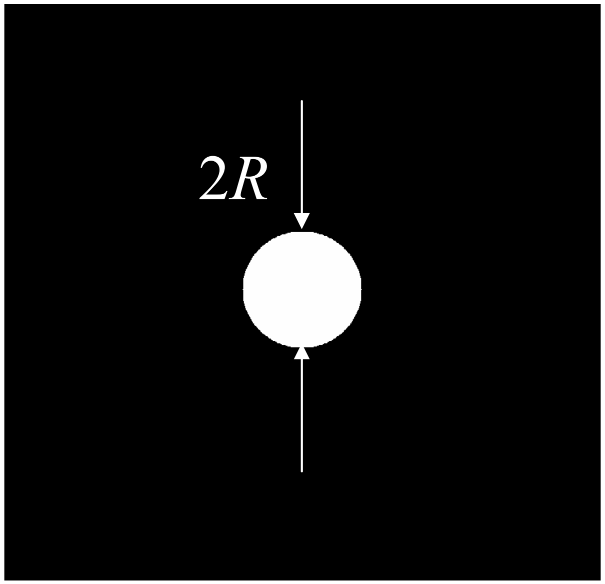
More commonly, $h(x',y')$ is called the
Coherent Point Spread Function (Coherent PSF)

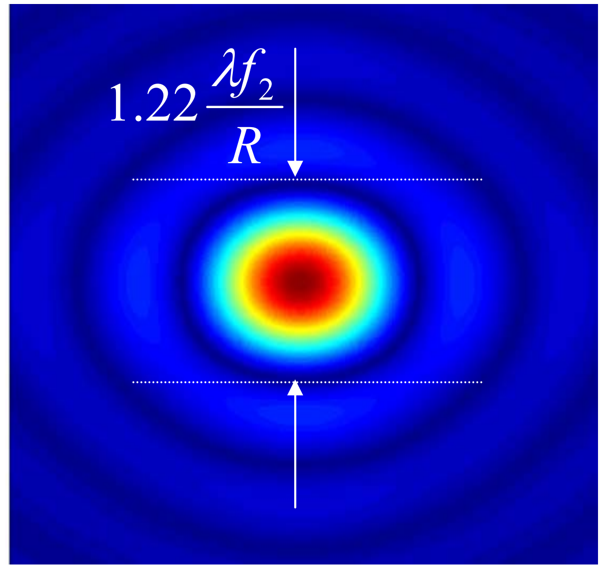
Coherent imaging
as a linear, shift-invariant system



transfer function $H(u,v)$: aka pupil function

Transfer function & impulse response of circular aperture





Transfer function:
circular aperture

$$\mathrm{circ}\left(\frac{r''}{R}\right)$$

Impulse response:
Airy function

$$\mathrm{jinc}\left(\frac{r'R}{\lambda f_2}\right)$$

Coherent imaging
as a linear, shift-invariant system

Example: 4F system with *circular* aperture @ Fourier plane

Thin transparency $t(x,y)$

$g_1(x,y)$

illumi
nation

$$g_2(x,y) = g_1(x,y)\,t(x,y)$$

$$h(x,y) = \mathrm{jinc}\left(\frac{r'R}{\lambda f_2}\right)$$

impulse response

convolution

output amplitude

$$g_3(x',y') = g_2(x,y) * h(x,y)$$

Fourier transform

Fourier transform

(≡plane wave spectrum)

$$G_2(u,v)$$

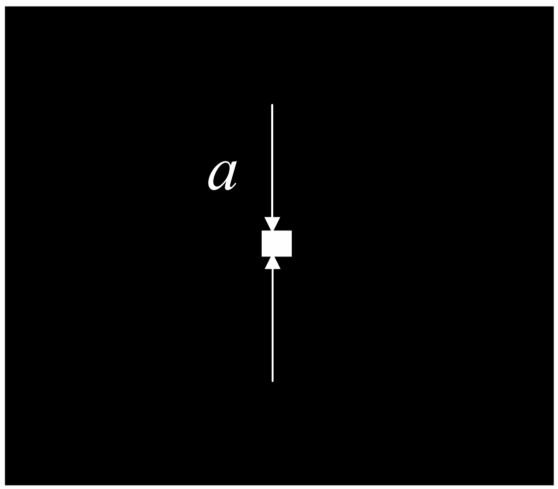
transfer function

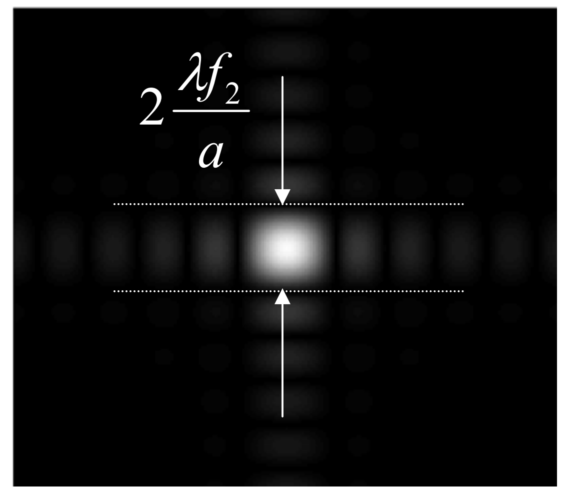
multiplication

$$G_3(u,v) = G_2(u,v)\,H(u,v)$$

$$H(u,v) = \mathrm{circ}\left(\frac{\lambda f\sqrt{u^2+v^2}}{R}\right)$$

Transfer function & impulse response of rectangular aperture





$$\mathrm{rect}\left(\frac{x''}{a}\right)\mathrm{rect}\left(\frac{y''}{b}\right)$$

$$\mathrm{sinc}\left(\frac{x'a}{\lambda f_2}\right)\mathrm{sinc}\left(\frac{y'b}{\lambda f_2}\right)$$

Coherent imaging
as a linear, shift-invariant system

<u>Example: 4F system with *rectangular* aperture @ Fourier plane</u>

Thin transparency $t(x,y)$

$g_1(x,y)$ illumination

$$g_2(x,y) = g_1(x,y)\,t(x,y)$$

$$h(x,y) = \mathrm{sinc}\left(\frac{x'a}{\lambda f_2}\right)\mathrm{sinc}\left(\frac{y'b}{\lambda f_2}\right)$$

impulse response

convolution

output amplitude

$$g_3(x',y') = g_2(x,y) * h(x,y)$$

Fourier transform

Fourier transform

(≡plane wave spectrum)

$$G_2(u,v)$$

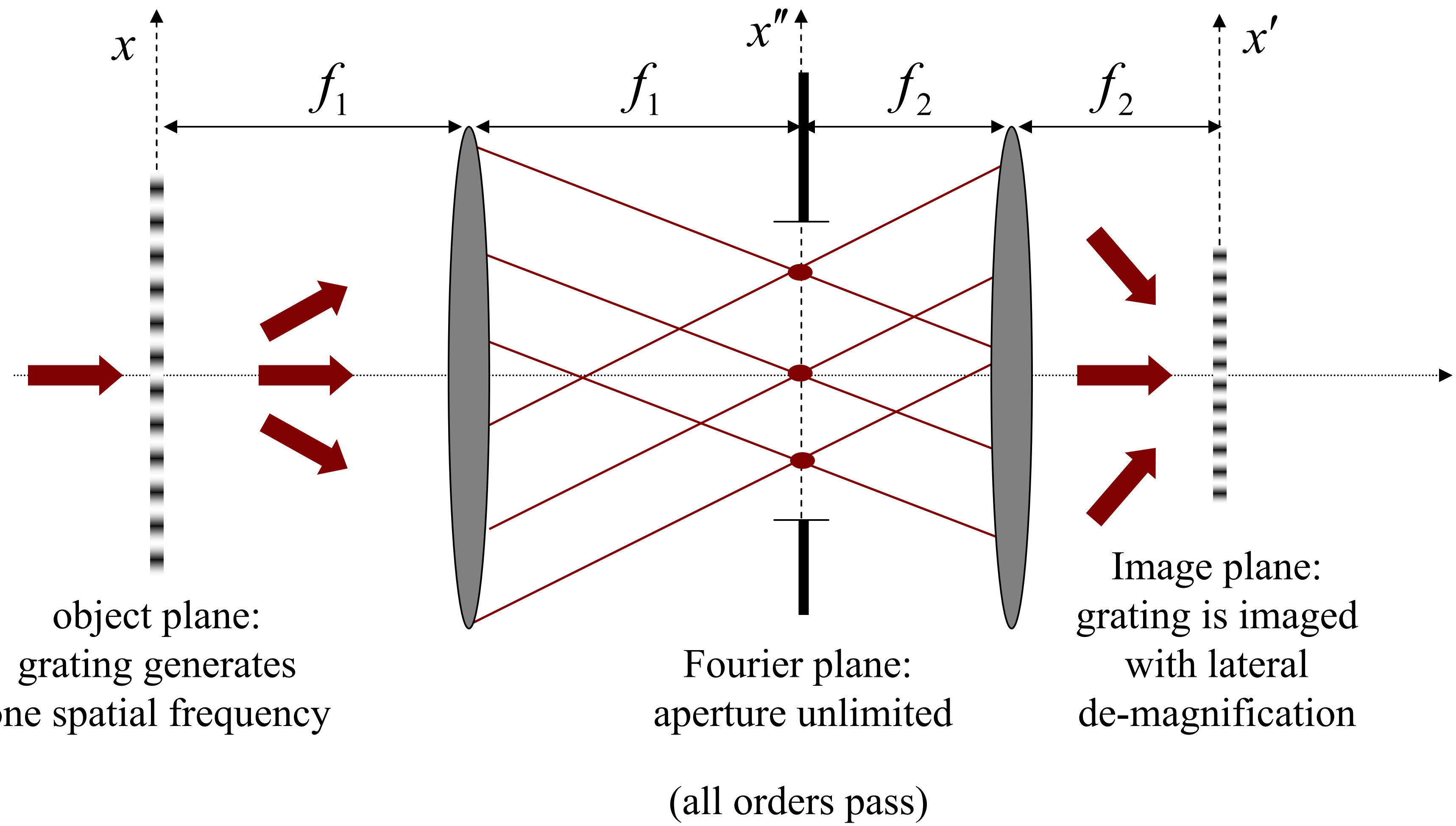
transfer function

multiplication

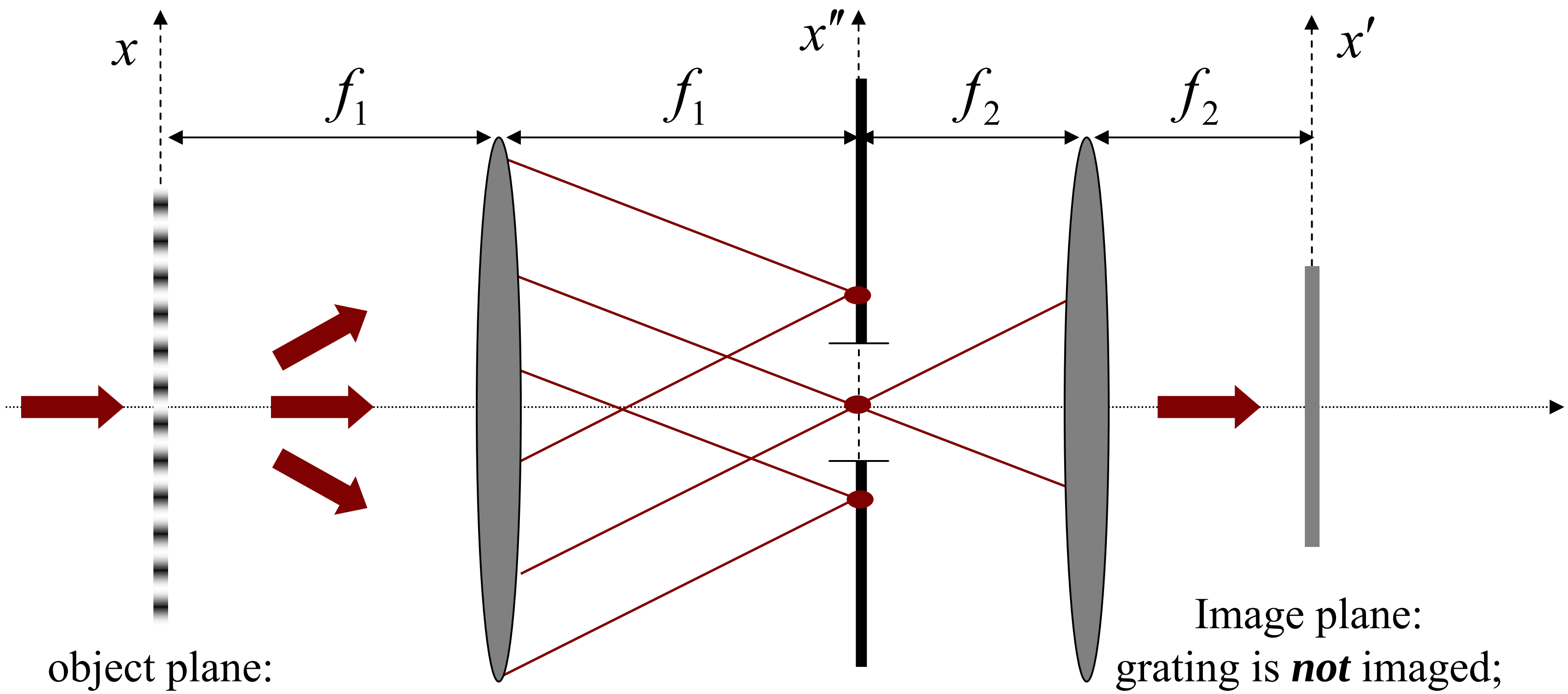
$$G_3(u,v) = G_2(u,v)\,H(u,v)$$

$$H(u,v) = \mathrm{rect}\left(\frac{\lambda f u}{a}\right)\mathrm{rect}\left(\frac{\lambda f v}{b}\right)$$

Aperture–limited spatial filtering



Aperture–limited spatial filtering



object plane:
grating generates
one spatial frequency

Fourier plane:
aperture limited

(some orders cut off)

Image plane:
grating is ***not*** imaged;
only 0^{th} order
(DC component)
surviving

Spatial frequency clipping

field after input transparency

$$g_{\text{in}}(x) = \frac{1}{2}[1 + \cos(2\pi u_0 x)] \Rightarrow G_{\text{in}}(u) = \frac{1}{2}\left[\delta(u) + \frac{1}{2}\delta(u - u_0) + \frac{1}{2}\delta(u + u_0)\right]$$

field before filter

$$g_{\text{f}-}(x'') = \frac{1}{2}\left[\delta(x'') + \cancel{\frac{1}{2}\delta(x'' - \lambda f_1 u_0)} + \cancel{\frac{1}{2}\delta(x'' + \lambda f_1 v_0)}\right]$$

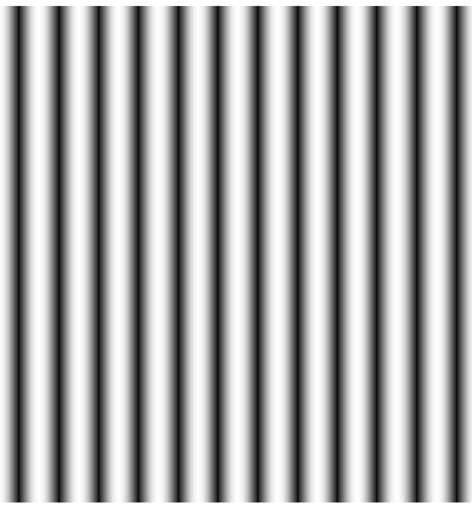
field after filter

$$g_{\text{f}+}(x'') = \frac{1}{2}\delta(x'')$$

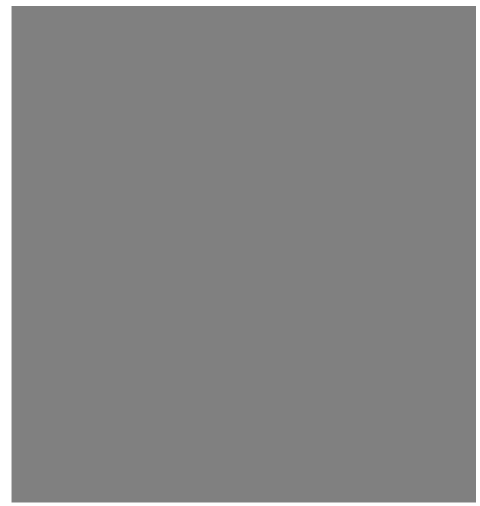
field at output (image plane)

$$G_{\text{out}}(u) = \frac{1}{2}\delta(u) \Rightarrow g_{\text{out}}(x') = \frac{1}{2}$$

Effect of spatial filtering



Fourier plane filter
with circ-aperture

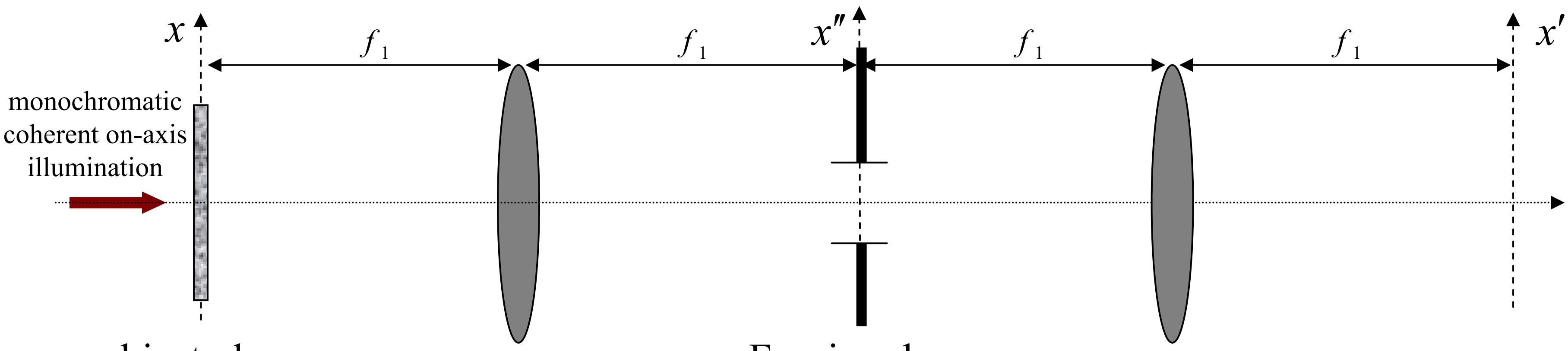


Original object
(sinusoidal spatial variation,
i.e. grating)

Frequency-filtered image
(spatial variation blurred out,
only average survives)

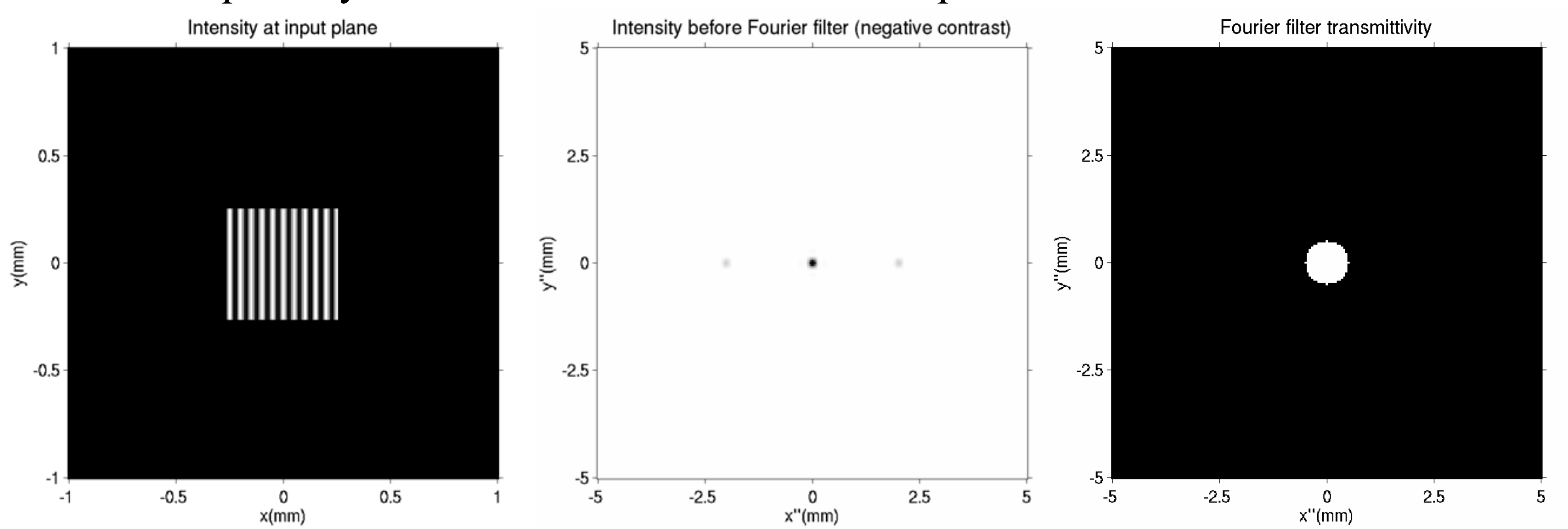
Spatial frequency clipping

f_1=20cm
λ=0.5μm

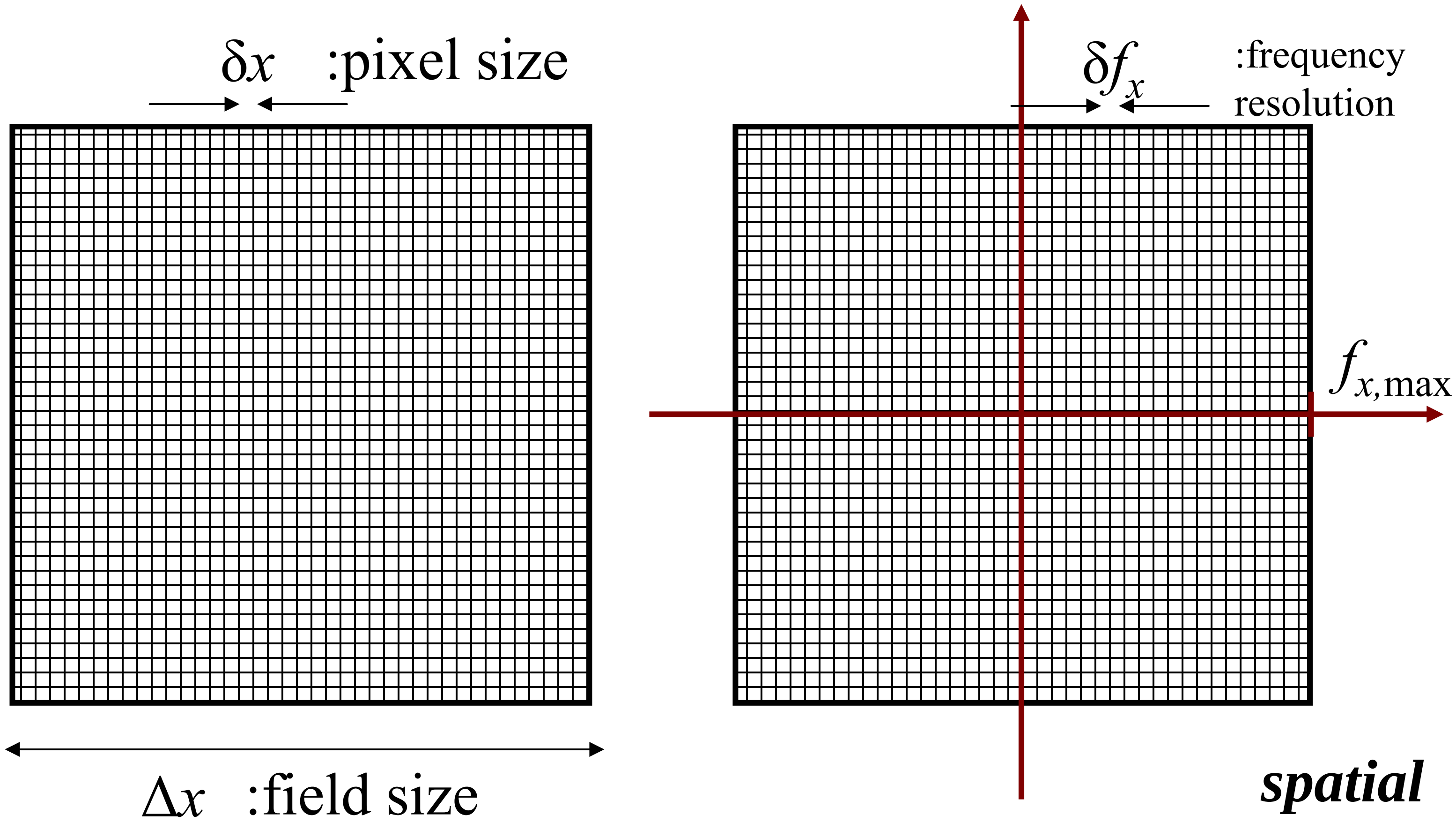


object plane
transparency

Fourier plane
circ-aperture



Space-Fourier coordinate transformations



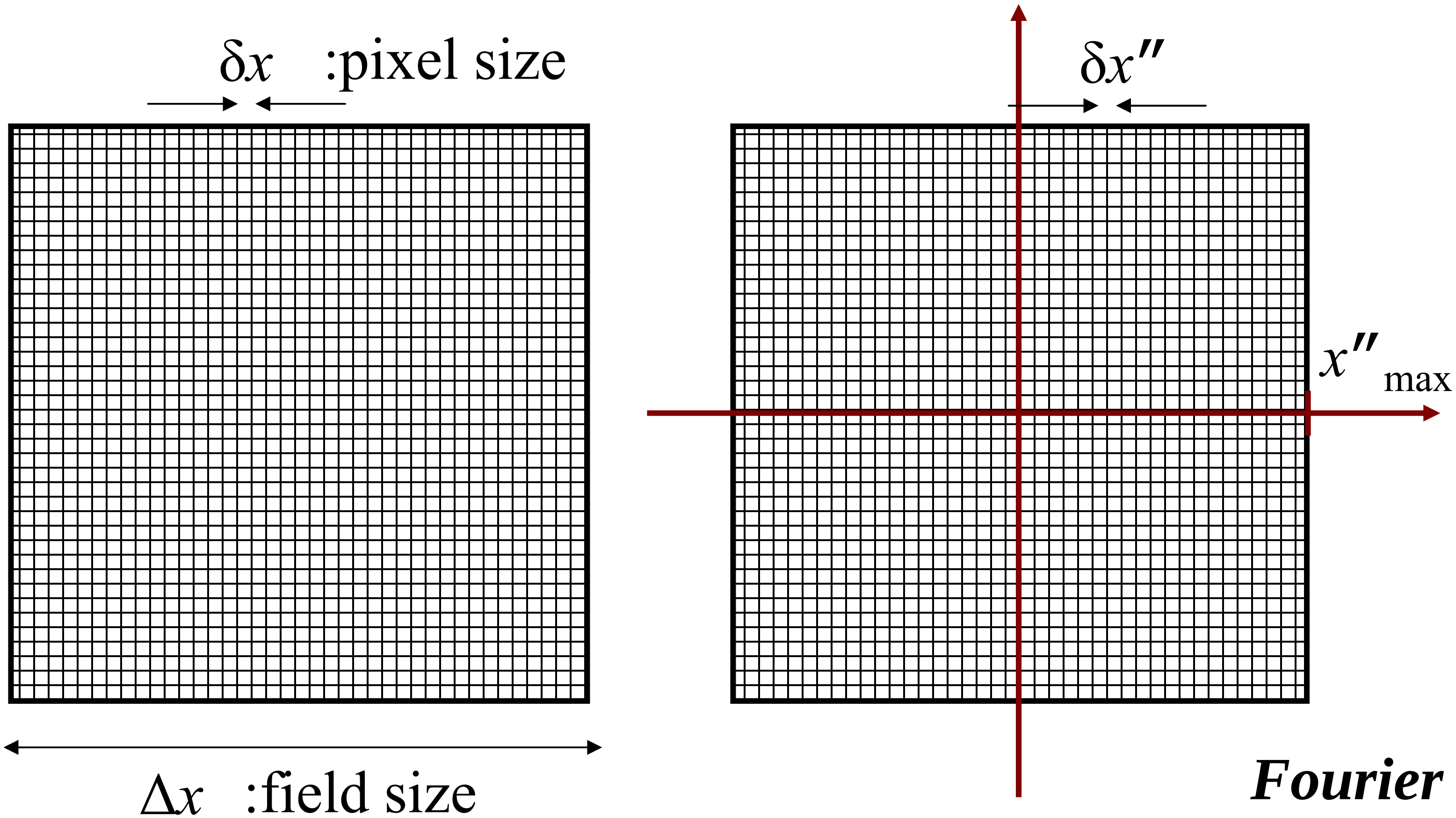
space domain

spatial frequency domain

Nyquist relationships:

$$f_{x,\max} = \frac{1}{2\delta x} \qquad \delta f_x = \frac{1}{2\Delta x}$$

4F coordinate transformations



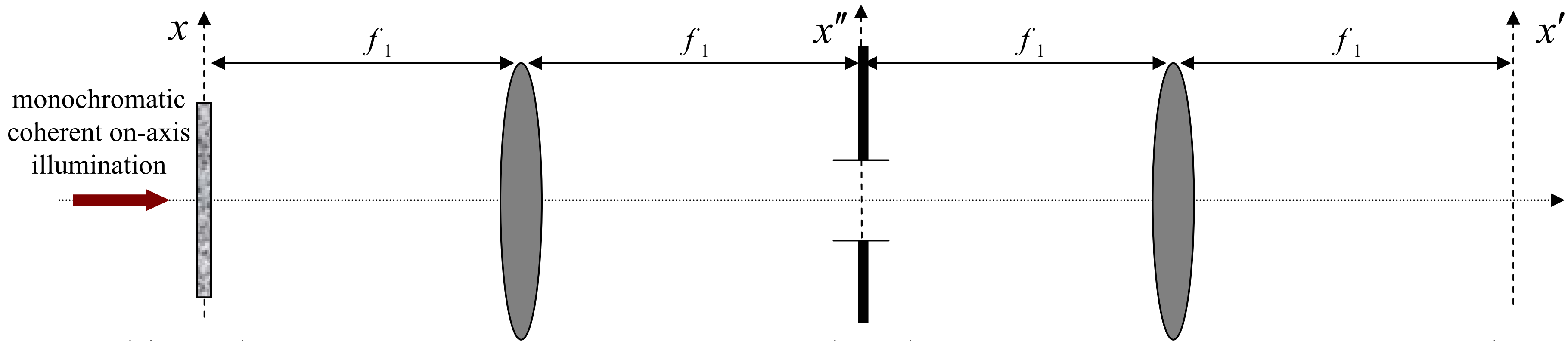
space domain

Fourier plane

Nyquist relationships: $x''_{\max} = \dfrac{\lambda f}{2\delta x}$ $\qquad \delta x'' = \dfrac{\lambda f}{2\Delta x}$

Spatial frequency clipping

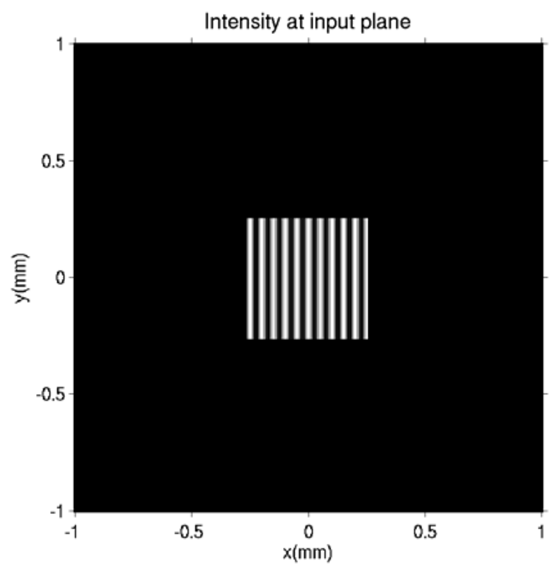
f_1=20cm
λ=0.5µm

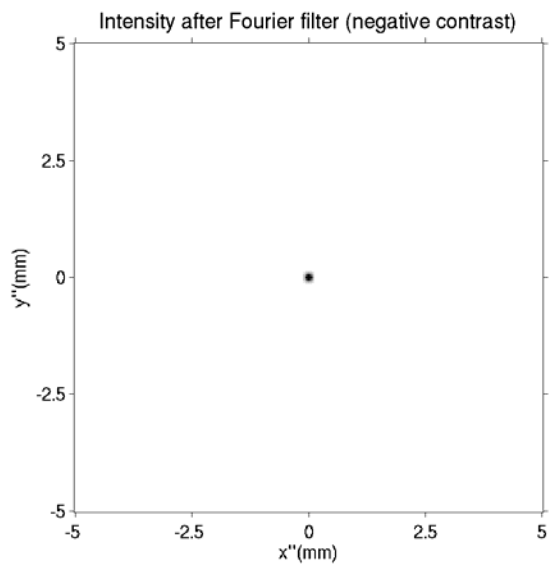


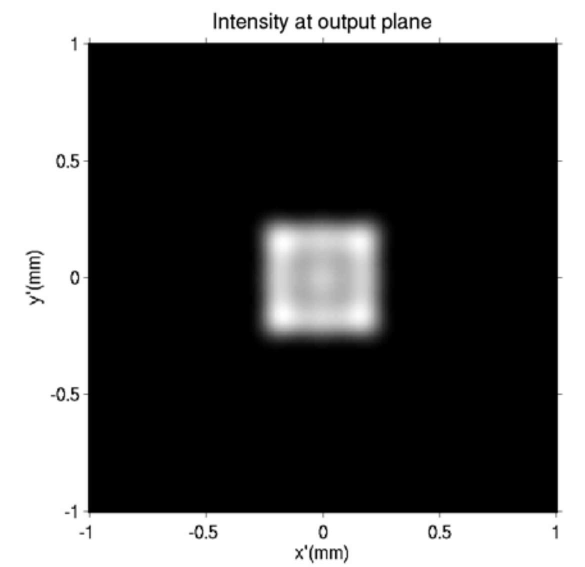
object plane
transparency

Fourier plane
circ-aperture

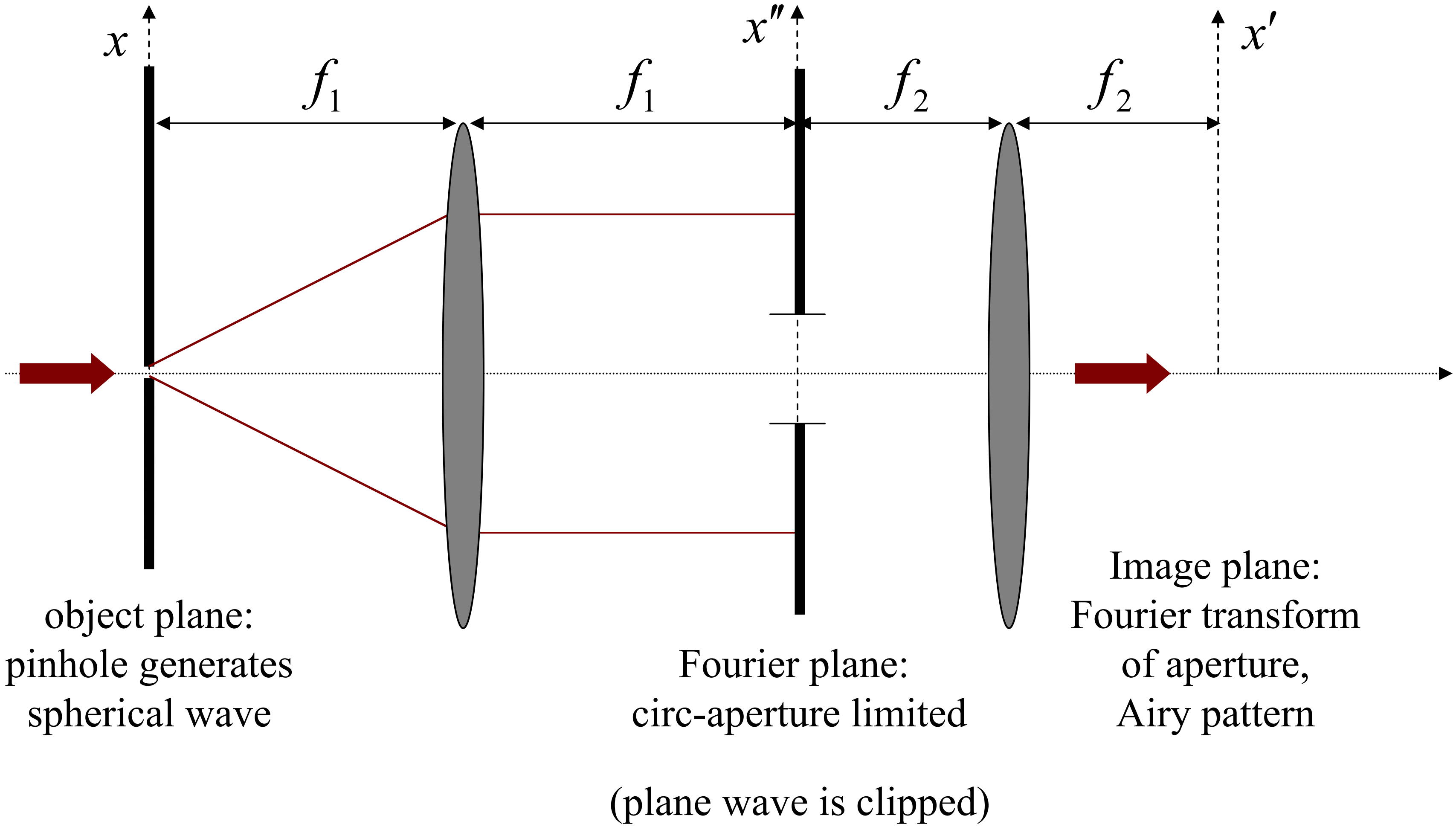
Image plane
observed field







Formation of the impulse response



Low–pass filtering

field after input transparency

$$g_{\text{in}}(x,y)=\delta(x)\delta(y)\Rightarrow G_{\text{in}}(u,v)=1$$

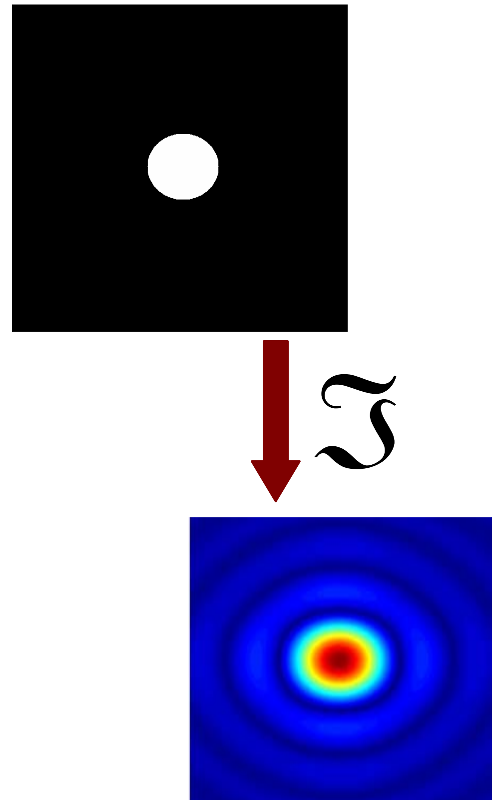
field before filter $\quad g_{\text{f}-}(x'',y'')=1$

field after filter $\quad g_{\text{f}+}(x'',y'')=\text{circ}\left(\dfrac{\sqrt{x''^2+y''^2}}{R}\right)$

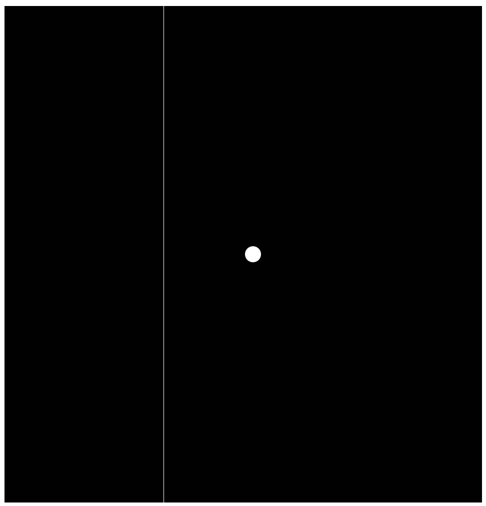
field at output
(image plane) $\quad G_{\text{out}}(u,v)=\text{circ}\left(\dfrac{\lambda f\sqrt{u^2+v^2}}{R}\right)\Rightarrow$

$$g_{\text{out}}(x',y')\propto\text{jinc}\left(\frac{R\sqrt{x'^2+y'^2}}{\lambda f}\right)$$

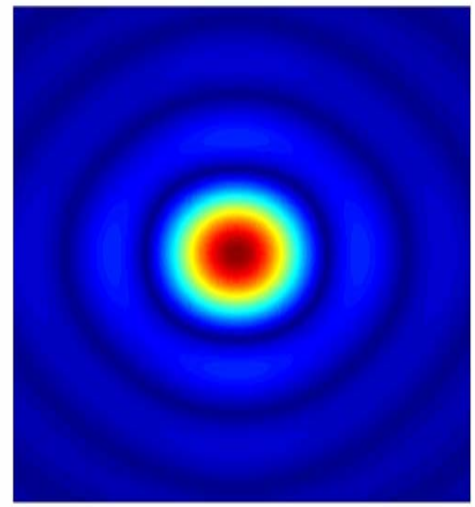
(Airy pattern)



Effect of spatial filtering



Fourier plane filter
with circ-aperture

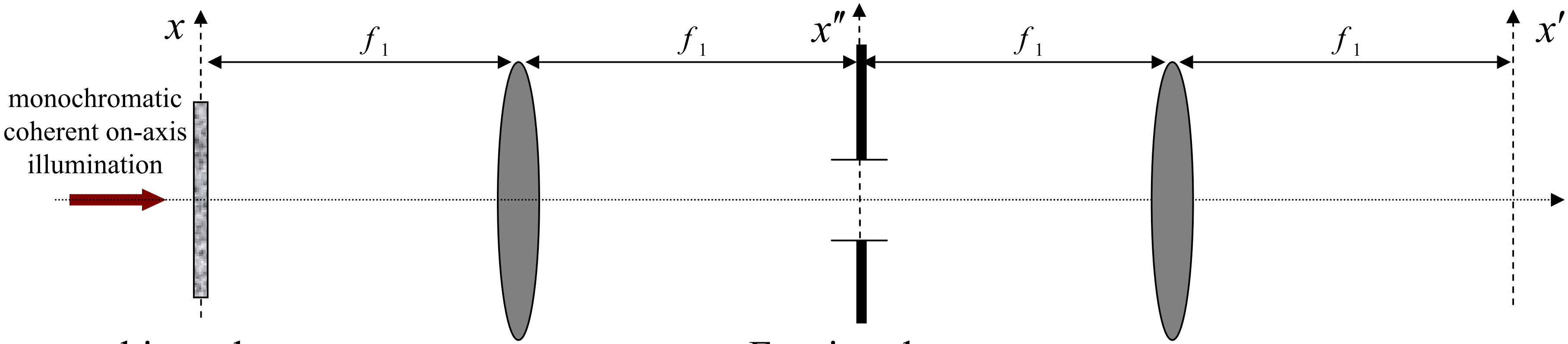


Original object
(small pinhole ⇔ impulse,
generating spherical wave
past the transparency)

Impulse reponse
(aka point-spread function,
original point has blurred to
an Airy pattern, or jinc)

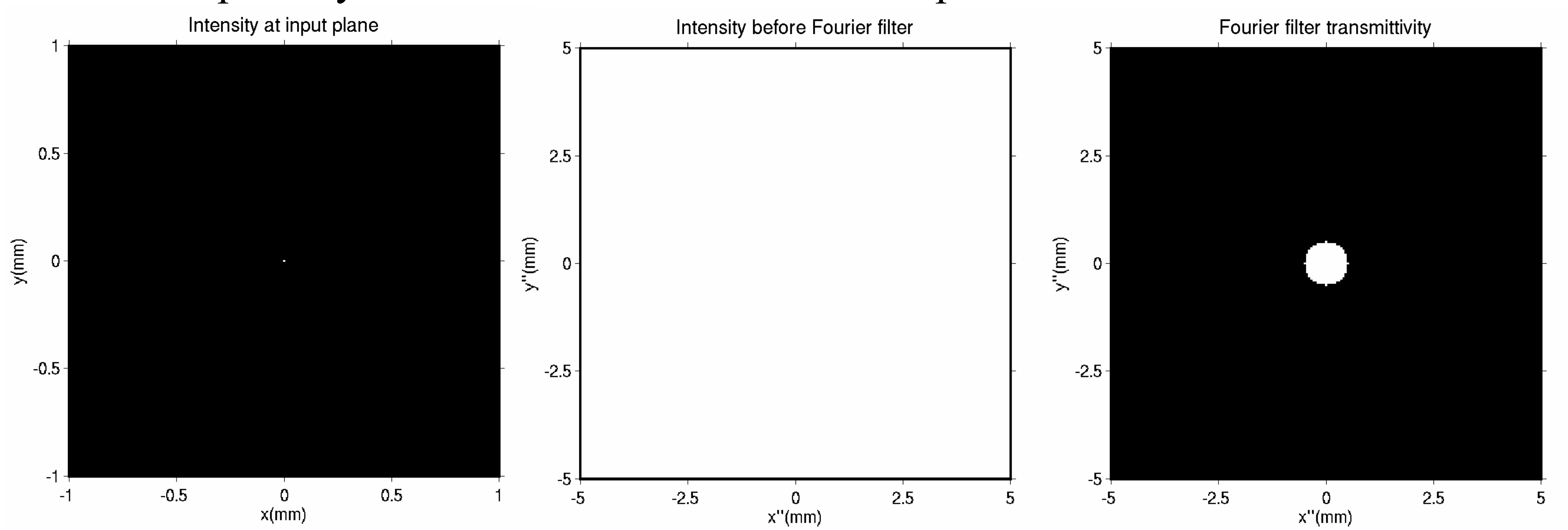
Low–pass filtering the impulse

f_1=20cm
λ=0.5μm



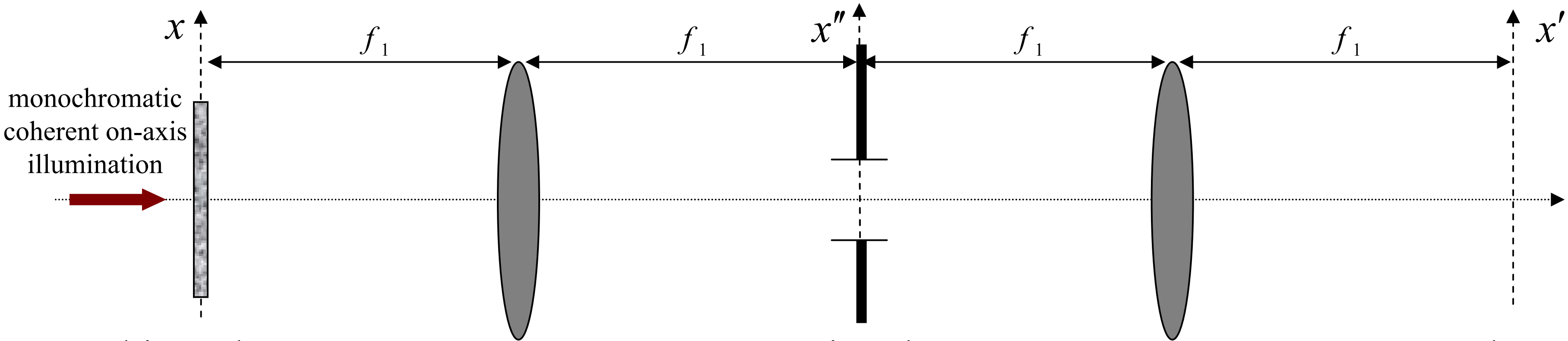
object plane transparency

Fourier plane circ-aperture



Spatial frequency clipping

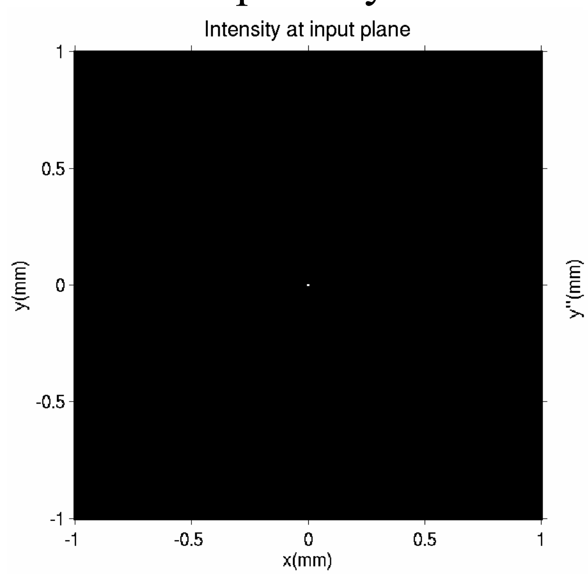
f_1=20cm
λ=0.5μm

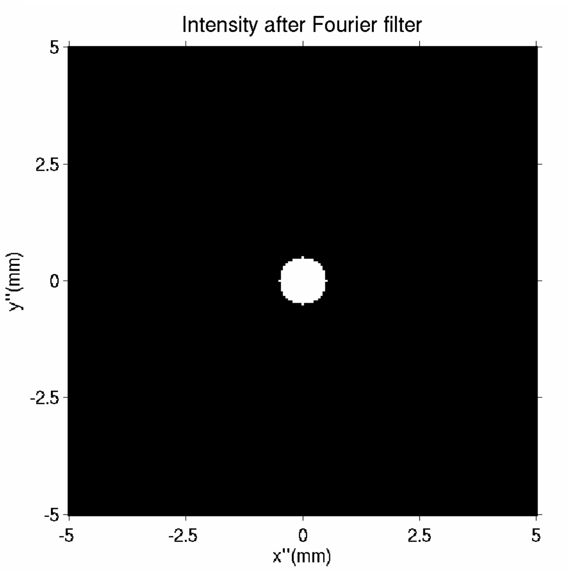


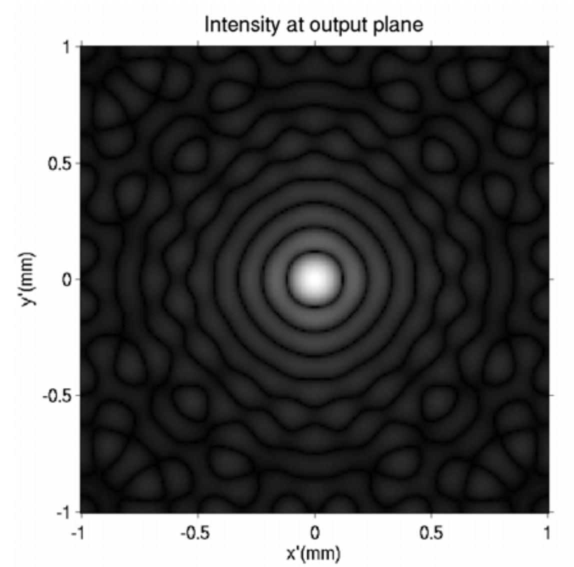
object plane
transparency

Fourier plane
circ-aperture

Image plane
observed field

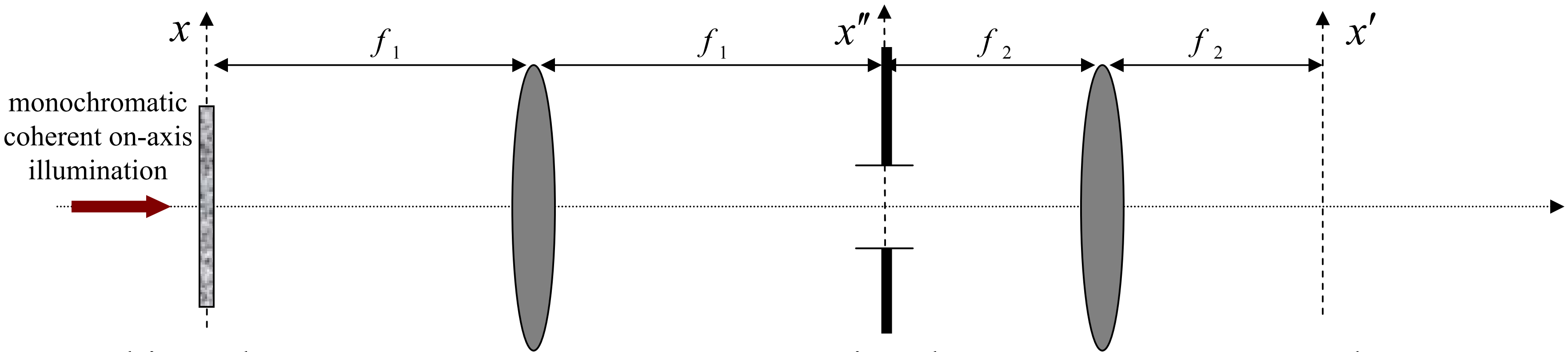






note: pseudo-accentuated
sidelobes

Low-pass filtering with the 4F system



object plane transparency

$g_{\text{in}}(x, y)$

Fourier plane circ-aperture

$$H(x'', y'') = \text{circ}\left(\frac{\sqrt{x''^2 + y''^2}}{R}\right)$$

Image plane observed field

$$(g_{\text{in}} * \text{jinc})\left(-\frac{f_1}{f_2}x', -\frac{f_1}{f_2}y'\right)$$

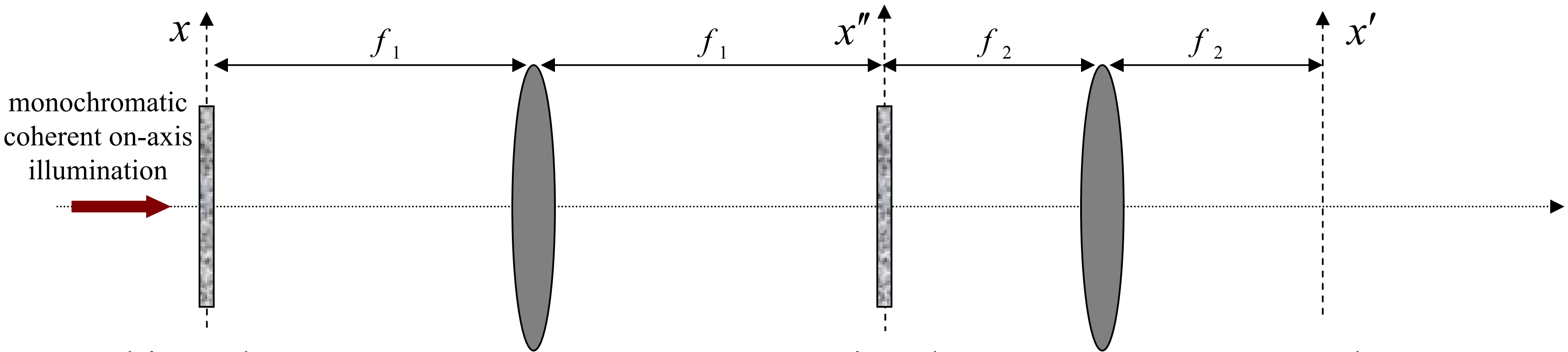
$\mathfrak{I}$ Fourier transform

field arriving at Fourier plane $G_{\text{in}}\left(\frac{x''}{\lambda f_1}, \frac{y''}{\lambda f_1}\right)$

field departing from Fourier plane $G_{\text{in}}\left(\frac{x''}{\lambda f_1}, \frac{y''}{\lambda f_1}\right) \times \text{circ}\left(\frac{\sqrt{x''^2 + y''^2}}{R}\right)$

$\mathfrak{I}$ Fourier transform

Spatial filtering with the 4F system



object plane transparency
$$g_{\text{in}}(x, y)$$

$\mathfrak{I}$ Fourier transform

field arriving at Fourier plane
$$G_{\text{in}}\left(\frac{x''}{\lambda f_1}, \frac{y''}{\lambda f_1}\right)$$

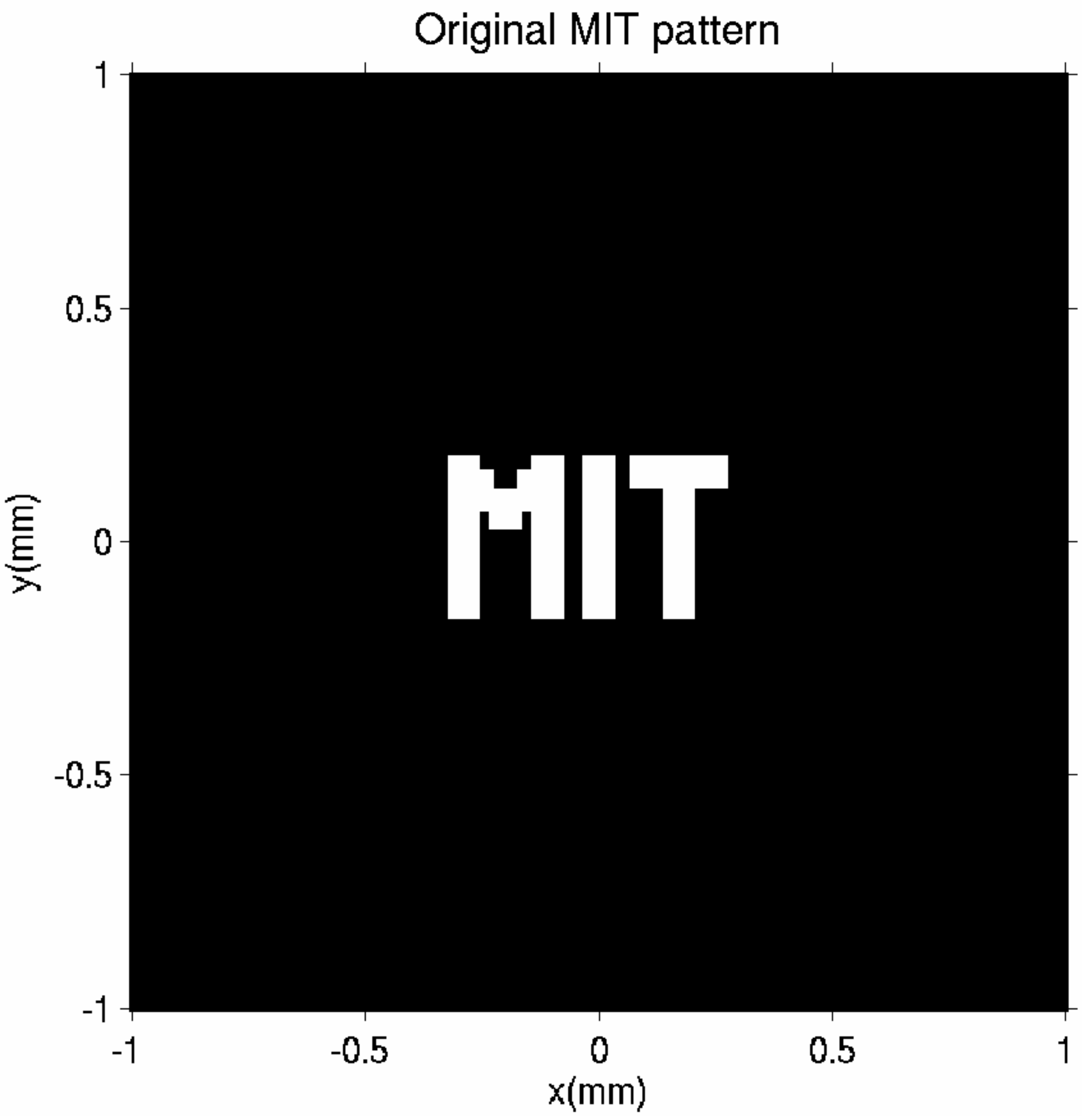
Fourier plane transparency
$$H(x'', y'')$$

field departing from Fourier plane
$$G_{\text{in}}\left(\frac{x''}{\lambda f_1}, \frac{y''}{\lambda f_1}\right) \times H(x'', y'')$$

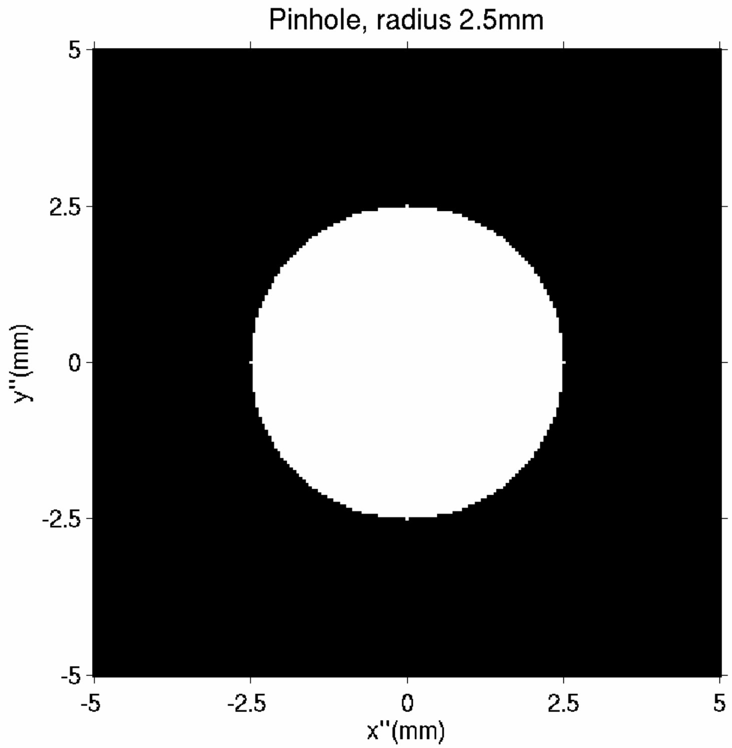
$\mathfrak{I}$ Fourier transform

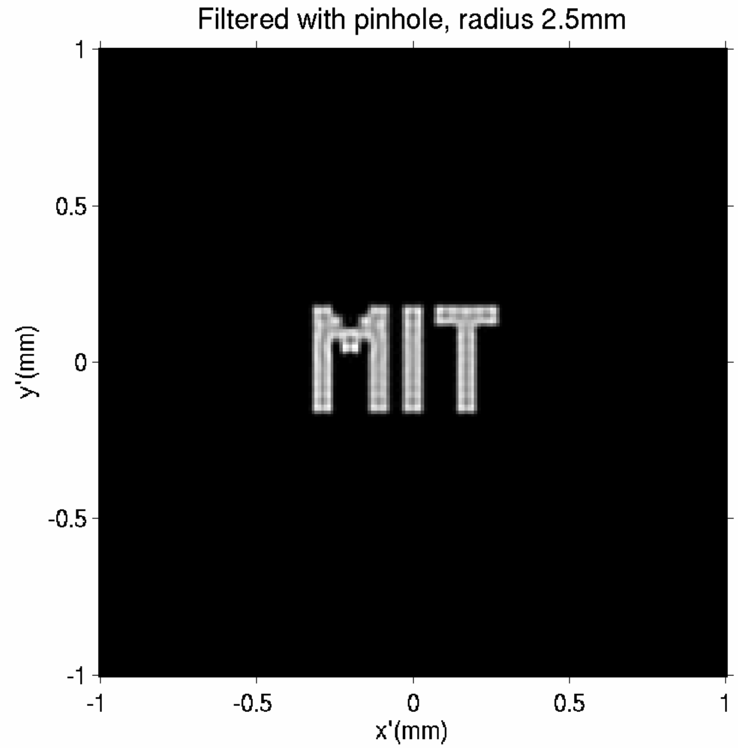
Image plane observed field
$$(g_{\text{in}} * h)\left(-\frac{f_1}{f_2}x', -\frac{f_1}{f_2}y'\right)$$

Examples: the amplitude MIT pattern



Weak low–pass filtering





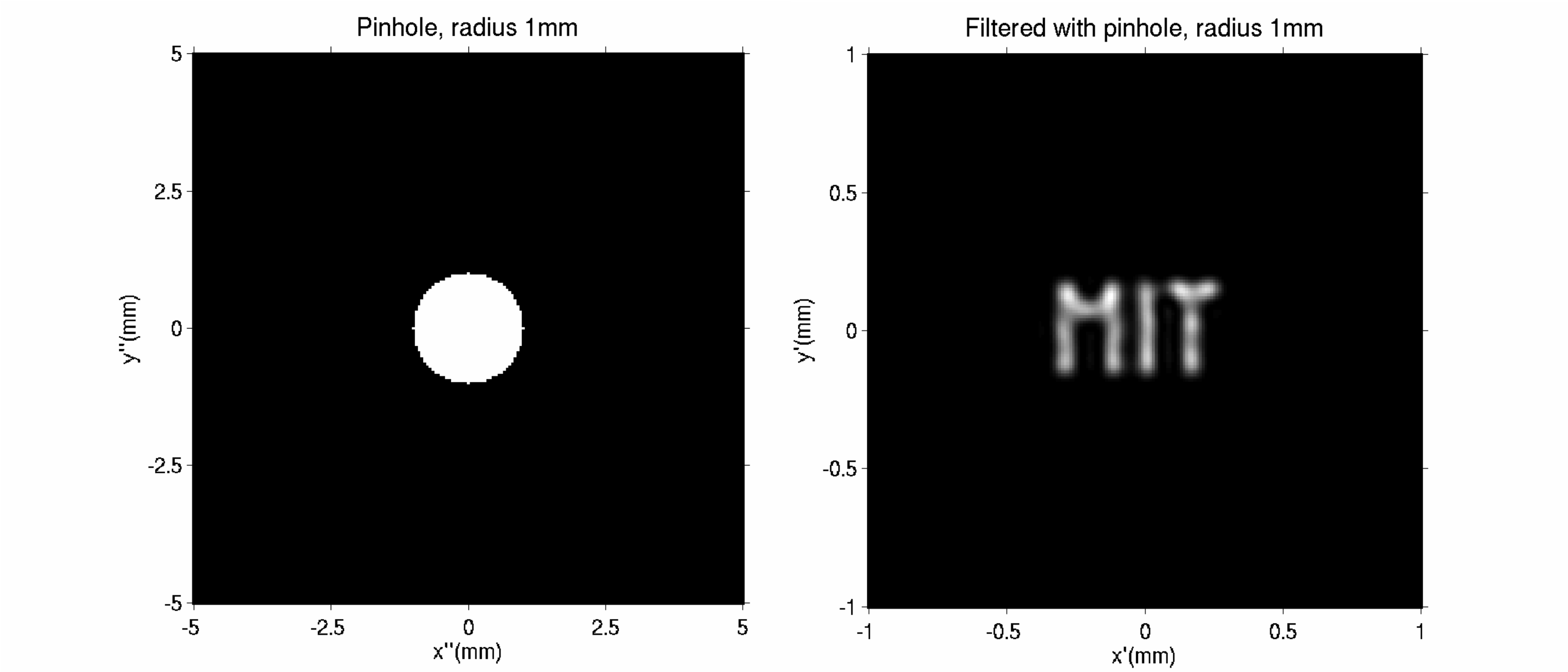
f_1=20cm
λ=0.5μm

Fourier filter

Intensity @ image plane

Moderate low–pass filtering

(*aka* blurring)

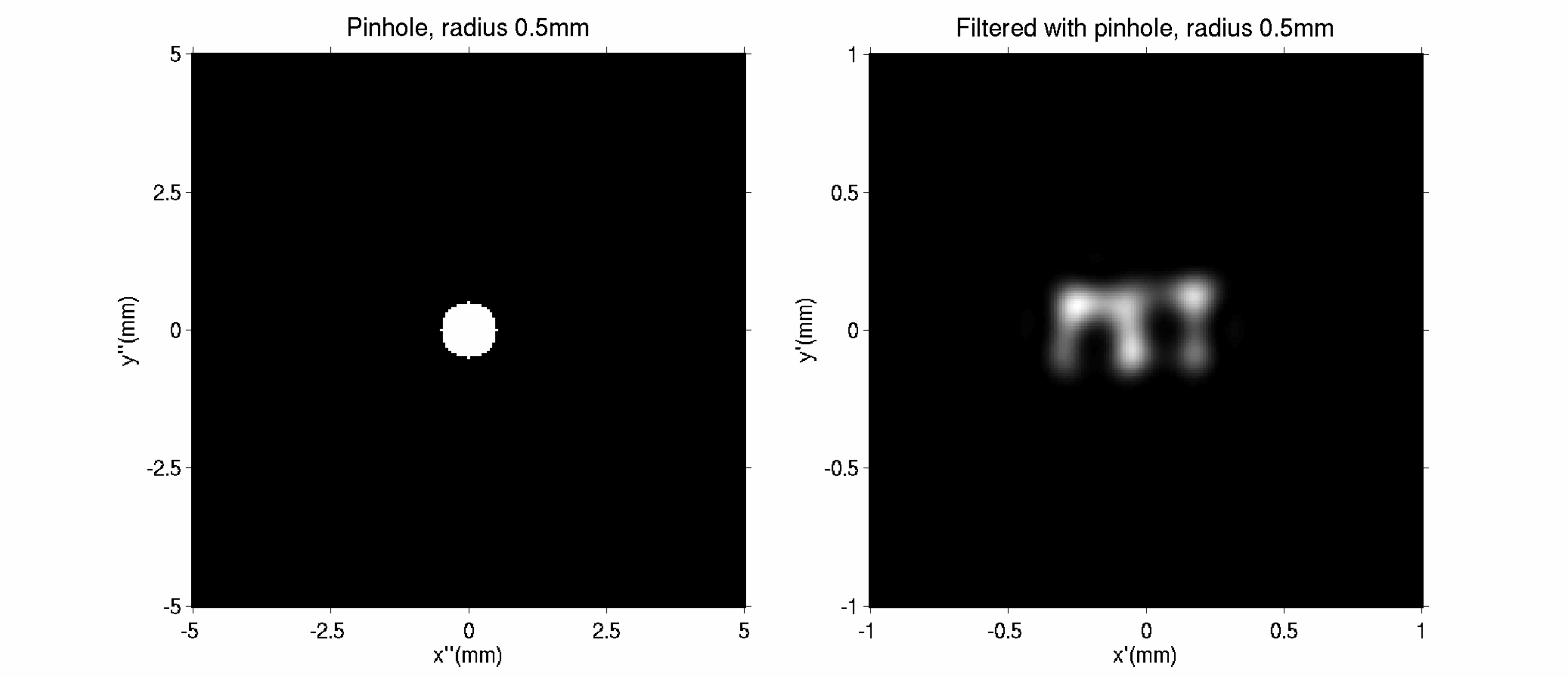


f_1=20cm
λ=0.5µm

Fourier filter

Intensity @ image plane

Strong low–pass filtering

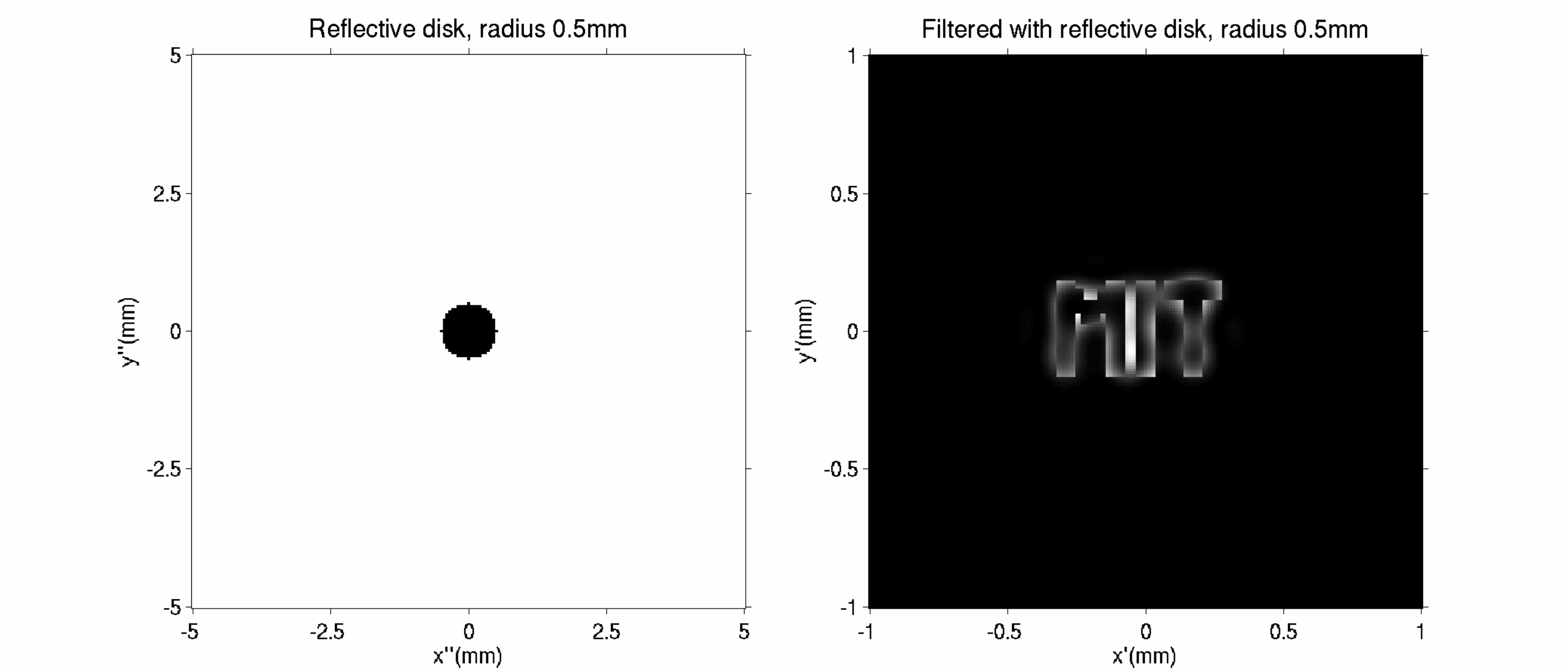


f_1=20cm
λ=0.5μm

Fourier filter

Intensity @ image plane

Moderate high–pass filtering



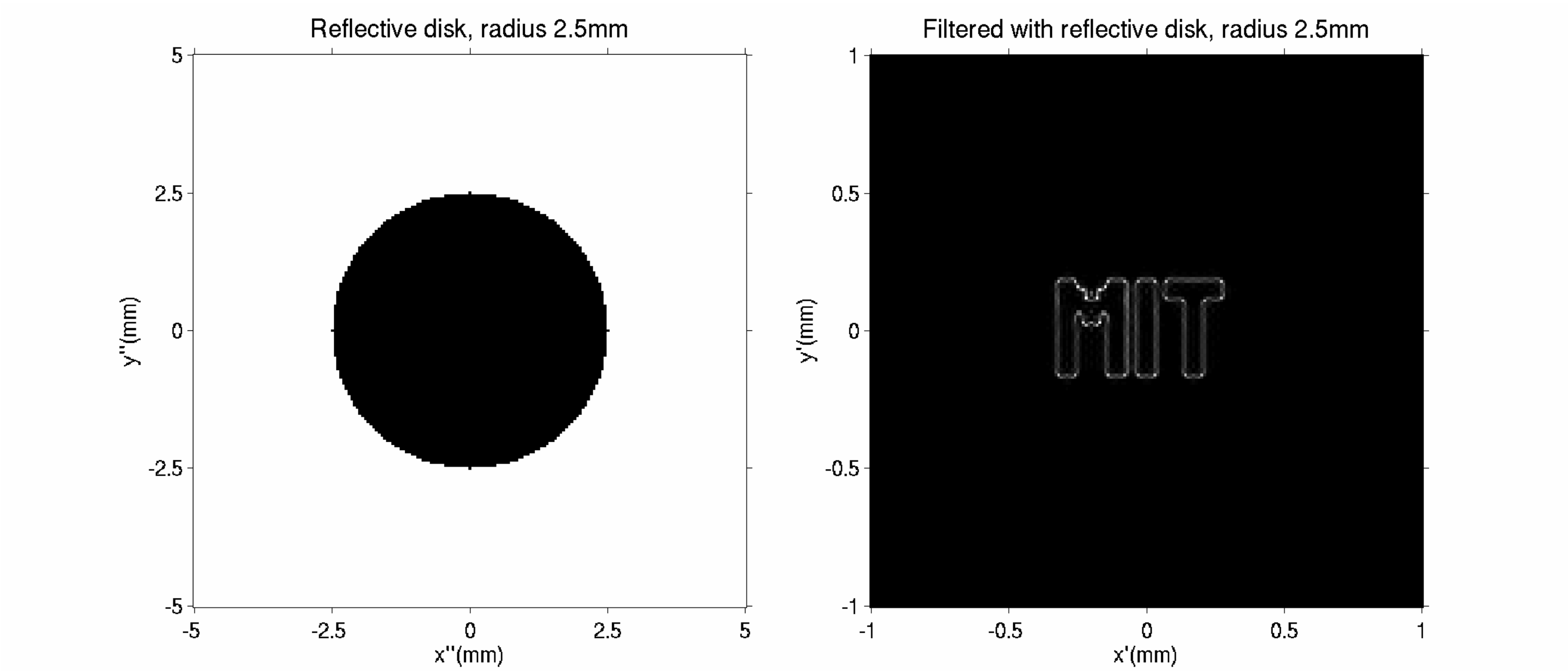
f_1=20cm
λ=0.5µm

Fourier filter

Intensity @ image plane

Strong high–pass filtering

(*aka* edge enhancement)

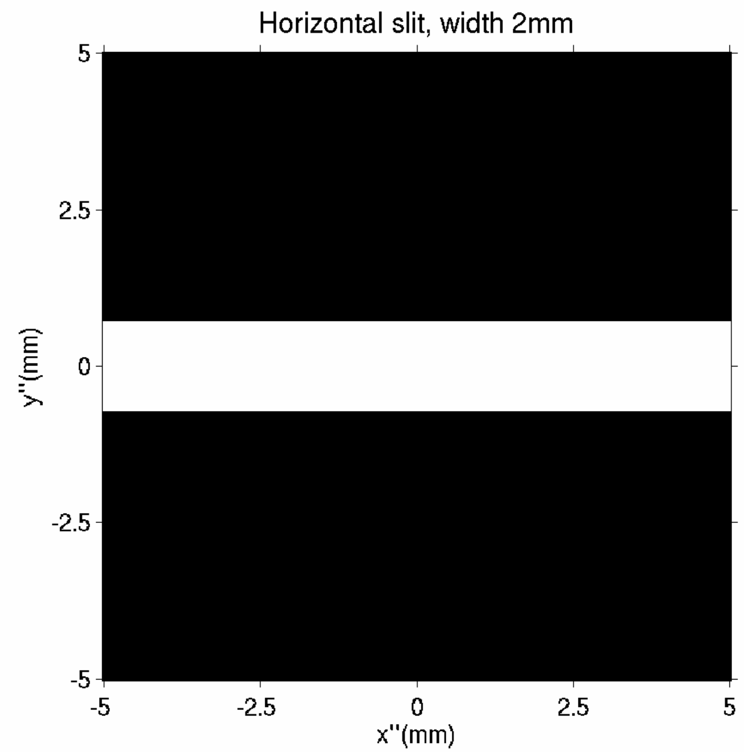


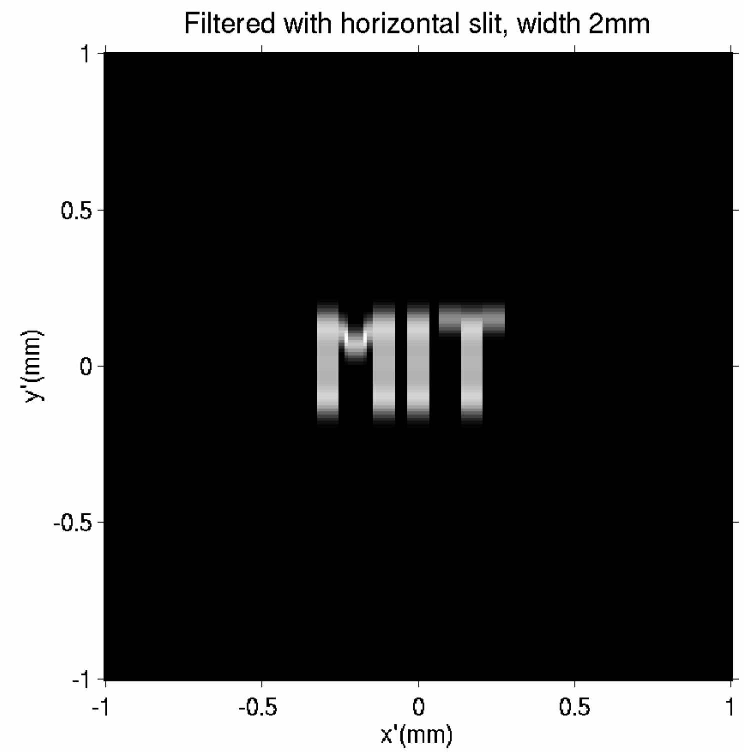
f_1=20cm
λ=0.5μm

Fourier filter

Intensity @ image plane

1-dimensional blurring



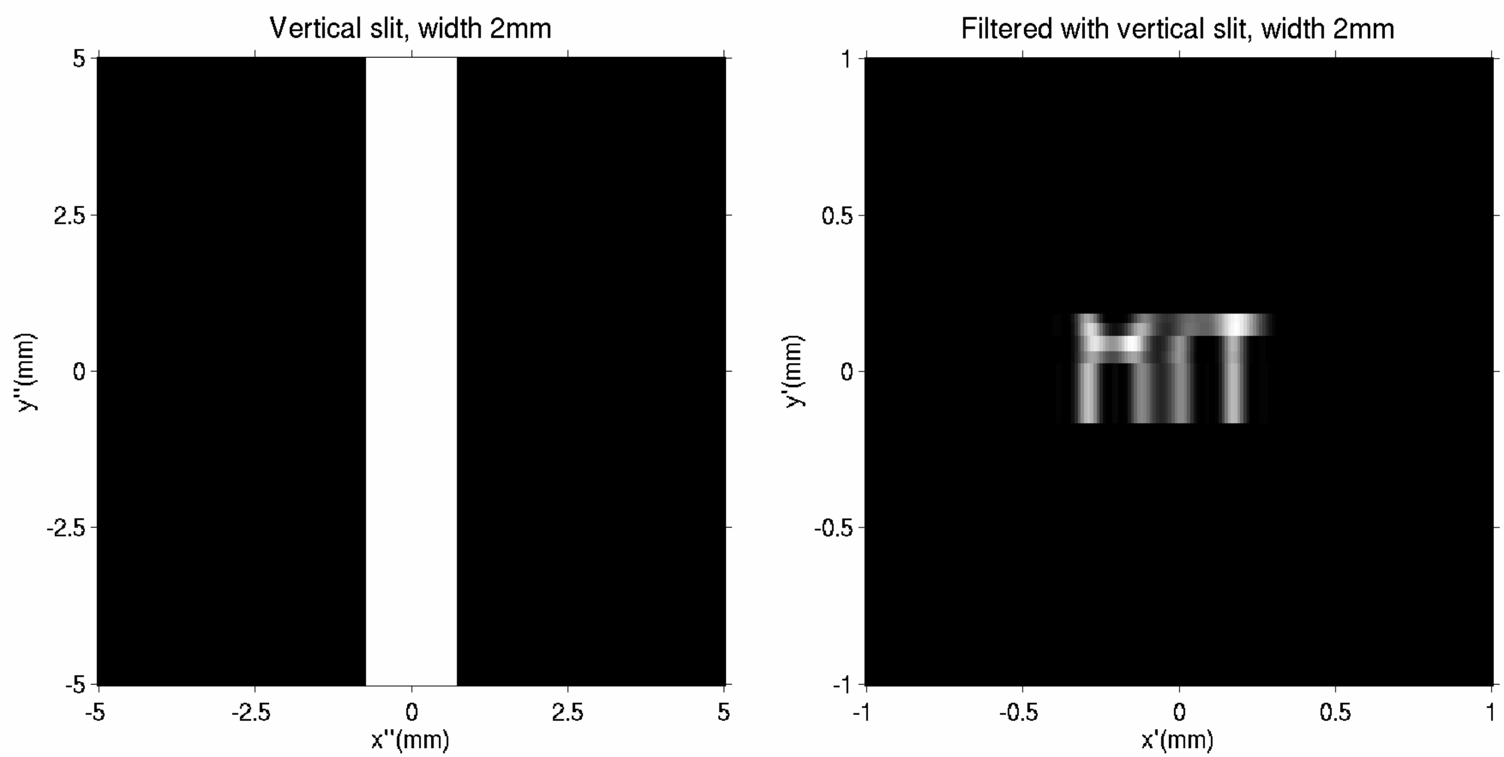


f_1=20cm
λ=0.5µm

Fourier filter

Intensity @ image plane

1-dimensional blurring

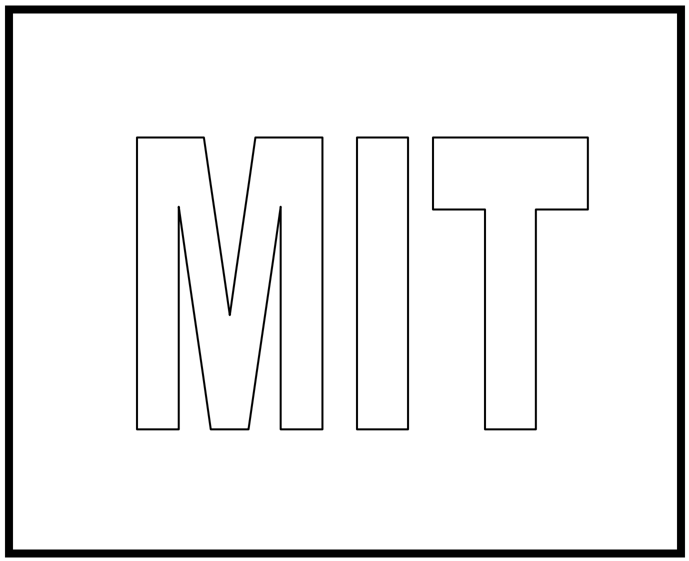


f_1=20cm
λ=0.5μm

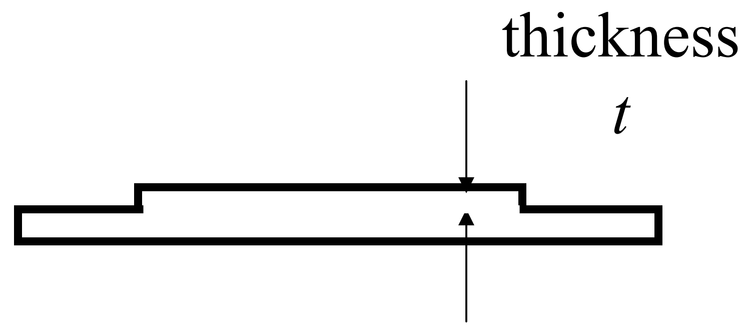
Fourier filter

Intensity @ image plane

Phase objects



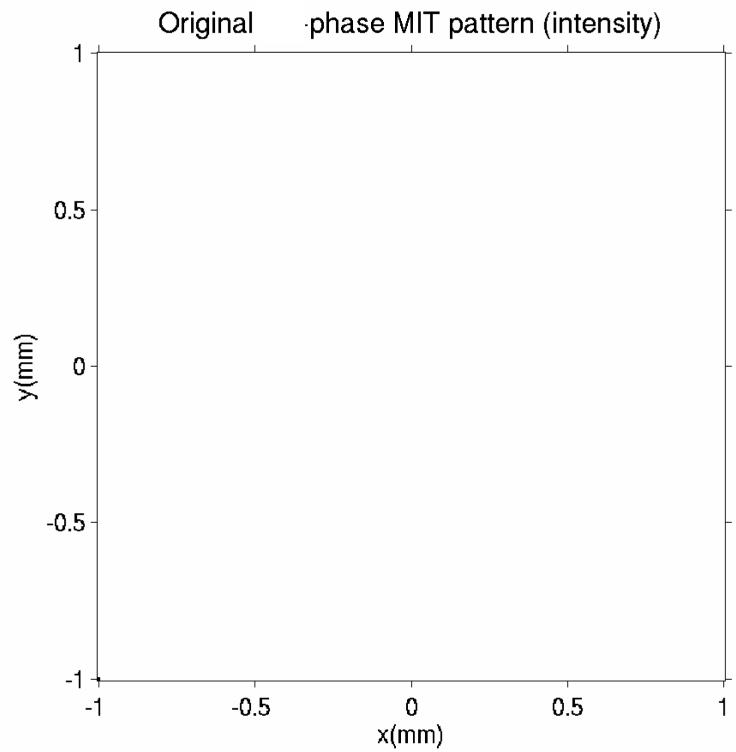
glass plate
(transparent)

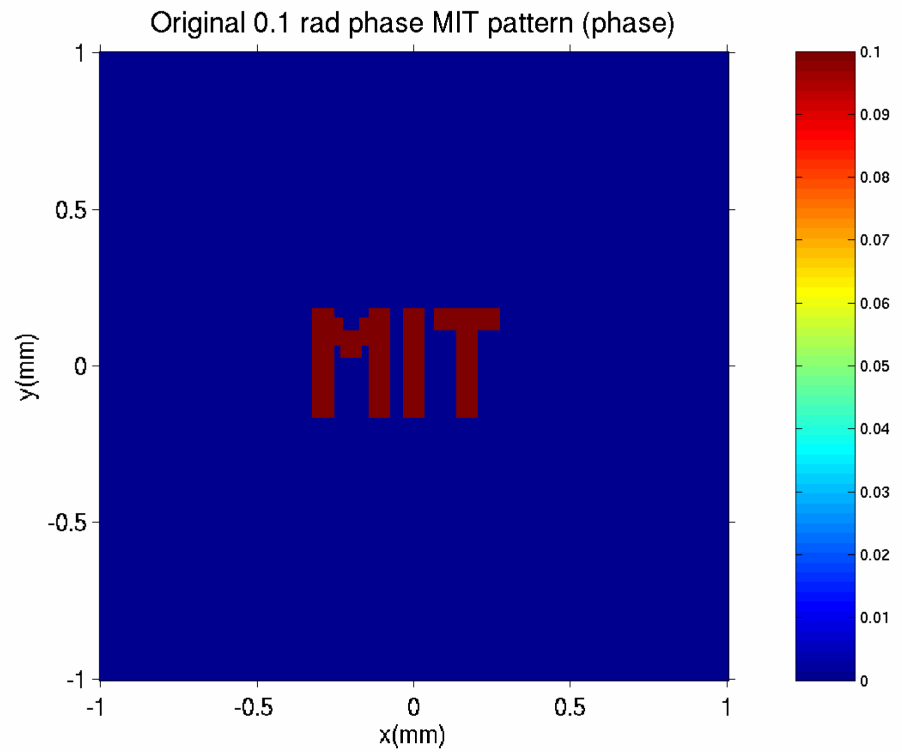


protruding part
phase-shifts
coherent illumination
by amount $\varphi=2\pi(n-1)t/\lambda$

Often useful in imaging biological objects (cells, etc.)

Viewing phase objects

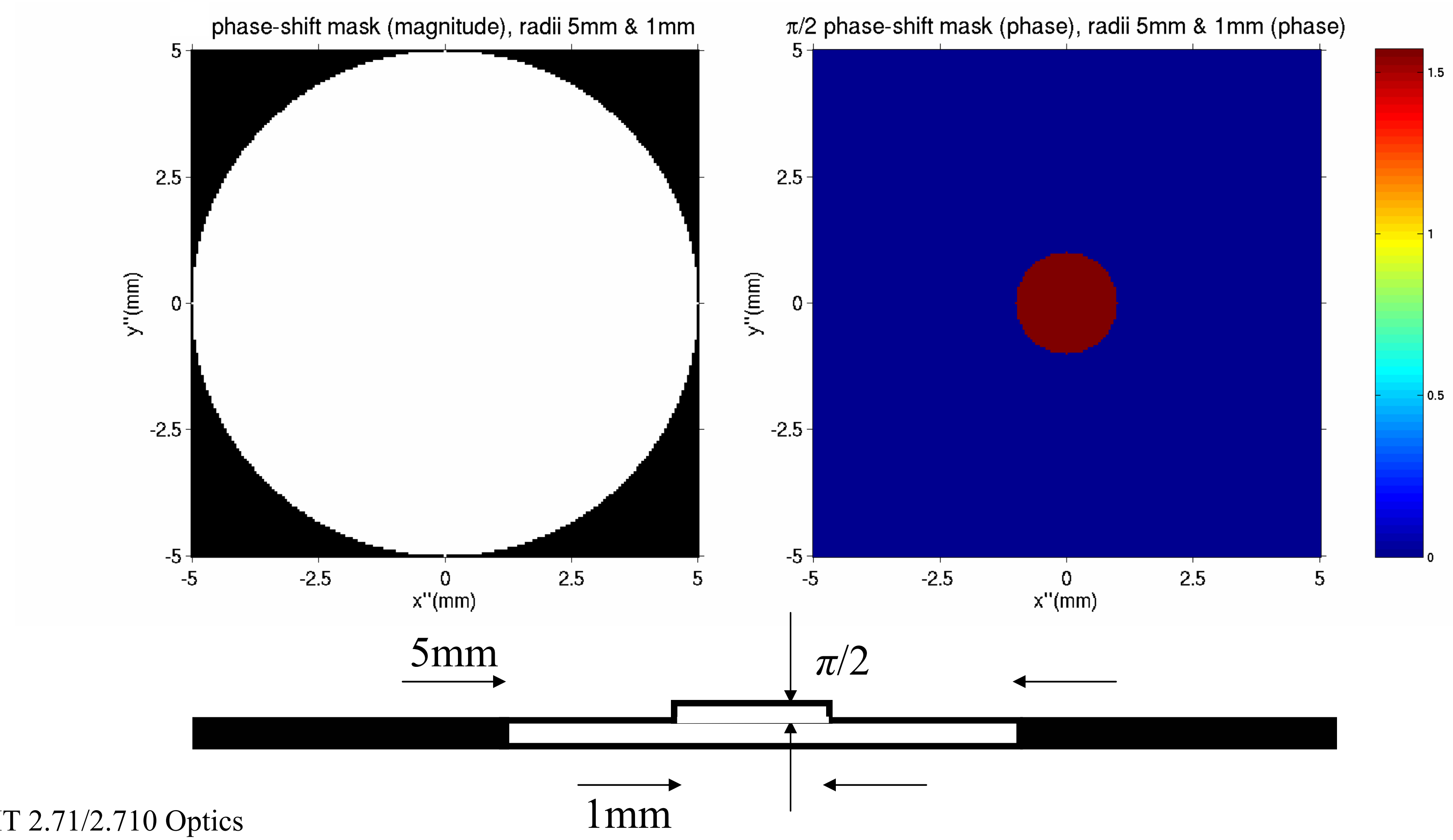




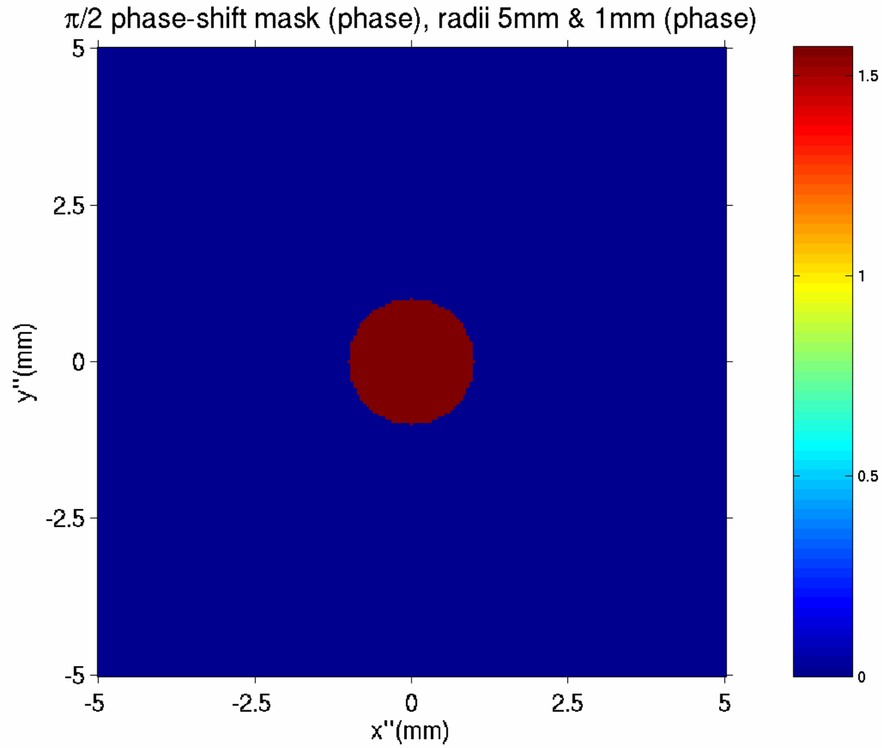
Intensity
(object is invisible)

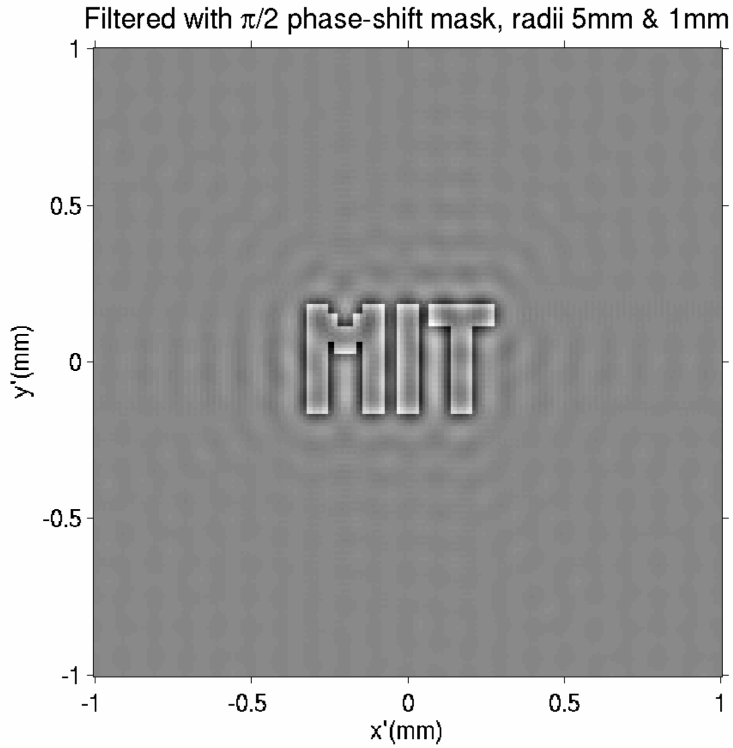
Amplitude
(need interferometer)

Zernicke phase-shift mask



Imaging with Zernicke mask



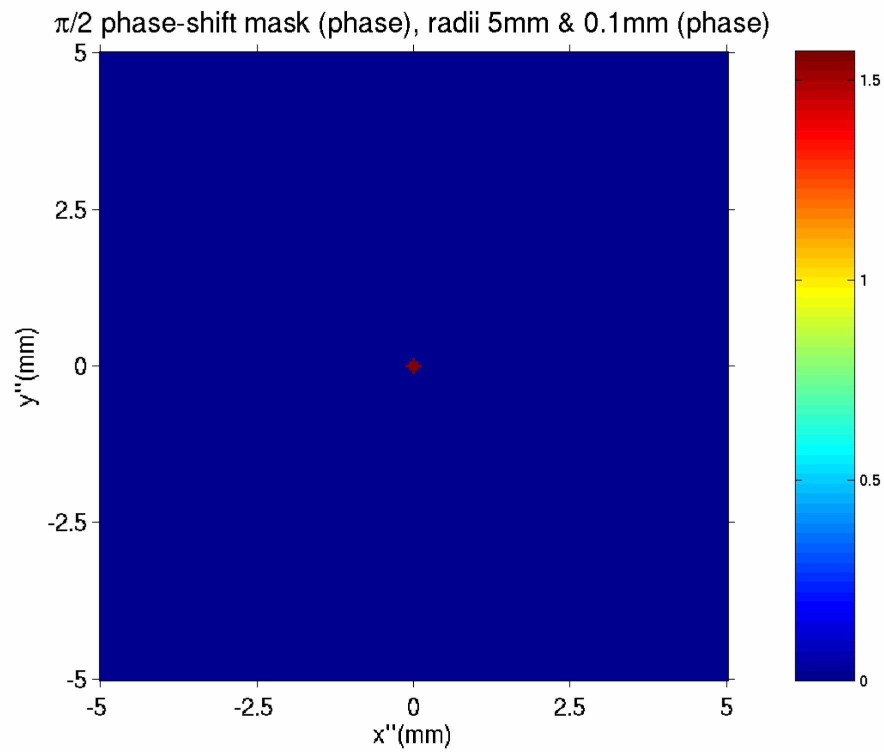


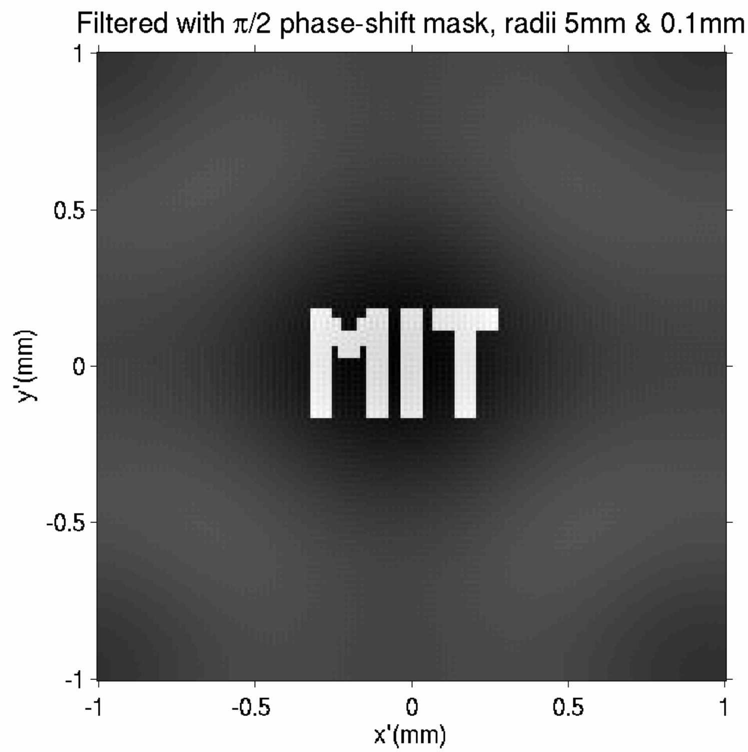
f_1=20cm
λ=0.5µm

Fourier filter

Intensity @ image plane

Imaging with Zernicke mask





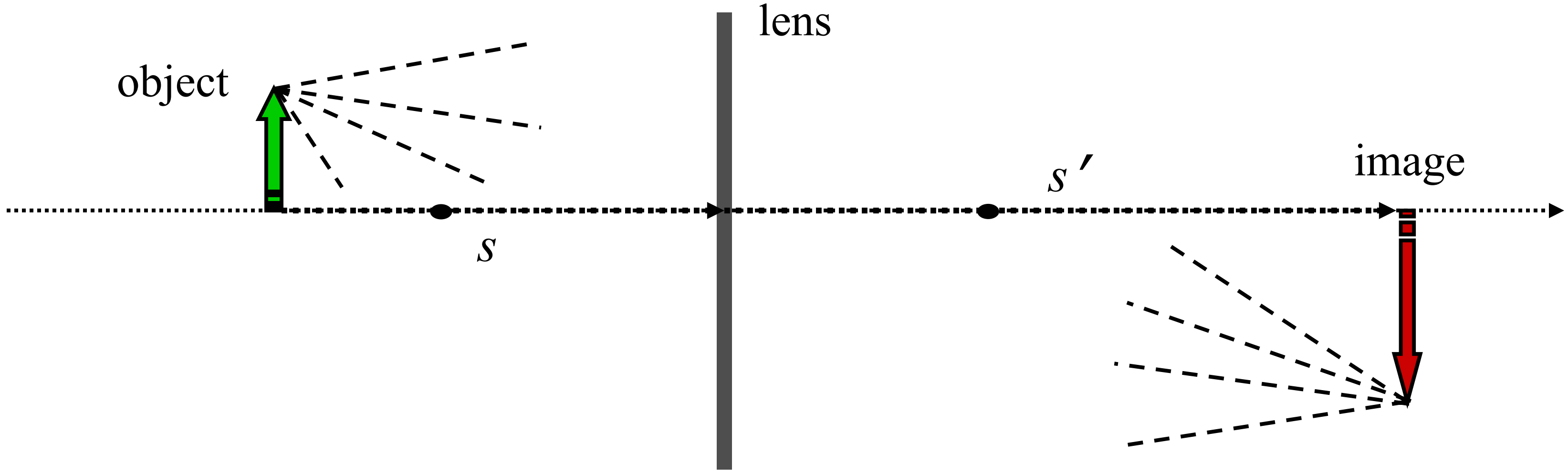
f_1=20cm
λ=0.5μm

Fourier filter

Intensity @ image plane

Coherent imaging with a single lens

Single-lens imaging condition



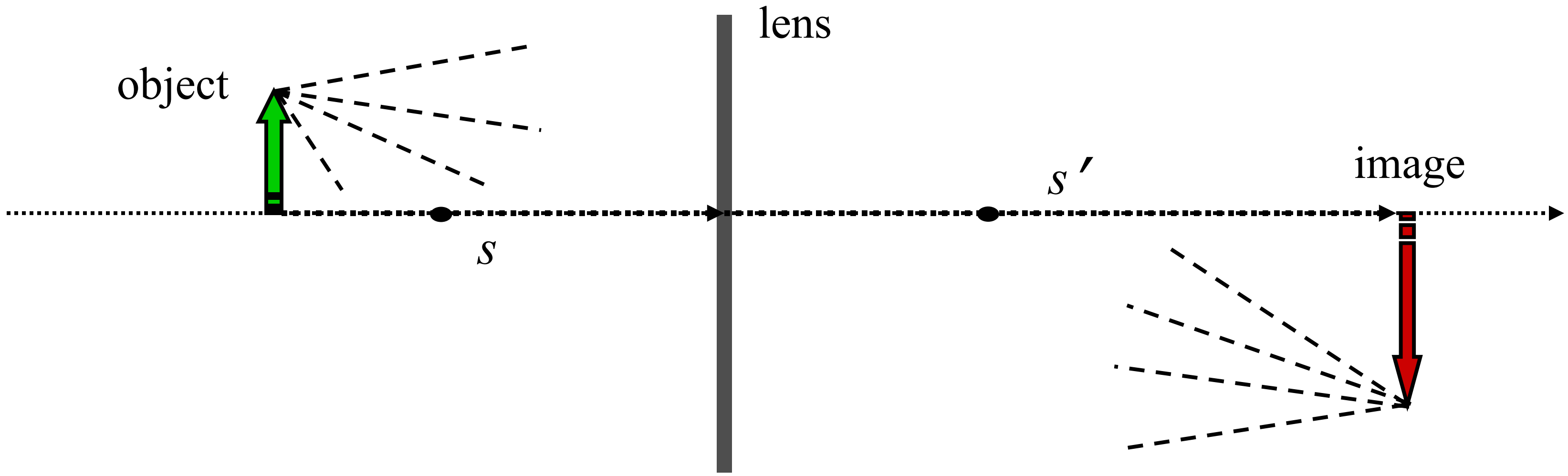
$$\boxed{\begin{aligned} \frac{1}{s}+\frac{1}{s'}&=\frac{1}{f} \\ m_{\text{lateral}}&=-\frac{s'}{s} \end{aligned}}$$

Imaging condition
(*aka* Lens Law)

Magnification

Derivation using wave optics ?!?

Single-lens imaging system



Single-lens imaging system

Impulse response (PSF)

Ideal PSF: $$h(x,y;x',y') = \delta(x'-mx)\,\delta(y'-my)$$

Diffraction-
-limited
PSF: $$h(x,y;x',y') = \operatorname{jinc}\left(\frac{R}{\lambda}\sqrt{\left(\frac{x'}{s'}-\frac{x}{s}\right)^2+\left(\frac{y'}{s'}-\frac{y}{s}\right)^2}\right)$$