Resolution

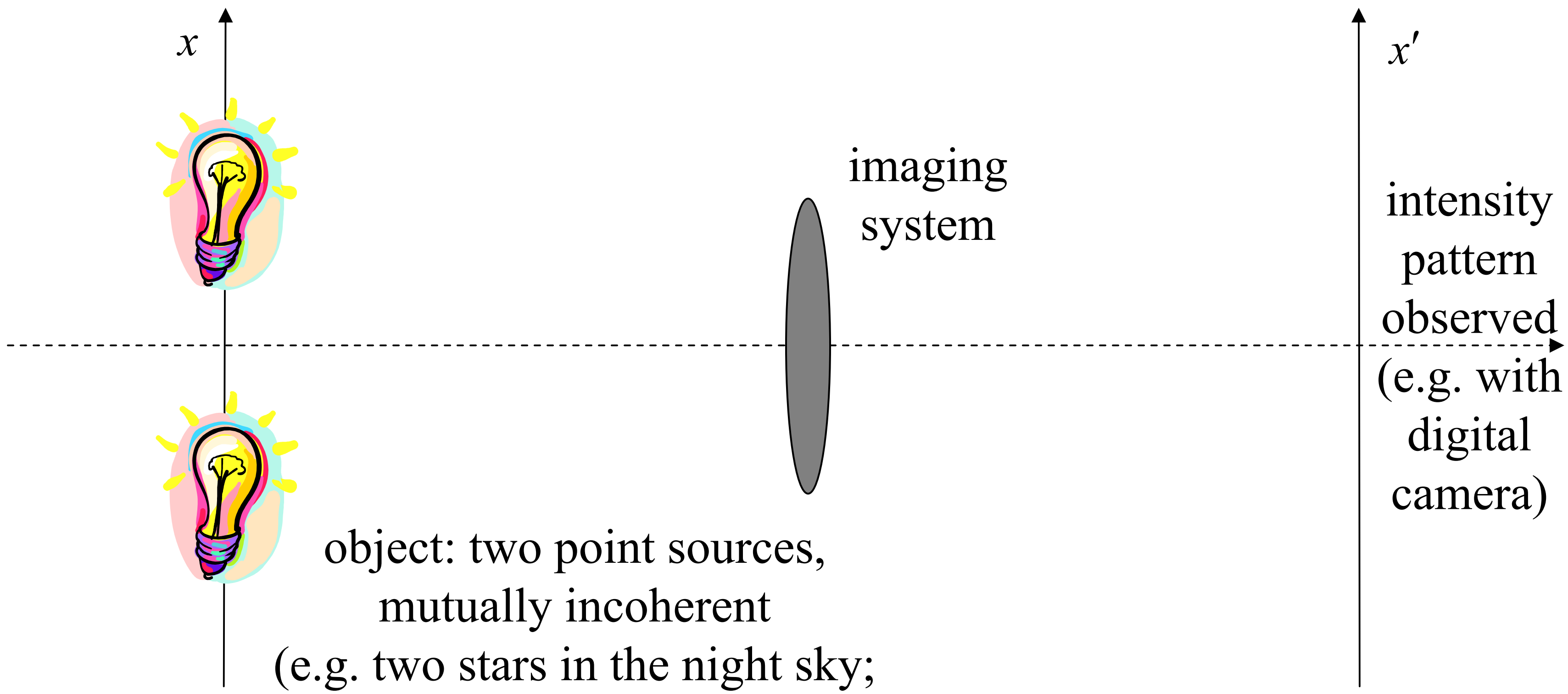
The meaning of “resolution”

[from the New Merriam-Webster Dictionary, 1989 ed.]:

resolve *v* : **1** to break up into constituent parts: ANALYZE; **2** to find an answer to : SOLVE; **3** DETERMINE, DECIDE; **4** to make or pass a formal resolution

resolution *n* : **1** the act or process of resolving **2** the action of solving, *also* : SOLUTION; **3** the quality of being resolute : FIRMNESS, DETERMINATION; **4** a formal statement expressing the opinion, will or, intent of a body of persons

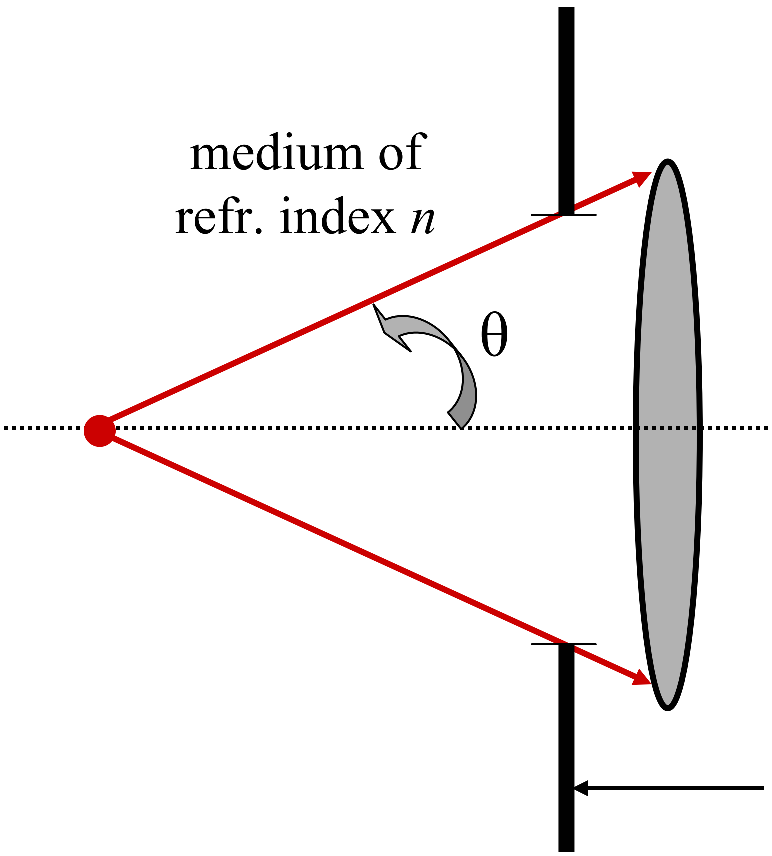
The two–point resolution problem



object: two point sources,
mutually incoherent
(e.g. two stars in the night sky;
two fluorescent beads in a solution)

<u>The resolution question</u> [Rayleigh, 1879]: when do we cease to be able to <u>resolve</u> the two point sources (*i.e.*, tell them apart) due to the blurring introduced in the image by the finite (NA)?

Numerical Aperture and Speed (or F–Number)



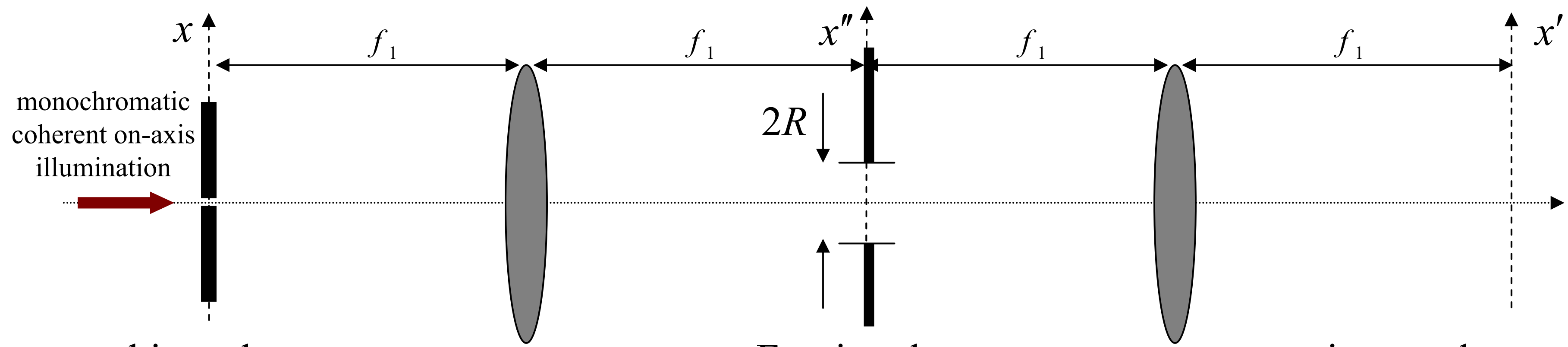
θ: half-angle subtended by the imaging system from an *axial* object

Numerical Aperture
(NA) = $n \sin\theta$

Speed (f/#)=1/2(NA)
pronounced f-number, e.g.
f/8 means (f/#)=8.

Aperture stop
the physical element which limits the angle of acceptance of the imaging system

PSF *vs* NA



object plane
impulse

Fourier plane
circ-aperture

image plane
observed field
(PSF)

$$g_{\text{in}}(x,y) = \delta(x)\delta(y)$$

$$H(x'',y'') = \text{circ}\left(\frac{r''}{R}\right)$$

$\mathfrak{I}$ Fourier transform

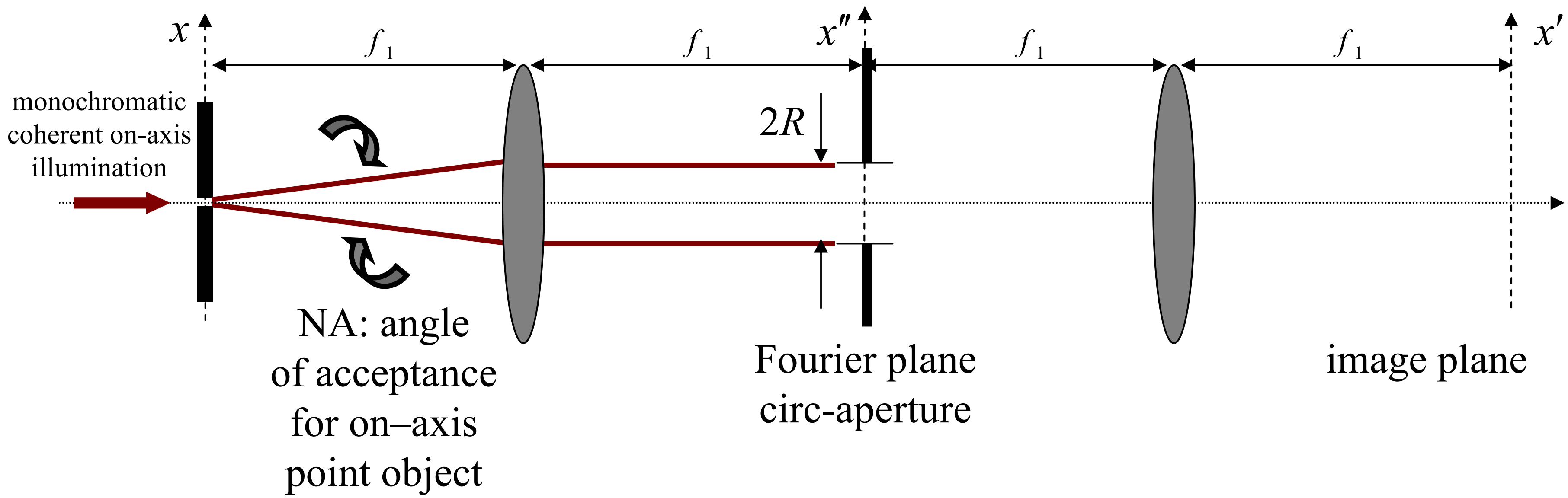
$$\text{jinc}(.,.) \equiv 2\frac{J_1\left(2\pi\frac{R}{f_1}\frac{r'}{\lambda}\right)}{2\pi\frac{R}{f_1}\frac{r'}{\lambda}}$$

(unit magnification)

$$r'' = \sqrt{x''^2 + y''^2}$$ radial coordinate @ Fourier plane

$$r' = \sqrt{x'^2 + y'^2}$$ radial coordinate @ image plane

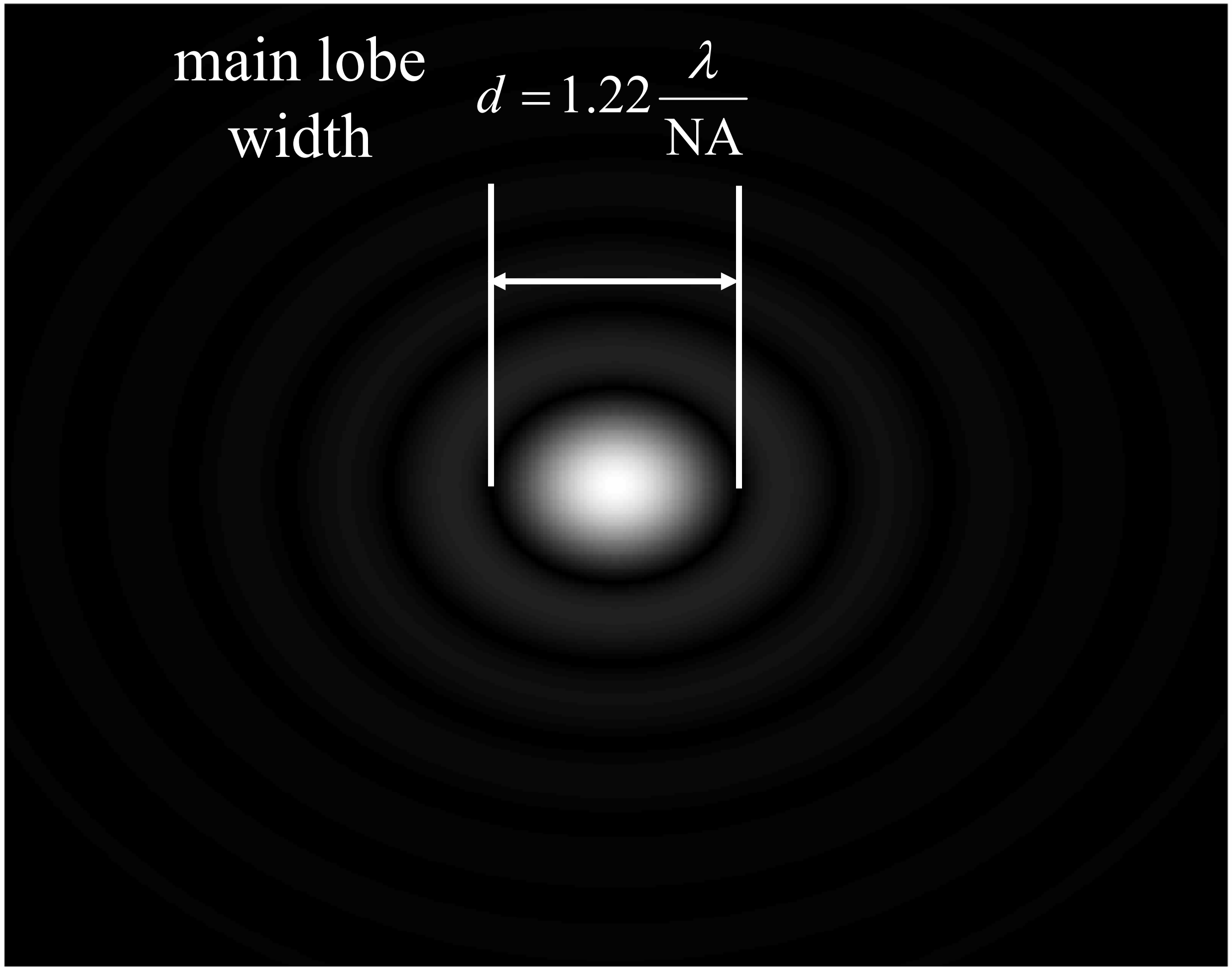
PSF *vs* NA



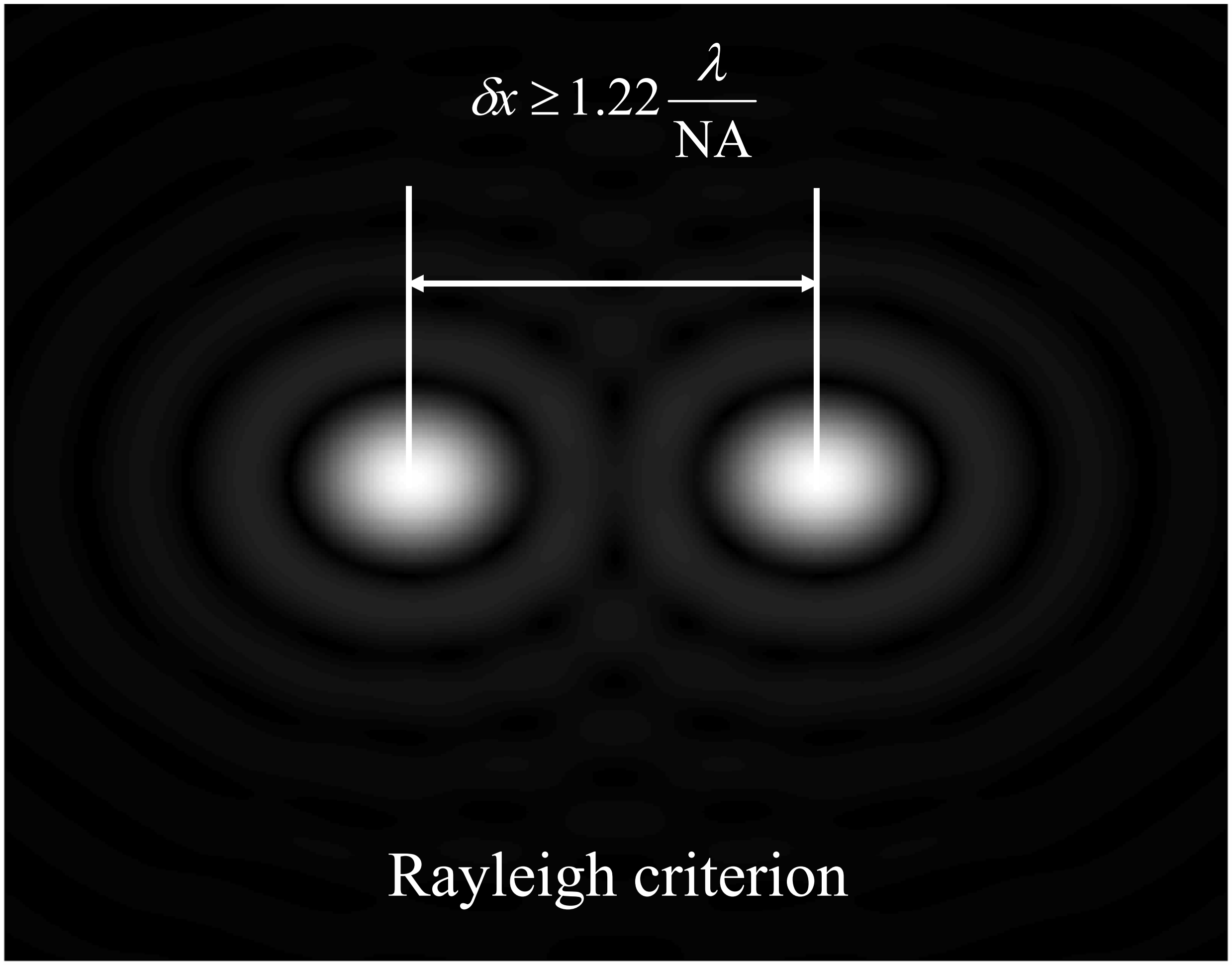
$$\mathrm{jinc}\left(-2\frac{R}{f_1}\frac{x'}{\lambda}, -2\frac{R}{f_1}\frac{y'}{\lambda}\right) \equiv 2\frac{\mathrm{J}_1\left(2\pi\frac{R}{f_1}\frac{r'}{\lambda}\right)}{2\pi\frac{R}{f_1}\frac{r'}{\lambda}} = 2\frac{\mathrm{J}_1\left(2\pi(\mathrm{NA})\frac{r'}{\lambda}\right)}{2\pi(\mathrm{NA})\frac{r'}{\lambda}}$$

Numerical Aperture (NA) by definition: $(\mathrm{NA}) \equiv \dfrac{R}{f_1}$

Diffraction

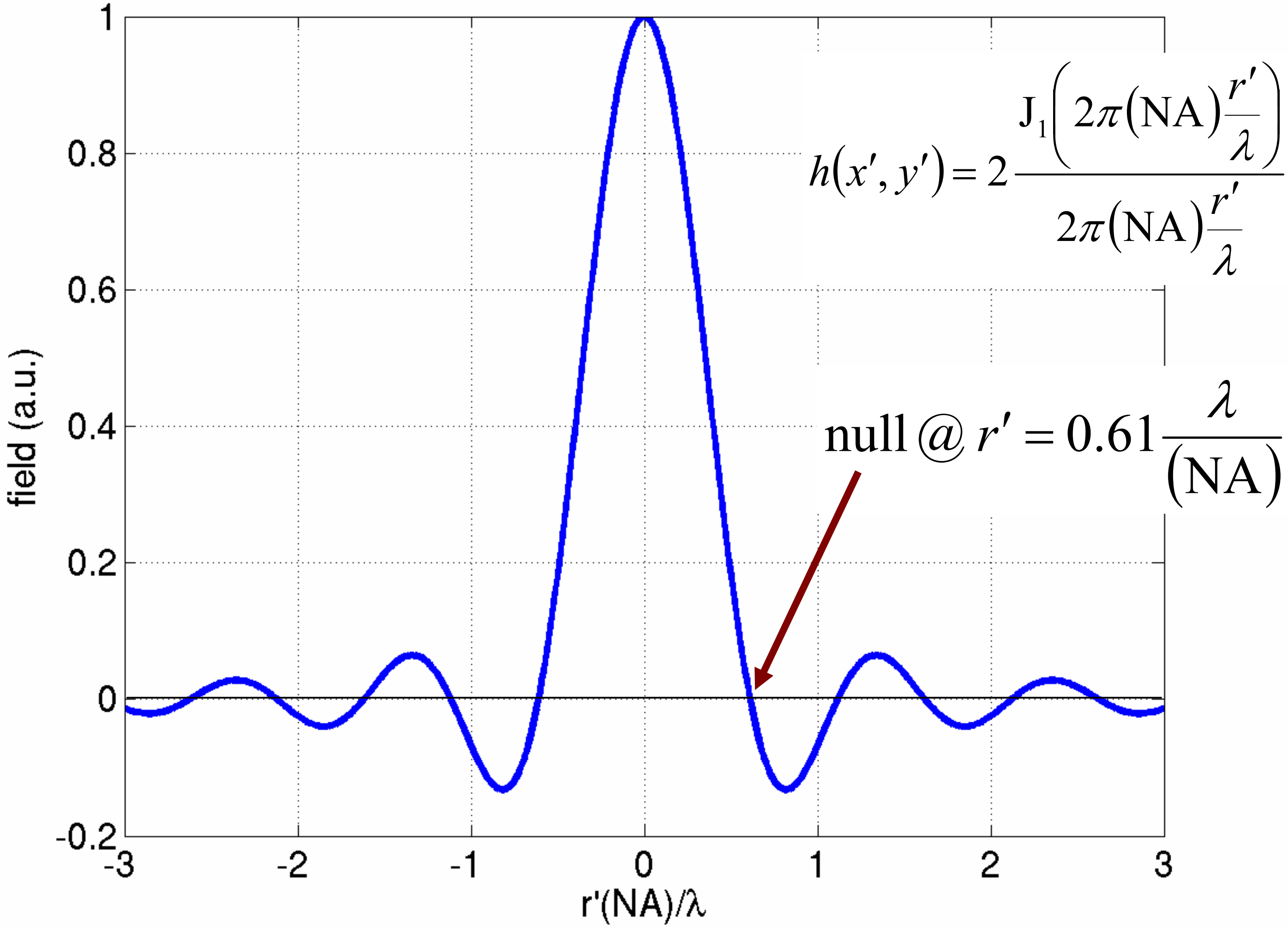


Diffraction-limited resolution



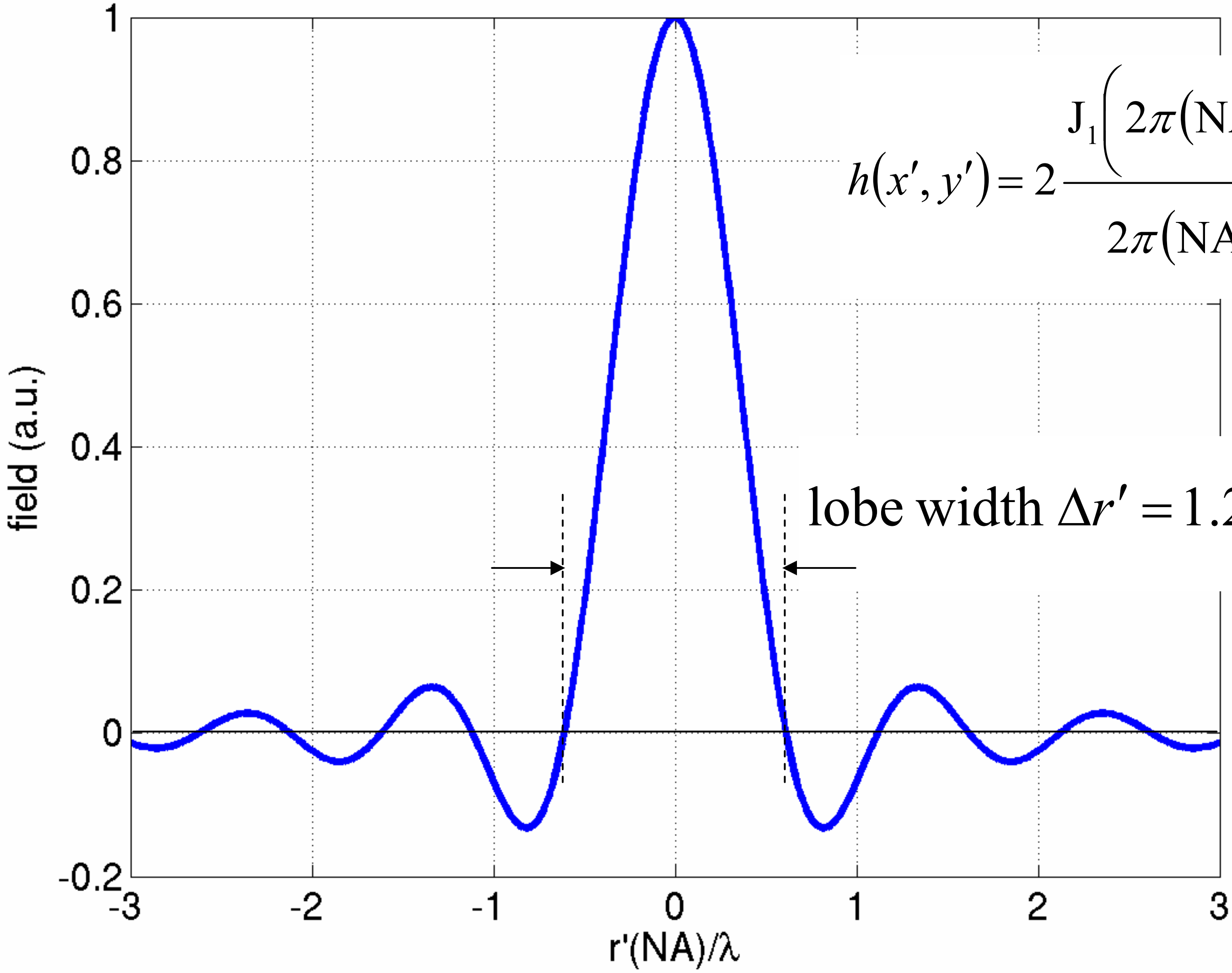
(incoherent imaging)

PSF *vs* NA



1
0.8
0.6
0.4
0.2
0
-0.2
field (a.u.)
-3
-2
-1
0
1
2
3
r'(NA)/λ
$h(x', y') = 2\frac{J_1\left(2\pi(\mathrm{NA})\frac{r'}{\lambda}\right)}{2\pi(\mathrm{NA})\frac{r'}{\lambda}}$
null @ $r' = 0.61\frac{\lambda}{(\mathrm{NA})}$

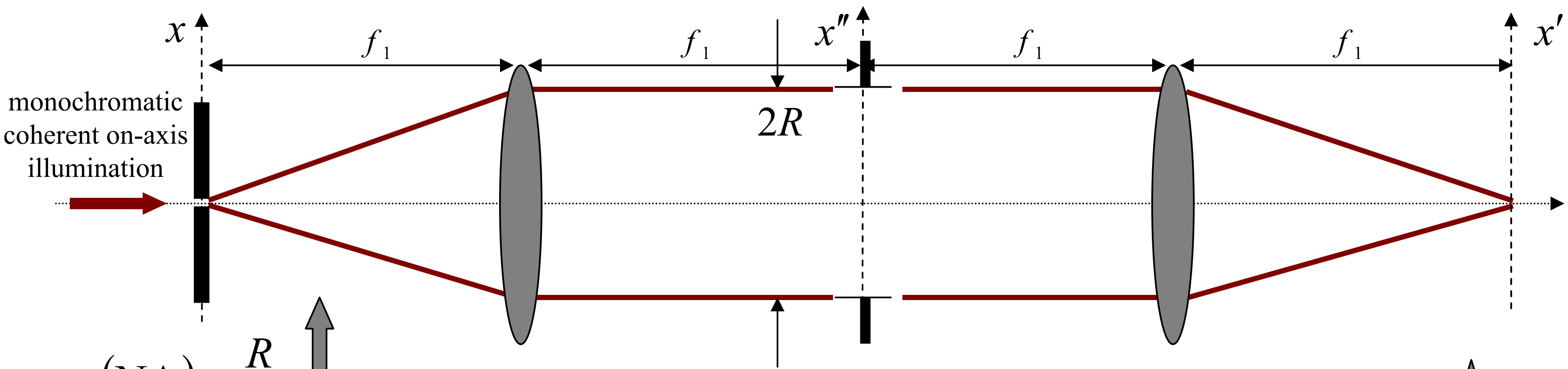
PSF *vs* NA



$$h(x',y') = 2\frac{J_1\left(2\pi(\mathrm{NA})\frac{r'}{\lambda}\right)}{2\pi(\mathrm{NA})\frac{r'}{\lambda}}$$

$$\text{lobe width } \Delta r' = 1.22\frac{\lambda}{(\mathrm{NA})}$$

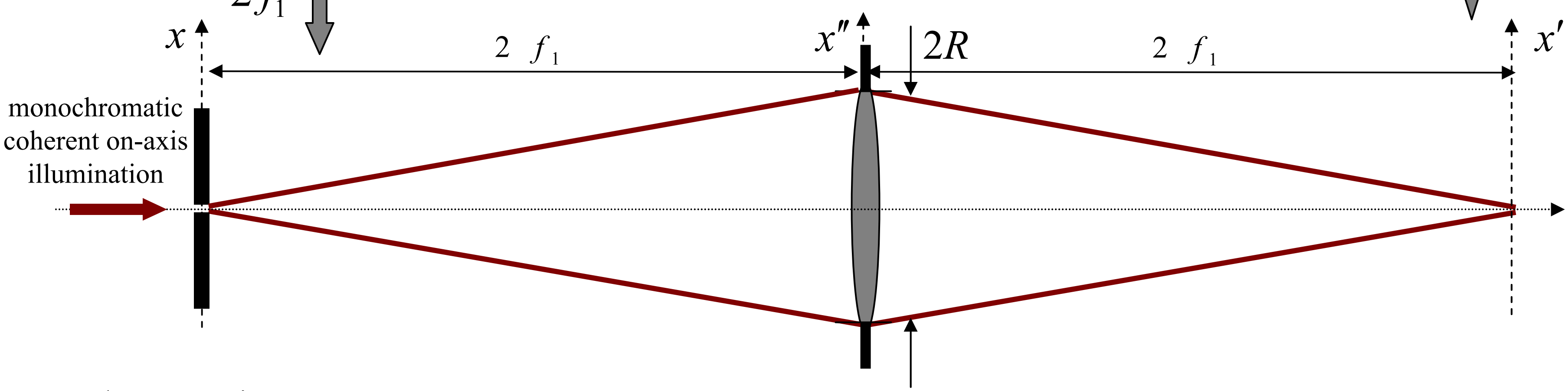
NA in unit–mag imaging systems



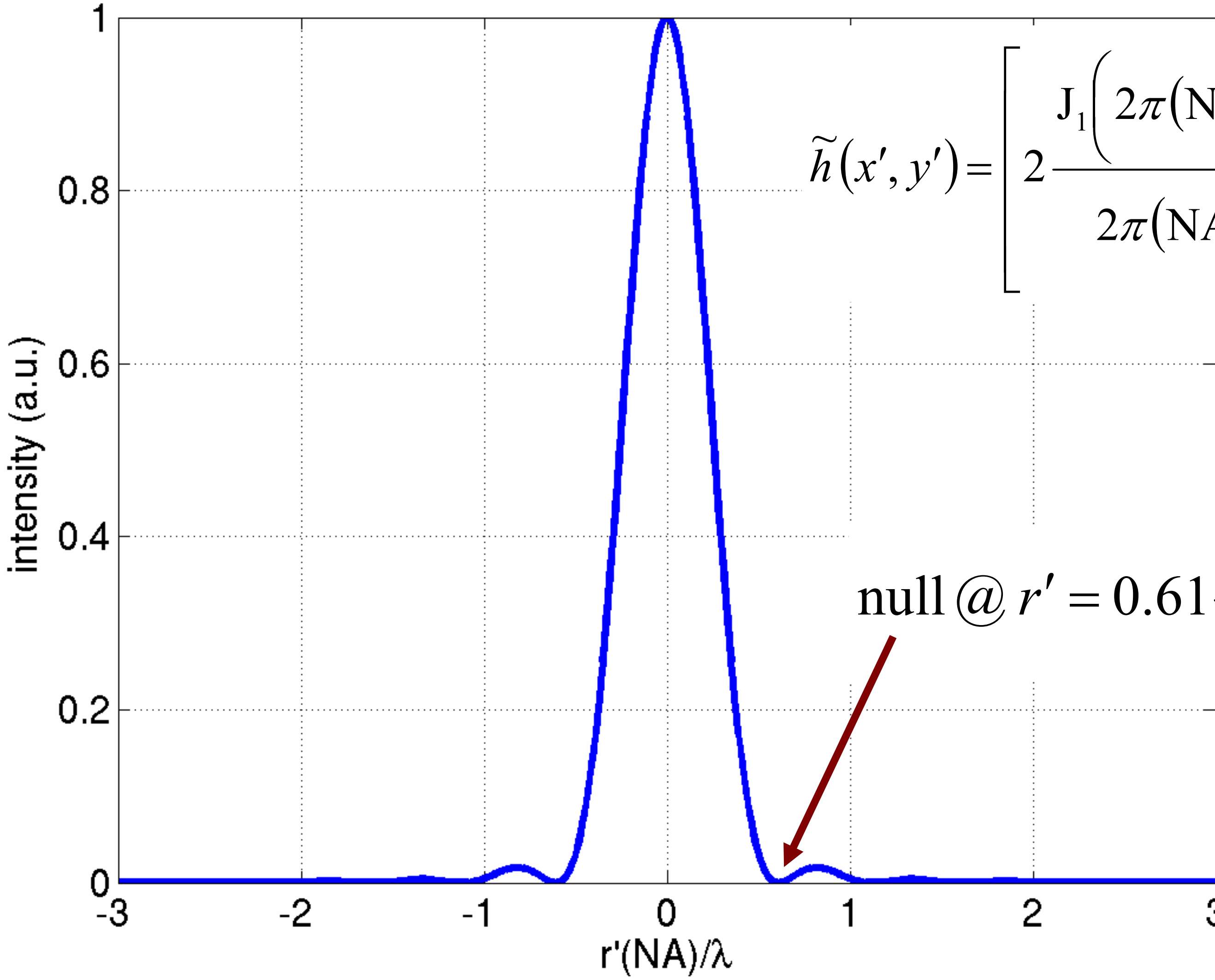
$$(\mathrm{NA}) \equiv \frac{R}{f_1}$$

in both cases, $\mathrm{PSF} = h(x', y') = h(r') = \mathrm{jinc}\left(2(\mathrm{NA})\frac{r'}{\lambda}\right)$

$$(\mathrm{NA}) \equiv \frac{R}{2f_1}$$



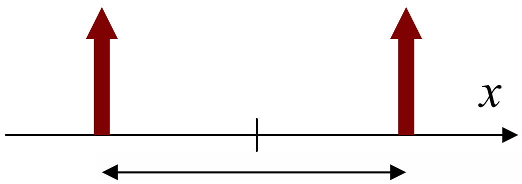
The incoherent case: $\tilde{h}(x', y') = |h(x', y')|^2$



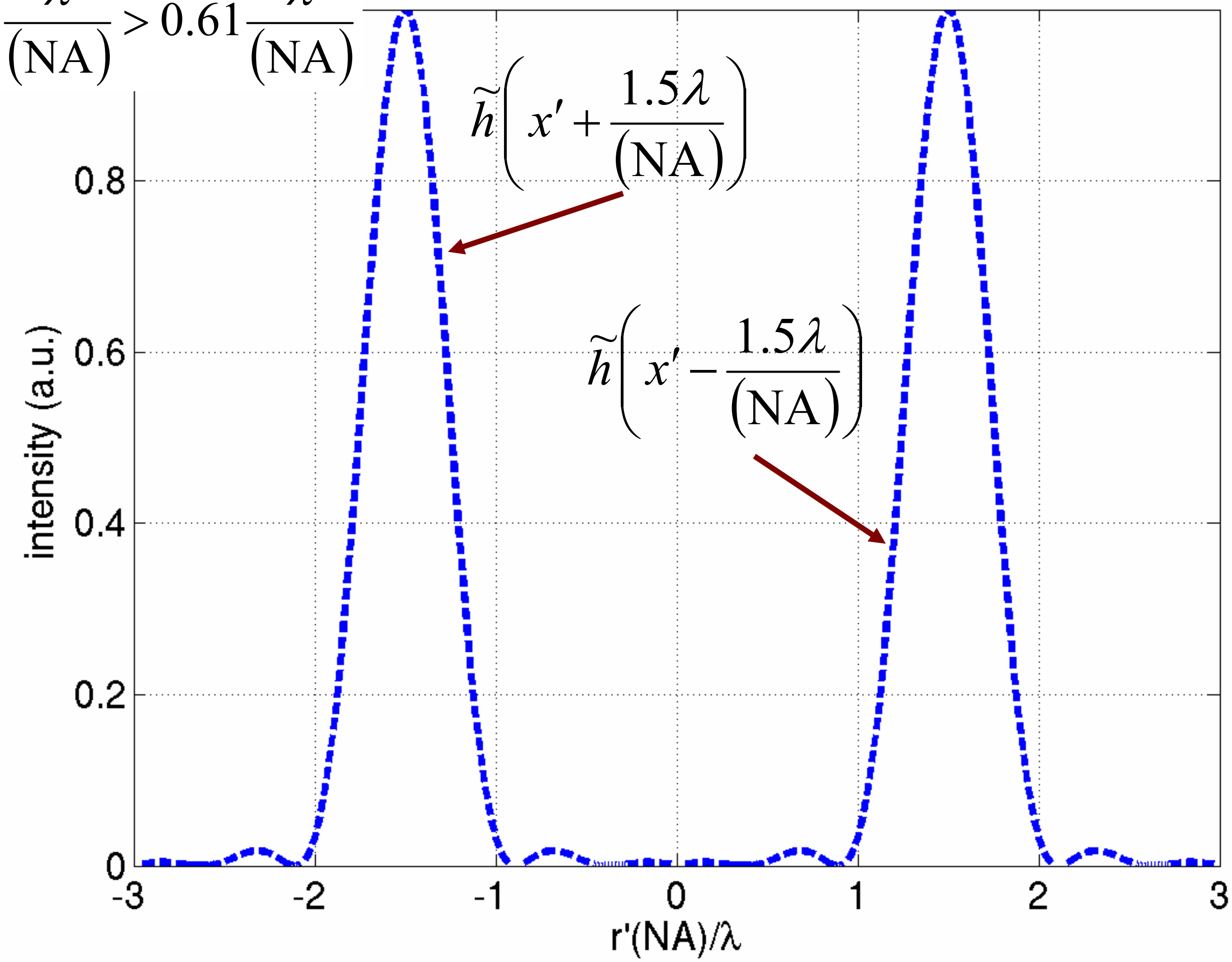
$$\tilde{h}(x', y') = \left[2 \frac{J_1\left(2\pi(\mathrm{NA})\frac{r'}{\lambda}\right)}{2\pi(\mathrm{NA})\frac{r'}{\lambda}} \right]^2$$

$$\text{null} @ r' = 0.61 \frac{\lambda}{(\mathrm{NA})}$$

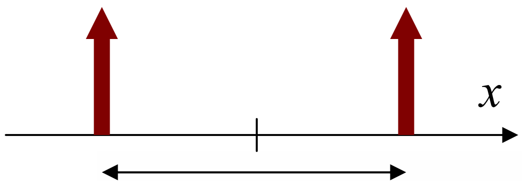
Resolution in optical systems



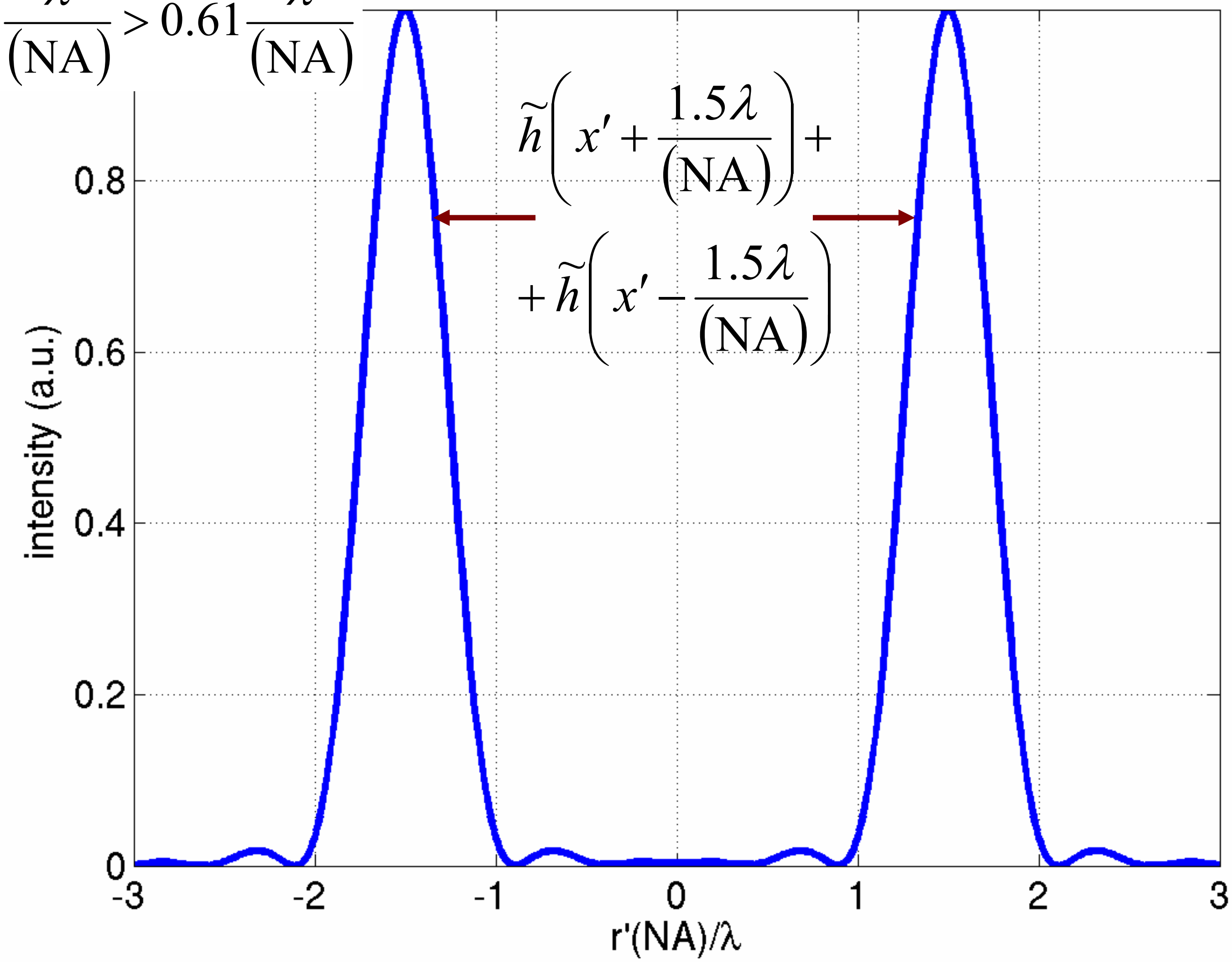
$$\Delta r = 3.0\frac{\lambda}{(\mathrm{NA})} > 0.61\frac{\lambda}{(\mathrm{NA})}$$



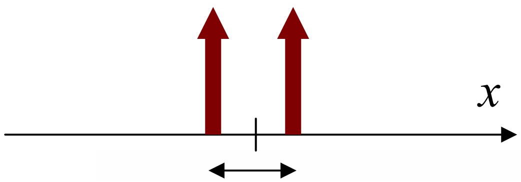
Resolution in optical systems



$$\Delta r = 3.0\frac{\lambda}{(\mathrm{NA})} > 0.61\frac{\lambda}{(\mathrm{NA})}$$

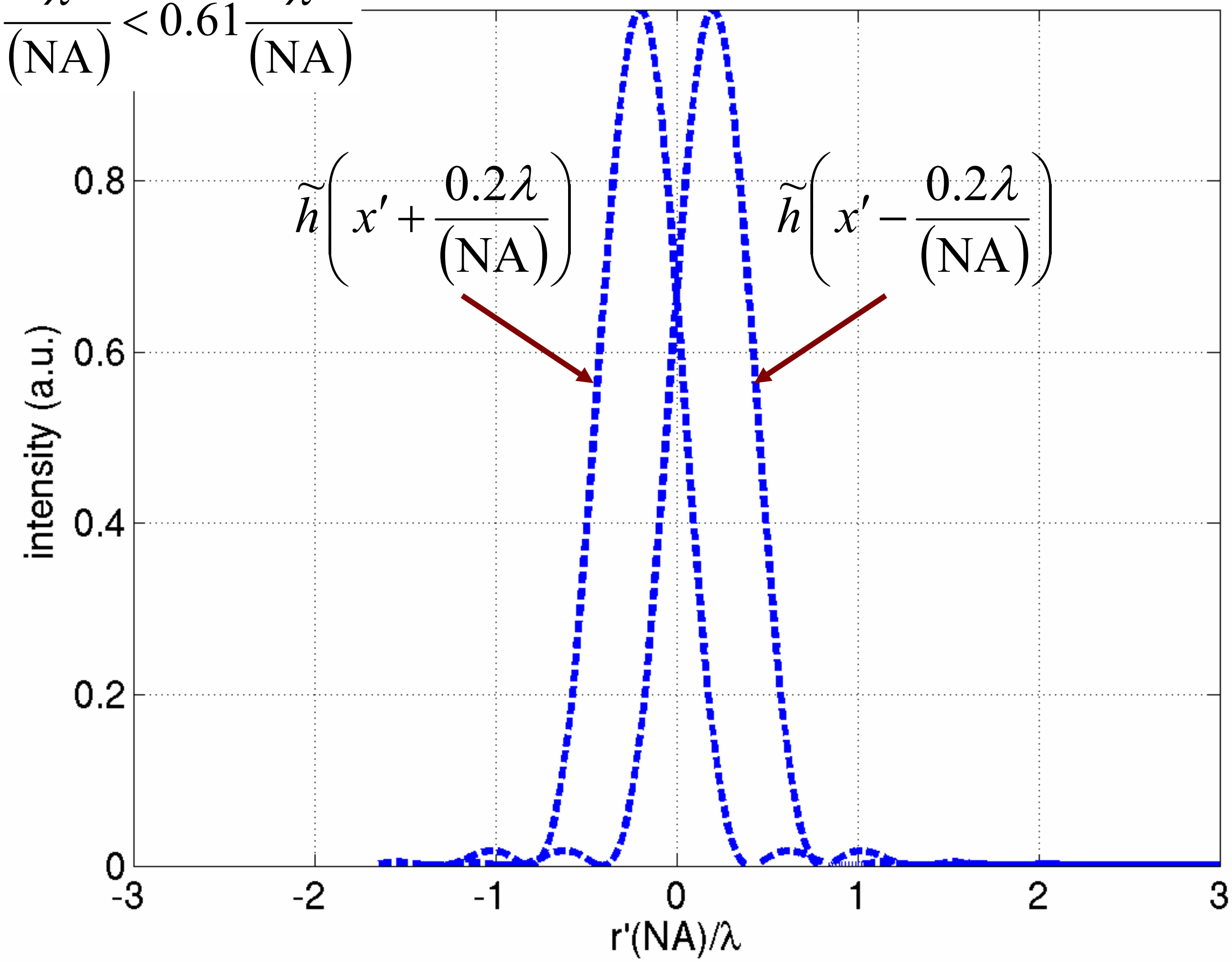


$$\tilde{h}\left(x' + \frac{1.5\lambda}{(\mathrm{NA})}\right) + \tilde{h}\left(x' - \frac{1.5\lambda}{(\mathrm{NA})}\right)$$

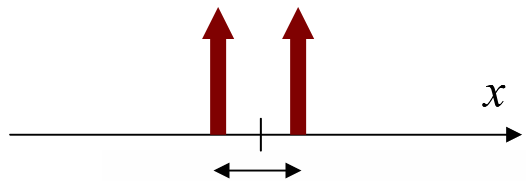


$$\Delta r = 0.4\frac{\lambda}{(\mathrm{NA})} < 0.61\frac{\lambda}{(\mathrm{NA})}$$

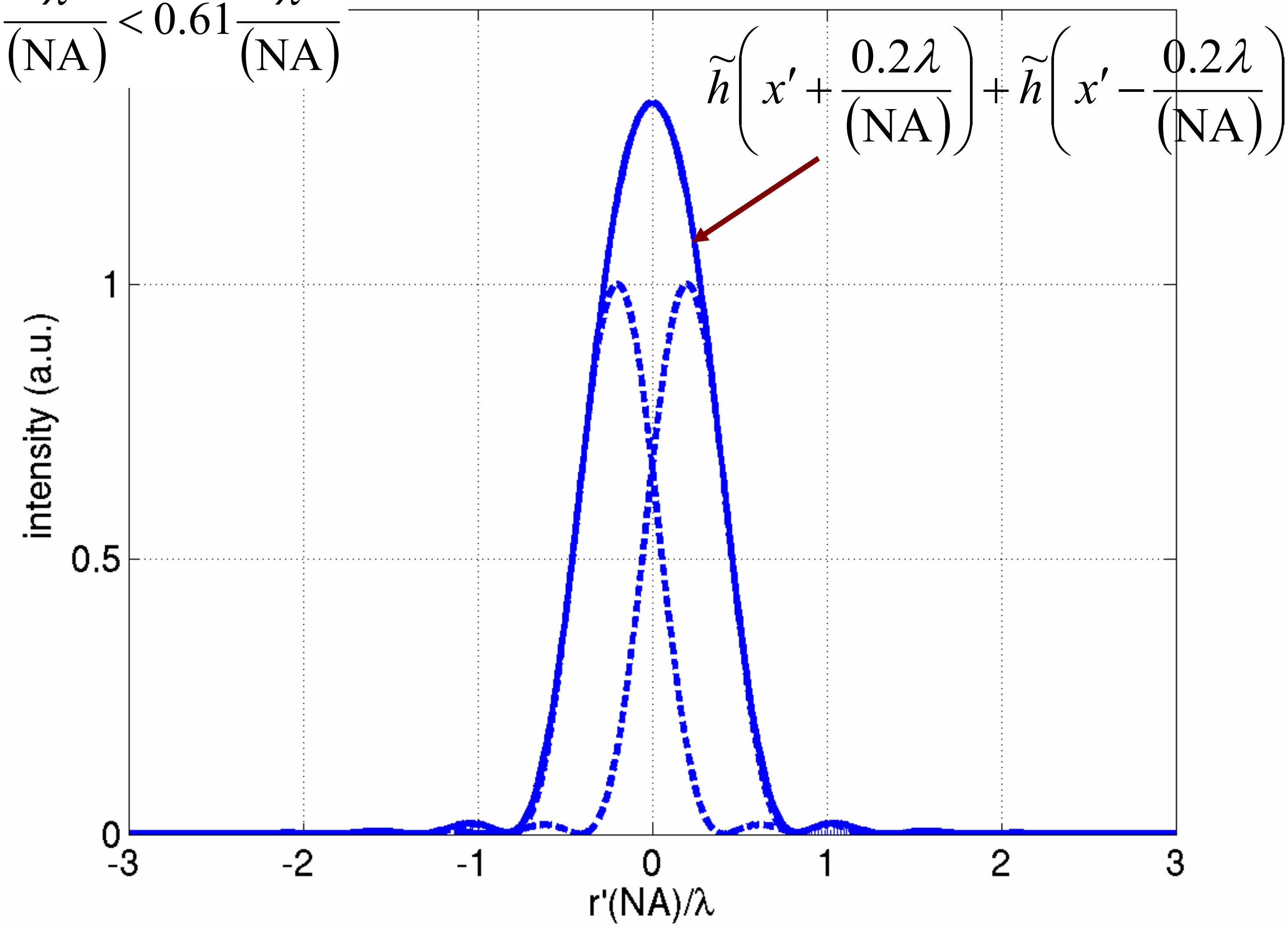
Resolution in optical systems



Resolution in optical systems

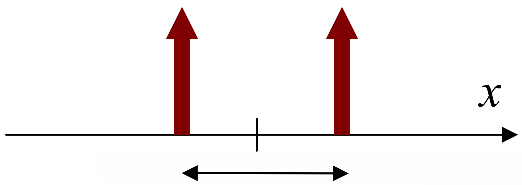


$$\Delta r = 0.4\frac{\lambda}{(\mathrm{NA})} < 0.61\frac{\lambda}{(\mathrm{NA})}$$

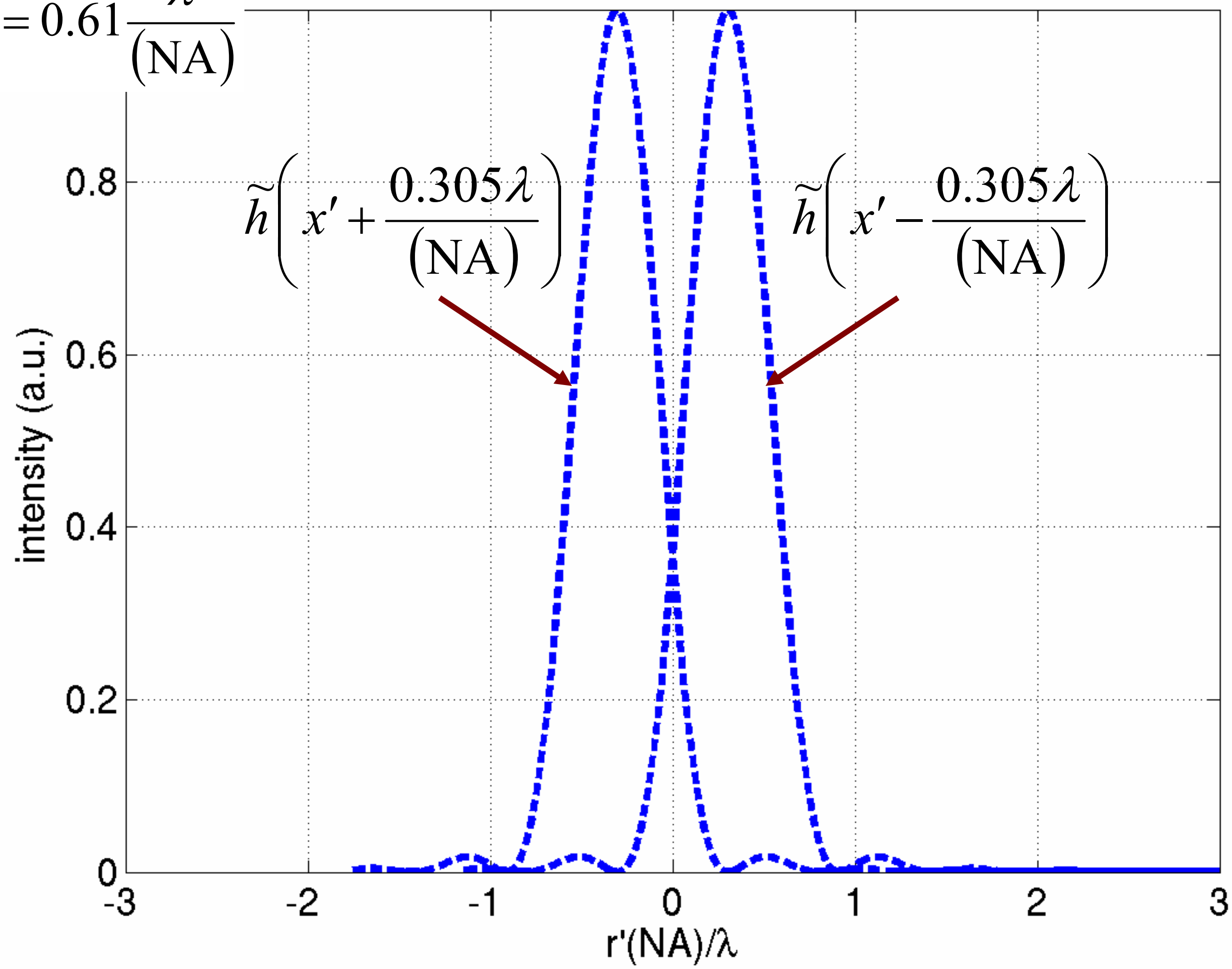


$$\tilde{h}\left(x' + \frac{0.2\lambda}{(\mathrm{NA})}\right) + \tilde{h}\left(x' - \frac{0.2\lambda}{(\mathrm{NA})}\right)$$

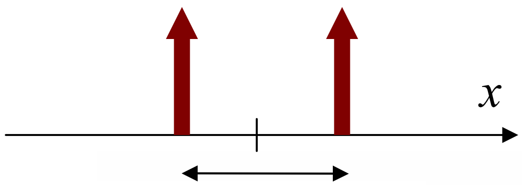
Resolution in optical systems



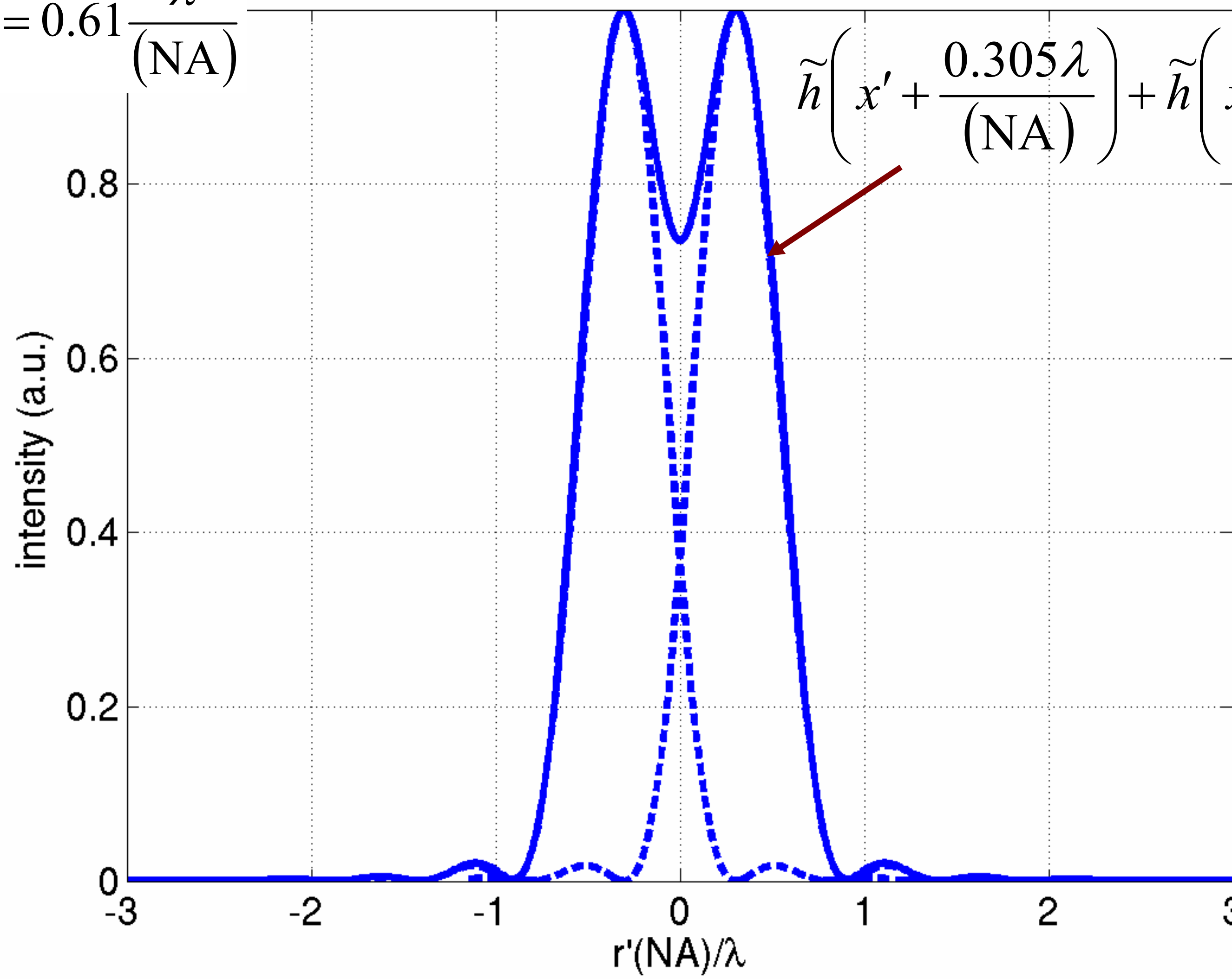
$$\Delta r = 0.61\frac{\lambda}{(\mathrm{NA})}$$



Resolution in optical systems

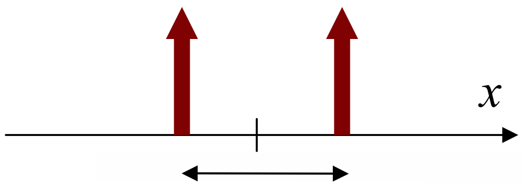


$$\Delta r = 0.61 \frac{\lambda}{(\mathrm{NA})}$$

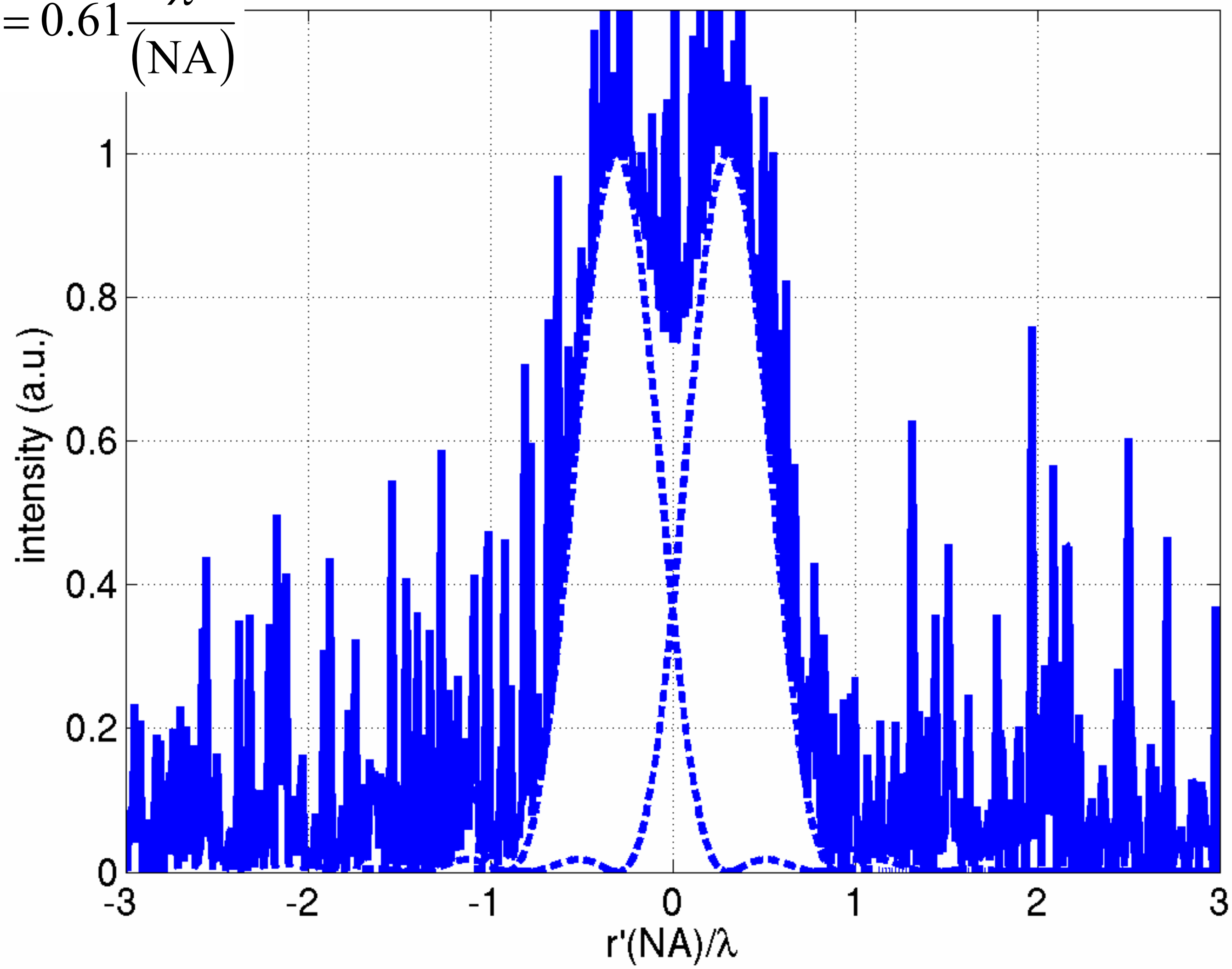


$$\tilde{h}\left(x' + \frac{0.305\lambda}{(\mathrm{NA})}\right) + \tilde{h}\left(x' - \frac{0.305\lambda}{(\mathrm{NA})}\right)$$

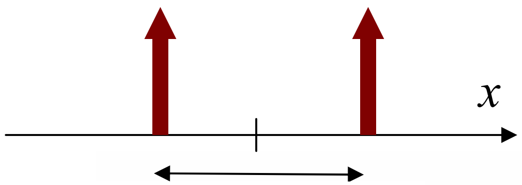
Resolution in noisy optical systems



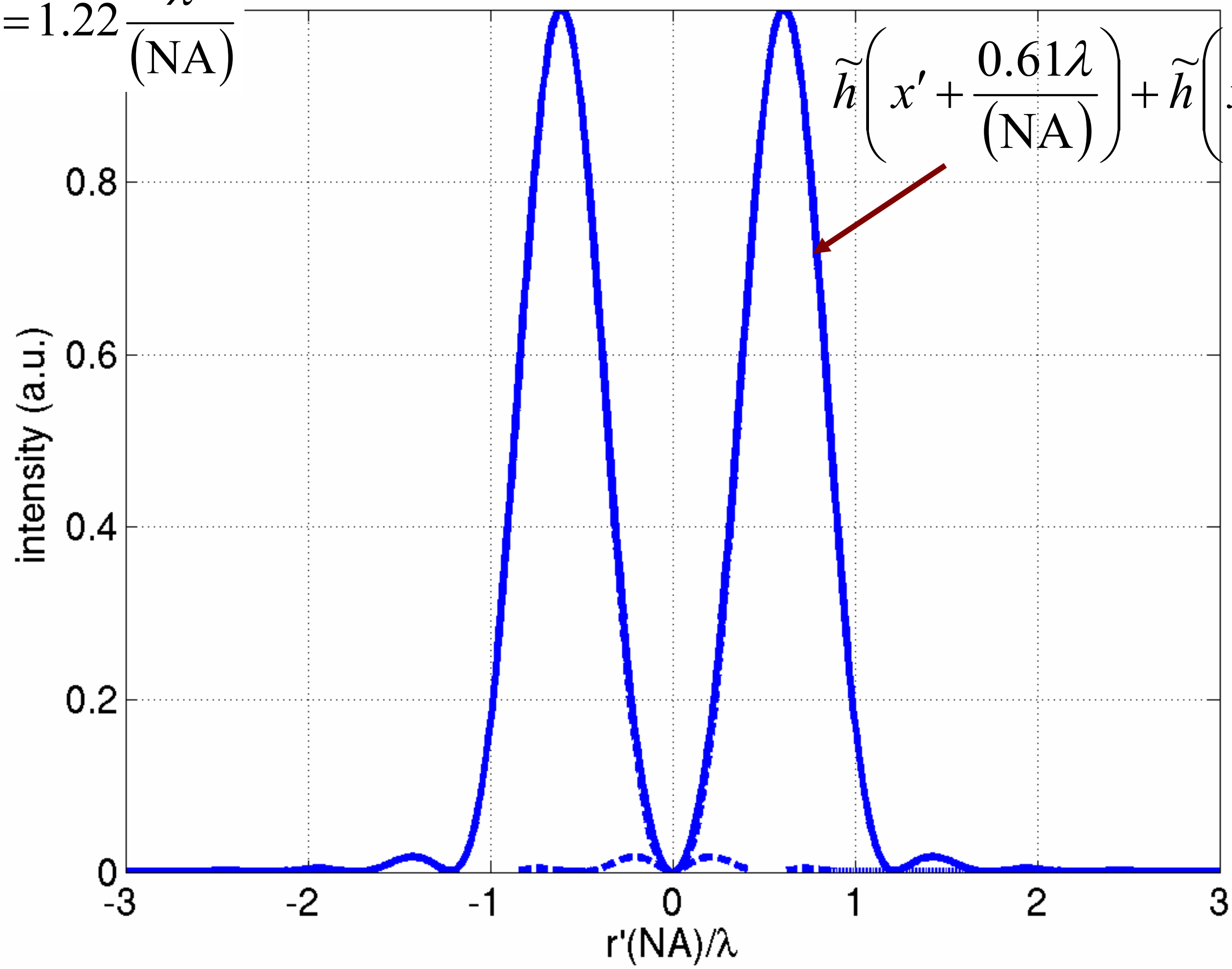
$$\Delta r = 0.61 \frac{\lambda}{(\mathrm{NA})}$$



“Safe” resolution in optical systems



$$\Delta r = 1.22\frac{\lambda}{(\mathrm{NA})}$$



$$\tilde{h}\left(x' + \frac{0.61\lambda}{(\mathrm{NA})}\right) + \tilde{h}\left(x' - \frac{0.61\lambda}{(\mathrm{NA})}\right)$$

Diffraction–limited resolution (safe)

Two point objects are "**just resolvable**" (limited by diffraction only)
if they are separated by:

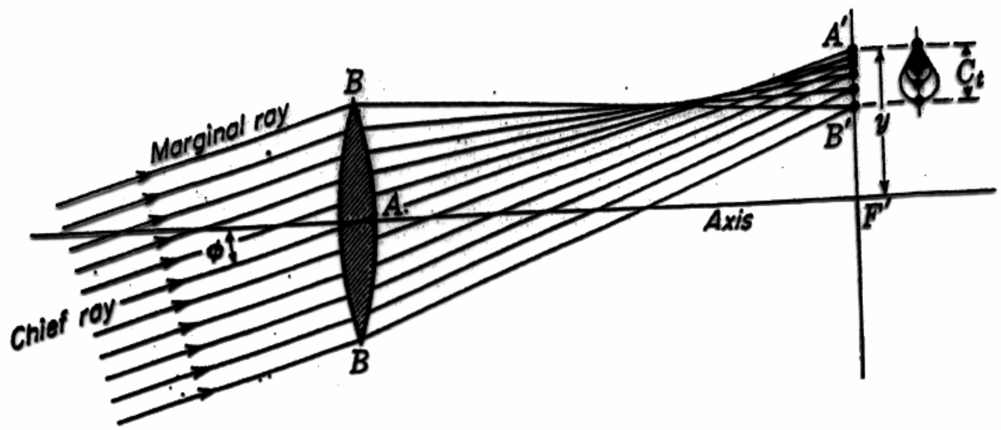
Two–dimensional systems (rotationally symmetric PSF)	One–dimensional systems (e.g. slit–like aperture)
Safe definition: (one–lobe spacing) $\Delta r' = 1.22\dfrac{\lambda}{(\mathrm{NA})}$	$\Delta x' = \dfrac{\lambda}{(\mathrm{NA})}$
Pushy definition: (1/2–lobe spacing) $\Delta r' = 0.61\dfrac{\lambda}{(\mathrm{NA})}$	$\Delta x' = 0.5\dfrac{\lambda}{(\mathrm{NA})}$

You will see different authors giving different definitions.
Rayleigh in his original paper (1879) noted the issue of noise
and warned that the definition of "just–resolvable" points
is system– or application –dependent

Also affecting resolution: aberrations

All our calculations have assumed "geometrically perfect" systems, i.e. we calculated the wave–optics behavior of systems which, in the paraxial geometrical optics approximation would have imaged a point object onto a perfect point image.

The effect of aberrations (calculated with non–paraxial geometrical optics) is to blur the "geometrically perfect" image; including the effects of diffraction causes additional blur.



geometrical optics description

Lens aberrations

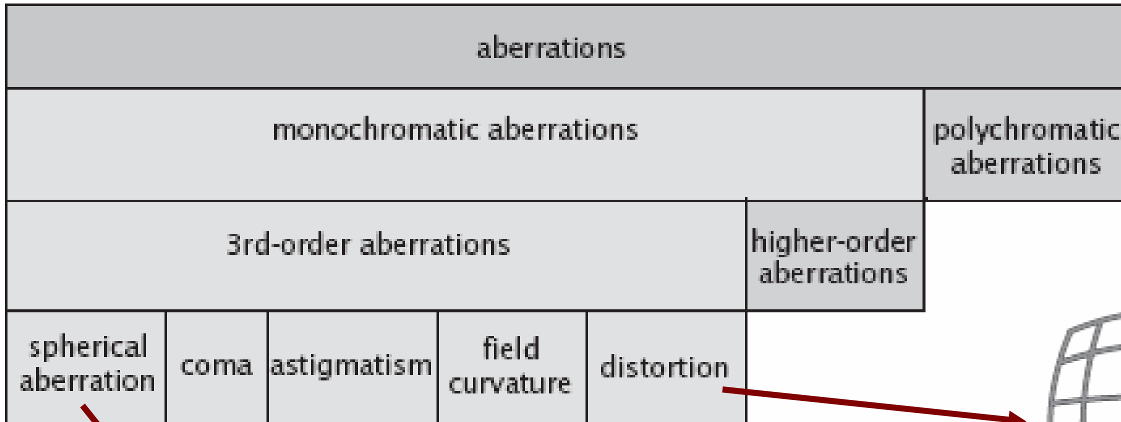
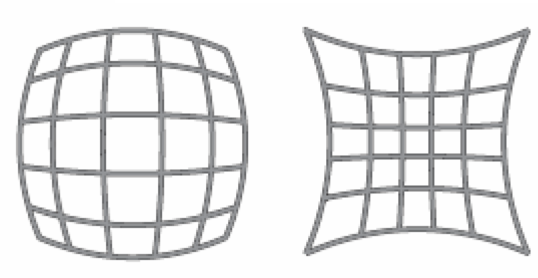


Figure 3.9: Classification of aberrations.



barrel

pincushion

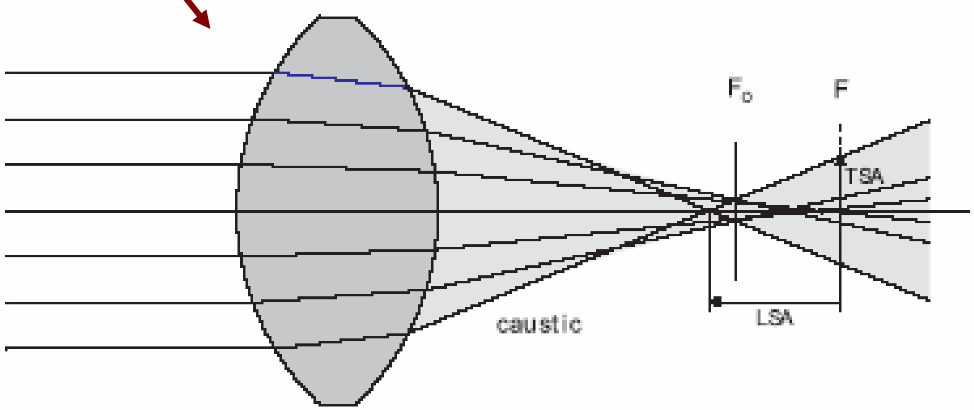
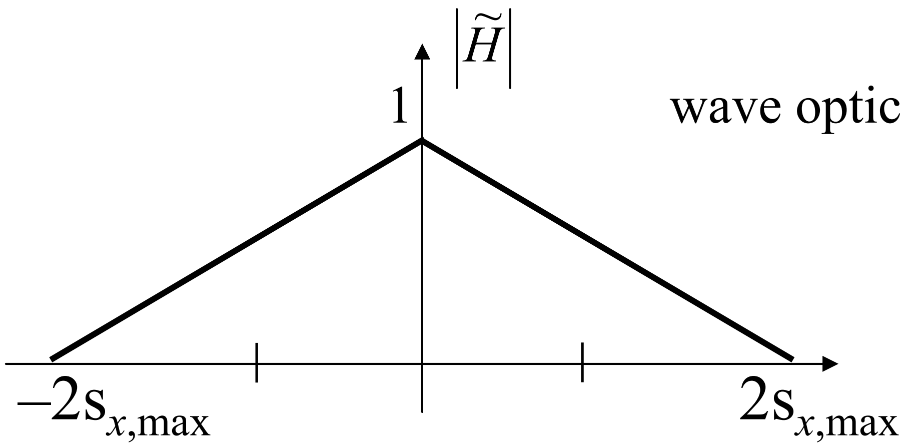
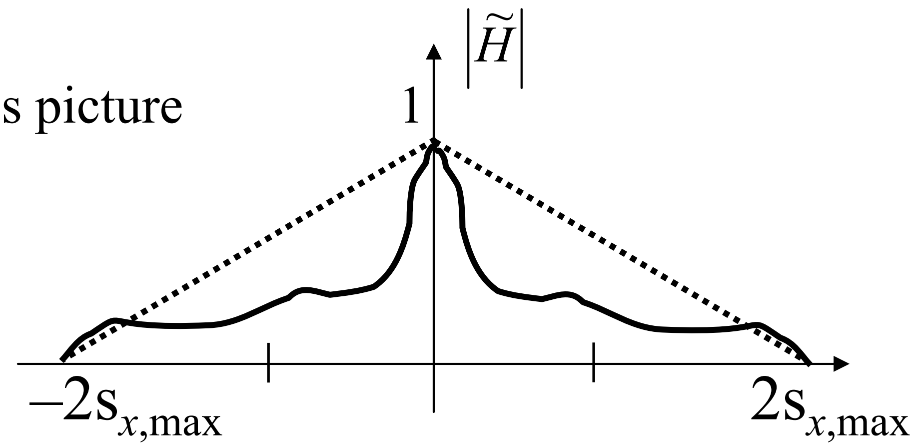


Figure 3.10: Spherical aberration of a convex lens. To obtain the best image quality, the image plane has to be moved from the paraxial focal plane F to the optimal position F_o. The caustic is the envelope of the outgoing ray bundle.

Effect of aberrations on resolution

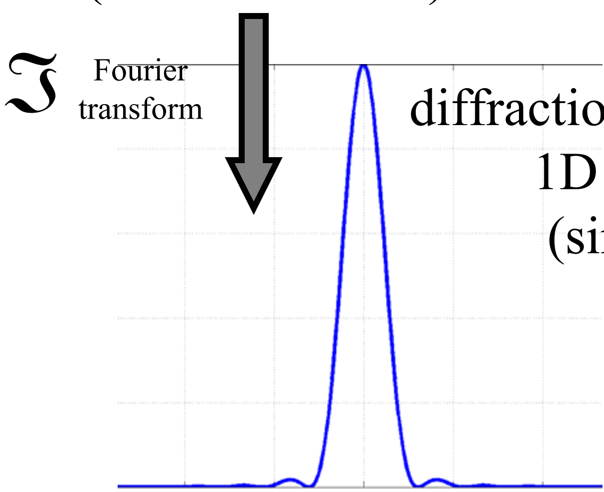


wave optics picture

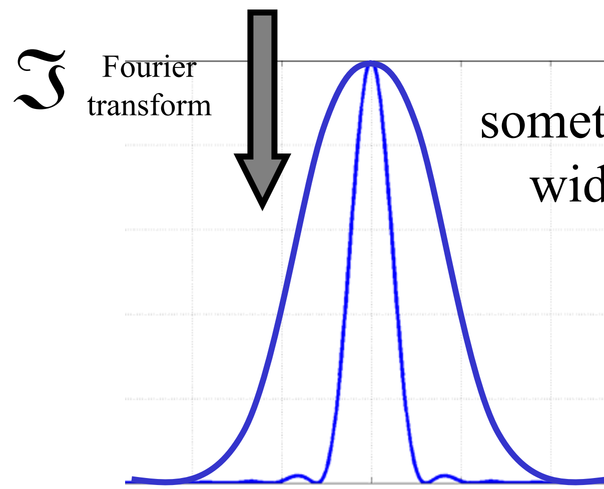


"**diffraction–limited**"
(aberration–free) 1D MTF

1D MTF with aberrations



diffraction–limited
1D PSF
(sinc^2)

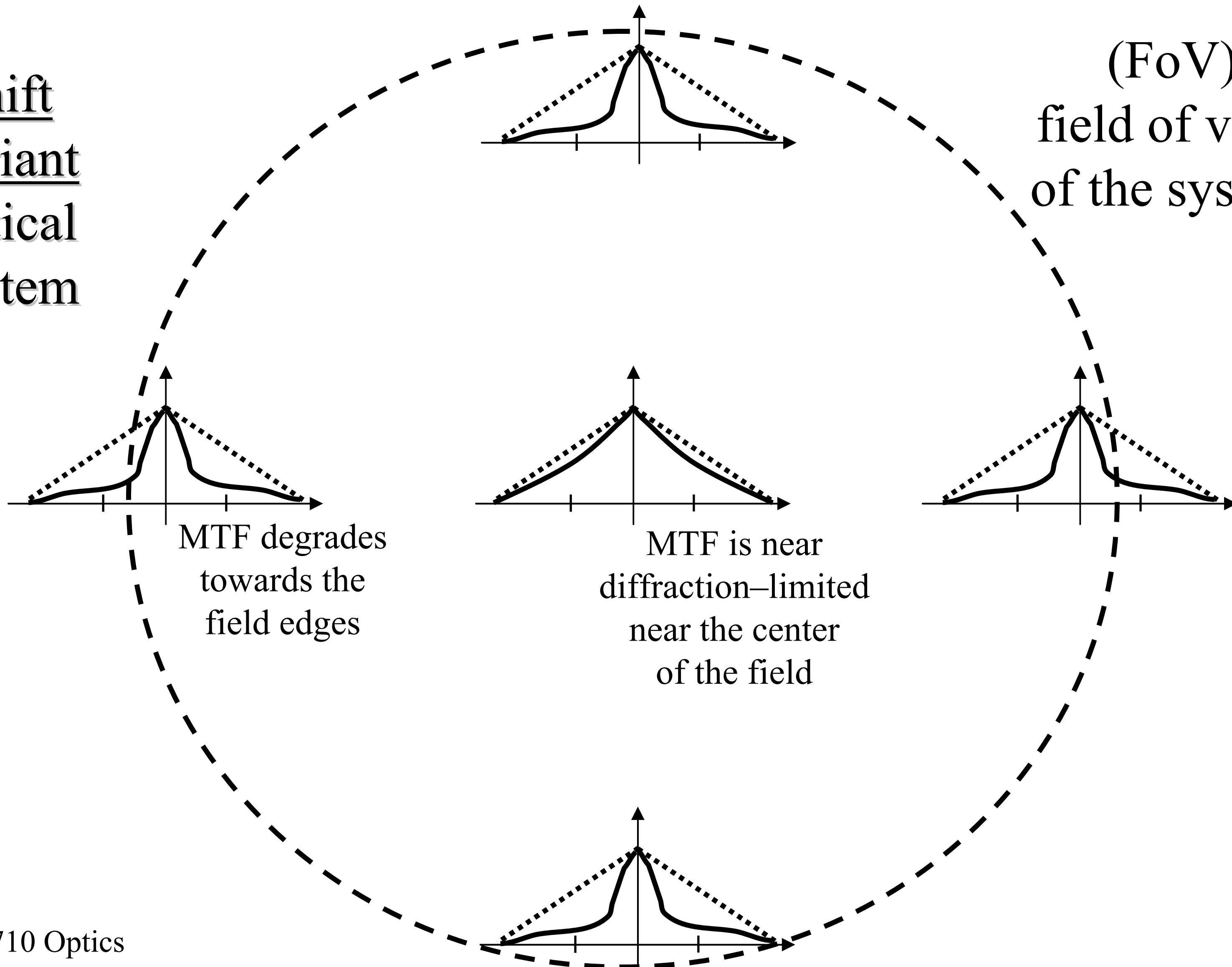


something
wider

Typical result of optical design

<u>shift</u>
<u>variant</u>
optical
system

(FoV)
field of view
of the system



The limits of our approximations

- Real–life MTFs include aberration effects, whereas our analysis has been "diffraction–limited"
- Aberration effects on the MTF are FoV (field) location–dependent: typically we get more blur near the edges of the field (narrower MTF $\Leftrightarrow$ broader PSF)
- This, in addition, means that real–life optical systems are not shift invariant either!
- $\Rightarrow$ the concept of MTF is approximate, near the region where the system is approximately shift invariant (recall: transfer functions can be defined only for shift invariant linear systems!)

The utility of our approximations

- Nevertheless, within the limits of the paraxial, linear shift–invariant system approximation, the concepts of PSF/MTF provide
  – a useful way of *thinking* about the behavior of optical systems
  – an upper limit on the performance of a given optical system (diffraction–limited performance is the best we can hope for, in paraxial regions of the field; aberrations will only make worse non–paraxial portions of the field)

Common misinterpretations

Attempting to resolve object features smaller than the "resolution limit" (e.g. $1.22\lambda/NA$) is hopeless.

NO: Image quality degradation as object features become smaller than the resolution limit ("exceed the resolution limit") is noise dependent and gradual.

Common misinterpretations

Attempting to resolve object features smaller than the "resolution limit" (e.g. $1.22\lambda/NA$) is hopeless.

NO: Besides, digital processing of the acquired images (e.g. methods such as the CLEAN algorithm, Wiener filtering, expectation maximization, etc.) can be employed.

Common misinterpretations

<u>Super-resolution</u>

By engineering the pupil function ("apodizing") to result in a PSF with narrower side–lobe, one can "beat" the resolution limitations imposed by the angular acceptance (NA) of the system.

NO:

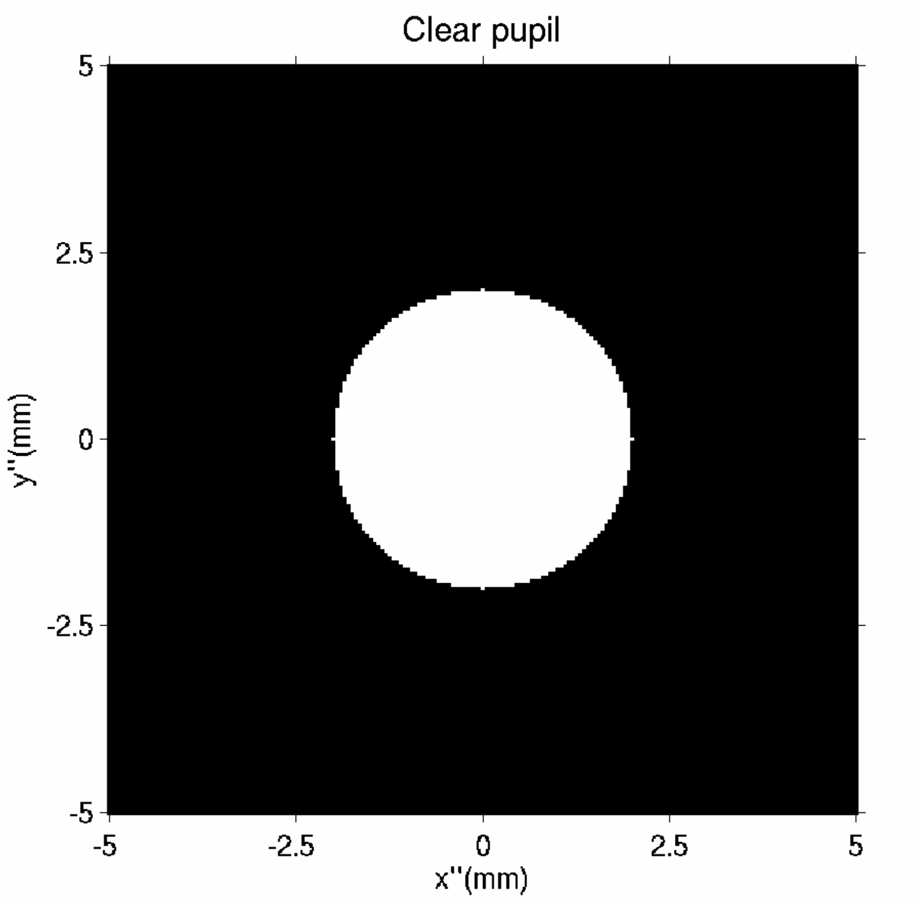
Pupil function design always results in

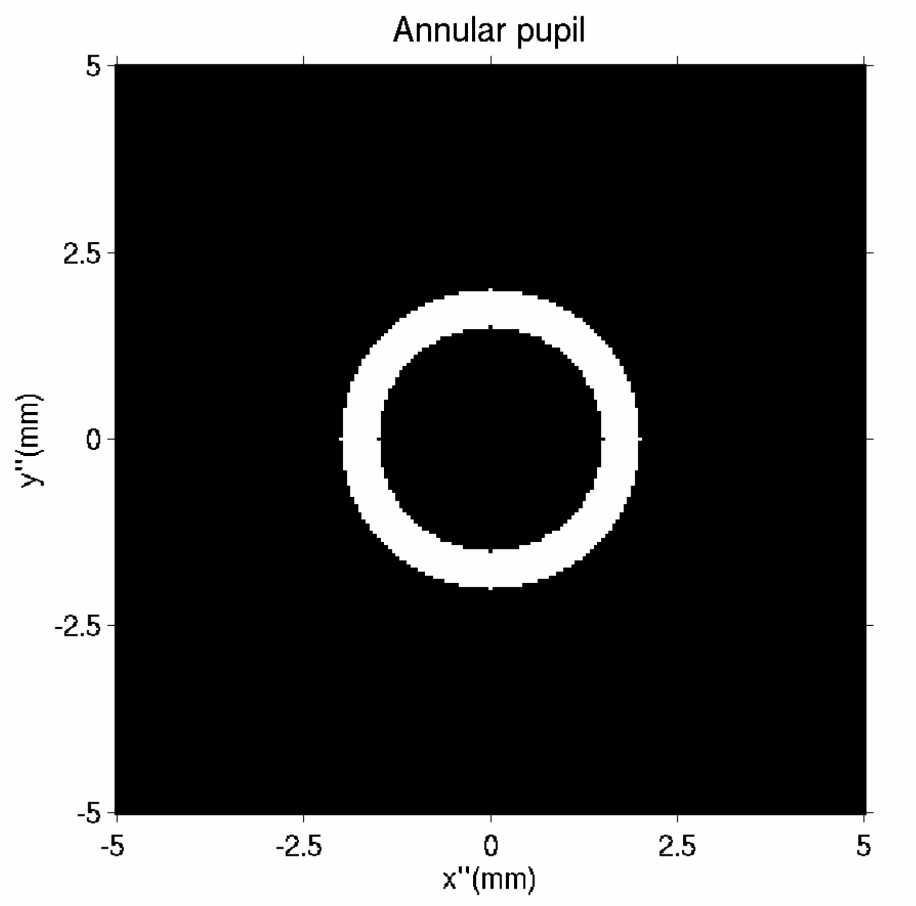
(i) narrower main lobe but accentuated side–lobes

(ii) lower power transmitted through the system

Both effects are **BAD** on the image

Apodization

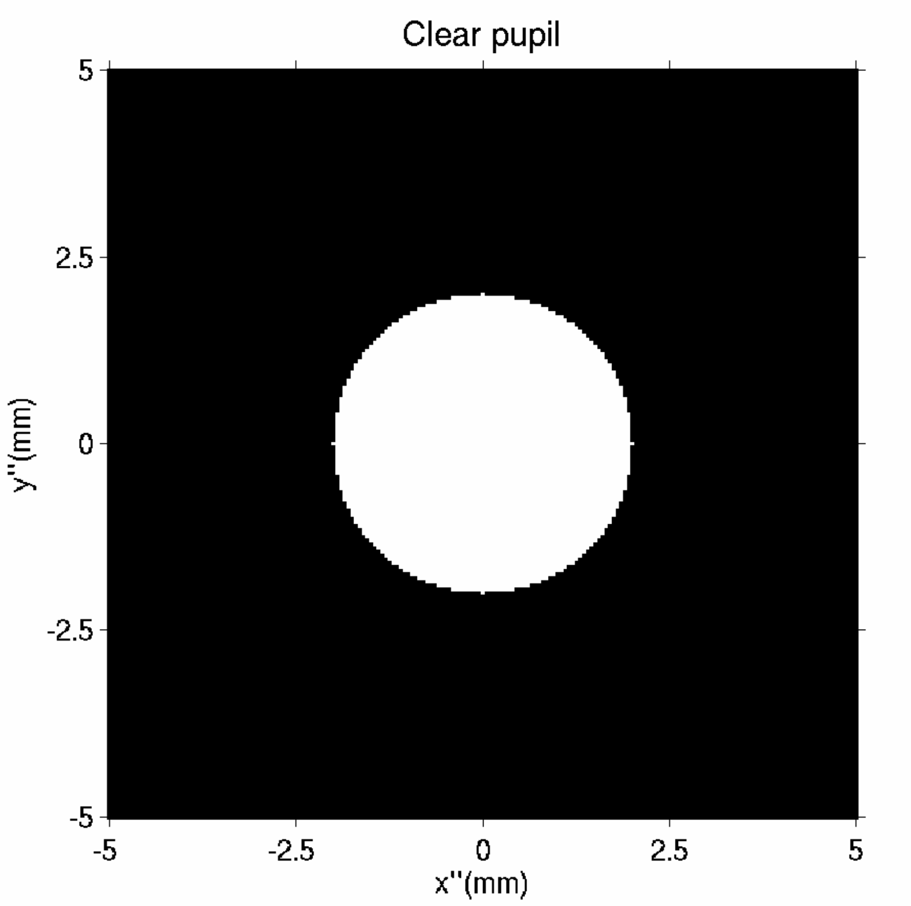


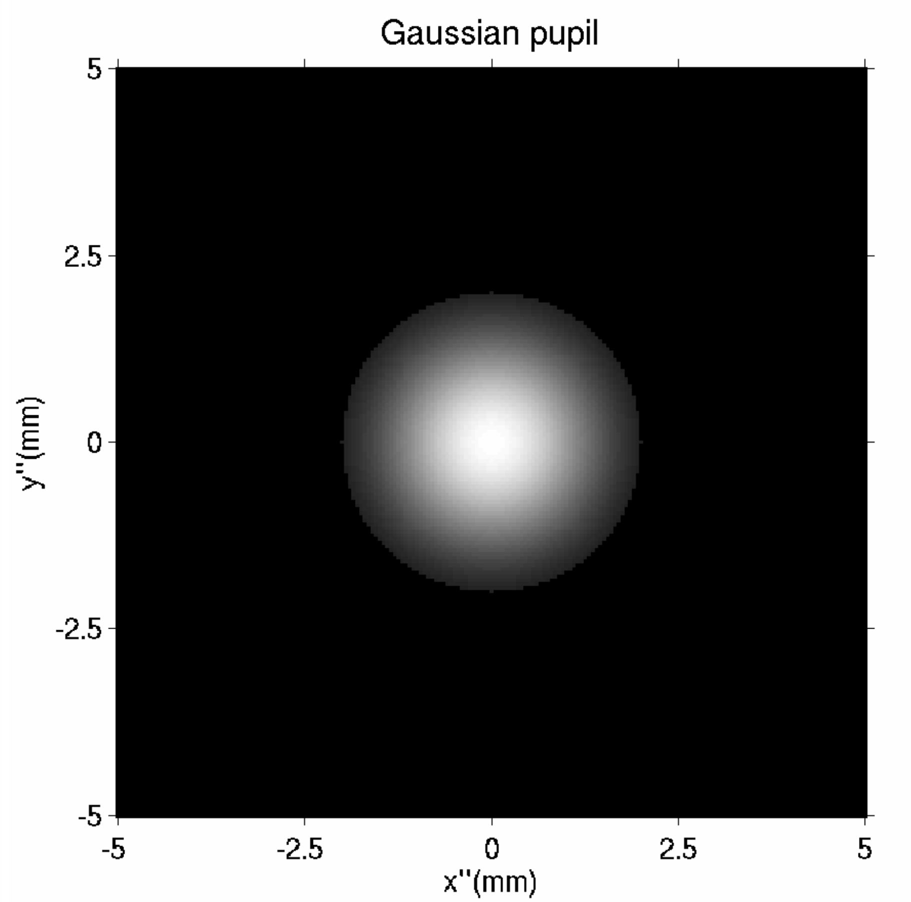


f_1=20cm
λ=0.5μm

$$H(r'') = \mathrm{circ}\left(\frac{r''}{R}\right) - \mathrm{circ}\left(\frac{r''}{R_2}\right)$$

Apodization

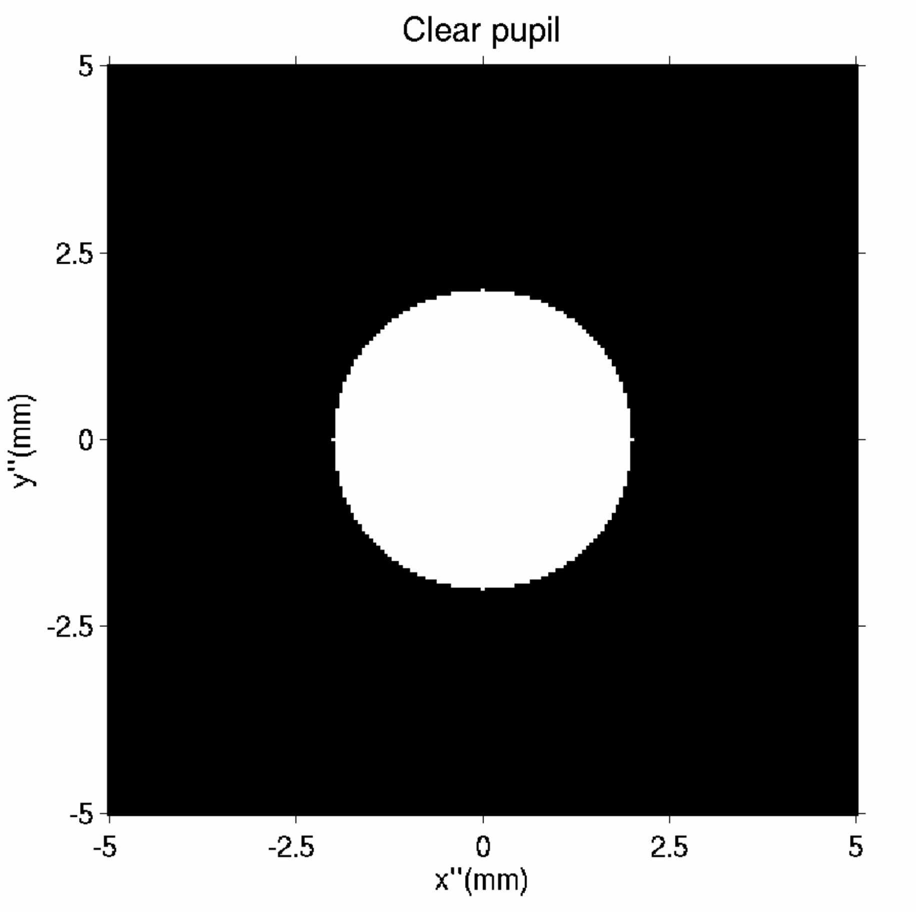


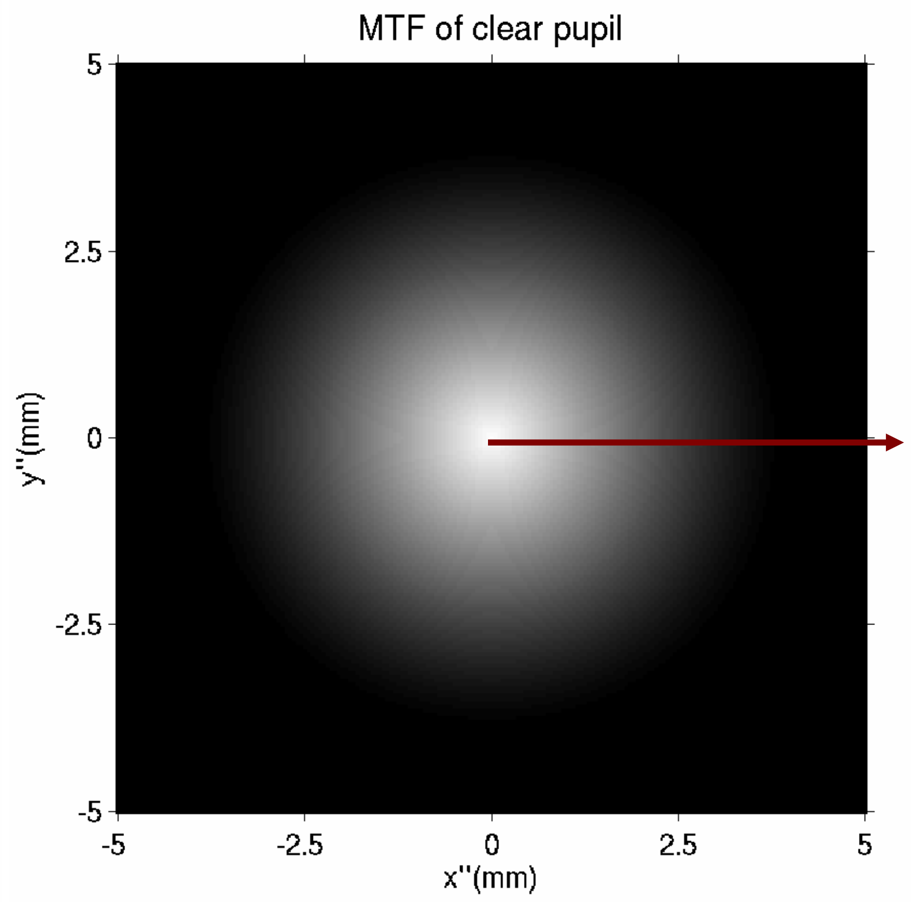


f_1=20cm
λ=0.5µm

$$H(r'') = \mathrm{circ}\left(\frac{r''}{R}\right) \times \exp\left(-\frac{r''^2}{2R_0^2}\right)$$

Unapodized (clear–aperture) MTF



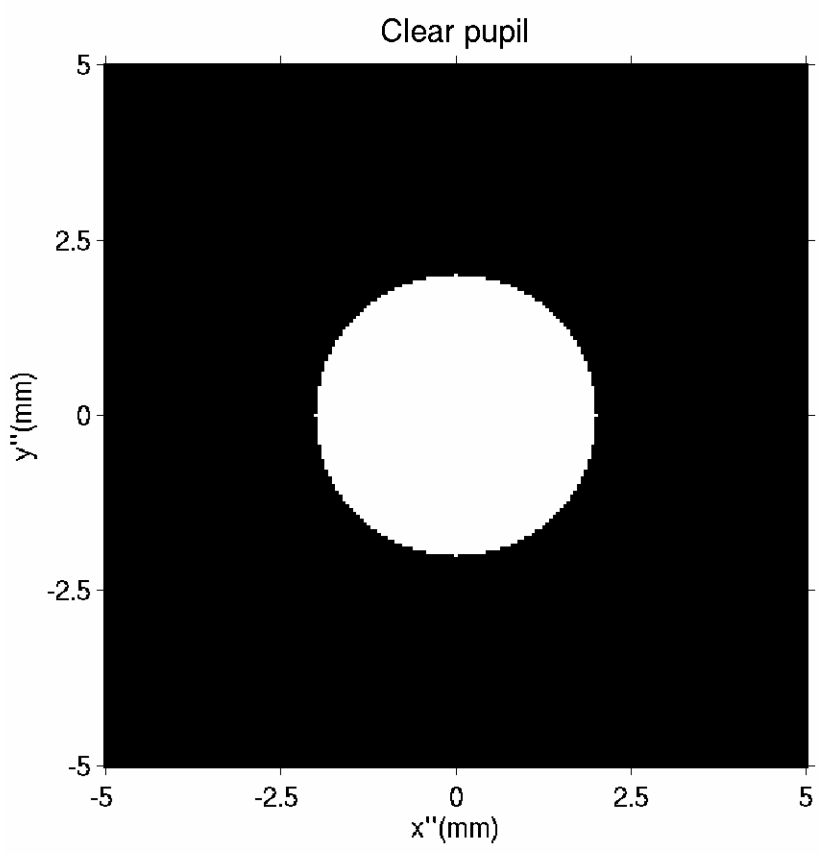


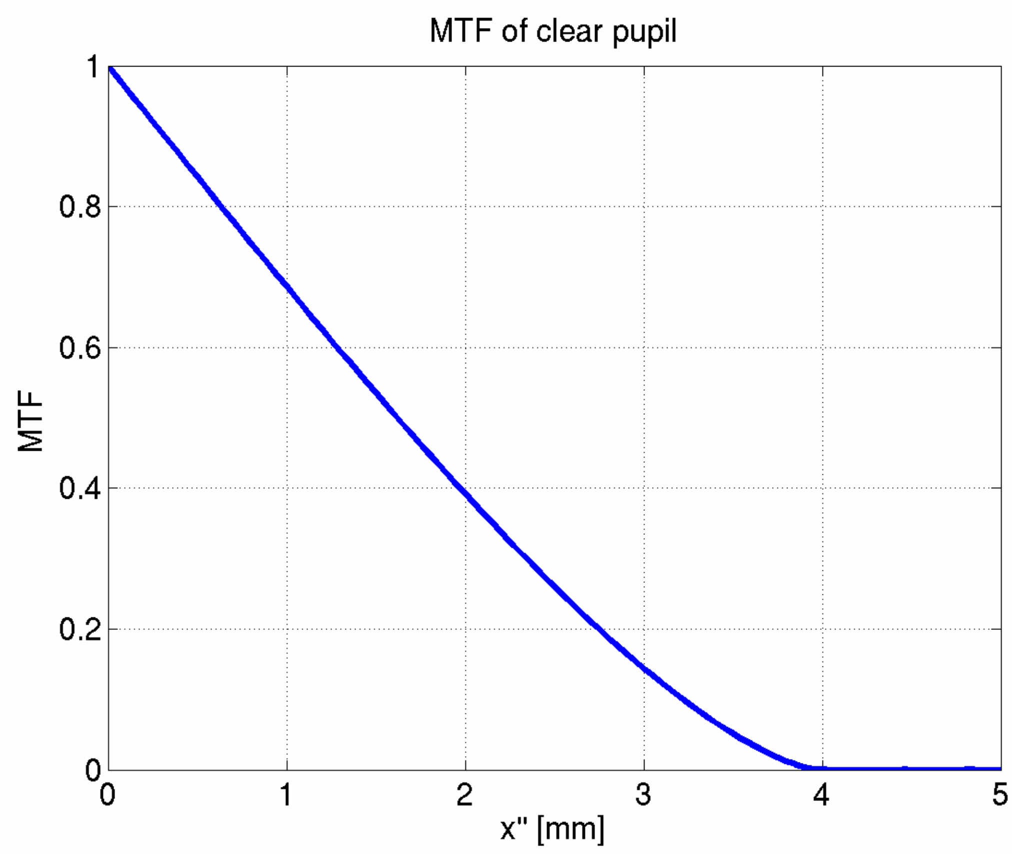
f_1=20cm
λ=0.5µm

$$\widetilde{H}(r'') = \mathrm{circ}\left(\frac{r''}{R}\right) \otimes \mathrm{circ}\left(\frac{r''}{R}\right)$$

auto-correlation

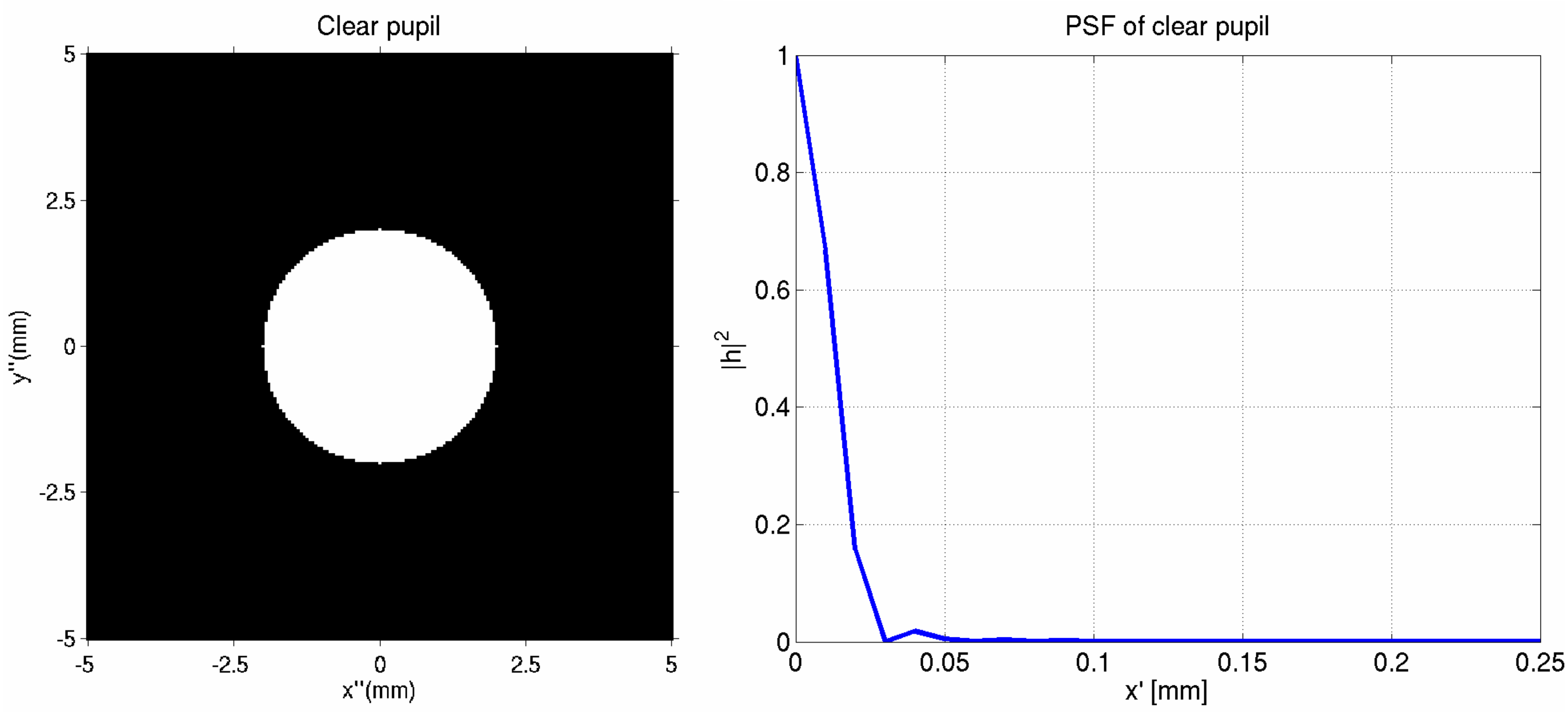
Unapodized (clear–aperture) MTF





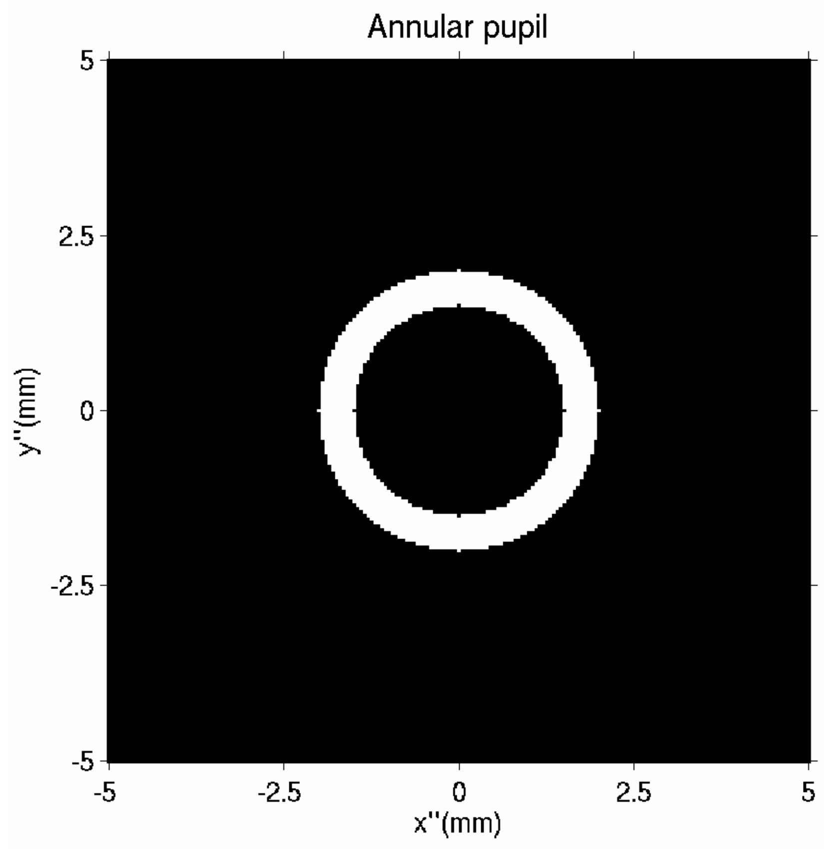
f_1=20cm
λ=0.5μm

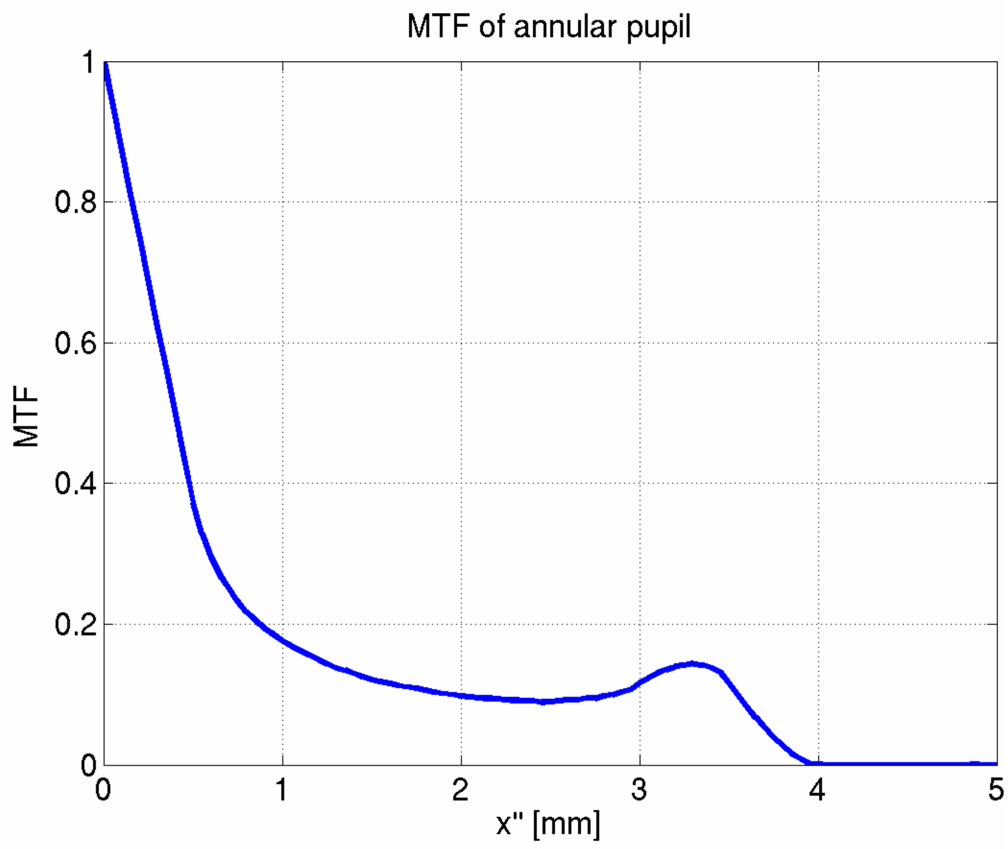
Unapodized (clear–aperture) PSF



f_1=20cm
λ=0.5μm

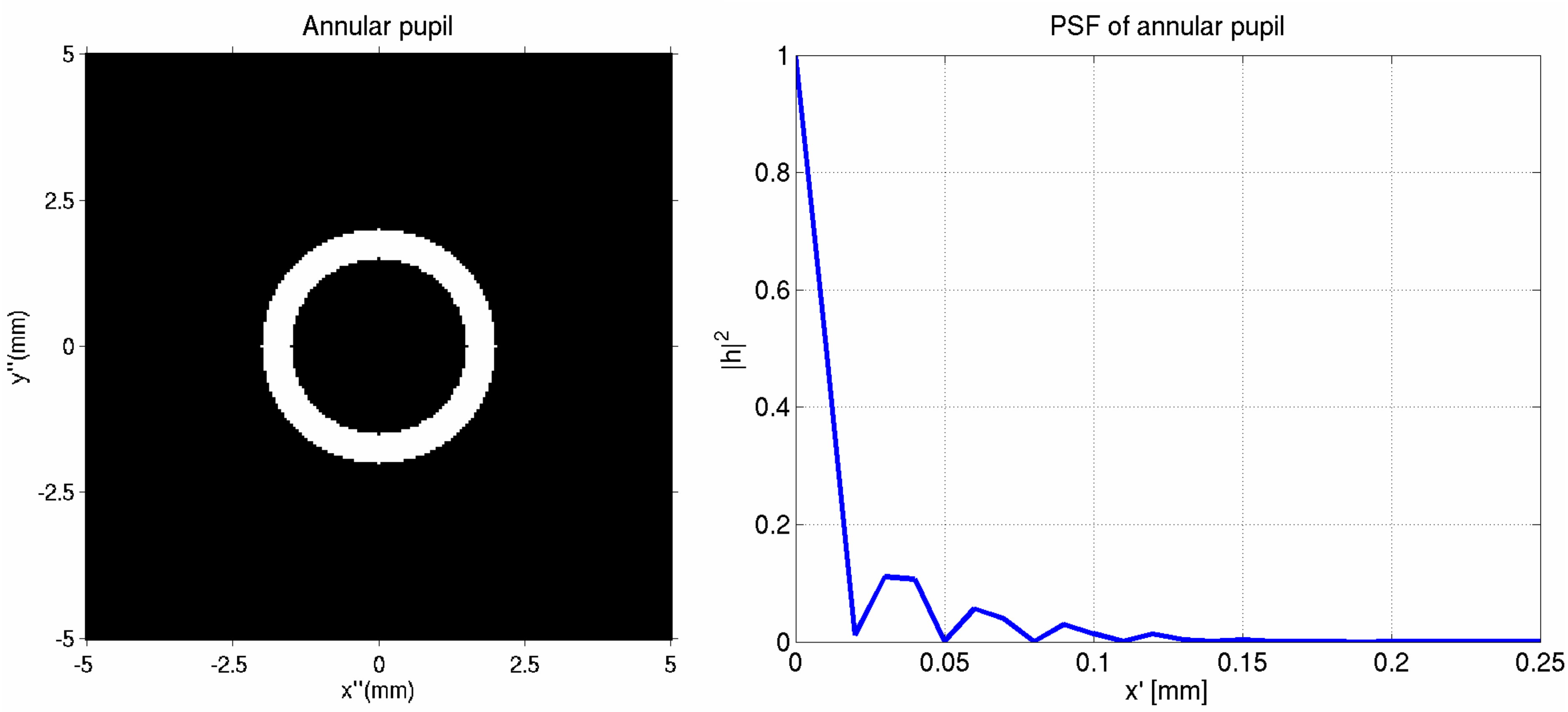
Apodized (annular) MTF





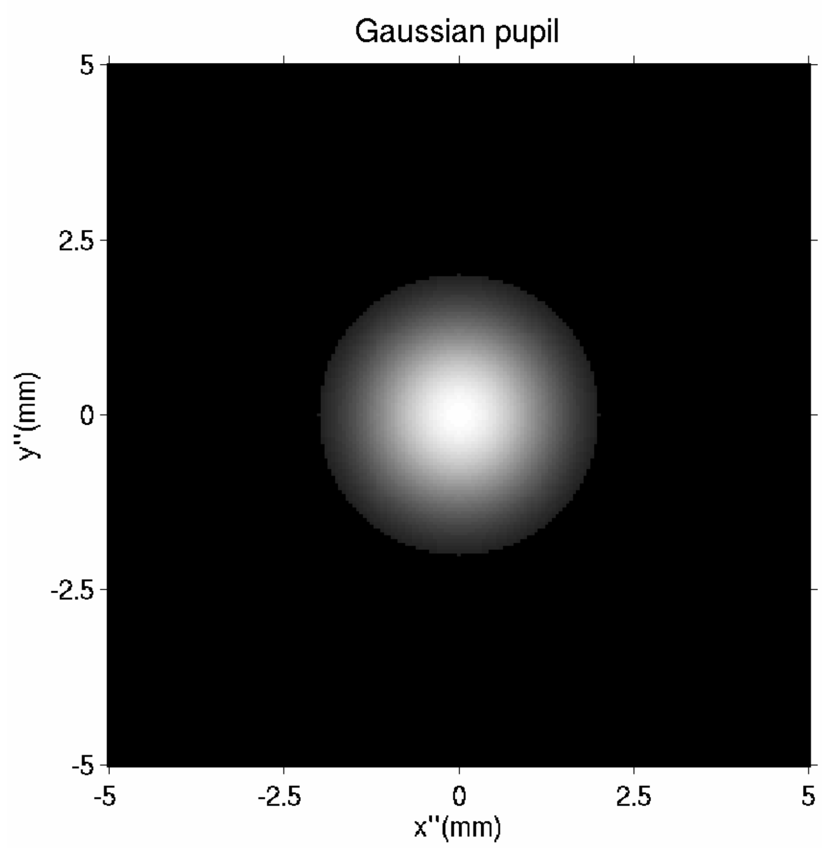
f_1=20cm
λ=0.5µm

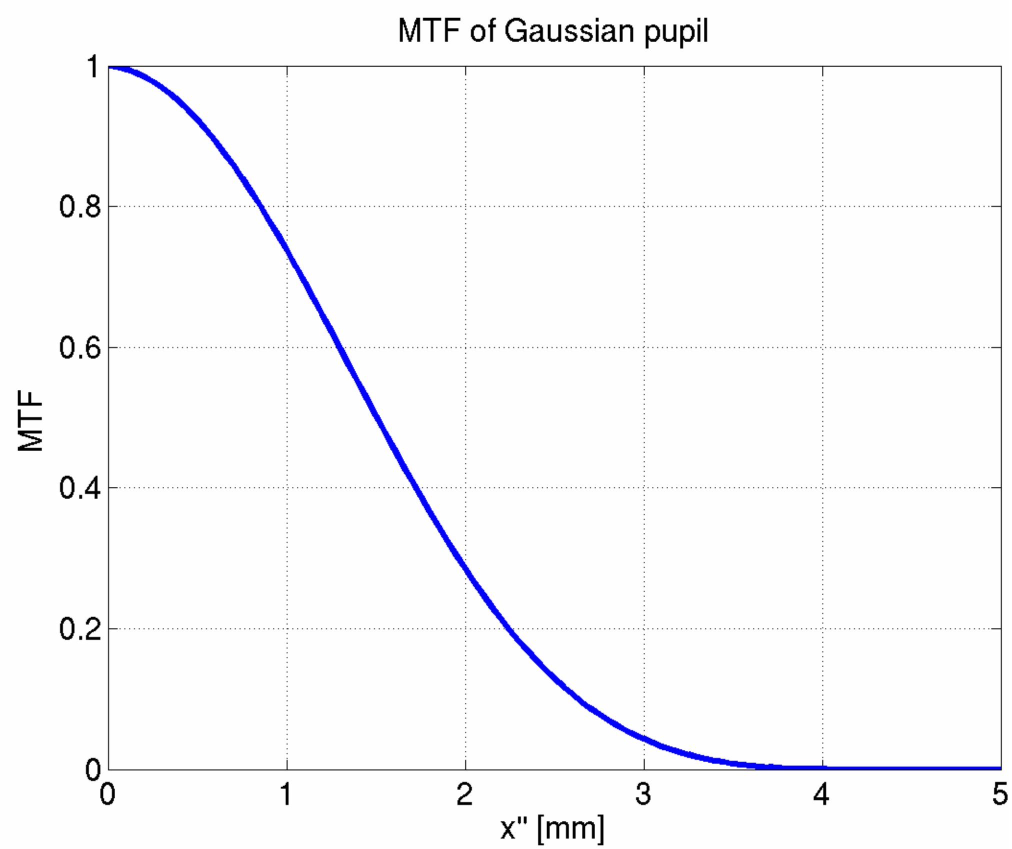
Apodized (annular) PSF



f_1=20cm
λ=0.5µm

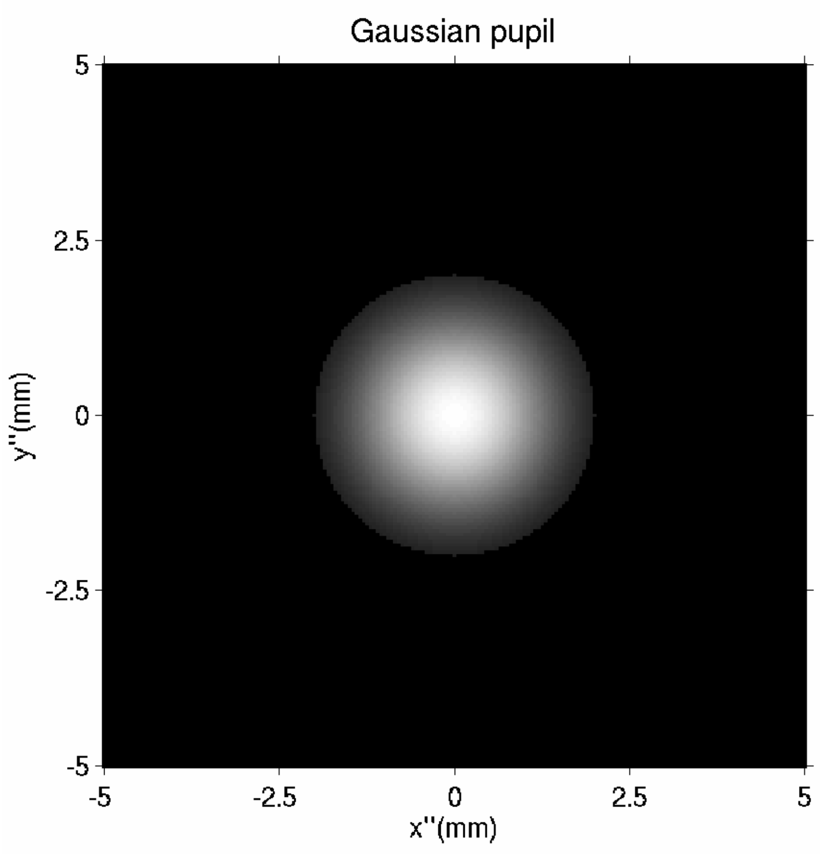
Apodized (Gaussian) MTF

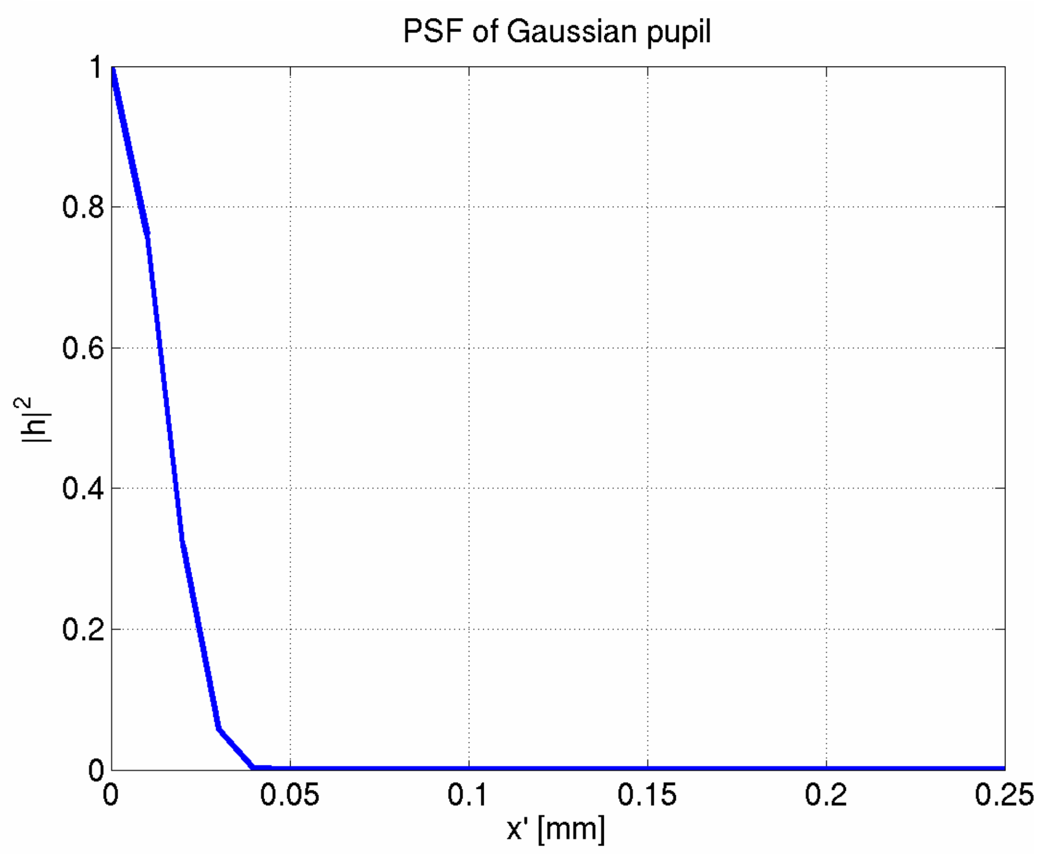




f_1=20cm
λ=0.5μm

Apodized (Gaussian) PSF





f_1=20cm
λ=0.5µm

Conclusions (?)

- Annular–type pupil functions typically narrow the main lobe of the PSF at the expense of higher side lobes
- Gaussian–type pupil functions typically suppress the side lobes but broaden the main lobe of the PSF
- Compromise? $\rightarrow$ application dependent
  - for point–like objects (e.g., stars) annular apodizers may be a good idea
  - for low–frequency objects (e.g., diffuse tissue) Gaussian apodizers may image with fewer artifacts
- Caveat: Gaussian amplitude apodizers very difficult to fabricate and introduce energy loss $\Rightarrow$ binary phase apodizers (lossless by nature) are used instead; typically designed by numerical optimization

Common misinterpretations

Super-resolution

By engineering the pupil function ("apodizing") to result in a PSF with narrower side–lobe, one can "beat" the resolution limitations imposed by the angular acceptance (NA) of the system.

NO:

main lobe size $\downarrow \Leftrightarrow$ sidelobes $\uparrow$
and *vice versa*
main lobe size $\uparrow \Leftrightarrow$ sidelobes $\downarrow$

power loss an important factor

compromise application dependent

Common misinterpretations

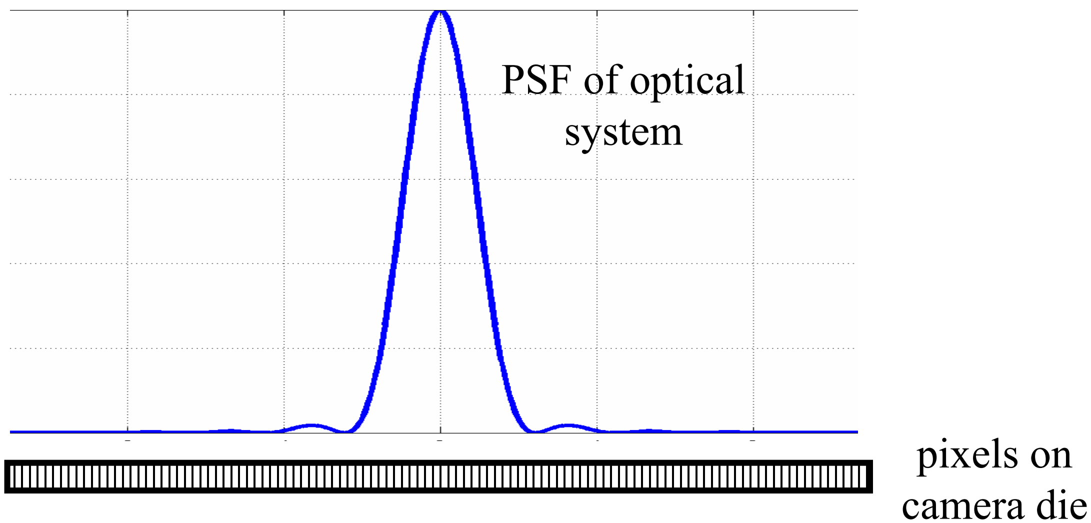
"This super cool digital camera has resolution of 5 Mega pixels (5 million pixels)."

NO:

This is the most common and worst misuse of the term "resolution."
They are actually referring to the **space–bandwidth product (SBP)** of the camera

What *can* a camera resolve?

Answer depends on the magnification and
PSF of the optical system attached to the camera



Pixels significantly smaller than the system PSF
are somewhat underutilized (the effective SBP is reduced)

Summary of misinterpretations of "resolution" and their refutations

- It is pointless to attempt to resolve beyond the Rayleigh criterion (however defined)
  - NO: difficulty increases gradually as feature size shrinks, and difficulty is noise dependent
- Apodization can be used to beat the resolution limit imposed by the numerical aperture
  - NO: watch sidelobe growth and power efficiency loss
- The resolution of my camera is $N \times M$ pixels
  - NO: the maximum possible SBP of your system may be $N \times M$ pixels but you can easily underutilize it by using a suboptimal optical system

So, what *is* resolution?

- Our ability to resolve two point objects (in general, two distinct features in a more general object) based on the image
- It is *related* to the NA but *not exclusively* limited by it
- Resolution, as it relates to NA:
  - **Resolution improves as NA increases**
- Other factors affecting resolution:
  - **aberrations / apodization** (i.e., the exact shape of the PSF)
  - **NOISE!**
- Is there an easy answer?
  - No ……

  but when in doubt quote $0.61\lambda/(\mathrm{NA})$ as an *estimate* (not as an exact limit).