Holography

- Preamble: modulation and demodulation
- The principle of wavefront reconstruction
- The Leith-Upatnieks hologram
- The Gabor hologram
- Image locations and magnification
- Holography of three-dimensional scenes
- Transmission and reflection holograms
- Rainbow hologram

Modulation & Demodulation

- Principle borrowed from radio telecommunications
- Idea is to take **baseband signal** (e.g. speech, music, with maximum frequencies up to ~20kHz) and **modulate** it onto a **carrier signal** which is a simple tone at the frequency where the radio station emits, e.g. 104.3 MHz (that's Boston's WBCN station)
- One of the benefits of modulation is that radio stations can be **multiplexed** by using a different emission frequency each
- After selecting the desired station, the receiver follows a process of **demodulation** which recovers the baseband signal and sends it to the speakers.

Types of modulation

- Amplitude modulation (AM)
- Frequency modulation (FM)
- Phase modulation (PM)
- Digital methods (Amplitude Shift Keying – ASK, Frequency Shift Keying – FSK, Phase Shift Keying – PSK, etc.)

used in radio at low frequencies only ("AM band" = 535kHz to 1.7MHz) ; as we will see, it is an almost-exact analog of holography

dominant in commercial radio ("FM band" = 88MHz to 108MHz) ; there is an analog in optics, called "spectral holography," but it is beyond the scope of the class

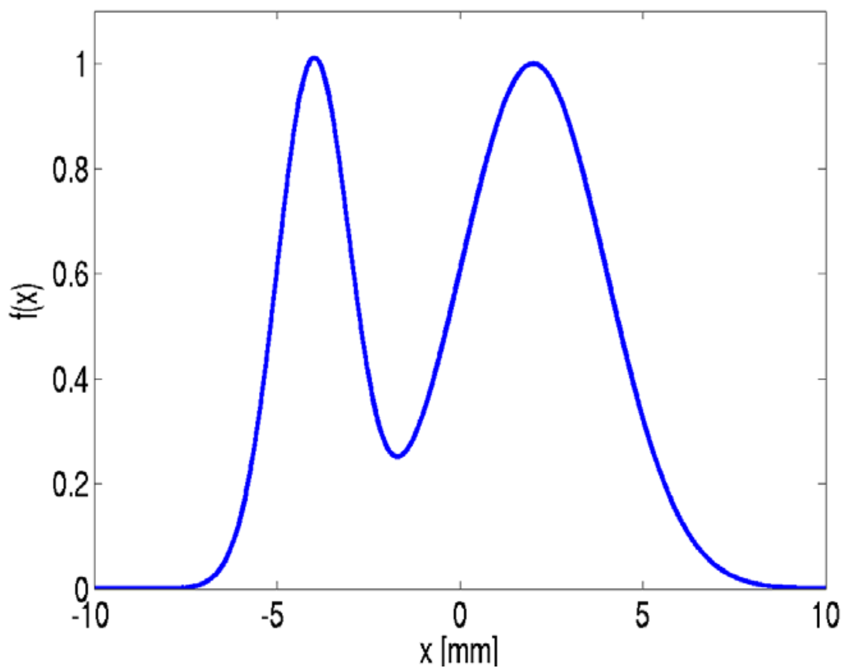
Amplitude modulation

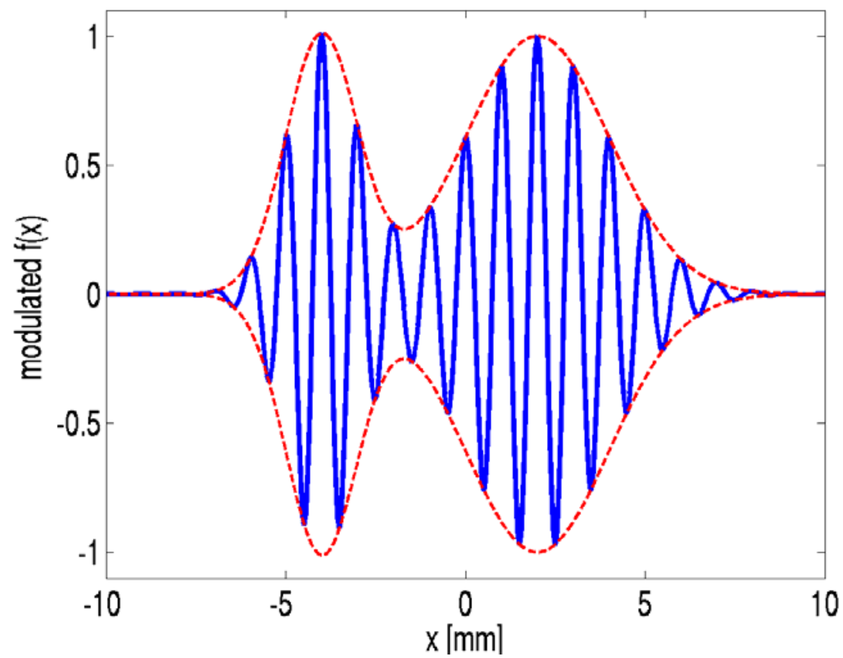
$f(x)$

(baseband)

modulated $f(x) =$

$= f(x) \times \cos(2\pi u_c x)$

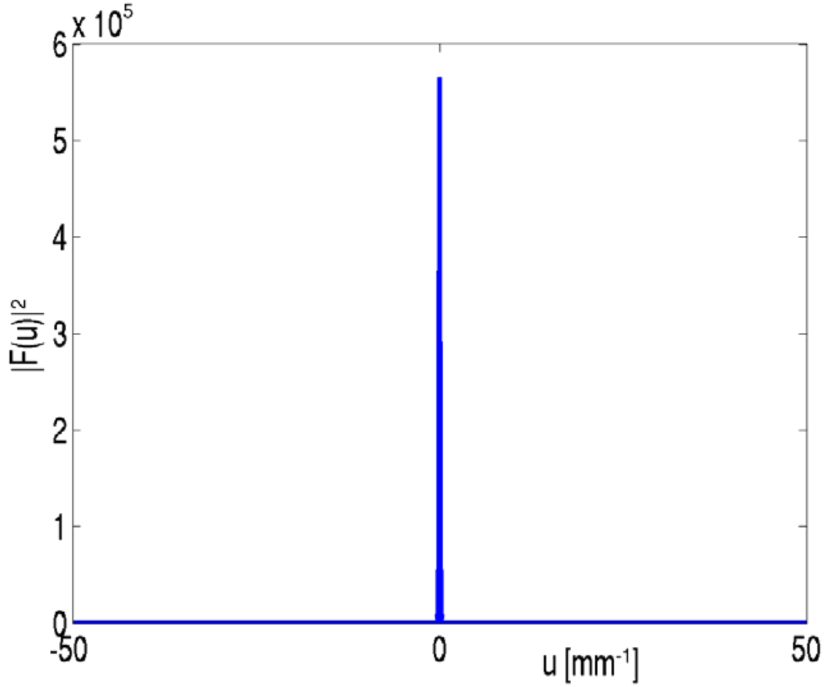




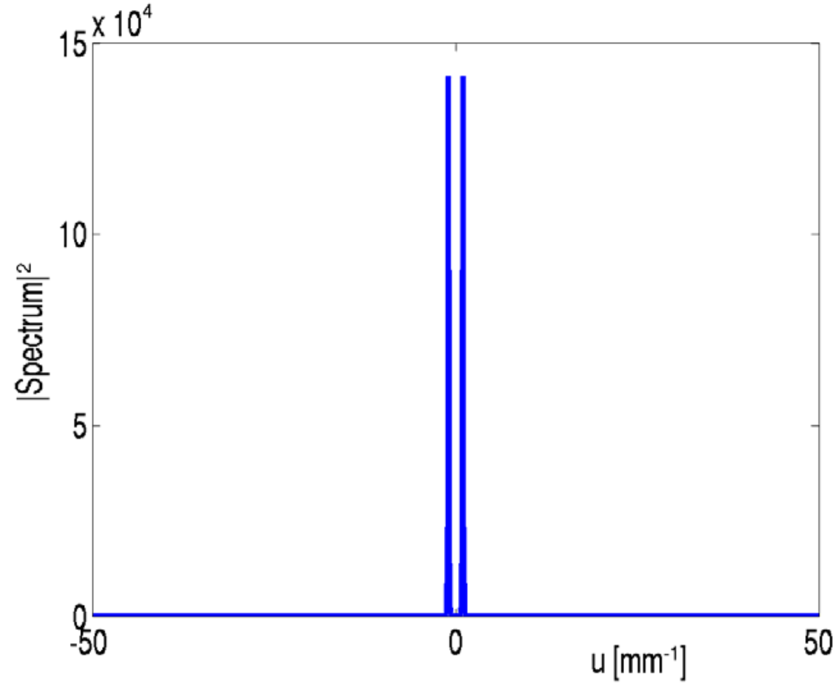
u_c: carrier frequency

AM in the frequency domain

spectrum of $f(x)$



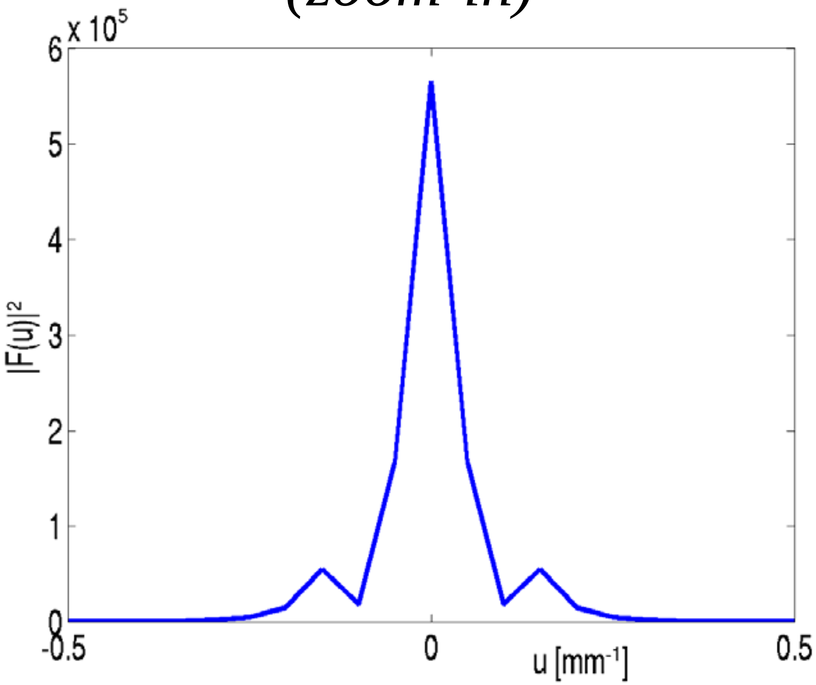
spectrum of modulated $f(x)$



AM in the frequency domain

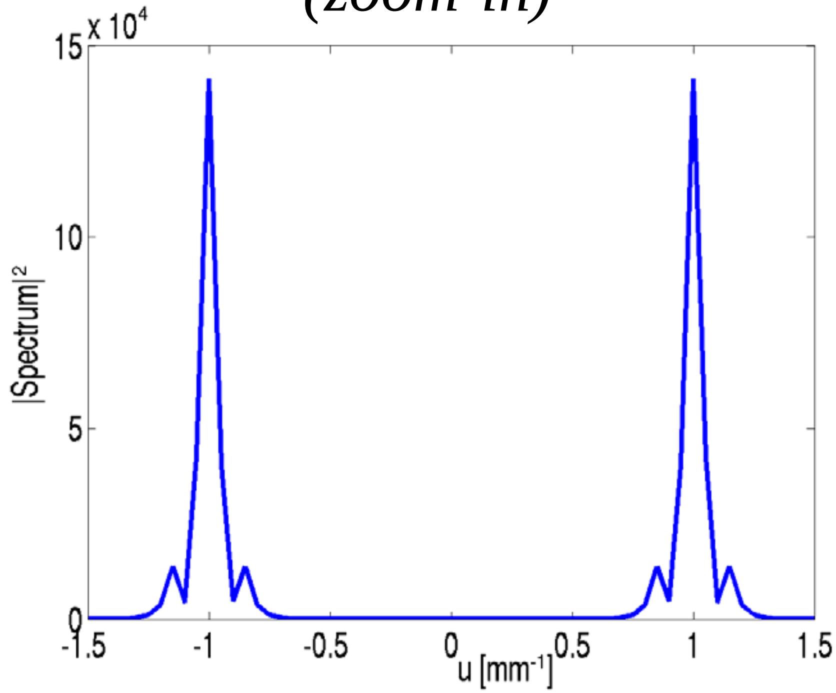
spectrum of $f(x)$

(zoom-in)



spectrum of modulated $f(x)$

(zoom-in)



AM in the frequency domain

$$f(x) \rightarrow F(u)$$

$$f(x)\cos(2\pi u_c x) = f(x) \times \frac{1}{2}\left[e^{i2\pi u_c x} + e^{-i2\pi u_c x}\right]$$

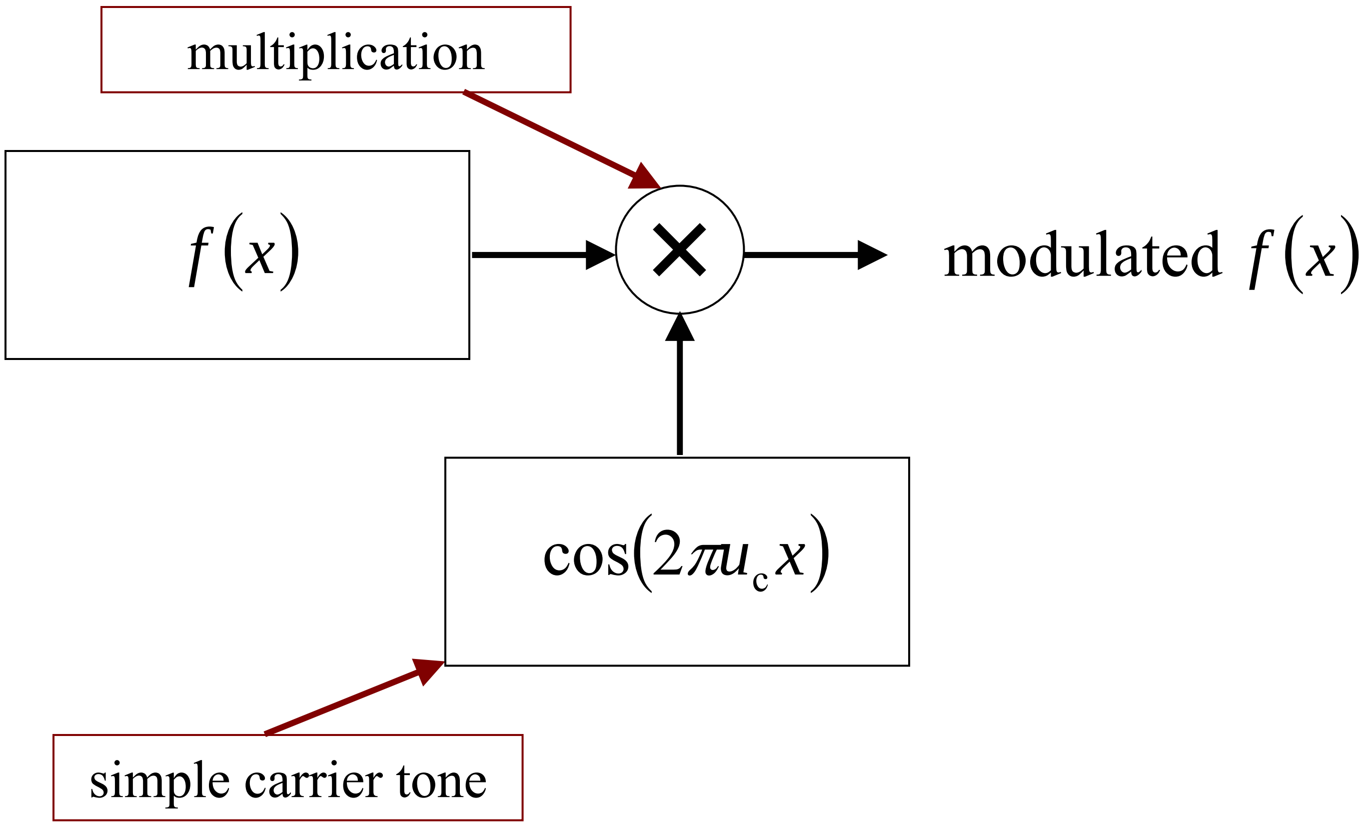
modulation in the space domain

$$\rightarrow F(u) * \frac{1}{2}\left[\delta(u - u_c) + \delta(u - u_c)\right] =$$

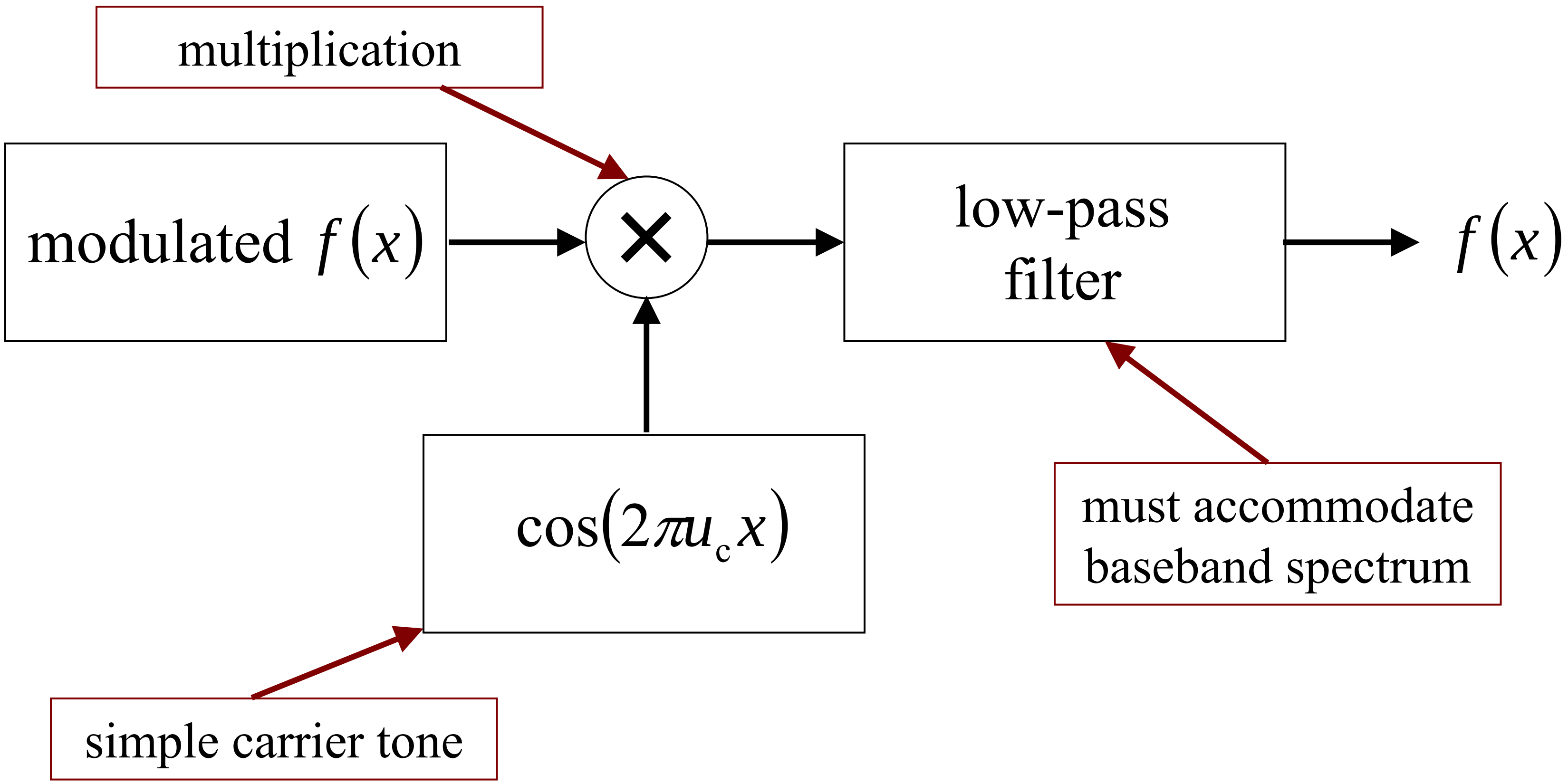
$$= \frac{1}{2}\left[F(u - u_c) + F(u - u_c)\right]$$

modulation in the frequency domain: two replicas of the baseband spectrum, centered on the carrier frequency

Modulation

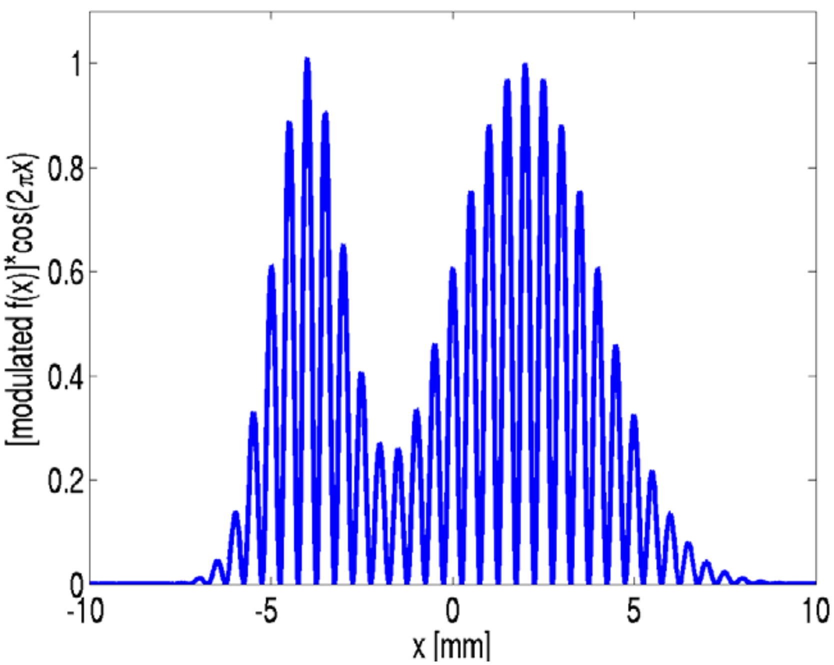

multiplication
$f(x)$
$\times$
modulated $f(x)$
$\cos(2\pi u_c x)$
simple carrier tone

Demodulation

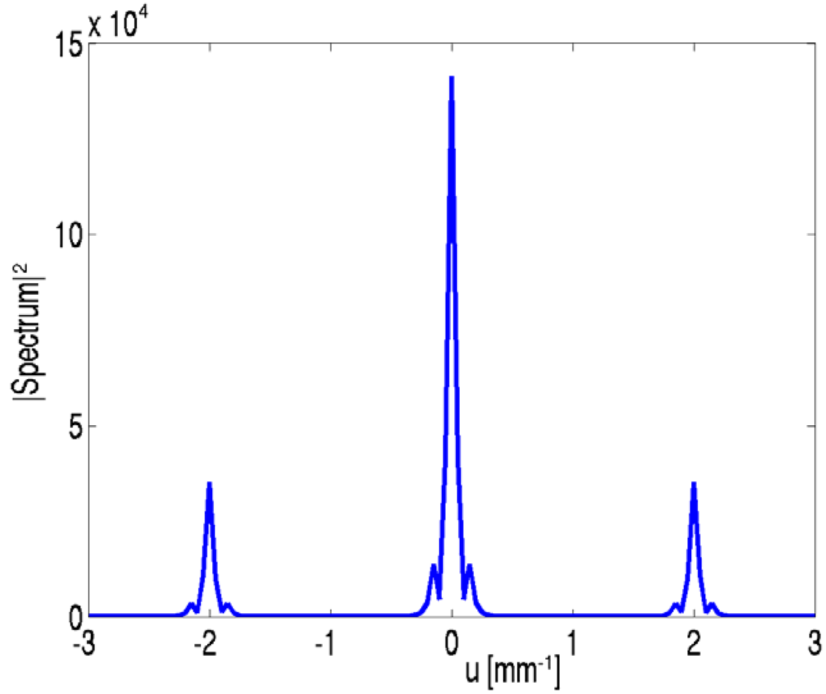


Demodulation

$f(x) \times \cos^2(2\pi u_c x)$

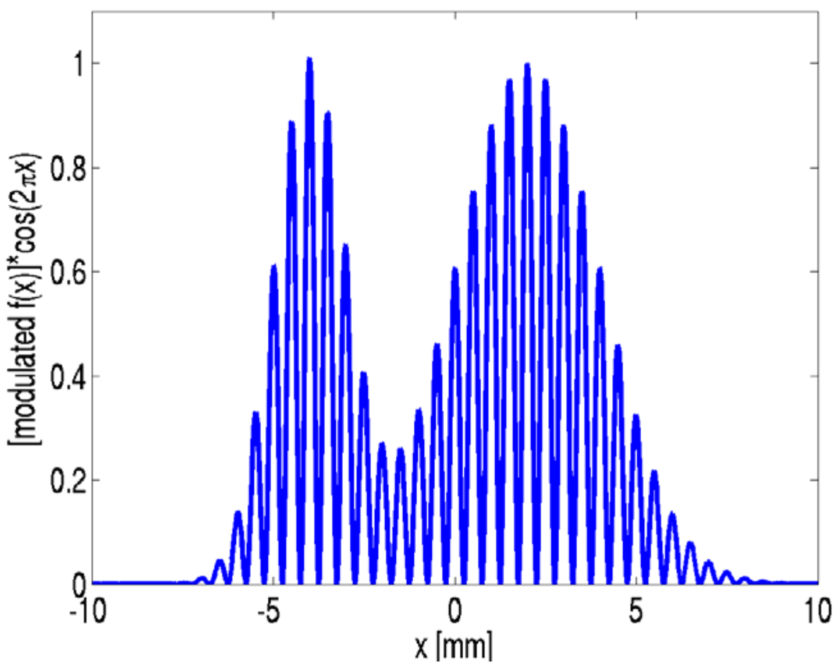


spectrum of $f(x) \times \cos^2(2\pi u_c x)$

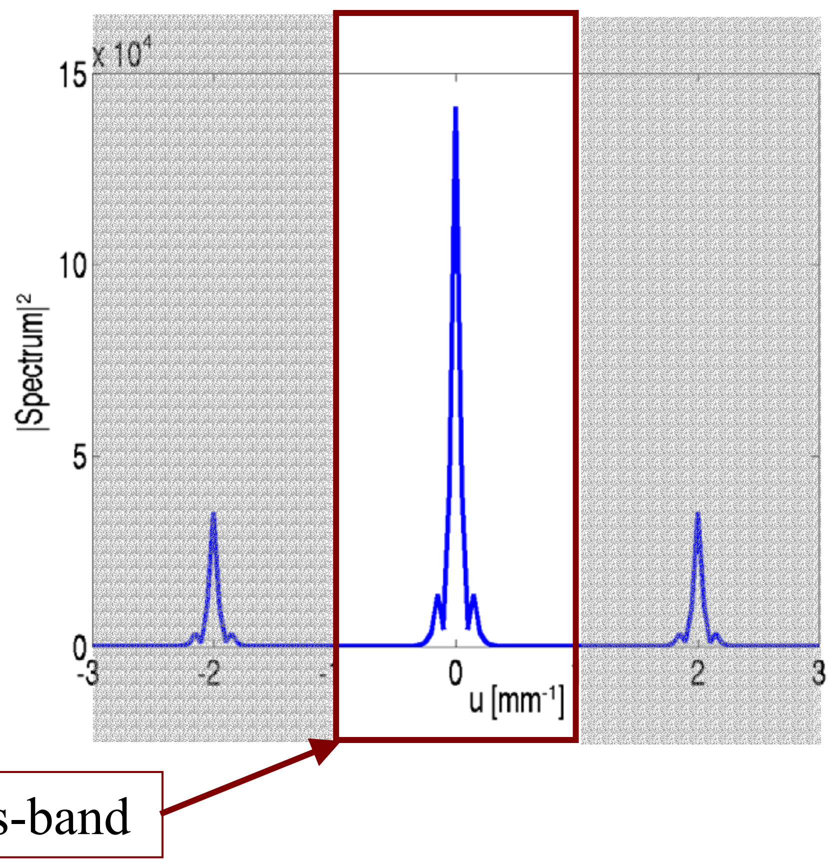


Demodulation

$f(x) \times \cos^2(2\pi u_c x)$



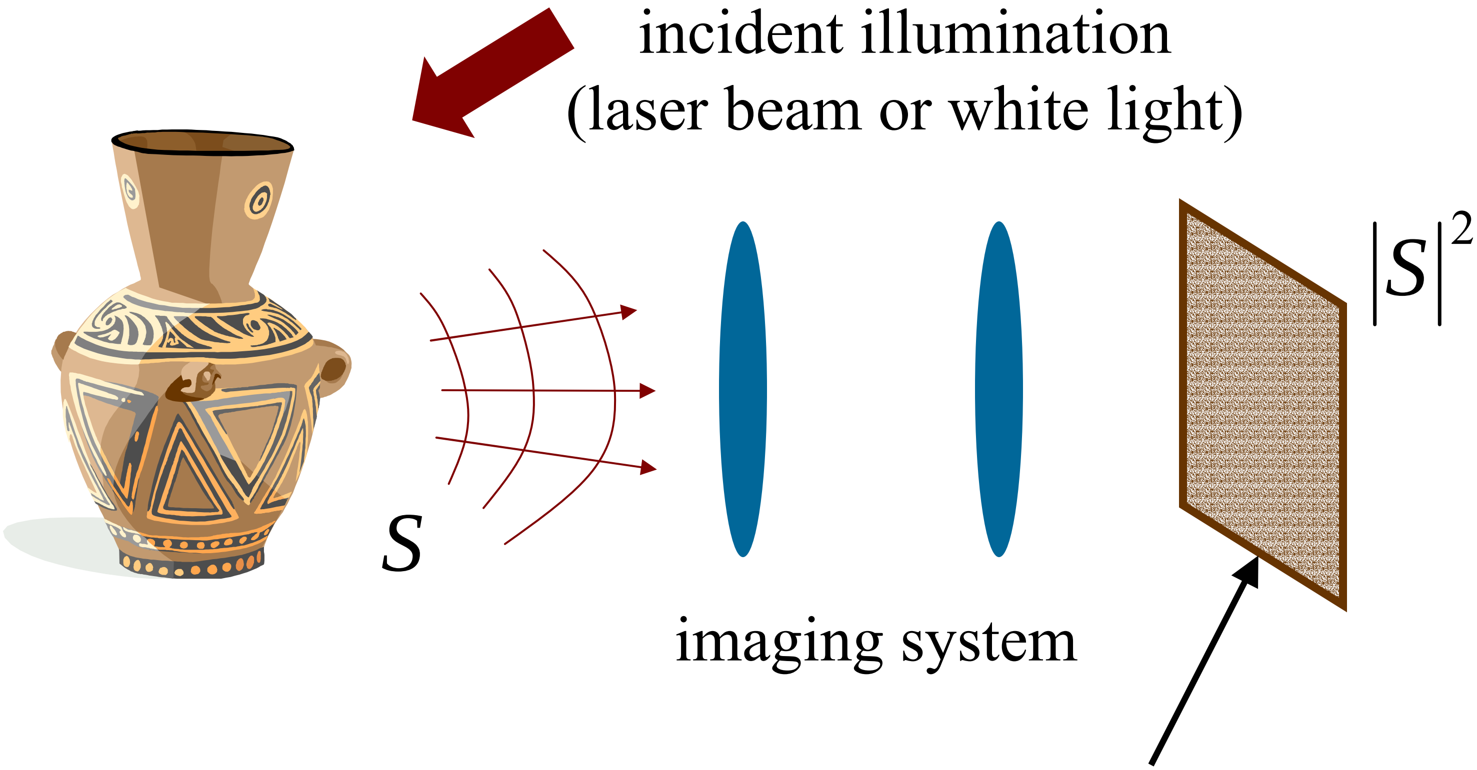
spectrum of $f(x) \times \cos^2(2\pi u_c x)$



The wavefront reconstruction problem

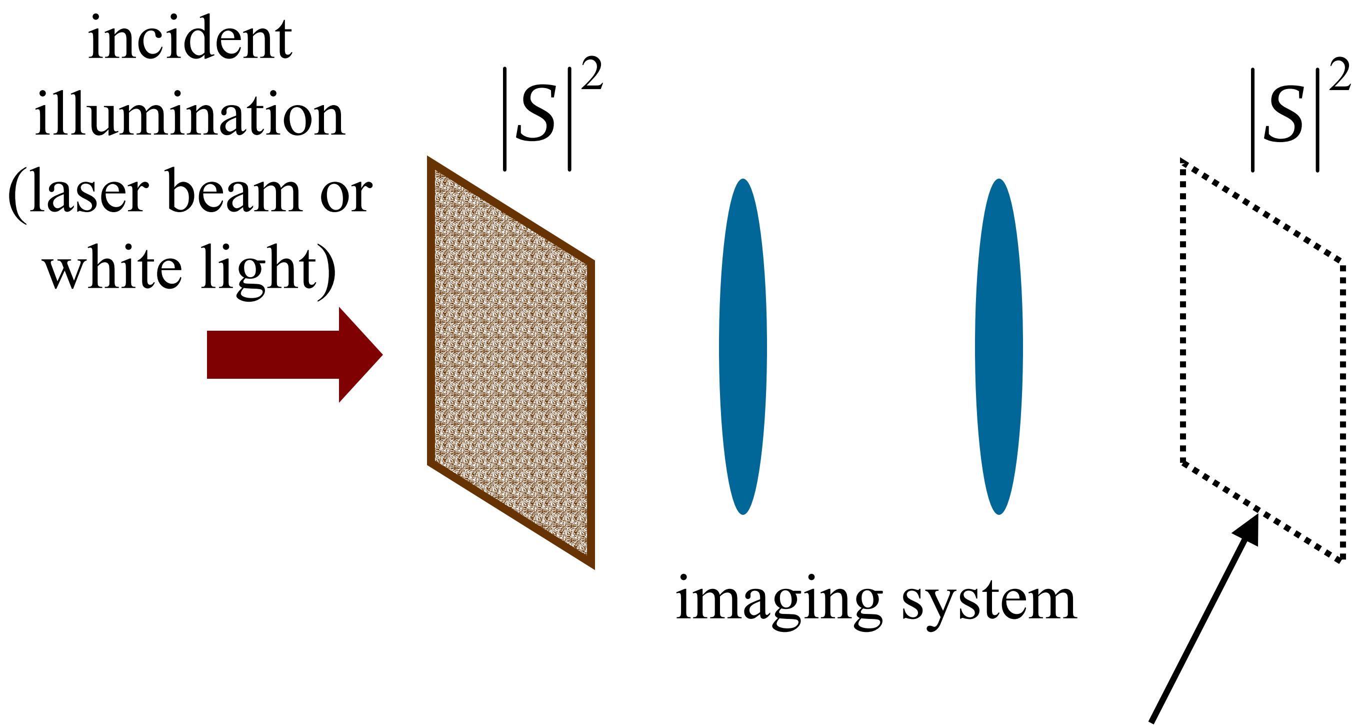
- Wavefront is the amplitude (i.e. magnitude and phase) of the electric field as function of position
- Traditional coherent imaging results in intensity images (because detectors do not respond fast enough at optical frequencies) → magnitude information is recovered but phase information is lost
- Can we imprint ***intensity*** information on an optical wave? YES → **photography** (known since the 1840's)
- Can we imprint ***wavefront*** information on an optical wave? YES → **holography** (Gabor, late 1940s)

Photography: recording



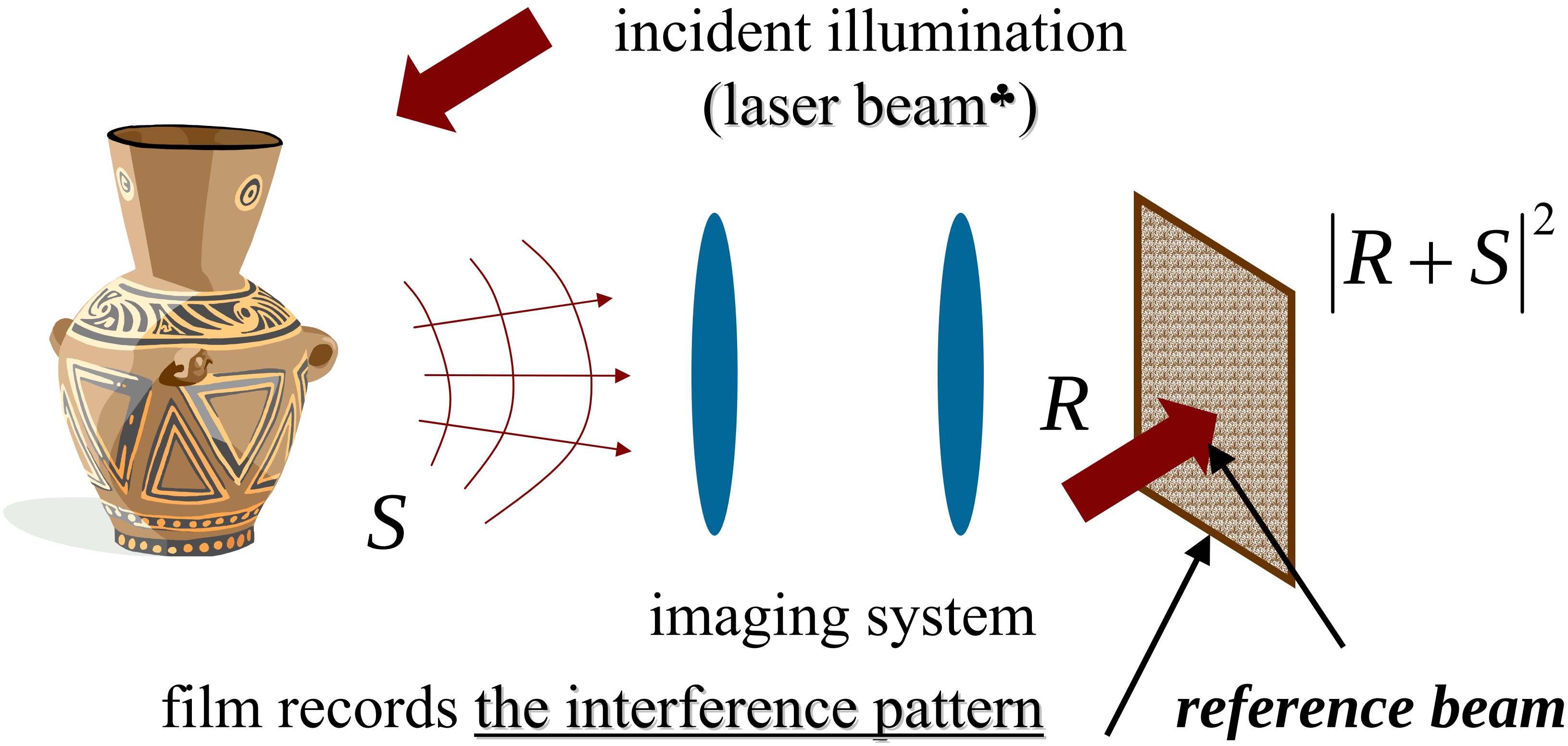
film records ***intensity*** information

Photography: reconstructing the <u>intensity</u>



at the image plane, an intensity pattern is formed that replicates the originally recorded intensity

Holography: recording

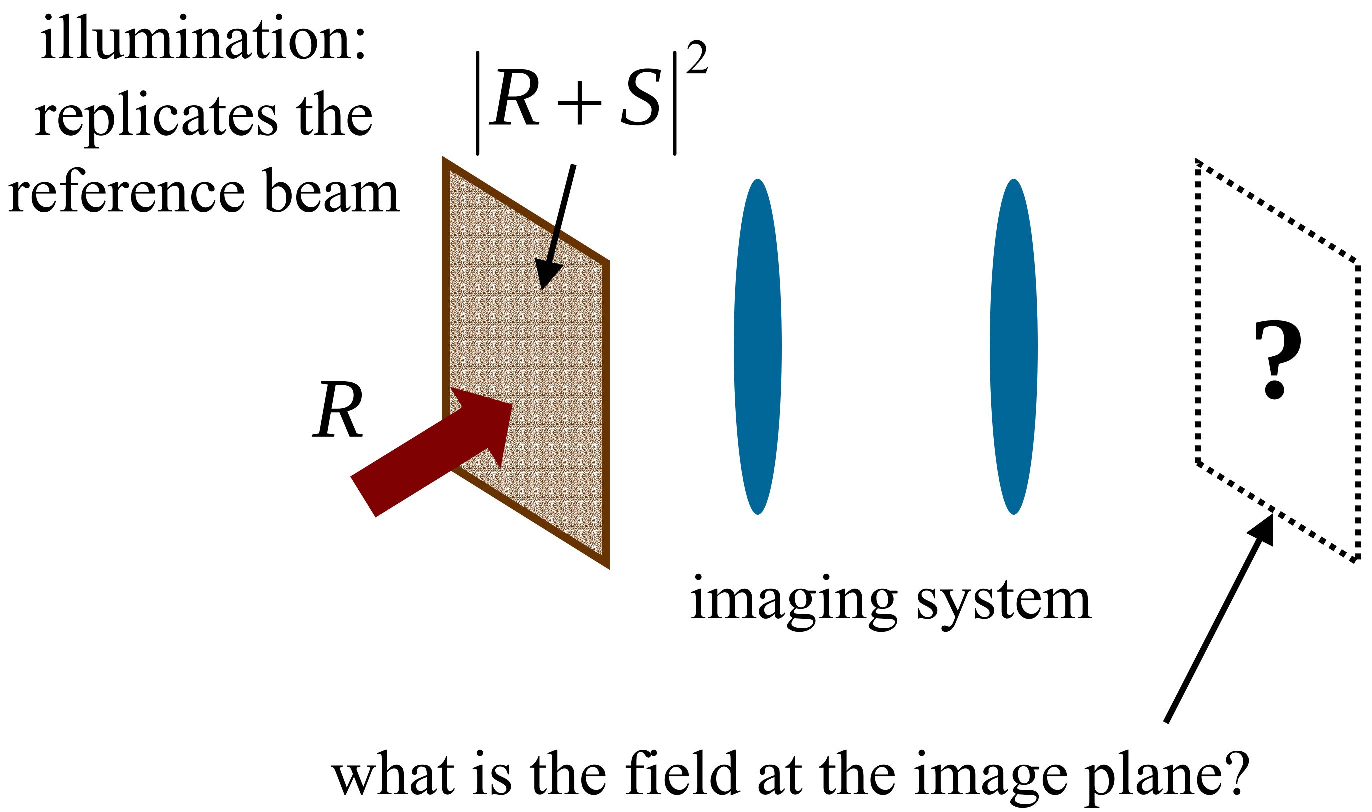


film records <u>the interference pattern</u> ***(interferogram)*** of the object wavefront and the reference wavefront

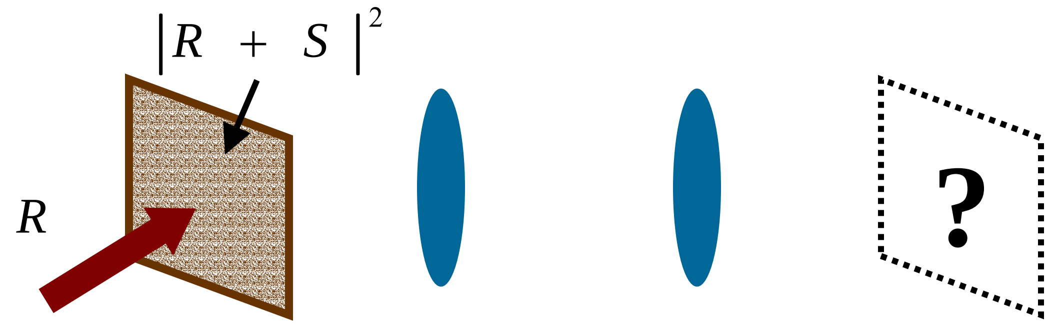
reference beam
(split from the same laser)

♣in general, the illumination must be quasi-monochromatic, and spatially mutually coherent with the reference beam throughout the wavefront

Holography: reconstructing the <u>wavefront</u>



Holography: reconstructing the <u>wavefront</u>

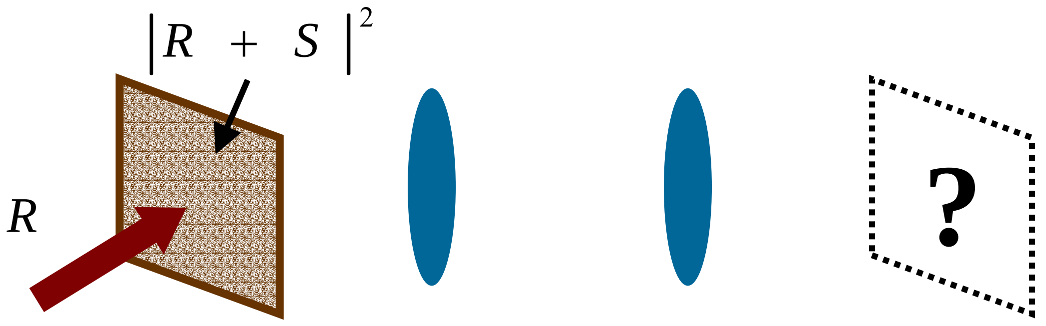


The field being imaged is:

$$R \times \left|R+S\right|^2 = R \times \left(\left|R\right|^2 + \left|S\right|^2 + R^* S + R S^* \right) =$$

$$= \underbrace{R \times \left(\left|R\right|^2 + \left|S\right|^2 \right)}_{1} + \underbrace{\left|R\right|^2 S}_{2} + \underbrace{R^2 S^*}_{3}$$

Holography: reconstructing the <u>wavefront</u>



take the simplest possible reference wave, a **plane wave**:

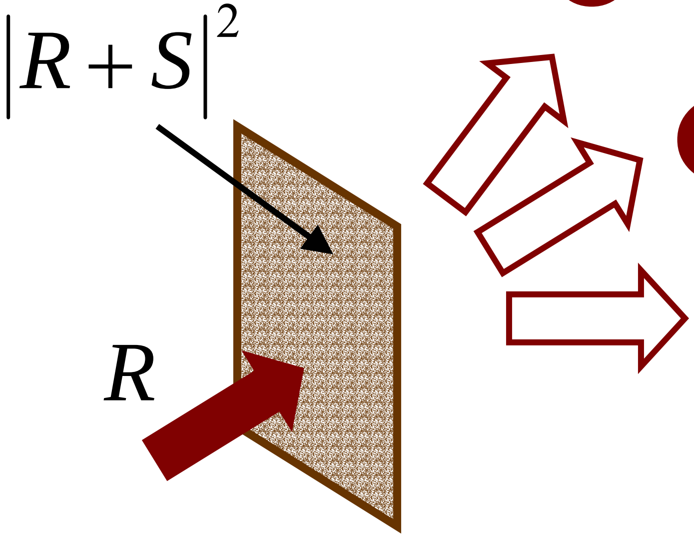
$$R = \mathrm{e}^{i2\pi u_0 x} \quad , \quad \text{spatial frequency} \quad u_0 = \frac{\sin\theta_0}{\lambda}$$

then the reconstructed field is:

$$= \underbrace{\mathrm{e}^{i2\pi u_0 x} \times \left(1 + |S|^2\right)}_{❶} + \underbrace{S}_{❷} + \underbrace{\mathrm{e}^{i2\pi(2u_0)x}\, S^*}_{❸}$$

Holography: reconstructing the <u>wavefront</u>

propagates at angle:



3 $e^{i2\pi(2u_0)x}\, S^* \longrightarrow 2\theta_0$

1 $e^{i2\pi u_0 x}\times\left(1+|S|^2\right) \longrightarrow \theta_0$

2 $S \longrightarrow$ on-axis

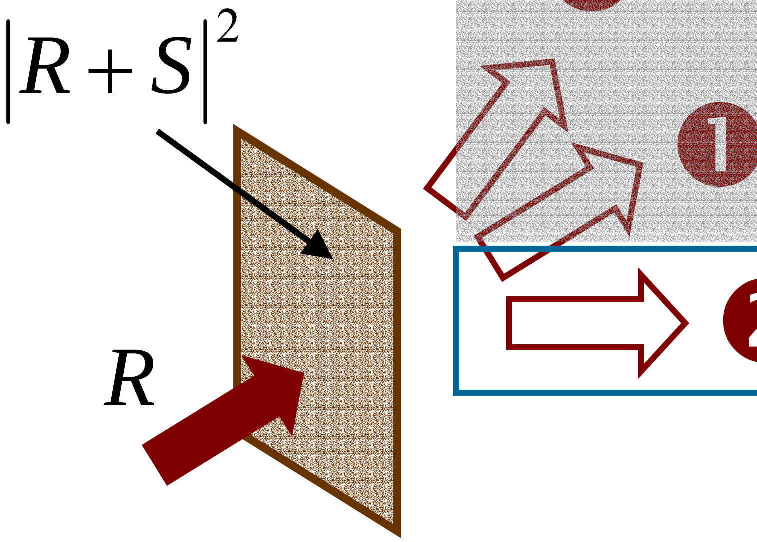
fields departing from the hologram

Holography: reconstructing the <u>wavefront</u>

not wanted

3 $e^{i2\pi(2u_0)x} S^* \longrightarrow 2\theta_0$

1 $e^{i2\pi u_0 x} \times \left(1+|S|^2\right) \longrightarrow \theta_0$

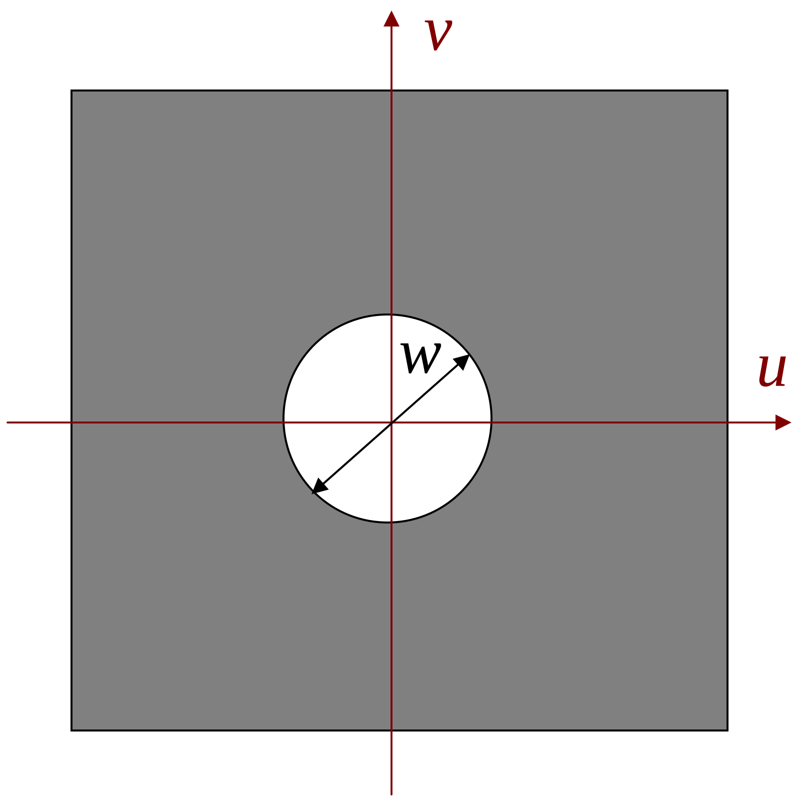


2 $S \longrightarrow$ on-axis

wanted

fields departing from the hologram

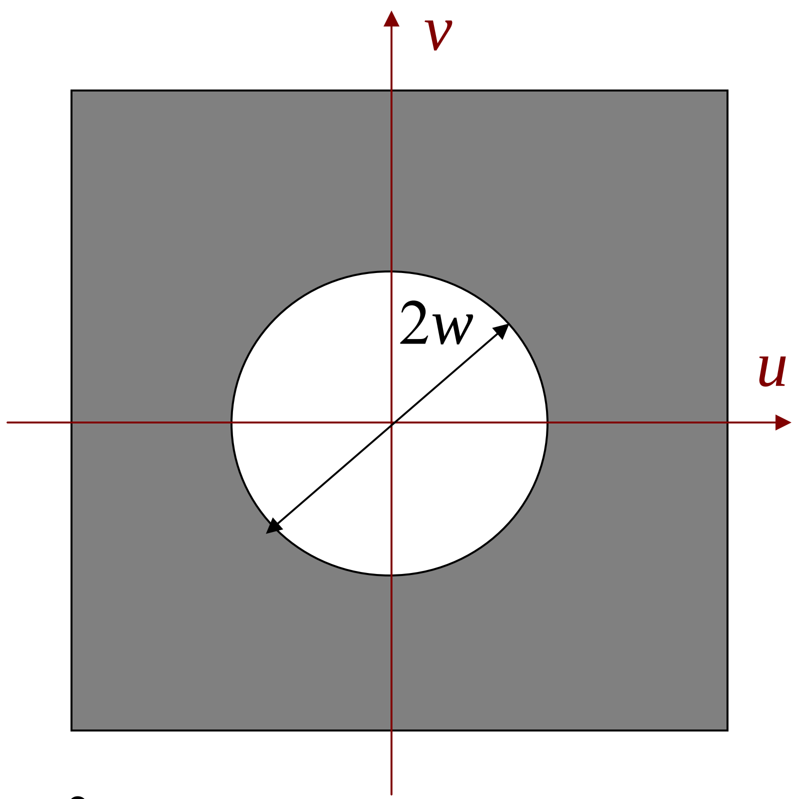
Filtering the wavefront: bandlimited signal



S has bandwidth w, i.e.

$$\Im\{S\} \neq 0 \quad \text{within circle of radius } w$$

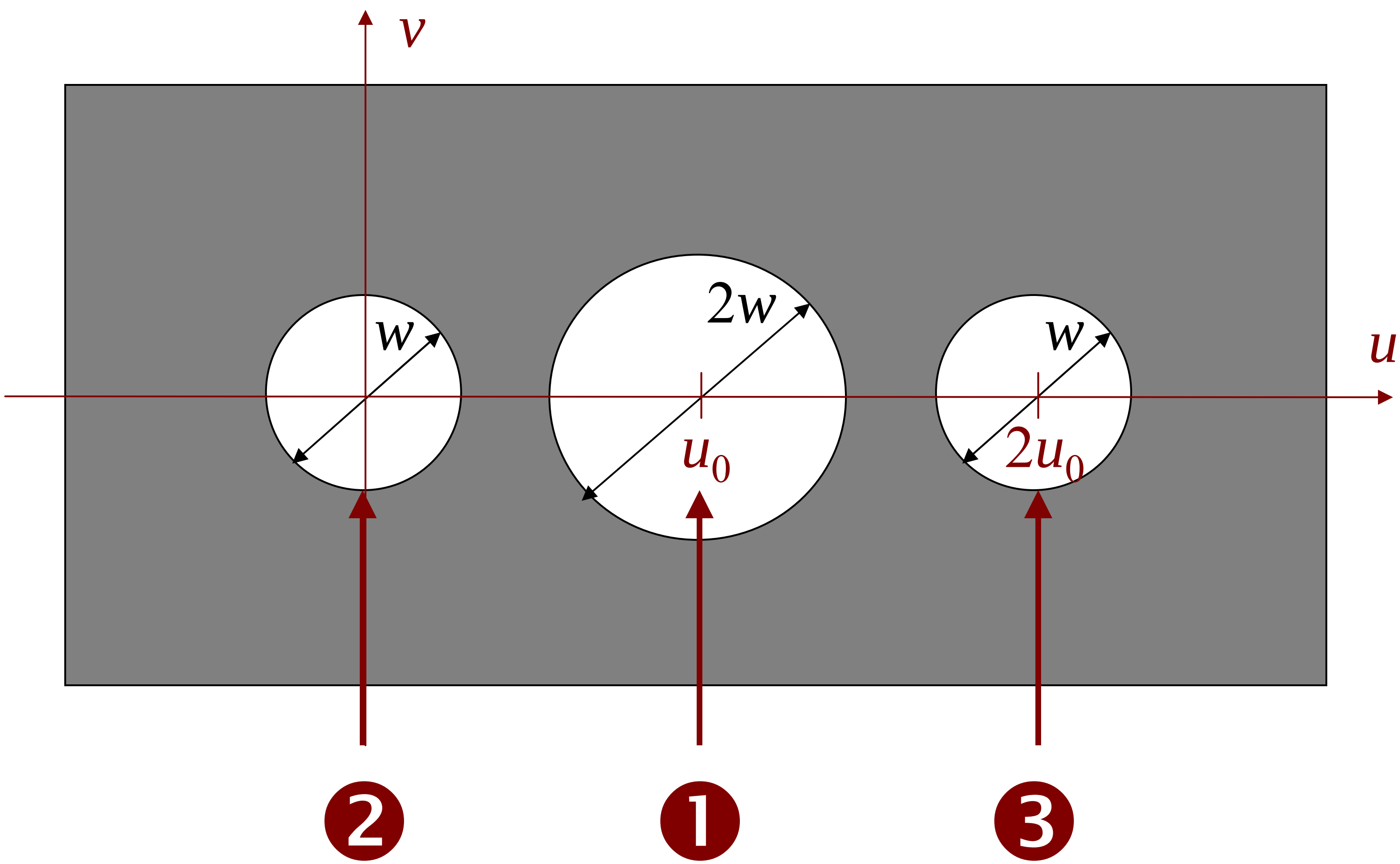
Filtering the wavefront: bandlimited signal



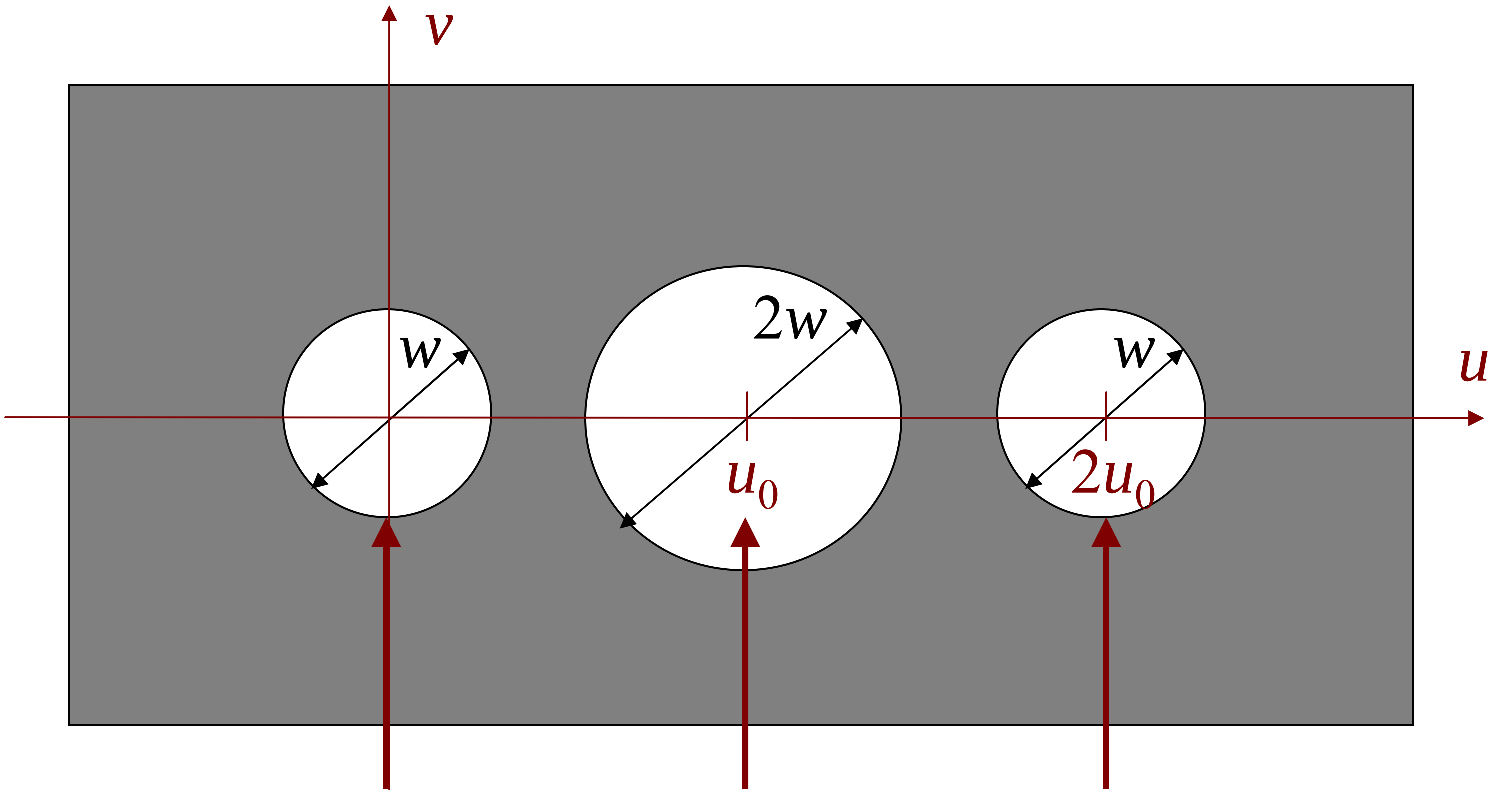
Term ❶ → $1+|S|^2$ has bandwidth $2w$, because

$$\Im\left\{|S|^2\right\}=\Im\{S\}*\Im\left\{S^*\right\}=\Im\{S\}\otimes\Im\{S\}$$

Filtering the wavefront: Fourier transform description



Filtering the wavefront: Fourier transform description

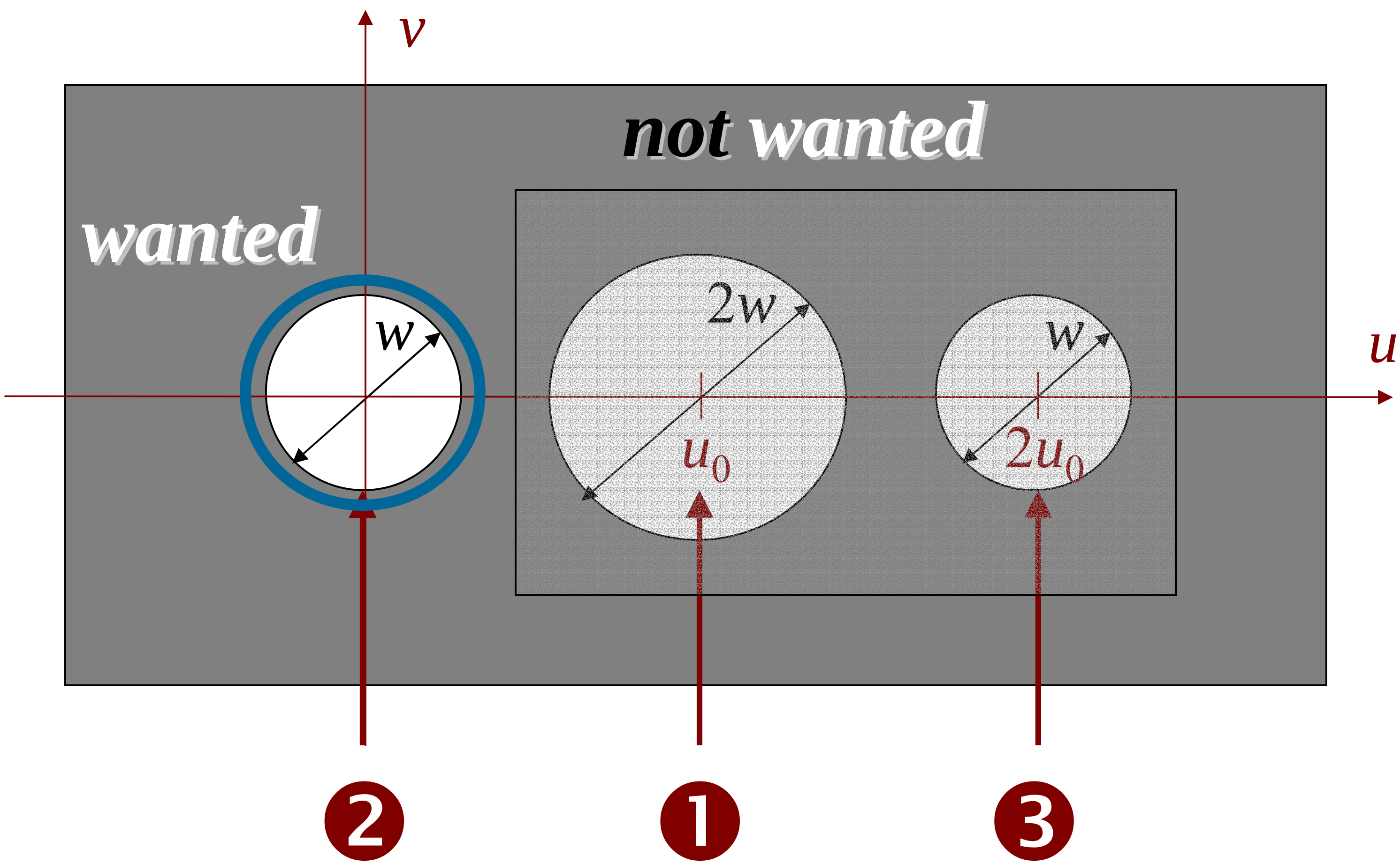


2 original spectrum

1 autocorrelation of the original spectrum

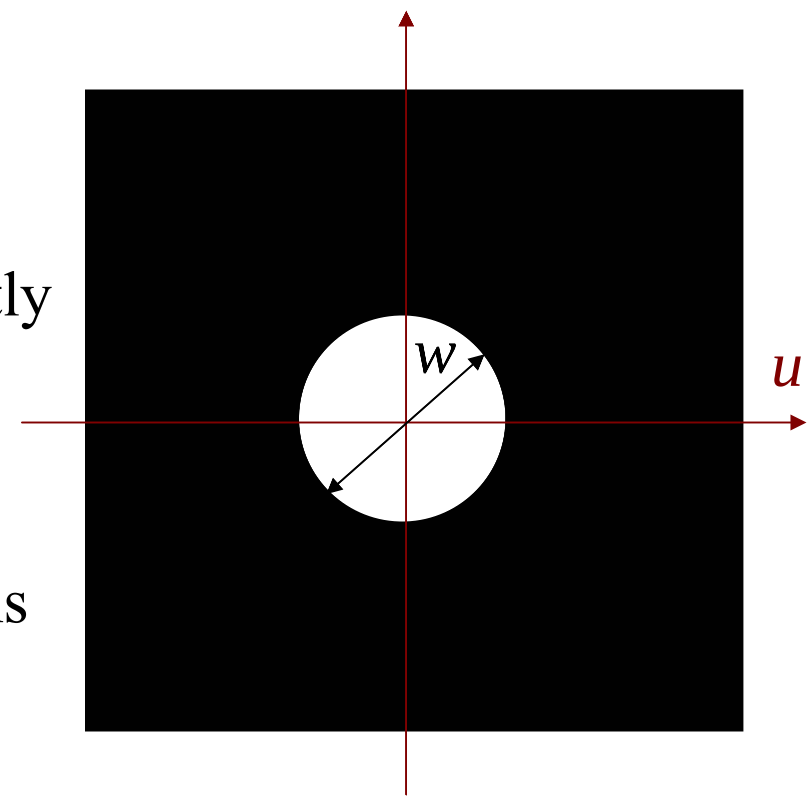
3 original spectrum but phase--conjugated: inside-out, or “pseudo-scopic”

Filtering the wavefront: Fourier transform description



Filtering the wavefront: Fourier transform description

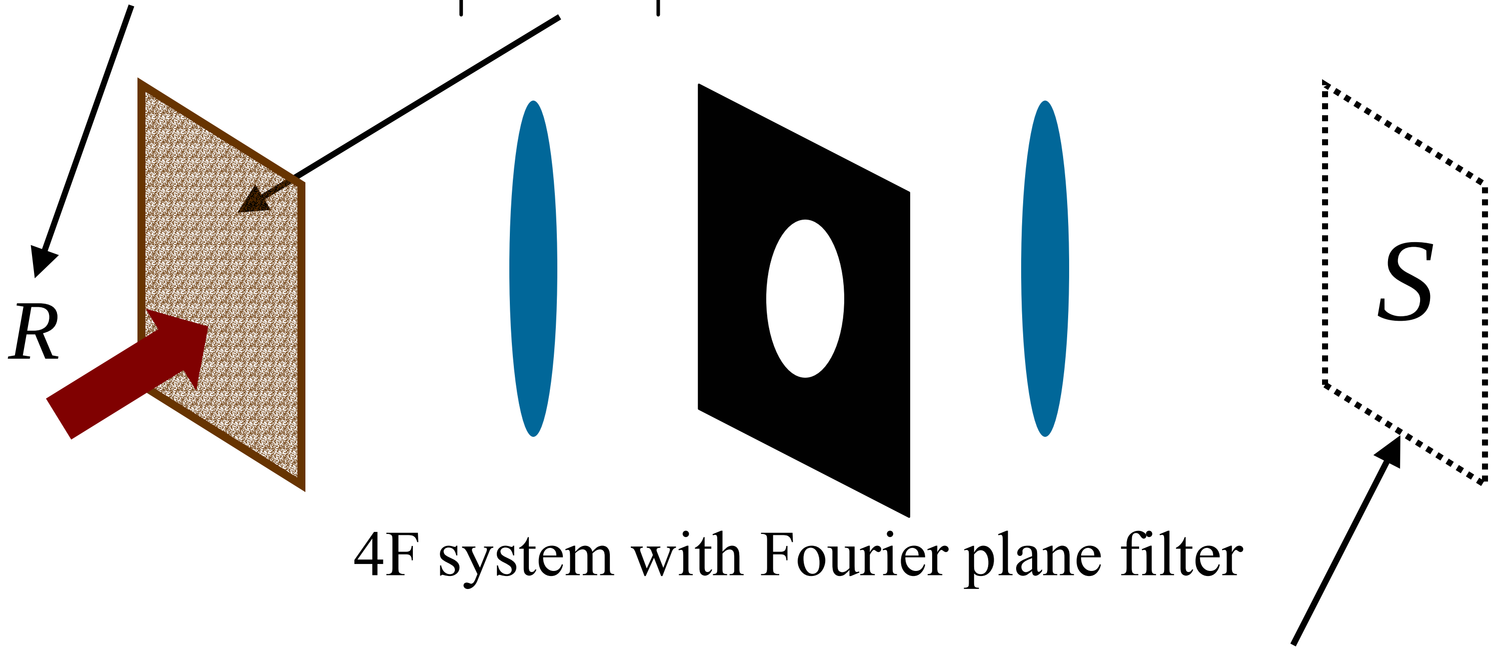
a low-pass filter of passband *w* or slightly greater permits the desired term ❷ to pass, and eliminates the undesirable terms ❶ and ❸.



Holography: reconstructing the <u>wavefront</u>

illumination:
replicates the
reference beam

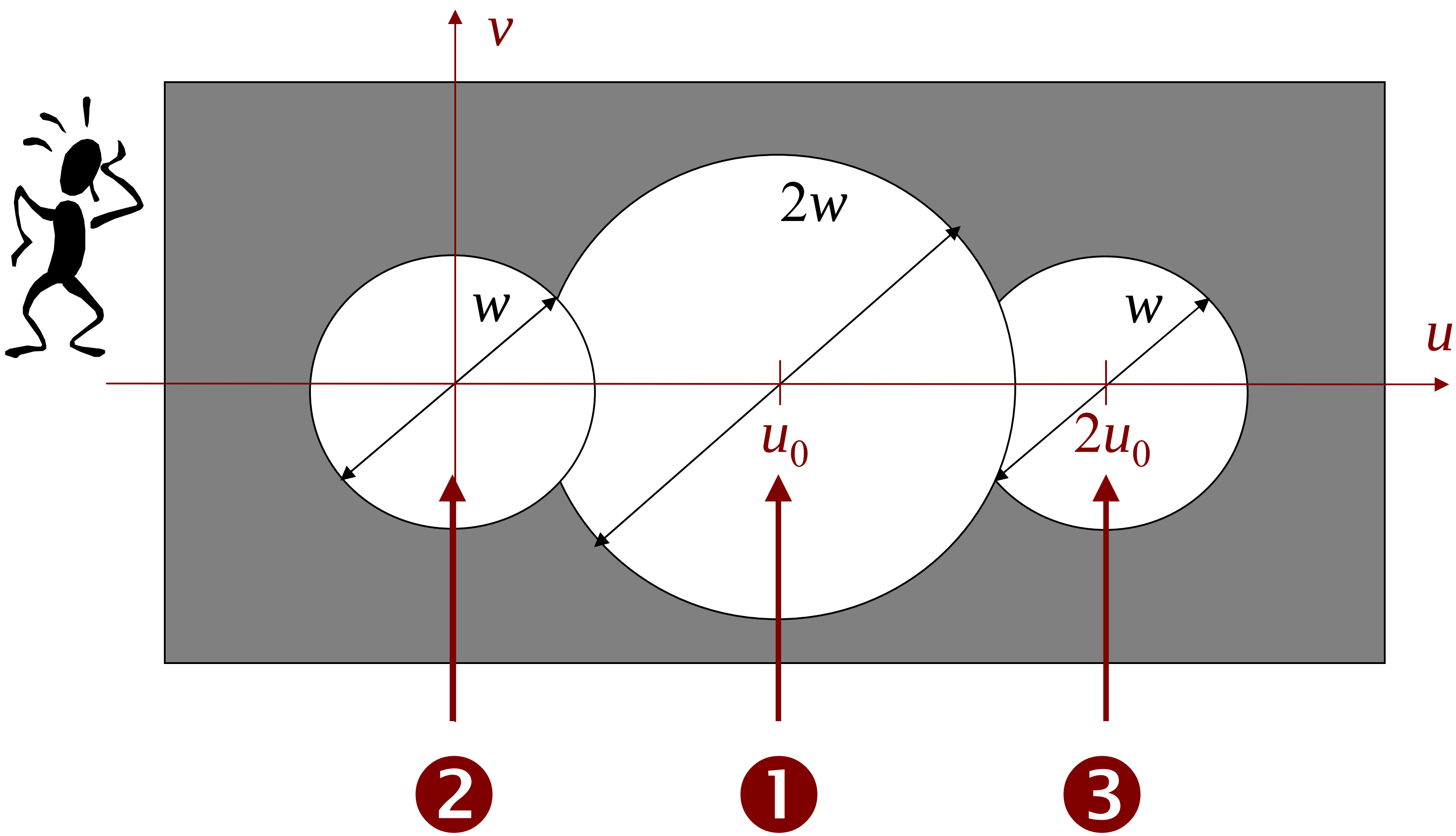
hologram:
$\left|R+S\right|^2$



4F system with Fourier plane filter

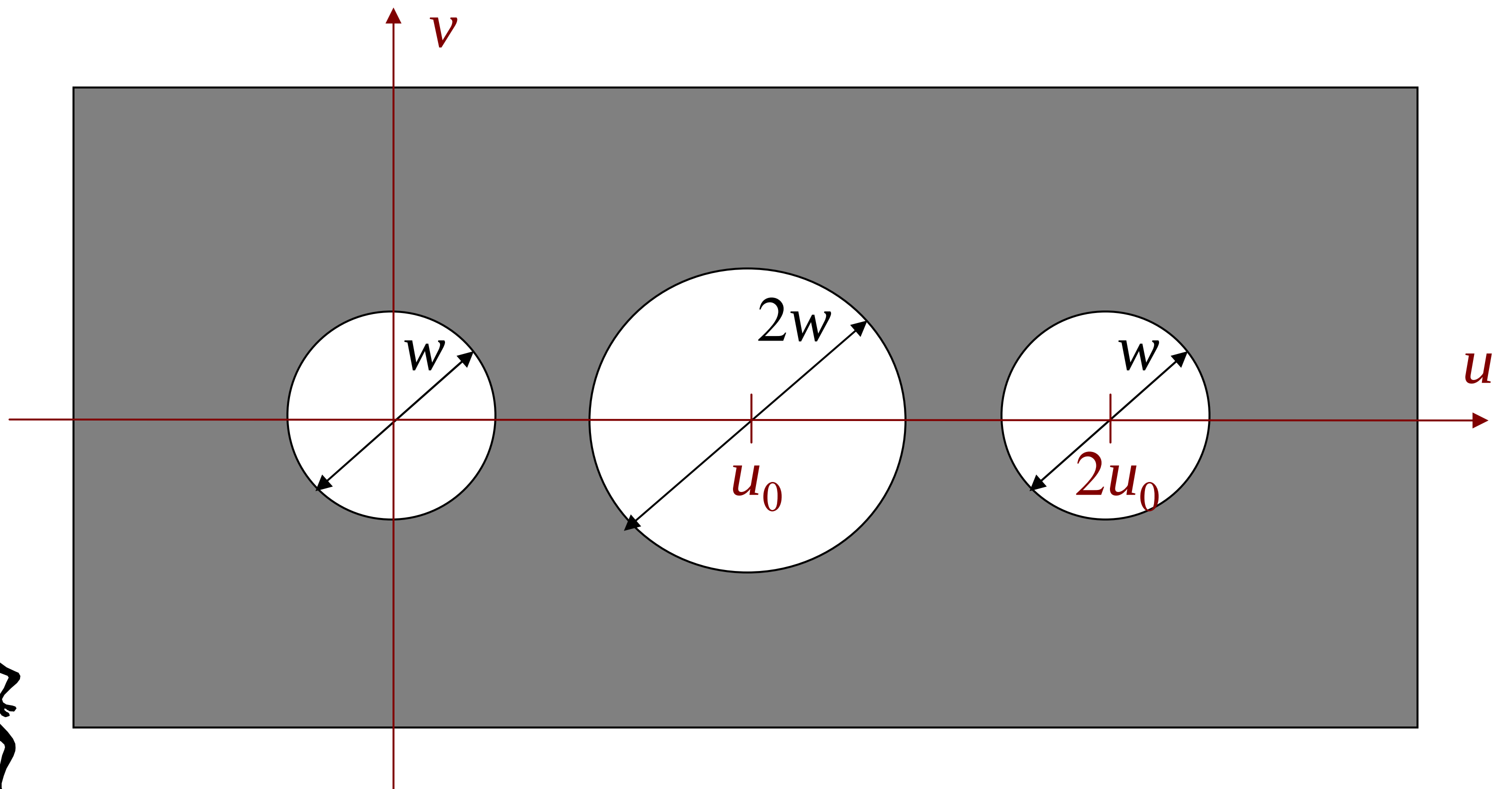
the field at the image plane
replicates the original *S* stored in the hologram

Filtering the wavefront: Fourier transform description



Potential problem: spectra overlap!

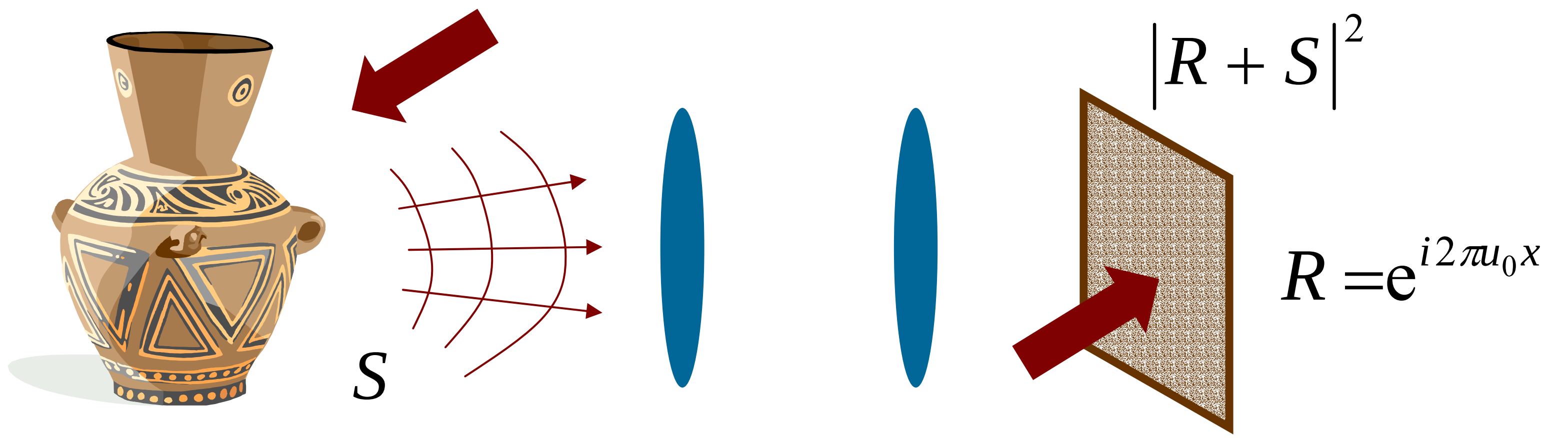
Filtering the wavefront: Fourier transform description



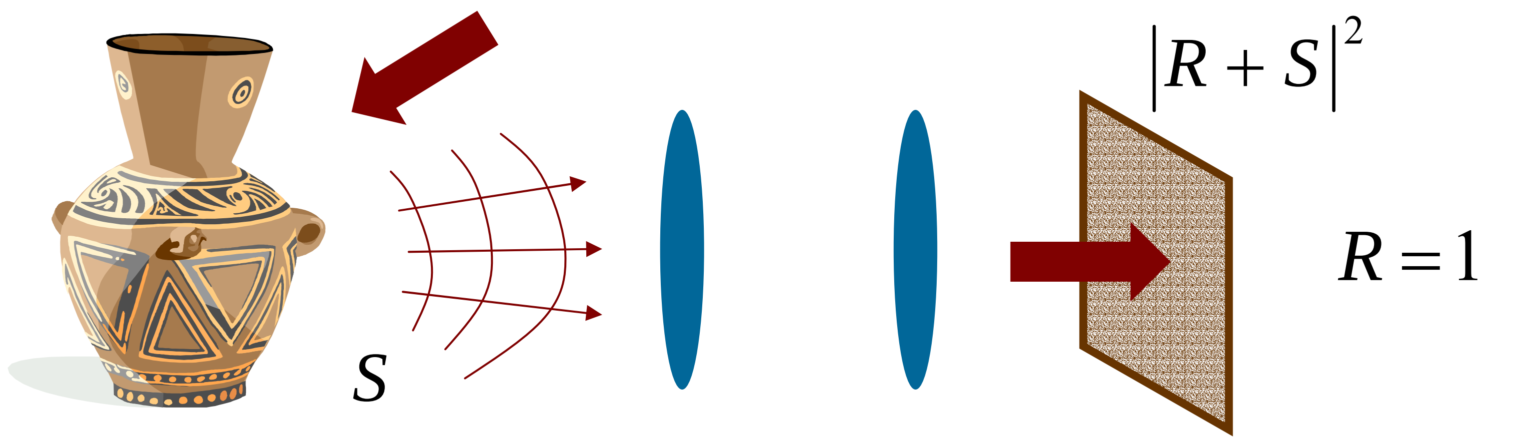
Spectra should not overlap, i.e. $u_0 - w > \dfrac{w}{2} \Leftrightarrow u_0 > \dfrac{3}{2} w$

Leith-Upatnieks *vs* Gabor hologram

Leith-Upatnieks



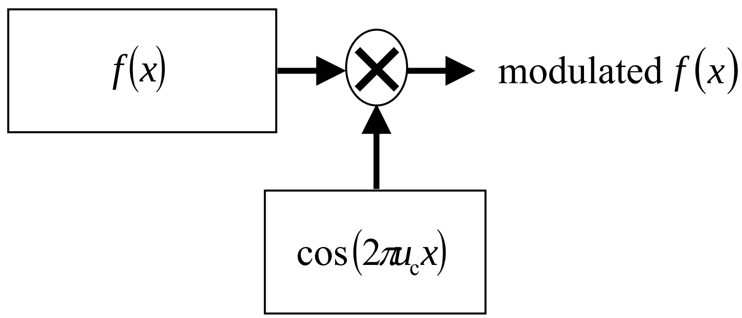
Gabor



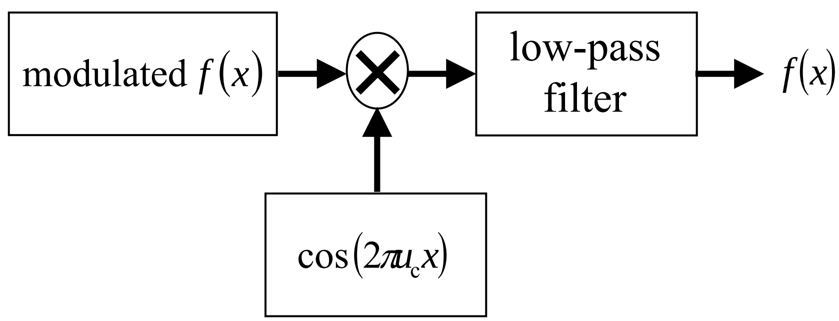
Analogy between the Leith-Upatnieks hologram and amplitude modulation (AM)

AM Radio

Modulation

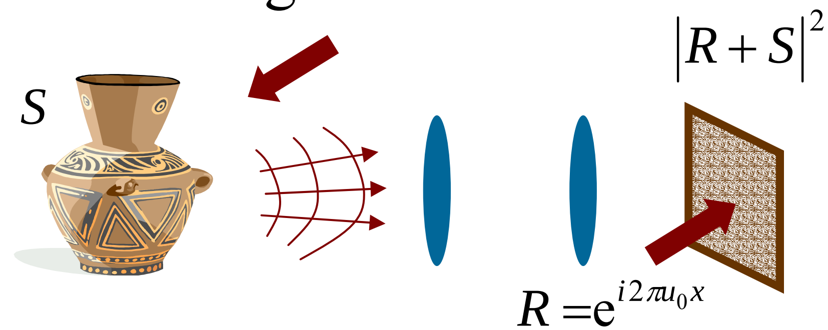


Demodulation



Holography

Recording



Reconstruction

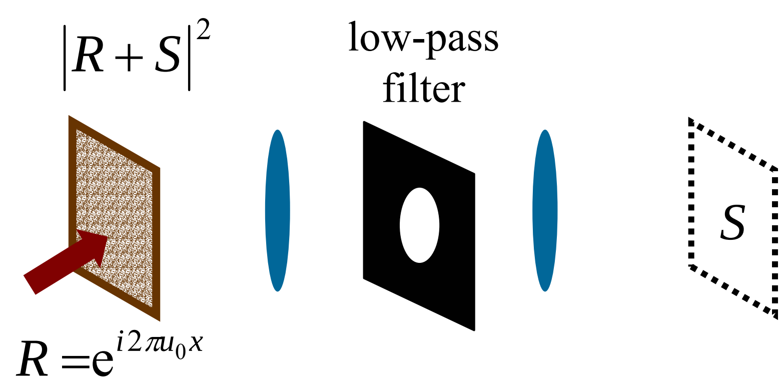


Image locations and magnification

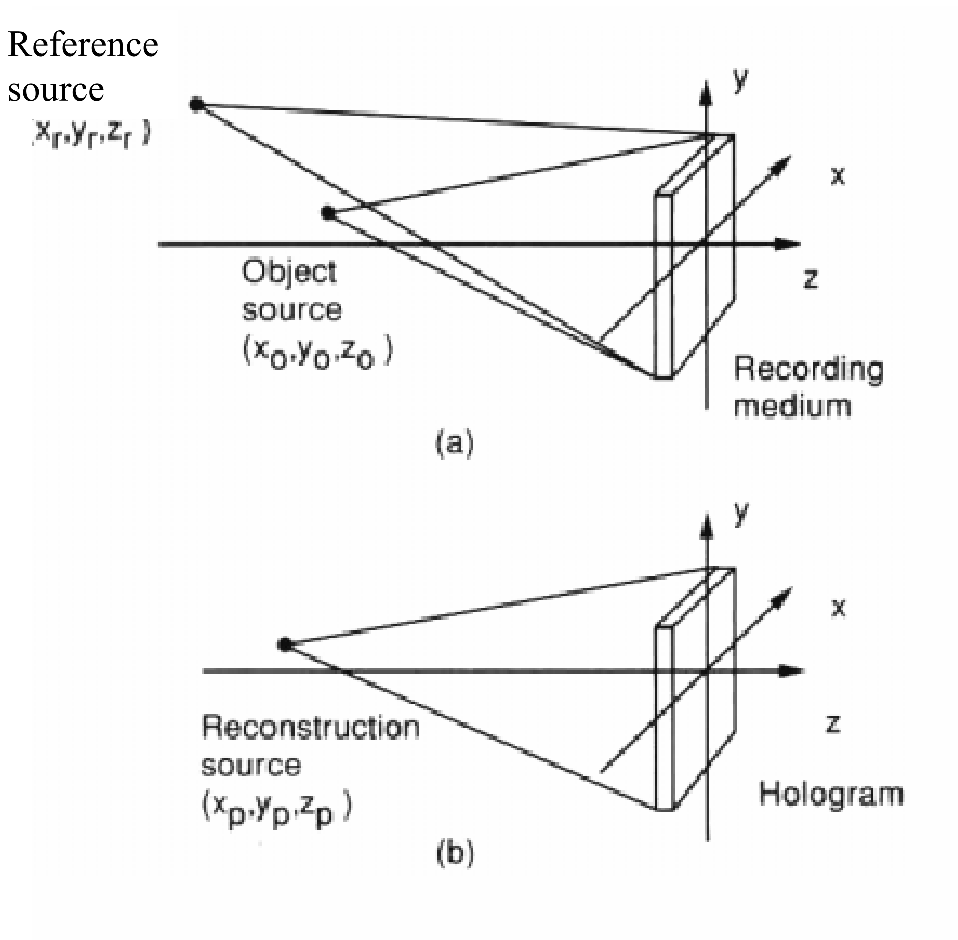


Image locations and magnification

$$z_i = \left(\frac{1}{z_p} \pm \frac{\lambda_2}{\lambda_1 z_r} \mp \frac{\lambda_2}{\lambda_1 z_0} \right)^{-1}$$

$$x_i = \mp \frac{\lambda_2 z_i}{\lambda_1 z_0} x_0 \pm \frac{\lambda_2 z_i}{\lambda_1 z_0} x_r + \frac{z_i}{z_p} x_p$$

$$y_i = \mp \frac{\lambda_2 z_i}{\lambda_1 z_0} y_0 \pm \frac{\lambda_2 z_i}{\lambda_1 z_0} y_r + \frac{z_i}{z_p} y_p$$

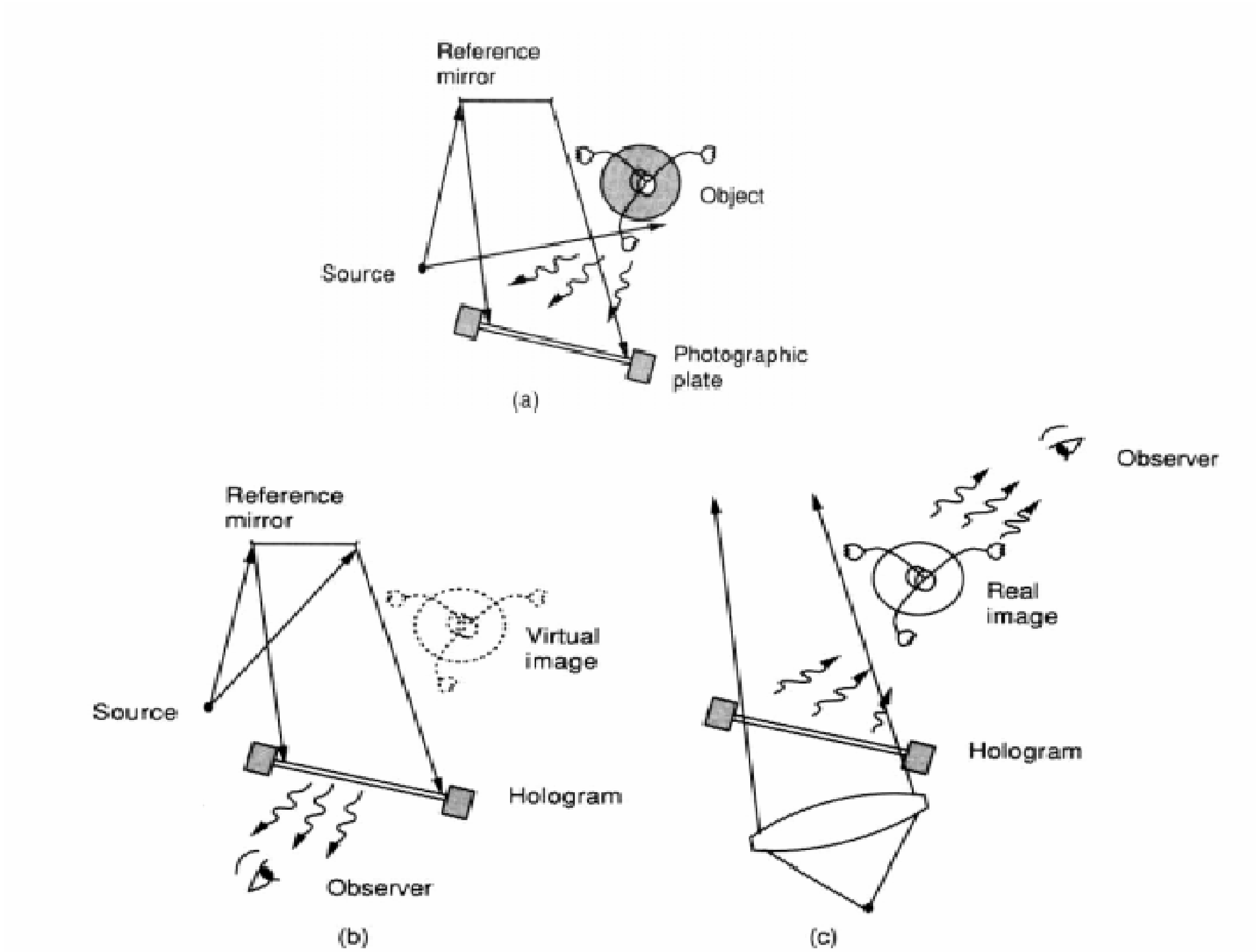
Transverse Magnification

$$M_t = \left| \frac{\partial x_i}{\partial x_0} \right| = \left| \frac{\partial y_i}{\partial y_0} \right| = \left| \frac{\lambda_2 z_i}{\lambda_1 z_0} \right| = \left| 1 - \frac{z_0}{z_r} \mp \frac{\lambda_1 z_0}{\lambda_2 z_p} \right|^{-1}$$

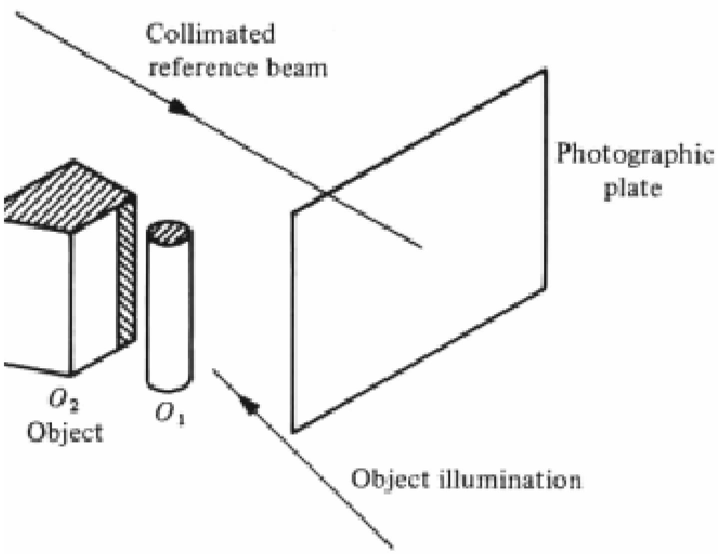
Axial Magnification

$$M_a = \left| \frac{\partial z_i}{\partial z_0} \right| = \left| \frac{\partial}{\partial z_0} \left(\frac{1}{z_p} \pm \frac{\lambda_2}{\lambda_1 z_r} \mp \frac{\lambda_2}{\lambda_1 z_0} \right)^{-1} \right| = \frac{\lambda_1}{\lambda_2} M_t^2$$

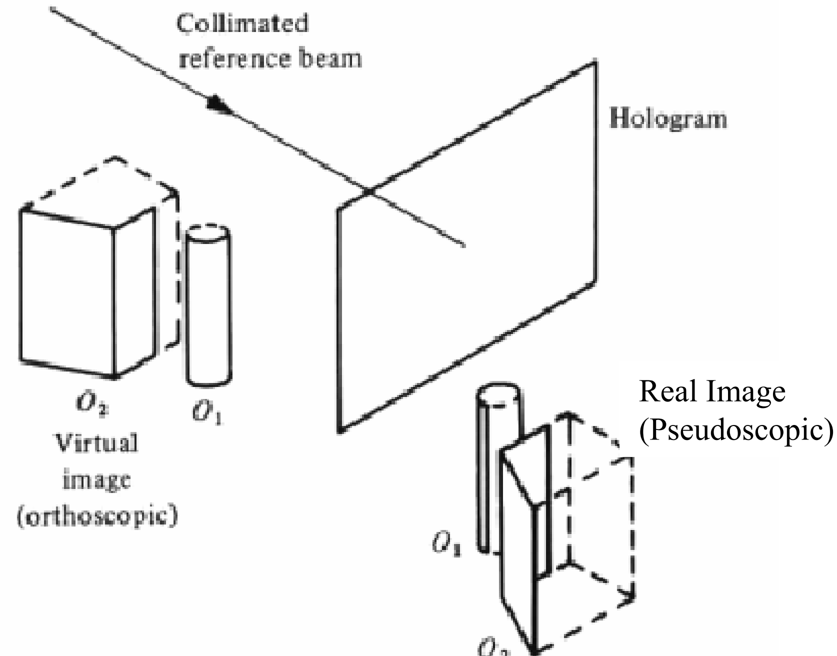
Holography of Three-Dimensional Scenes



Orthoscopic and Pesudoscopic

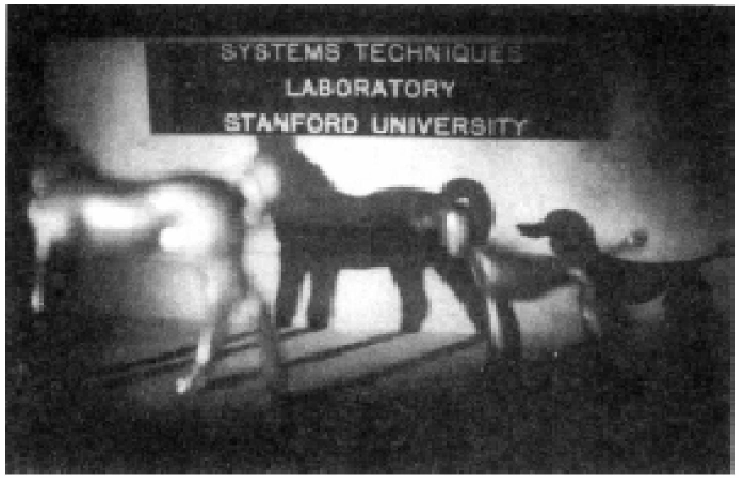


(a) Hologram recording

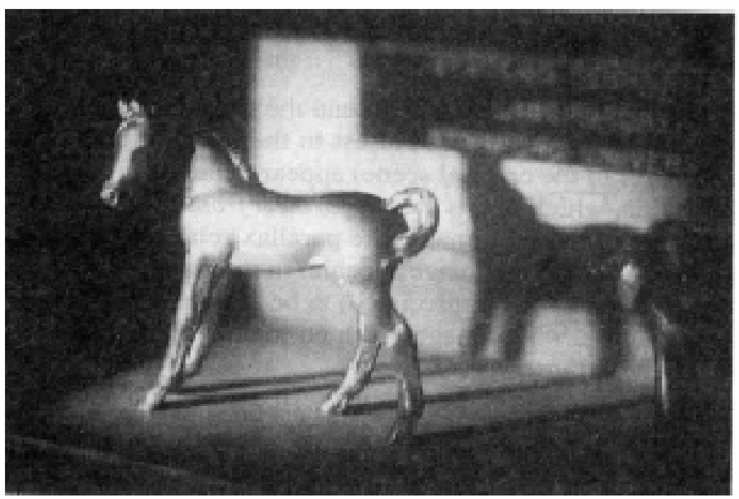


(b) Image reconstruction

Holography of Three-Dimensional Scenes



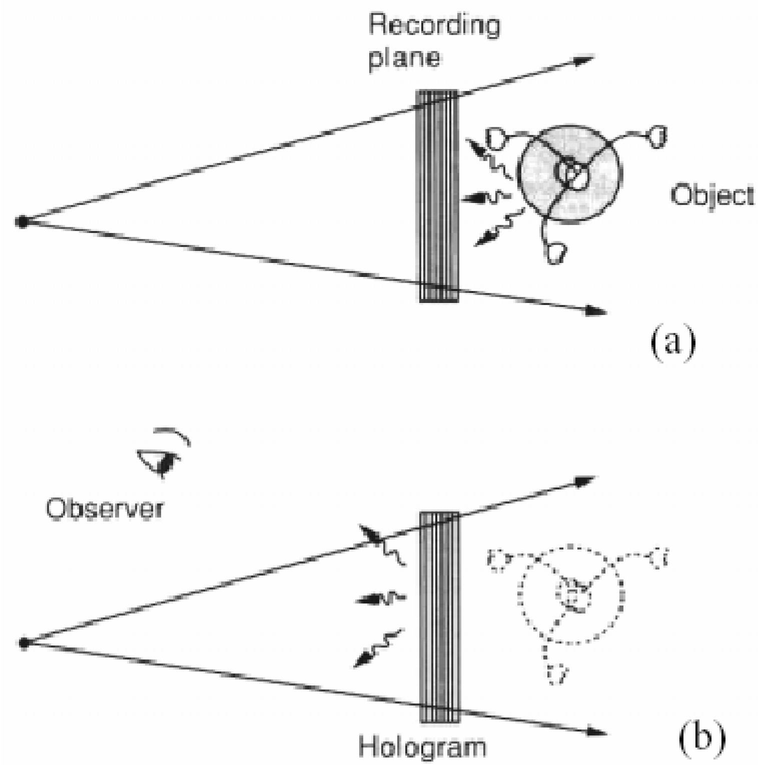
(a)



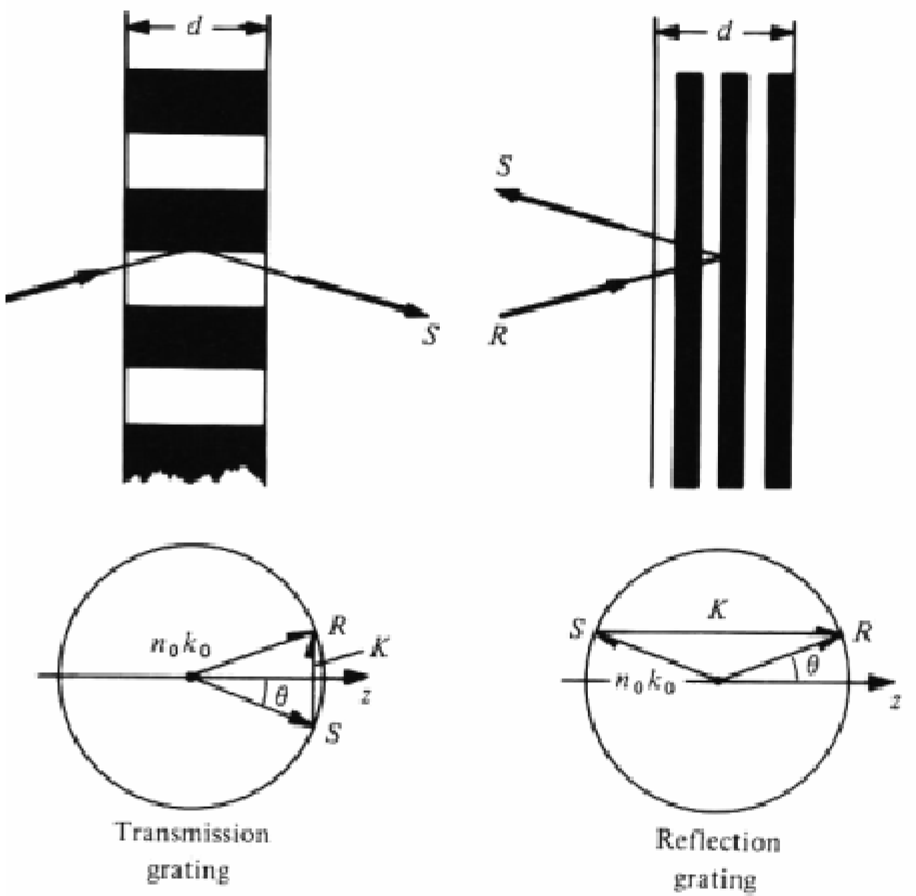
(b)

Transmission and Reflection Holograms

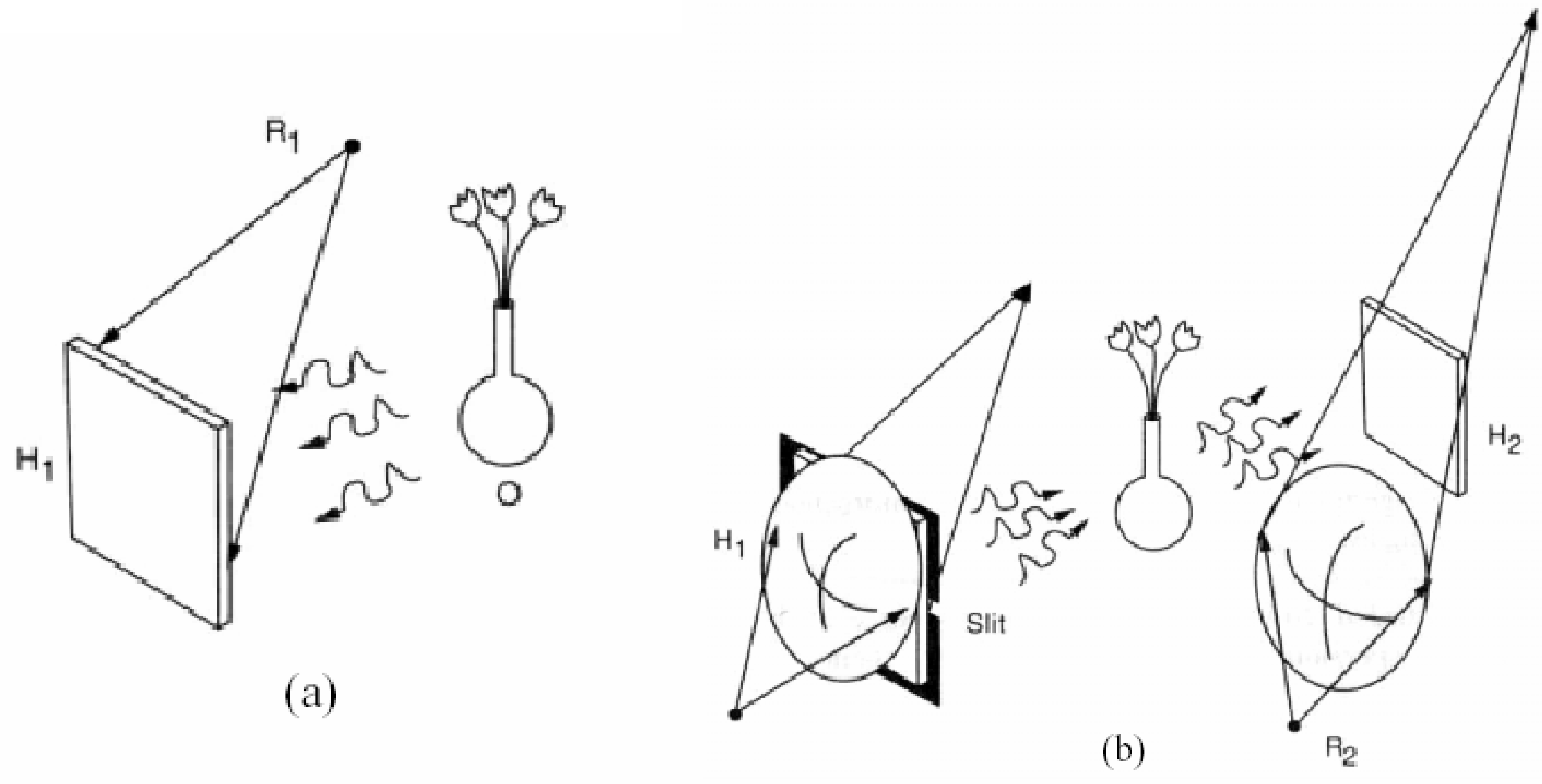
• Reflection hologram



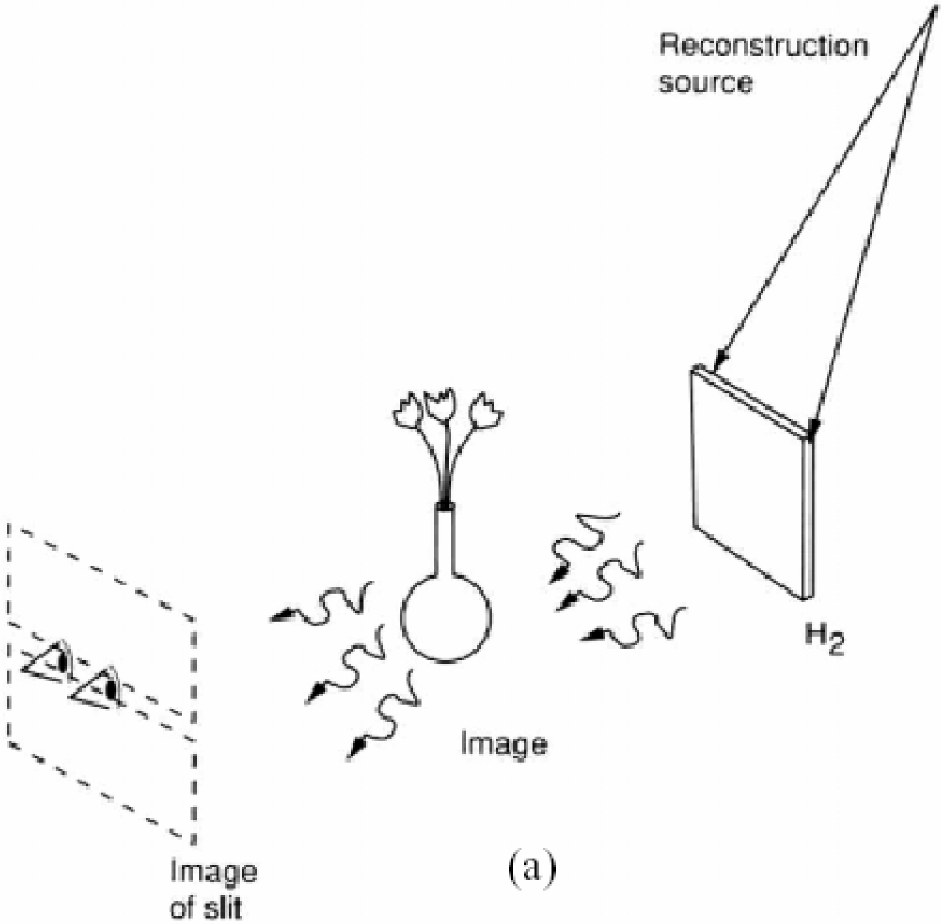
Transmission and Reflection Holograms



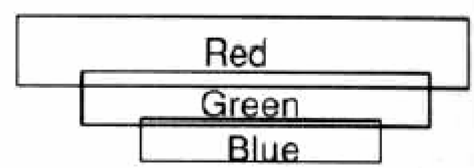
Rainbow hologram (Record)



Rainbow hologram (Reconstruct)



(a)



(b)