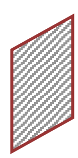
3D Optics

2D Optics

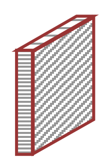
lenses



gratings

Light interacts with medium
on a sequence of
discrete surfaces

3D Optics

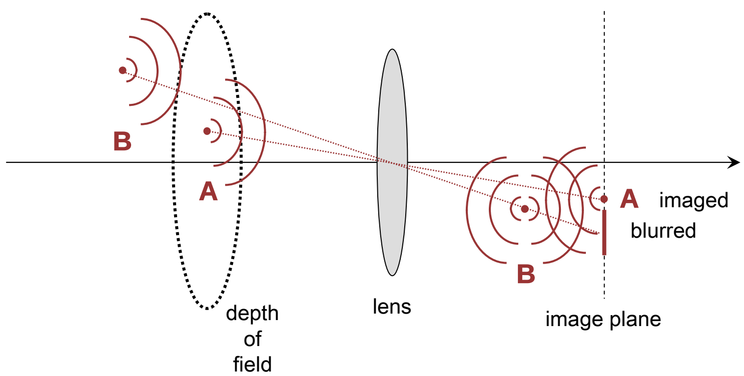


GRadient INdex optics
(GRIN)

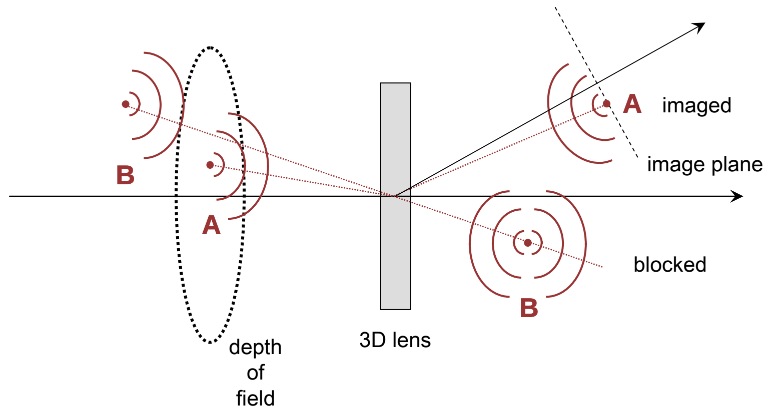
Volume
Holograms

Light interacts with medium
throughout a volume

Imaging with traditional lenses



Imaging with 3D lenses



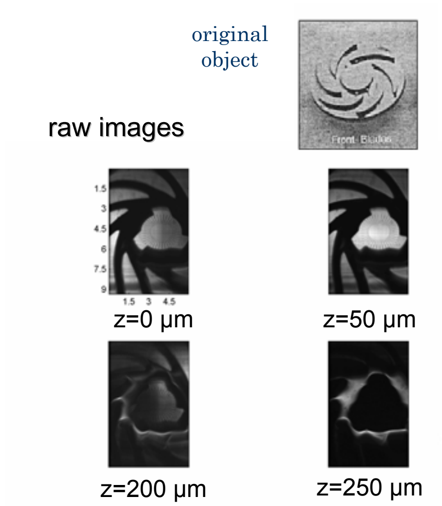
Defocus ⇔ Contrast

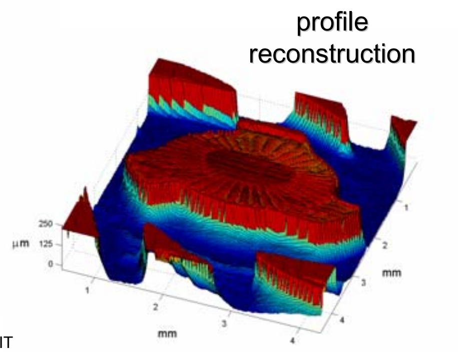
Shape recovery ("profilometry") with 3D lenses

Arnab Sinha, George Barbastathis
MIT 3D Optical Systems group
Optics Express **11**:3202, 2003

Object Distance = 46 cm

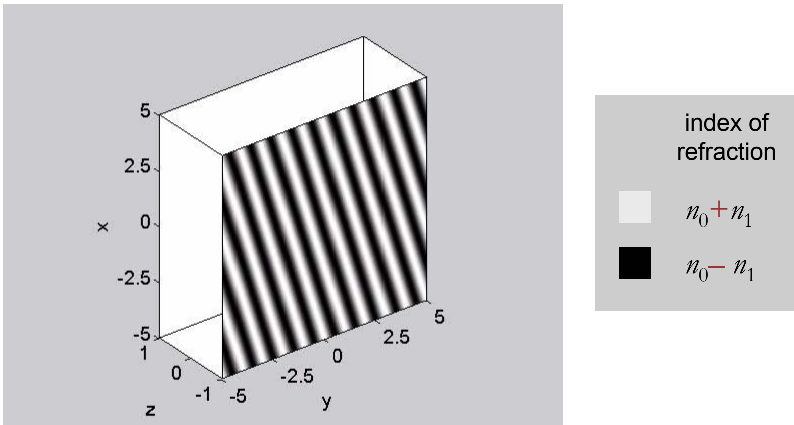
Resolution accomplished ≤ 100 μm





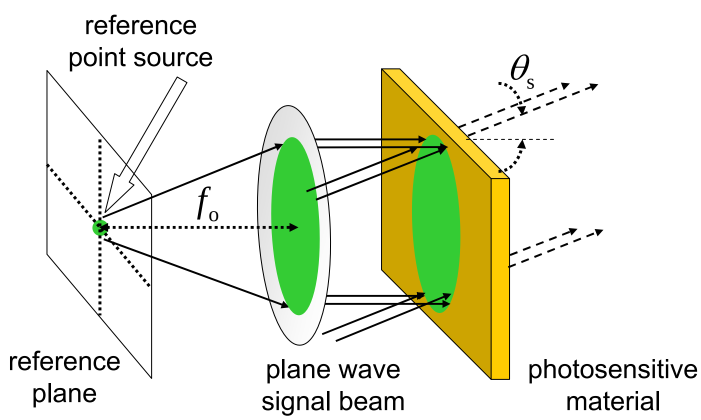
Microturbine provided by Chee Wei Wong, Alan Epstein, MIT

3D lens: plane-to-plane wave volume hologram



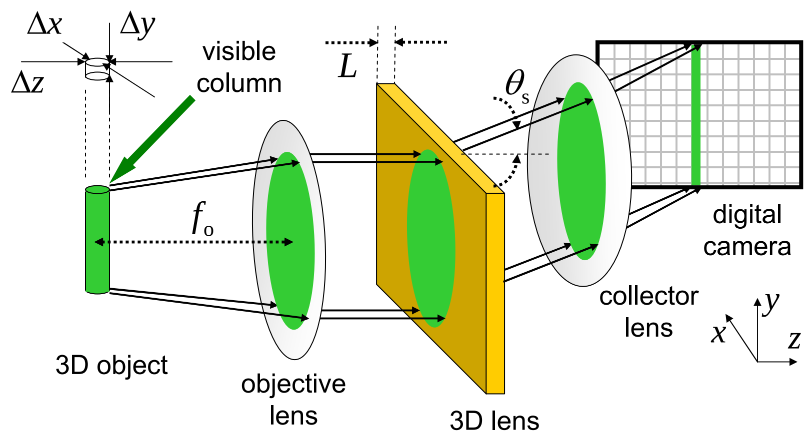
Making the (simplest) 3D lens

after exposure $n(\mathbf{r}) = n_0 + n_1 \cos\left[(\mathbf{k}_s - \mathbf{k}_r)\cdot \mathbf{r}\right]$

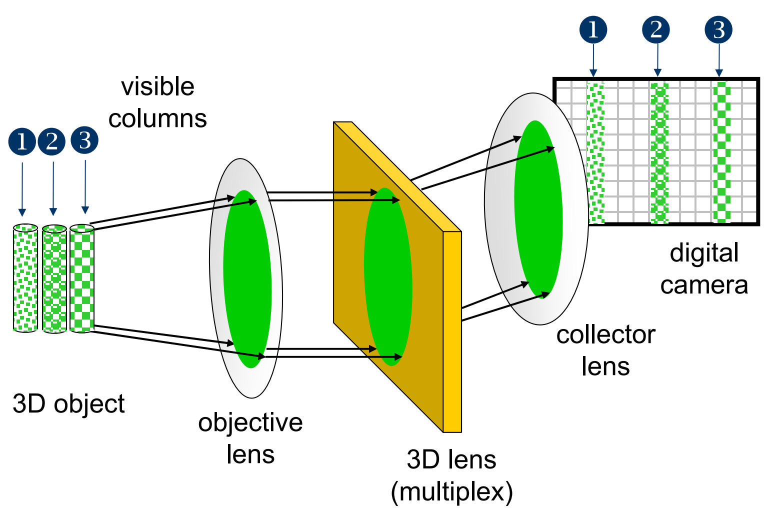


Operating the (simplest) 3D lens

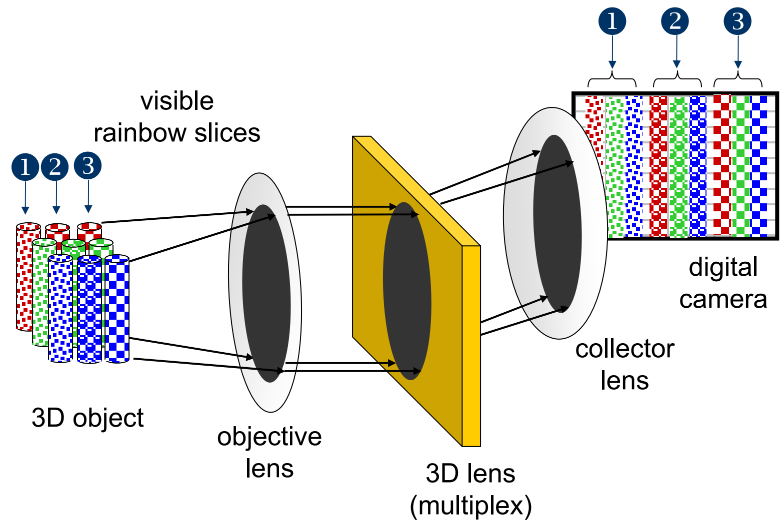
$$L \sim 100\mu\text{m} \div 1\text{mm}$$



Operating a multiplex 3D lens

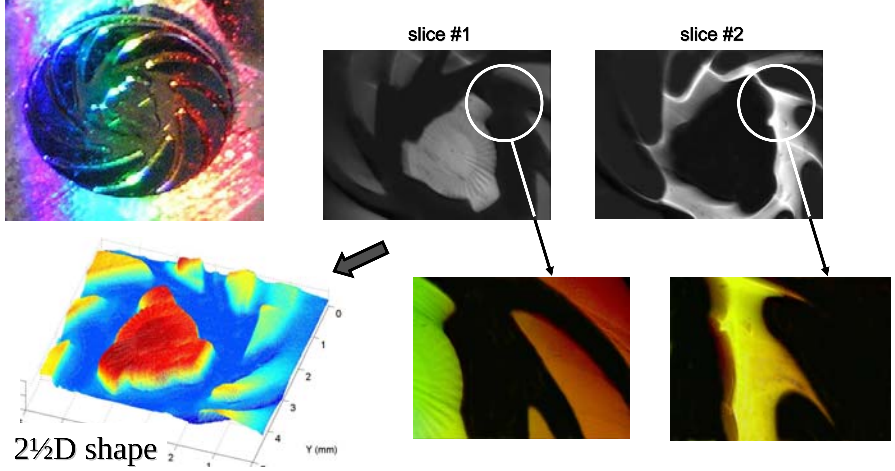


Operating a hyperspectral multiplex 3D lens



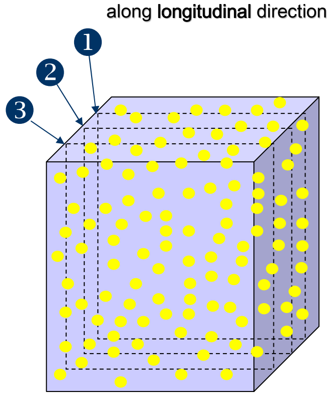
Rainbow volume holographic imaging

Wenyang Sun, George Barbastathis
MIT 3D Optical Systems group
Optics Letters **30**:976, 2005



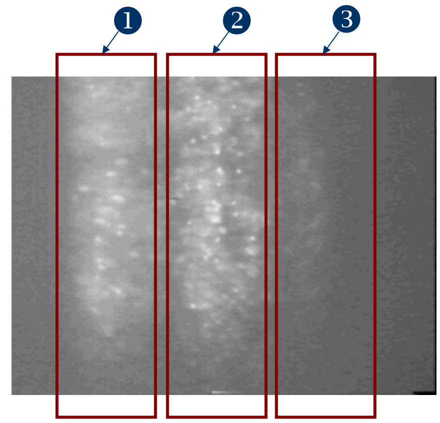
Multiplex imaging of 3D fluorescent object

three "slices" through fluorescent (3D) object, stacked along **longitudinal** direction …



3D object:
fluorescent beads in water

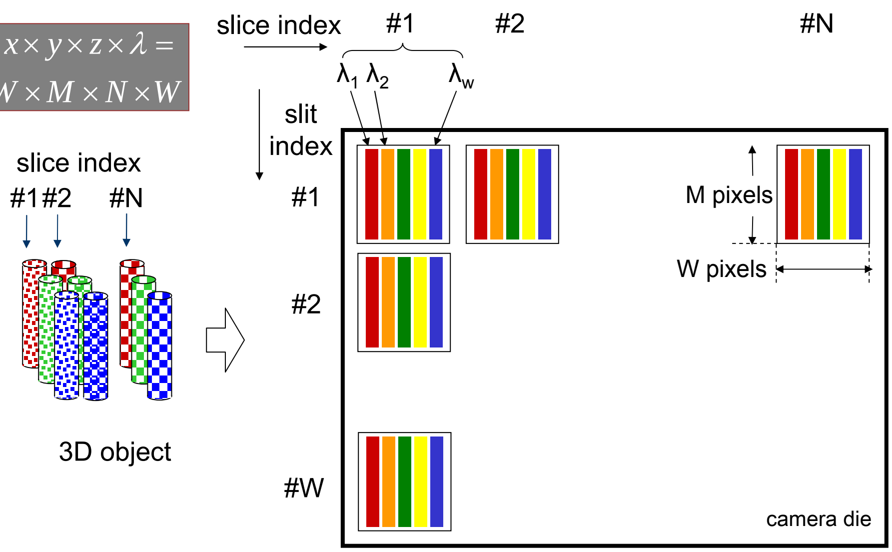
… are viewed **simultaneously** and **side-by-side** on the digital camera



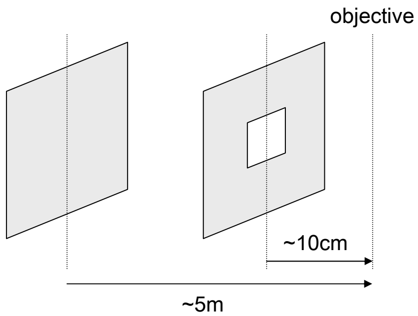
Wenhai Liu,* Demetri Psaltis,* George Barbastathis**
*Caltech Optical Info. Proc. group **MIT 3D Optical Systems group
Optics Letters **27**:854, 2002

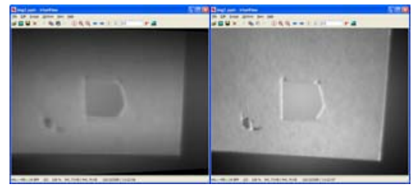
Camera pixel layout for 4D (3D spatial + spectral) imaging

$$x \times y \times z \times \lambda = W \times M \times N \times W$$



Sun Light (completely passive) illumination

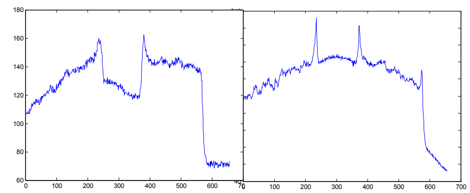




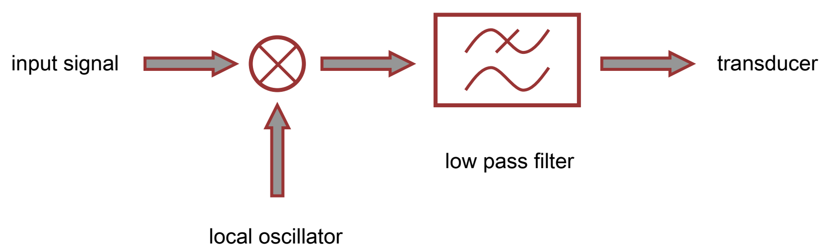
3D optical image

traditional camera

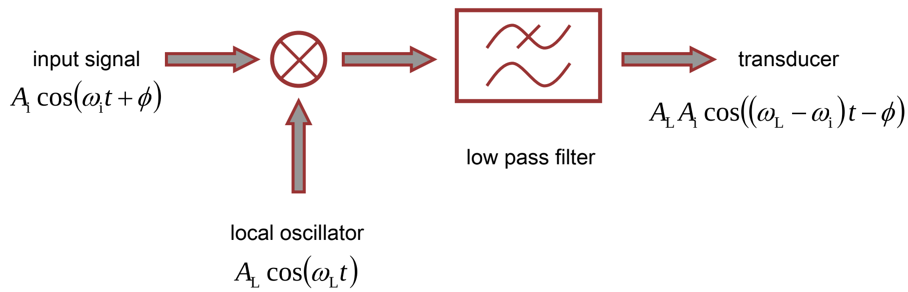
irradiance across image cross-section



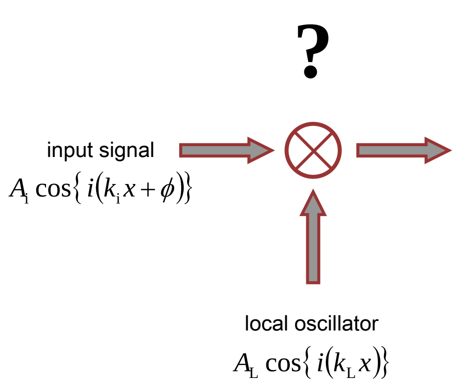
Temporal Heterodyning



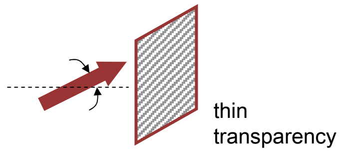
Temporal Heterodyning

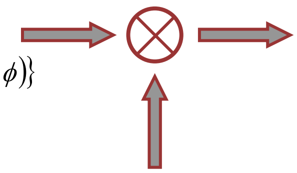


1D Spatial Heterodyning



1D Spatial Heterodyning





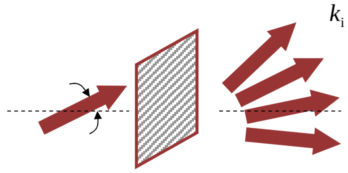
input signal

$A_i \exp\{ i(k_i x + \phi)\}$

local oscillator

$\left| A_R \exp(ik_R x) + A_S \exp(ik_S x + \phi_S) \right|^2$

1D Spatial Heterodyning

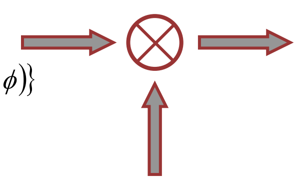


$k_i + k_R + k_S$

$k_i + k_R - k_S$

$k_i - k_R + k_S$

$k_i - k_R - k_S$



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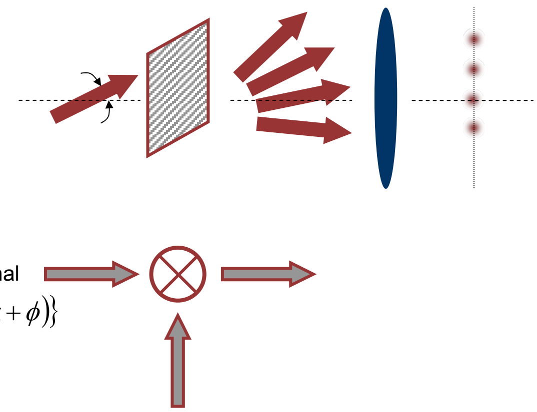


input signal

$$A_i \exp\{i(k_i x+\phi)\}$$

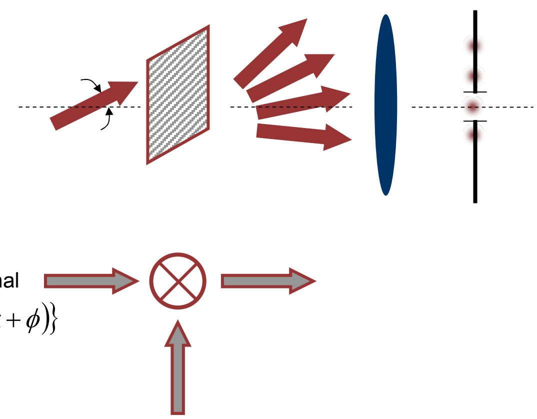
local oscillator

$$\left|A_R \exp(ik_R x)+A_S \exp(ik_S x+\phi_S)\right|^2$$

Massachusetts Institute of Technology

MIT 2.71/2.710 Optics
12/07/05 wk14-b-22

1D Spatial Heterodyning



input signal

$$A_i \exp\{i(k_i x+\phi)\}$$

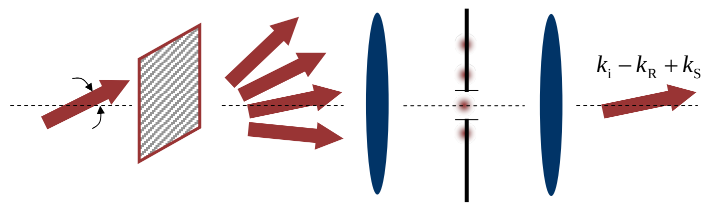
local oscillator

$$\left|A_R \exp(ik_R x)+A_S \exp(ik_S x+\phi_S)\right|^2$$

Massachusetts Institute of Technology

MIT 2.71/2.710 Optics
12/07/05 wk14-b-23

1D Spatial Heterodyning



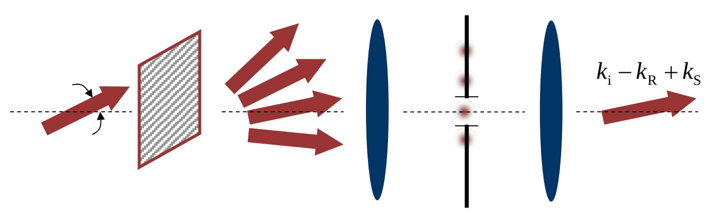
input signal

$A_i \exp\{ i(k_i x + \phi)\}$

local oscillator

$\left| A_R \exp(ik_R x) + A_S \exp(ik_S x + \phi_S)\right|^2$

1D Spatial Heterodyning



input signal

$A_i \exp\{ i(k_i x + \phi)\}$

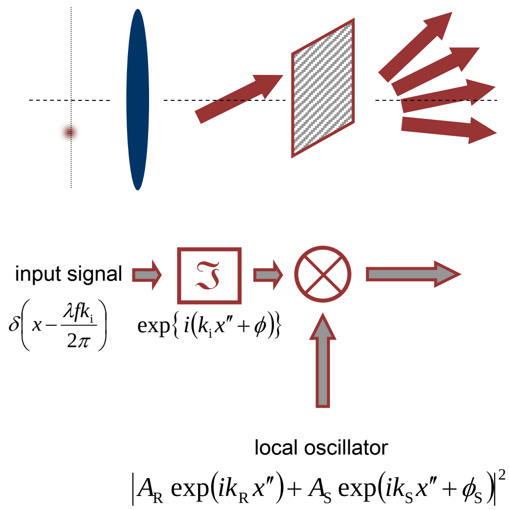
low pass filter

local oscillator

$\left| A_R \exp(ik_R x) + A_S \exp(ik_S x + \phi_S)\right|^2$

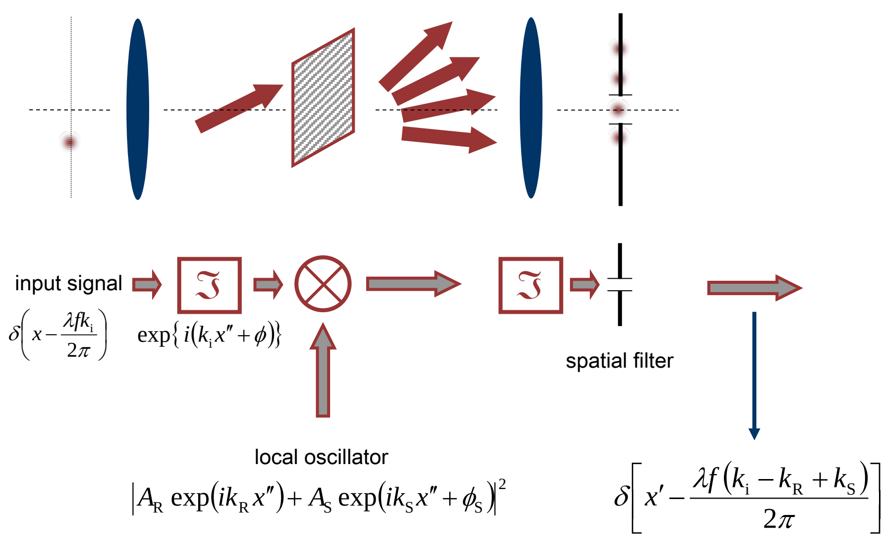
$$A_i A_R A_S \exp\left\{ i\begin{bmatrix} (k_i - k_R + k_S)x' \\ +(\phi + \phi_S) \end{bmatrix} \right\}$$

1D Spatial Heterodyning in the Fourier domain



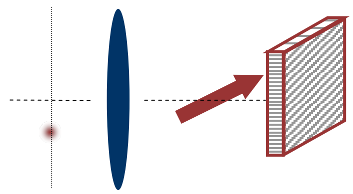
Massachusetts Institute of Technology

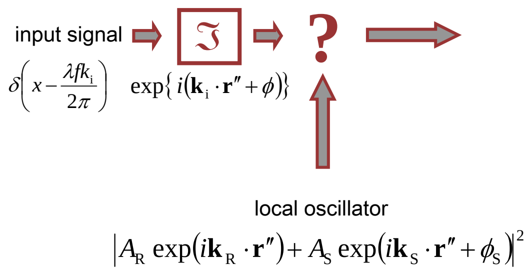
1D Spatial Heterodyning in the Fourier domain



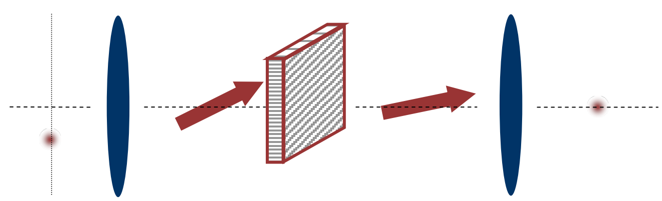
Massachusetts Institute of Technology

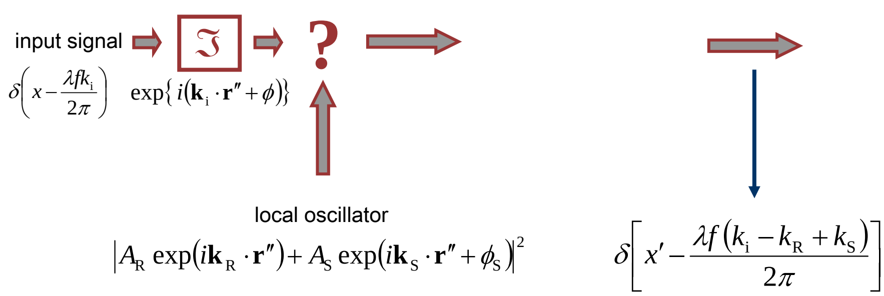
3D Spatial Heterodyning in the Fourier domain



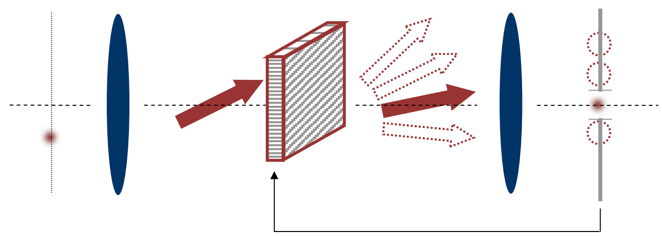


3D Spatial Heterodyning in the Fourier domain





3D Spatial Heterodyning in the Fourier domain



input signal $\Rightarrow$ $\boxed{\mathfrak{I}}$ $\Rightarrow$?

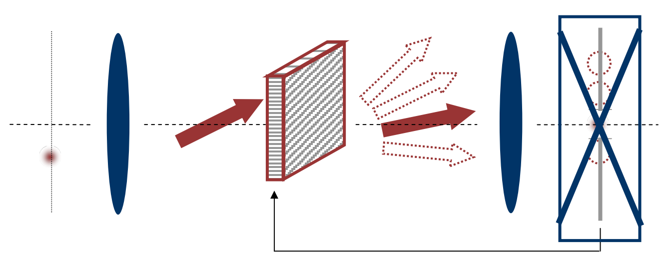
$\delta\left(x-\dfrac{\lambda f k_{\mathrm{i}}}{2\pi}\right)$ $\exp\{i(\mathbf{k}_{\mathrm{i}}\cdot\mathbf{r}''+\phi)\}$

local oscillator

$$\left|A_{\mathrm{R}}\exp(i\mathbf{k}_{\mathrm{R}}\cdot\mathbf{r}'')+A_{\mathrm{S}}\exp(i\mathbf{k}_{\mathrm{S}}\cdot\mathbf{r}''+\phi_{\mathrm{S}})\right|^2$$

$$\delta\left[x'-\frac{\lambda f\left(k_{\mathrm{i}}-k_{\mathrm{R}}+k_{\mathrm{S}}\right)}{2\pi}\right]$$

3D Spatial Heterodyning in the Fourier domain



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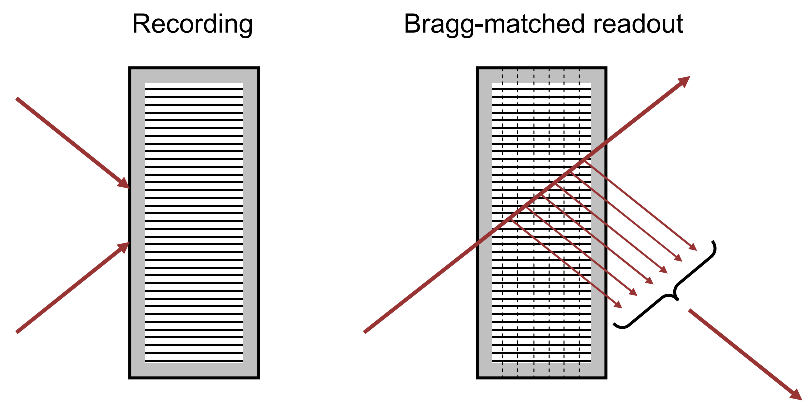
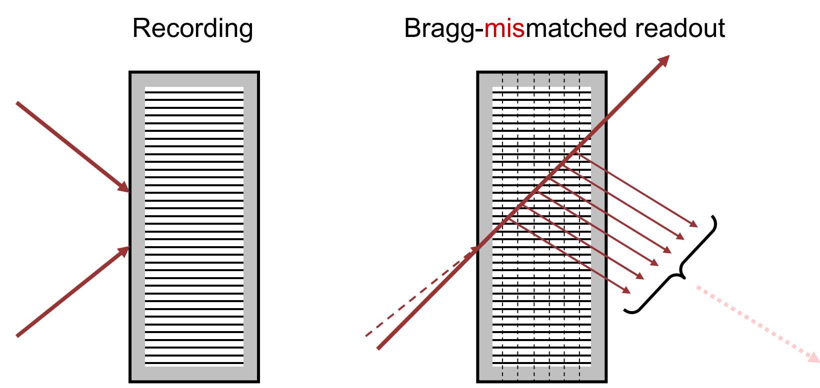
$$\delta\left[x'-\frac{\lambda f\left(k_{\mathrm{i}}-k_{\mathrm{R}}+k_{\mathrm{S}}\right)}{2\pi}\right]$$

Phase matching as a filter for 3D spatial heterodyning



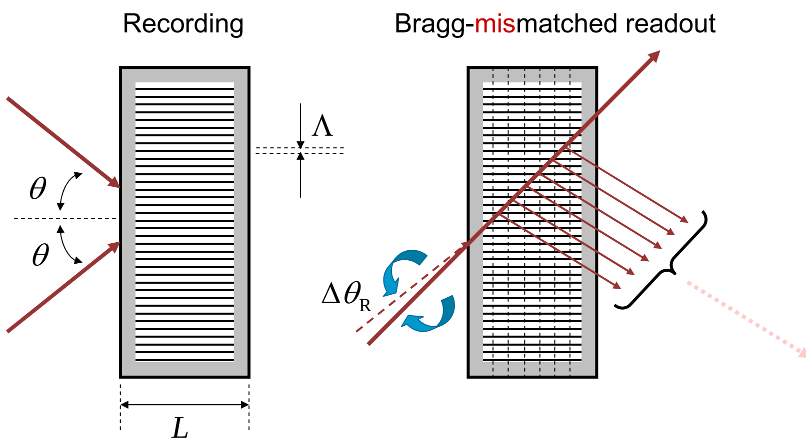
Diffracted beams from elemental thin slices are all **in phase** → strong diffraction

Phase matching as a filter for 3D spatial heterodyning



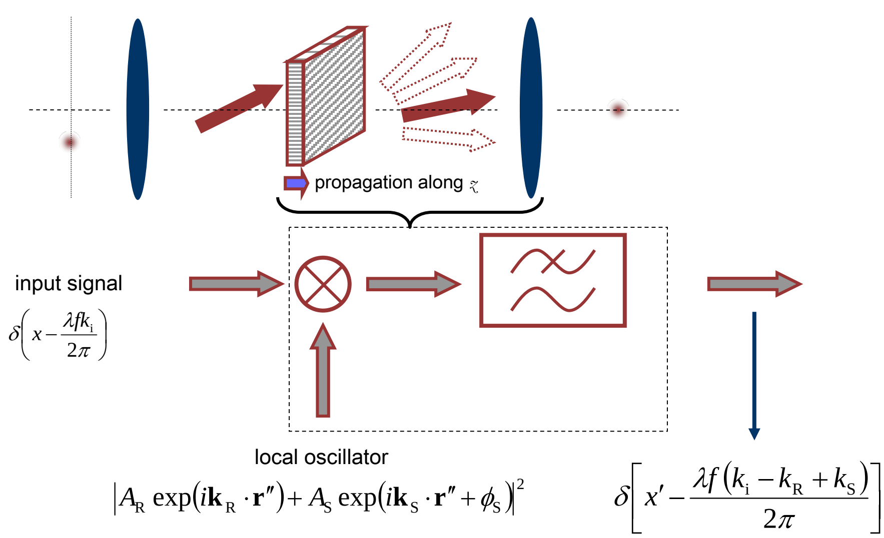
Diffracted beams from elemental thin slices are **out of phase** → no diffraction

Phase matching as a filter for 3D spatial heterodyning

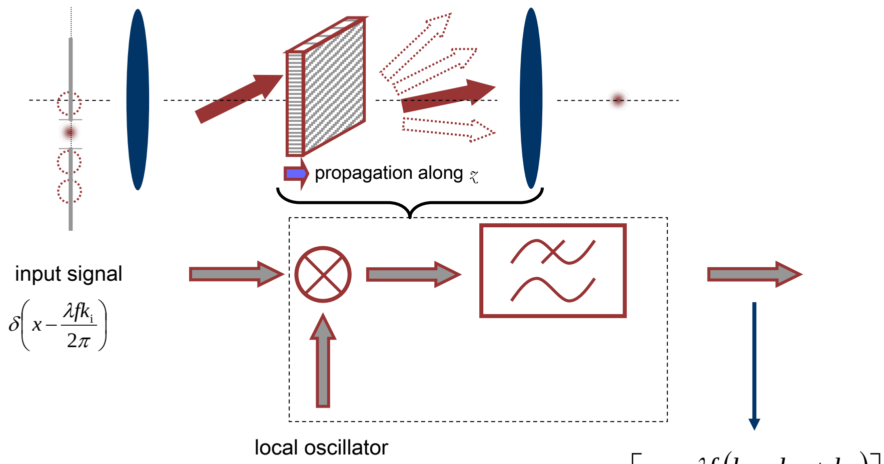


Angle Bragg selectivity $\Delta\theta_R = \dfrac{\Lambda}{L} = \dfrac{\lambda}{2L\sin\theta}$

3D Spatial Heterodyning in the Fourier domain



3D Spatial Heterodyning in the Fourier domain



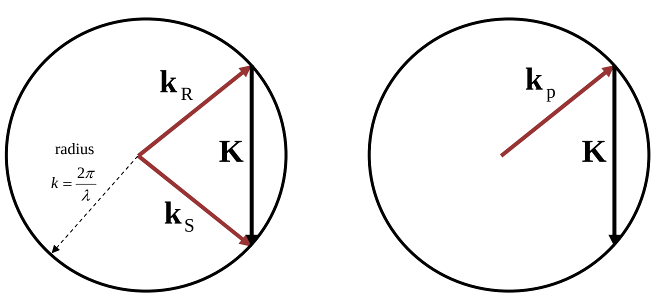
$$\delta\left(x - \frac{\lambda f k_{\mathrm{i}}}{2\pi}\right)$$

$$\left|A_{\mathrm{R}} \exp\left(i\mathbf{k}_{\mathrm{R}} \cdot \mathbf{r}''\right) + A_{\mathrm{S}} \exp\left(i\mathbf{k}_{\mathrm{S}} \cdot \mathbf{r}'' + \phi_{\mathrm{S}}\right)\right|^2$$

$$\delta\left[x' - \frac{\lambda f\left(k_{\mathrm{i}} - k_{\mathrm{R}} + k_{\mathrm{S}}\right)}{2\pi}\right]$$

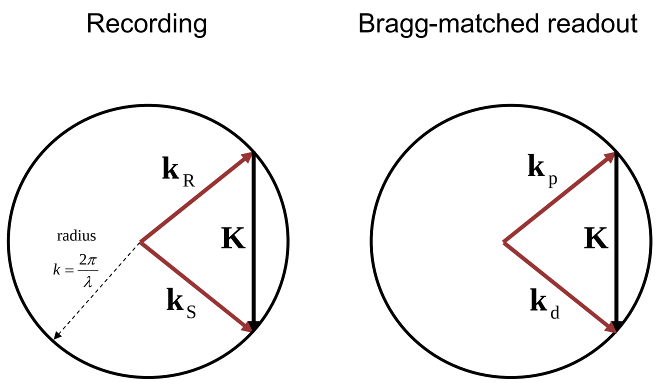
3D Spatial Heterodyning in *k*-space

Recording | Bragg-matched readout



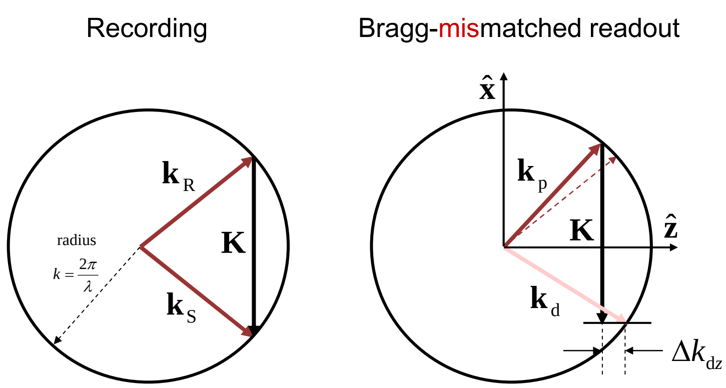
$$\mathbf{K} = \mathbf{k}_{\mathrm{S}} - \mathbf{k}_{\mathrm{R}}$$

3D Spatial Heterodyning in *k*-space



$$\mathbf{K} = \mathbf{k}_S - \mathbf{k}_R \qquad \mathbf{k}_d = \mathbf{k}_R + \mathbf{K}$$

3D Spatial Heterodyning in *k*-space



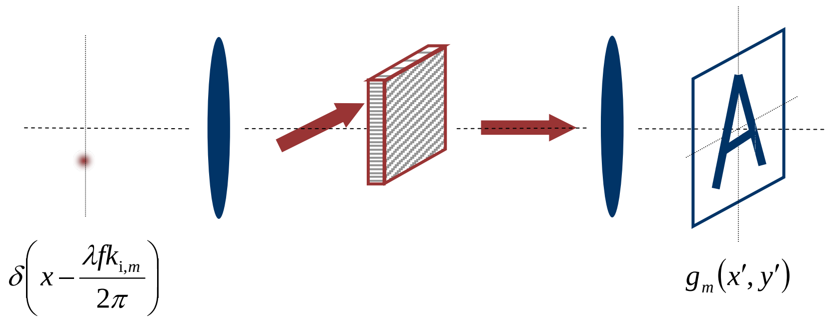
$$\mathbf{K} = \mathbf{k}_S - \mathbf{k}_R$$

$$\mathbf{k}_d = \left[\left(\mathbf{k}_R + \mathbf{K}\right)\cdot \hat{\mathbf{x}}\right]\hat{\mathbf{x}} + k_{dz}\hat{\mathbf{z}}$$

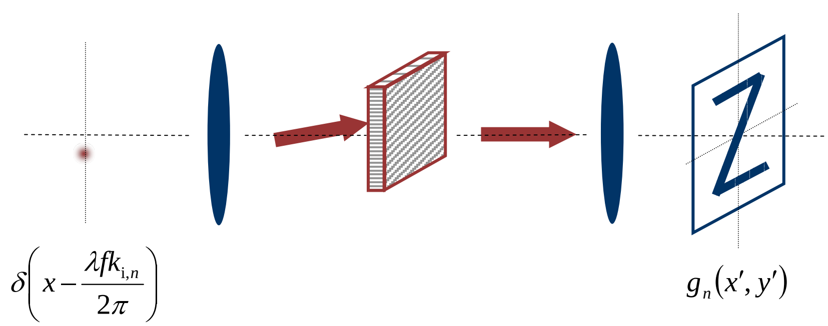
$$\left\| \mathbf{k}_d \right\| = k$$

$$I_{\text{diffracted}} = I_{\text{Bragg-matched}}\,\mathrm{sinc}^2\left(\frac{1}{2}\Delta k_{dz} L\right)$$

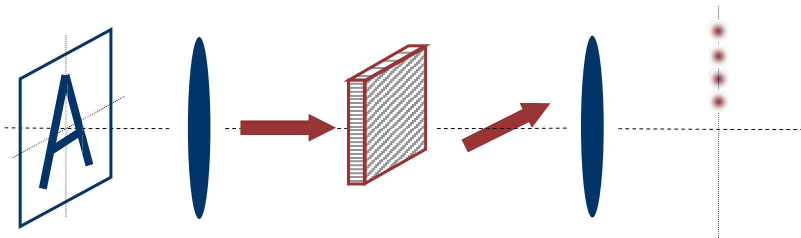
Applications: 3D holographic data storage



Applications: 3D holographic data storage



Applications: 3D holographic optical correlators

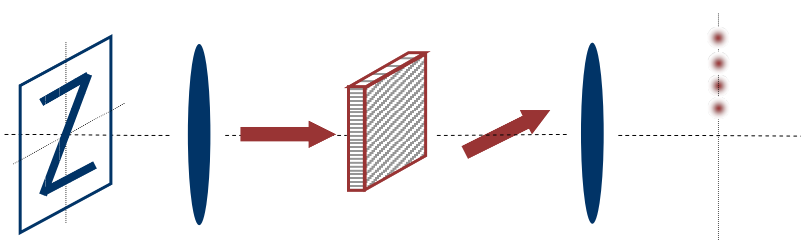


$g_m(x, y)$

$\delta\left(x' - \frac{\lambda f k_{\mathrm{i},m}}{2\pi}\right)$

strongest

Applications: 3D holographic optical correlators

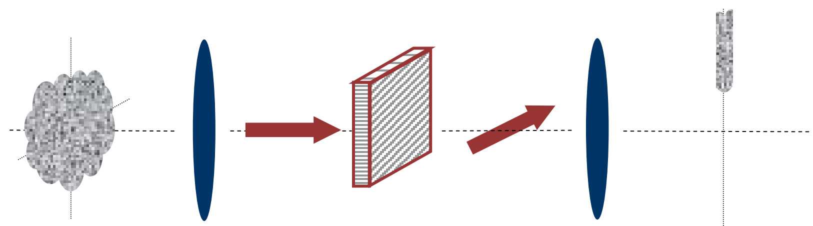


$g_n(x, y)$

$\delta\left(x' - \frac{\lambda f k_{\mathrm{i},n}}{2\pi}\right)$

strongest

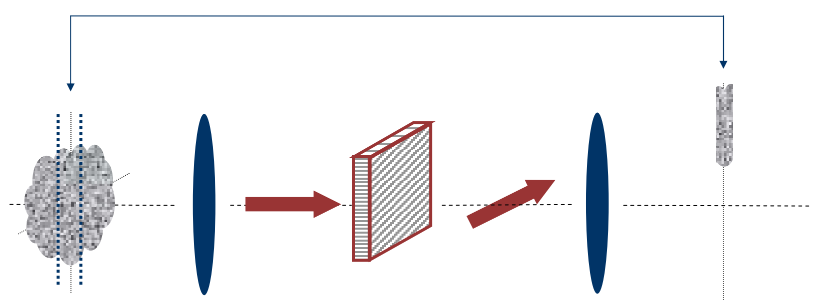
Applications: 3D holographic optical imaging



$$f(x, y, z)$$

$$f\left(x' - \frac{\lambda f k_0}{2\pi}, y', z\right) \times$$
$$\times \mathrm{slit}\left(\frac{x'}{\Delta x'}\right) \times \mathrm{slice}\left(\frac{z}{\Delta z}\right)$$

Applications: 3D holographic optical imaging



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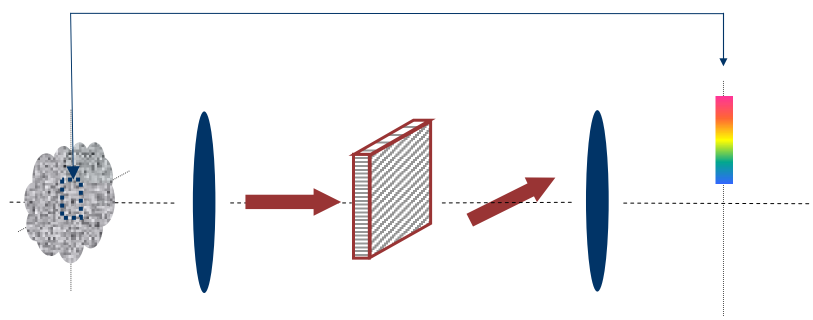
$$f\left(x' - \frac{\lambda f k_0}{2\pi}, y', z\right) \times$$
$$\times \mathrm{slit}\left(\frac{x'}{\Delta x'}\right) \times \mathrm{slice}\left(\frac{z}{\Delta z}\right)$$

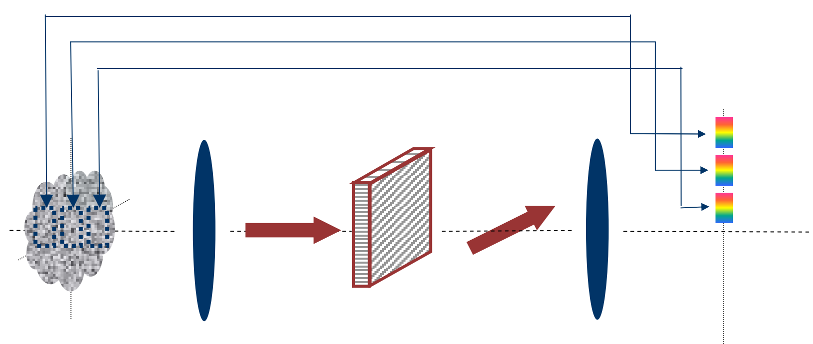
Applications: 3D holographic hyperspectral imaging



$f(x, y, z, \lambda)$

spectral components of selected voxel are spread out on detector plane

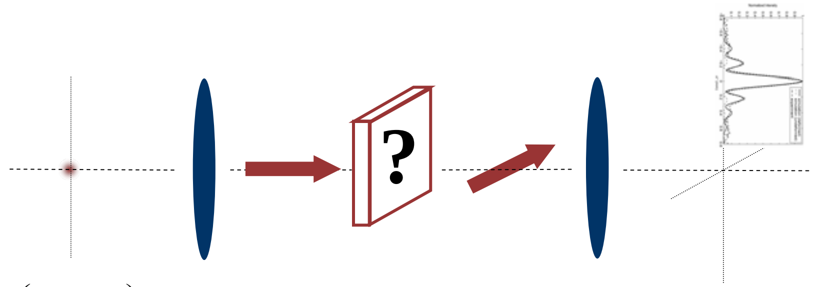
Applications: 3D multiplex hyperspectral imaging



$f(x, y, z, \lambda)$

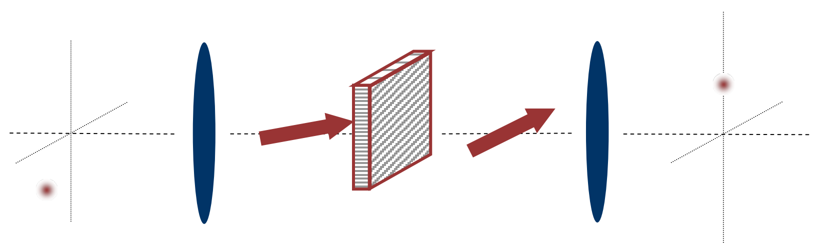
spectral components of multiple voxels are spread out on non-overlapping locations in detector plane

Applications: probing unknown 3D holograms



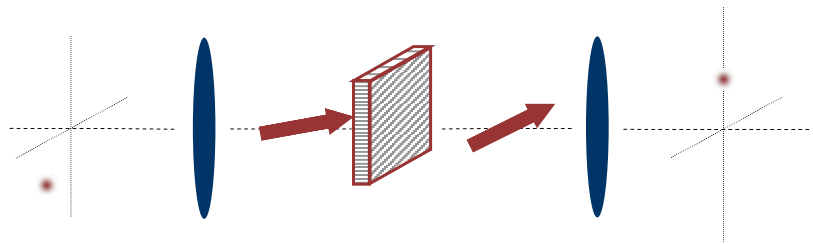
$\delta\left(x - \frac{\lambda f k_0}{2\pi}\right)$ $\quad\varepsilon(x'', y'', z'')$ $\quad I(x', y')$

Optical imaging with 3D pupil



$\delta(x - x_0, y - y_0)$ $\quad\varepsilon(x'', y'', z'')$ $\quad h(x', y'; x_0, y_0)$

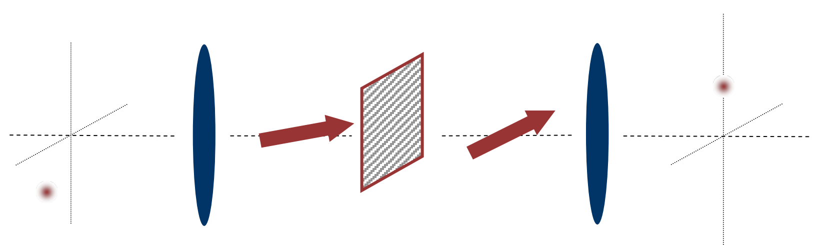
Optical imaging with 3D pupil



$$\delta(x-x_0, y-y_0) \qquad \varepsilon(x'', y'', z'') \qquad h(x', y'; x_0, y_0)$$

$$h(x', y'; x_0, y_0) = \Im\{\varepsilon\}\left[\frac{1}{\lambda}\left(\frac{x_0}{f_1}+\frac{x'}{f_2}\right), \frac{1}{\lambda}\left(\frac{y_0}{f_1}+\frac{y'}{f_2}\right), \frac{1}{\lambda}\left(\frac{x_0^2+y_0^2}{2f_1^2}-\frac{x'^2+y'^2}{2f_2^2}\right)\right]$$

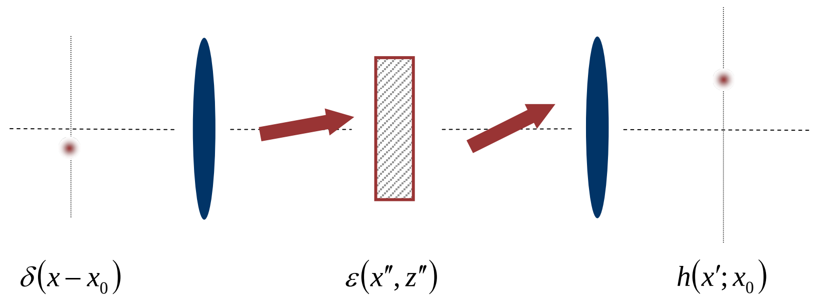
Optical imaging with $\boxed{2}$D pupil



$$\delta(x-x_0, y-y_0) \qquad \varepsilon(x'', y'', \quad) \qquad h(x', y'; x_0, y_0)$$

$$h(x', y'; x_0, y_0) = \Im\{\varepsilon\}\left[\frac{1}{\lambda}\left(\frac{x_0}{f_1}+\frac{x'}{f_2}\right), \frac{1}{\lambda}\left(\frac{y_0}{f_1}+\frac{y'}{f_2}\right), \quad\right]$$

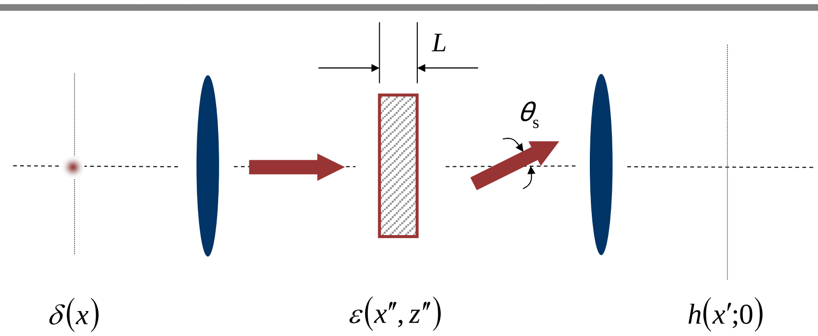
Optical imaging with 3D pupil



$$h(x';x_0)=\Im\{\varepsilon\}\left[\frac{1}{\lambda}\left(\frac{x_0}{f_1}+\frac{x'}{f_2}\right),\frac{1}{\lambda}\left(\frac{x_0^2}{2f_1^2}-\frac{x'^2}{2f_2^2}\right)\right]$$

simplify to 1D lateral dimension + longitudinal dimension

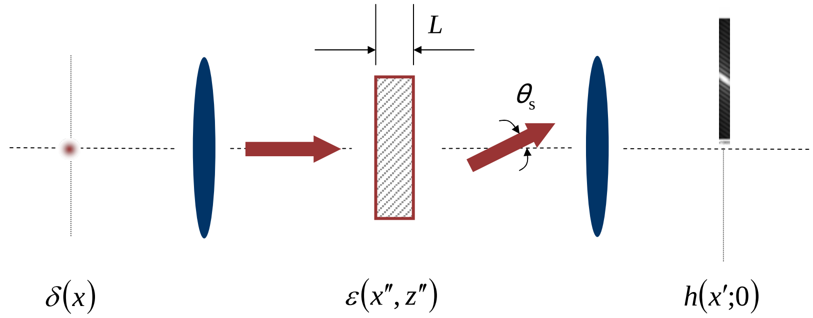
Optical imaging with 3D pupil: building the Hopkins matrix



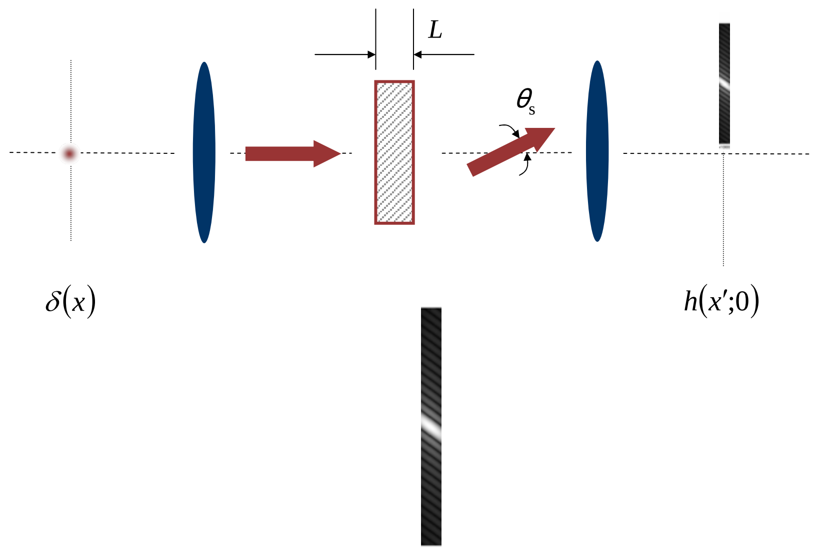
Plane-to-plane wave hologram
On-axis reference

$$\varepsilon(x'',z'')=\left|\exp(ikz)+\exp(ik[x\sin\theta_s+z\cos\theta_s])\right|^2$$

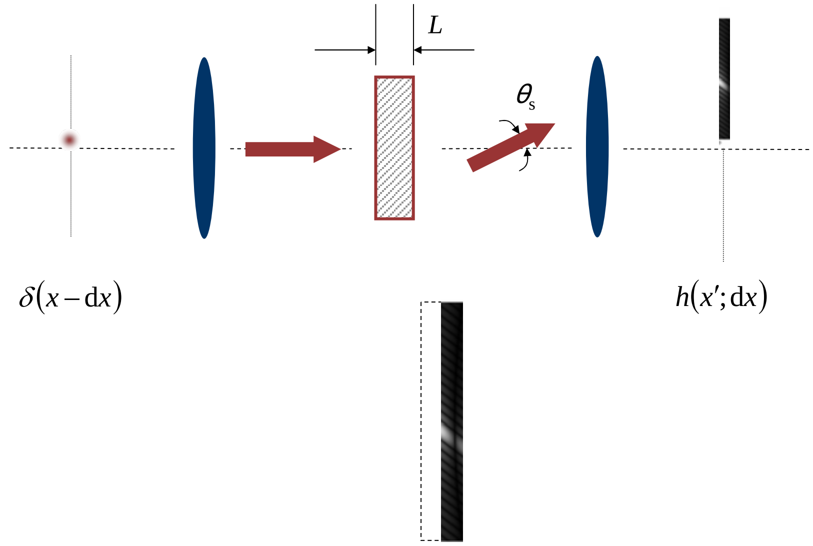
Optical imaging with 3D pupil: building the Hopkins matrix



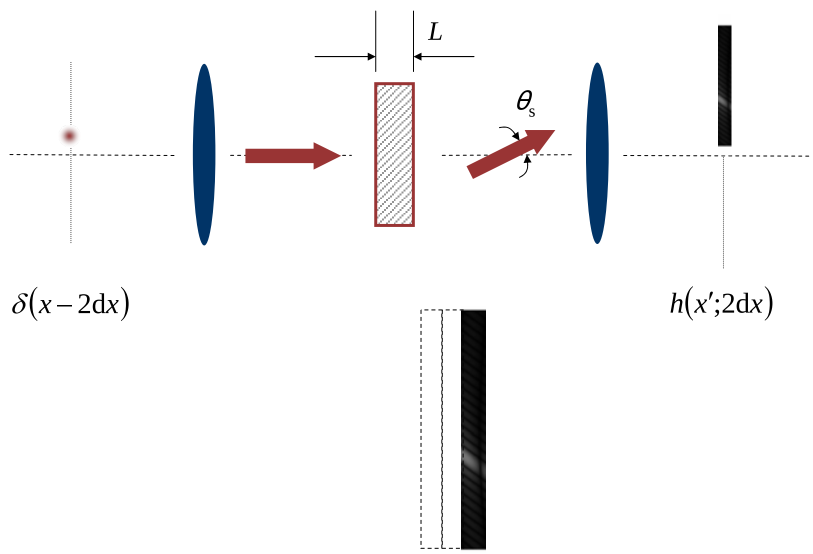
Optical imaging with 3D pupil: building the Hopkins matrix



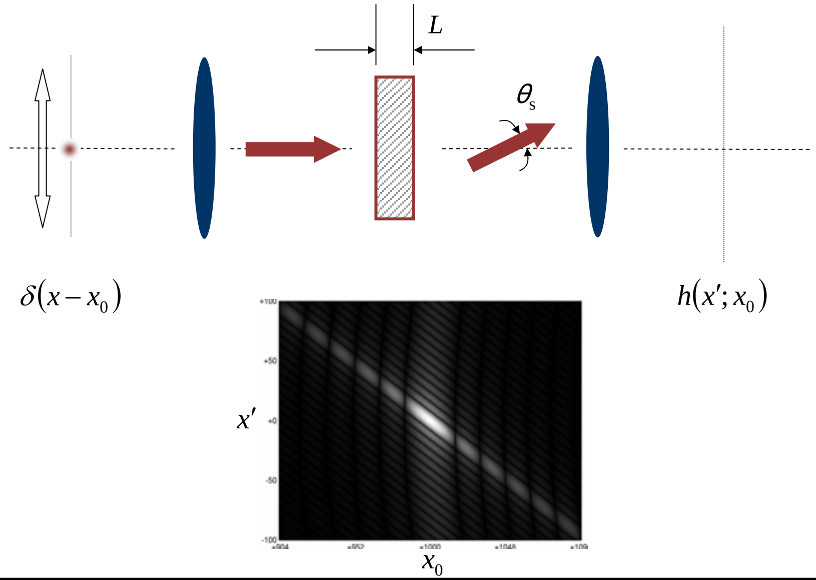
Optical imaging with 3D pupil: building the Hopkins matrix



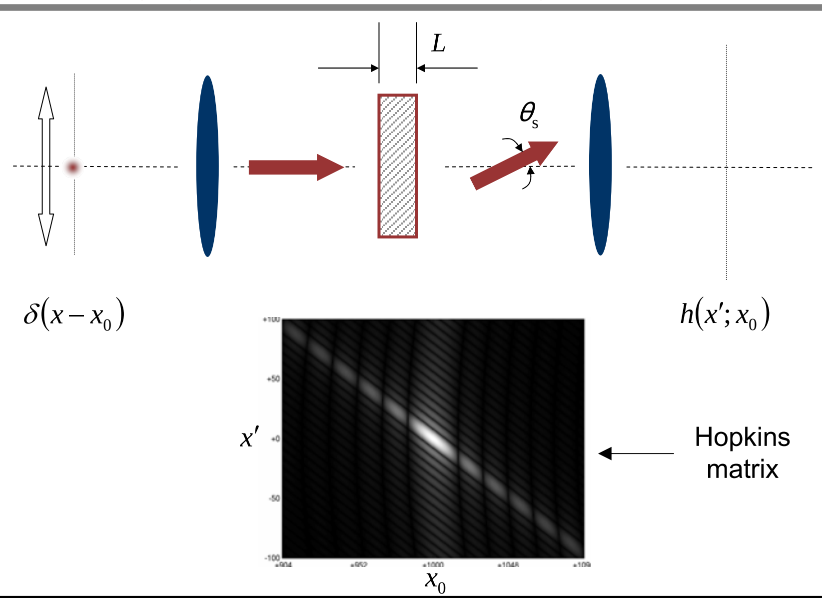
Optical imaging with 3D pupil: building the Hopkins matrix



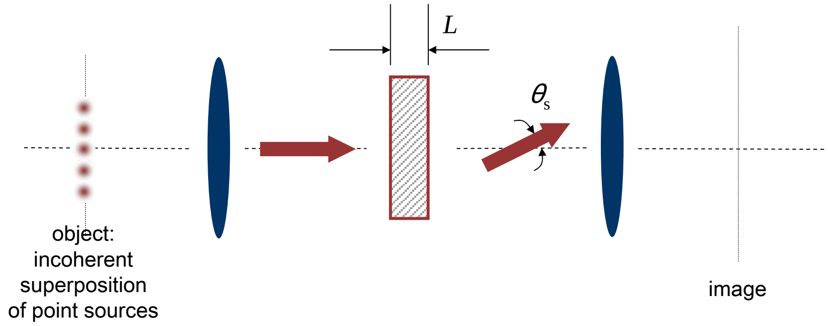
Optical imaging with 3D pupil: building the Hopkins matrix



Optical imaging with 3D pupil: building the Hopkins matrix

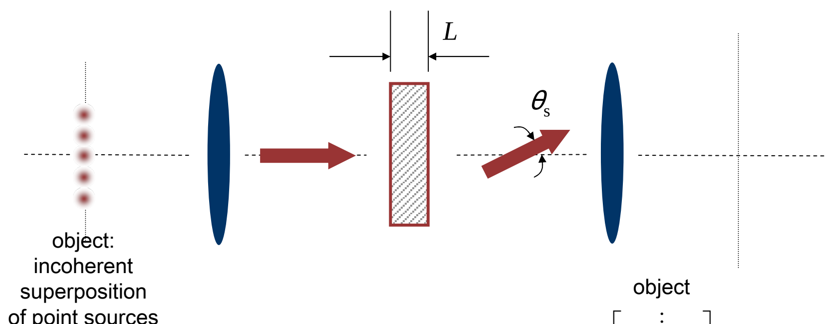


Incoherent imaging with 3D pupil



$$\begin{bmatrix} \vdots \\ f(2\mathrm{d}x) \\ f(\mathrm{d}x) \\ f(0) \\ f(-\mathrm{d}x) \\ f(-2\mathrm{d}x) \\ \vdots \end{bmatrix} \qquad \begin{bmatrix} \vdots \\ g(2\mathrm{d}x') \\ g(\mathrm{d}x') \\ g(0) \\ g(-\mathrm{d}x') \\ g(-2\mathrm{d}x') \\ \vdots \end{bmatrix} = ?$$

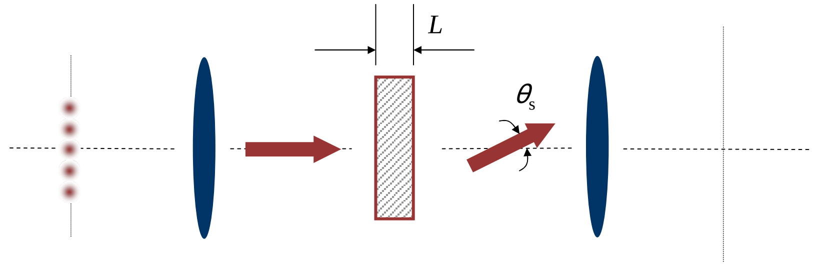
Incoherent imaging with 3D pupil



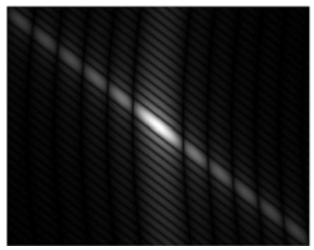
object

$$\begin{bmatrix} \vdots \\ f(2\mathrm{d}x) \\ f(\mathrm{d}x) \\ f(0) \\ f(-\mathrm{d}x) \\ f(-2\mathrm{d}x) \\ \vdots \end{bmatrix}$$

Incoherent imaging with 3D pupil



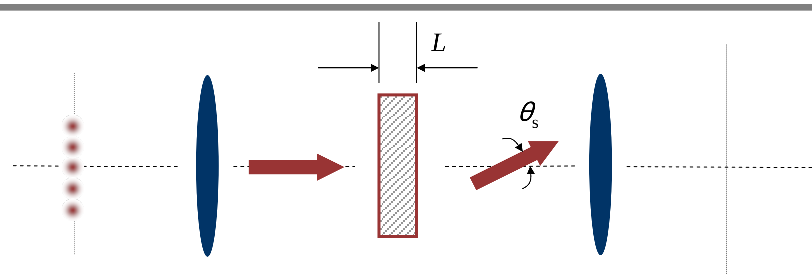
Hopkins matrix



object

$$\begin{bmatrix} \vdots \\ f(2dx) \\ f(dx) \\ f(0) \\ f(-dx) \\ f(-2dx) \\ \vdots \end{bmatrix}$$

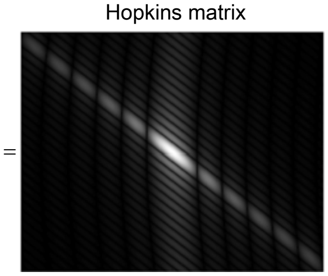
Incoherent imaging with 3D pupil



image

$$\begin{bmatrix} \vdots \\ g(2dx') \\ g(dx') \\ g(0) \\ g(-dx') \\ g(-2dx') \\ \vdots \end{bmatrix} =$$

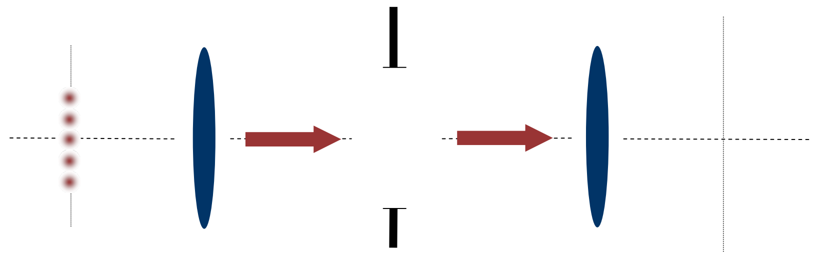
Hopkins matrix

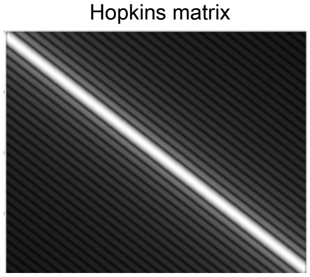


object

$$\begin{bmatrix} \vdots \\ f(2dx) \\ f(dx) \\ f(0) \\ f(-dx) \\ f(-2dx) \\ \vdots \end{bmatrix}$$

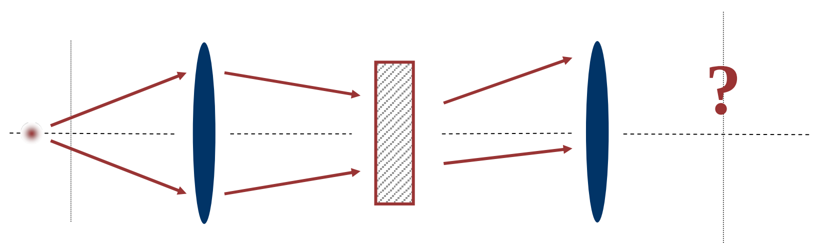
Incoherent imaging with clear (2D) pupil





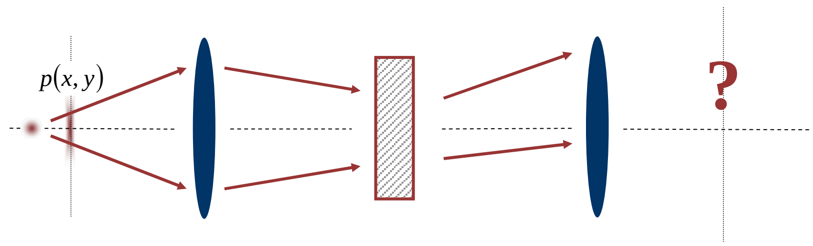
$$\begin{bmatrix} \vdots \\ g(2\mathrm{d}x') \\ g(\mathrm{d}x') \\ g(0) \\ g(-\mathrm{d}x') \\ g(-2\mathrm{d}x') \\ \vdots \end{bmatrix} = [\text{Hopkins matrix}] \begin{bmatrix} \vdots \\ f(2\mathrm{d}x) \\ f(\mathrm{d}x) \\ f(0) \\ f(-\mathrm{d}x) \\ f(-2\mathrm{d}x) \\ \vdots \end{bmatrix}$$

Defocused response of 3D pupils



$\delta(x-x_0, y, z-z_0)$ $\varepsilon(x'', y'', z'')$ $q(x', y'; x_0, z_0)$

Defocused response of 3D pupils



$\delta(x-x_0, y, z-z_0)$ $\quad\quad$ $\varepsilon(x'', y'', z'')$ $\quad\quad$ $q(x', y'; x_0, z_0)$

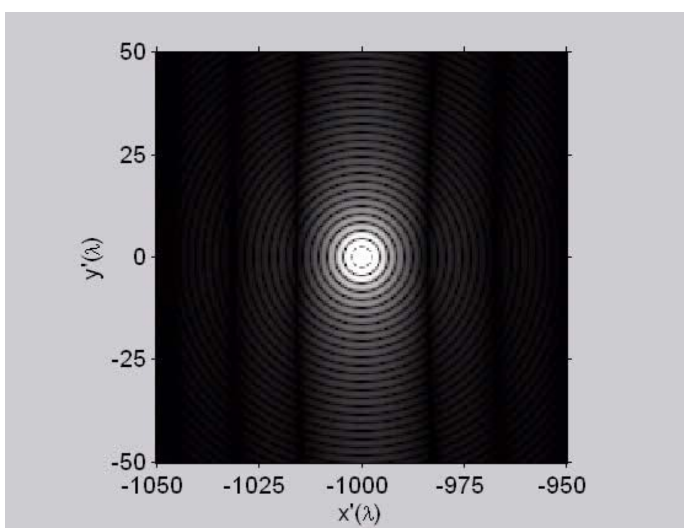
$$p(x,y) = \text{Fresnel}(z_0)$$

$$P(x'', y'', z'') = \exp\left\{i2\pi \frac{z}{\lambda}\right\} \iint \mathrm{d}x\mathrm{d}y\, p(x,y) \exp\left\{i2\pi \frac{xx'' + yy''}{\lambda f_1}\right\} \times \exp\left\{-i\pi \frac{(x^2+y^2)z''}{\lambda f_1^2}\right\}$$

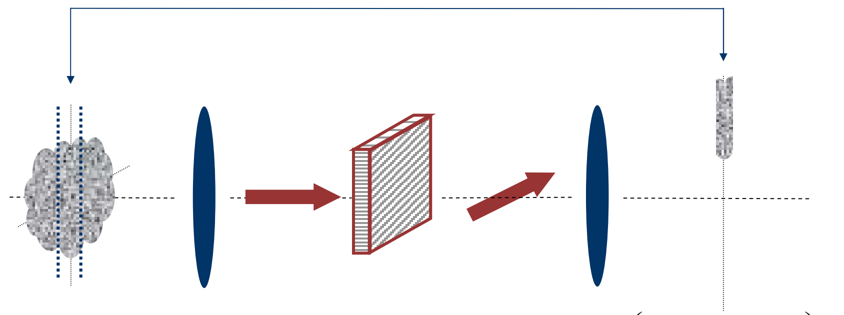
$$g(x'', y'', z'') = \varepsilon(x'', y'', z'') \times P(x'', y'', z'')$$

$$q(x', y') = G\left[\frac{x'}{\lambda f_2}, \frac{y'}{\lambda f_2}, \frac{1}{\lambda}\left(1 - \frac{x'^2 + y'^2}{2 f_2^2}\right)\right]$$

Blinking spot of 3D pupil



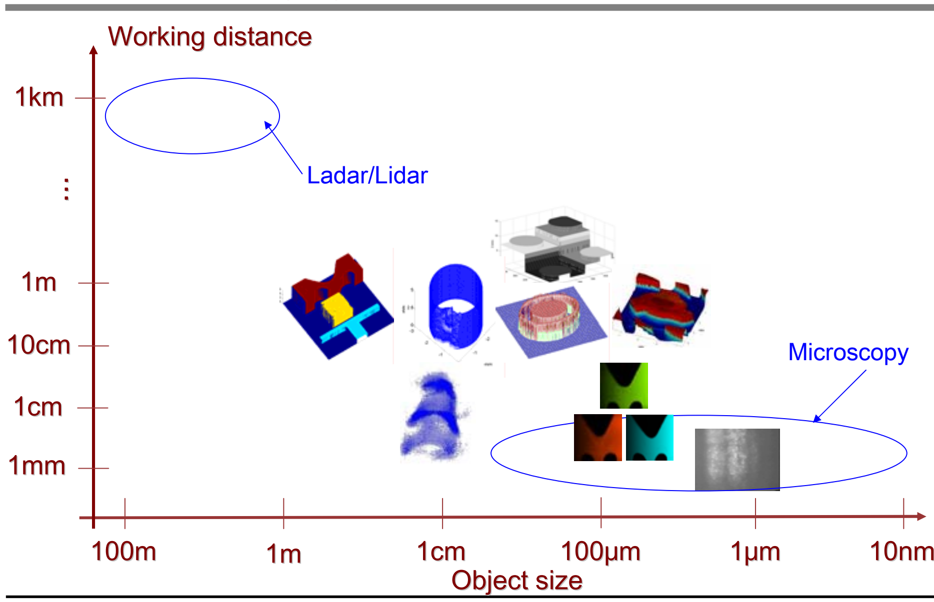
Applications: 3D holographic optical imaging



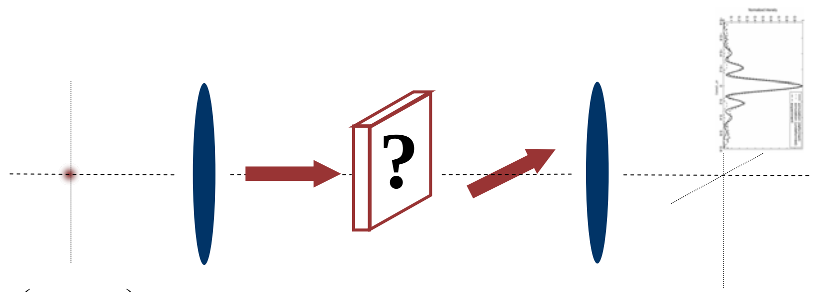
$$f(x,y,z)$$

$$f\left(x'-\frac{\lambda f k_0}{2\pi}, y', z\right)\times$$
$$\times\,\mathrm{slit}\left(\frac{x'}{\Delta x'}\right)\times\mathrm{slice}\left(\frac{z}{\Delta z}\right)$$

3D optics for imaging: summary

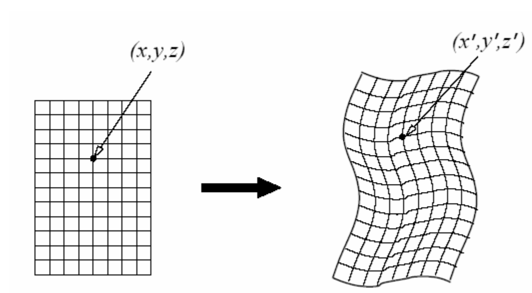


Probing unknown 3D holograms

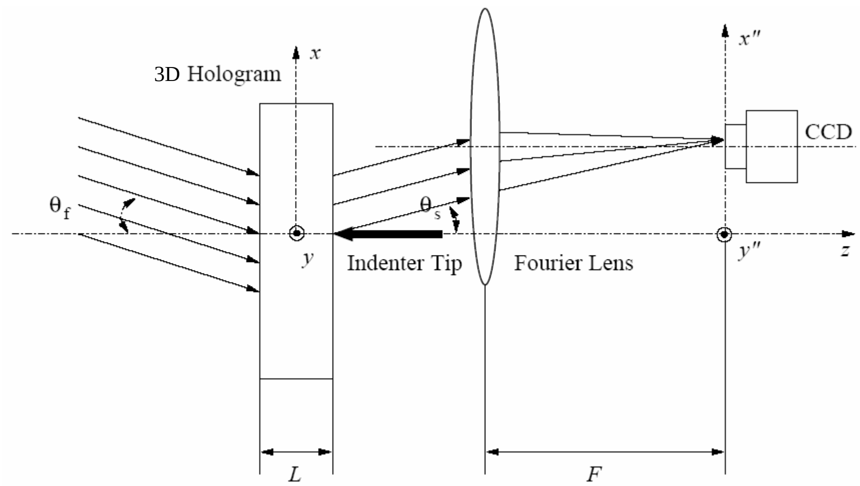


$$\delta\left(x-\frac{\lambda f k_0}{2\pi}\right) \qquad \varepsilon(x'', y'', z'') \qquad I(x', y')$$

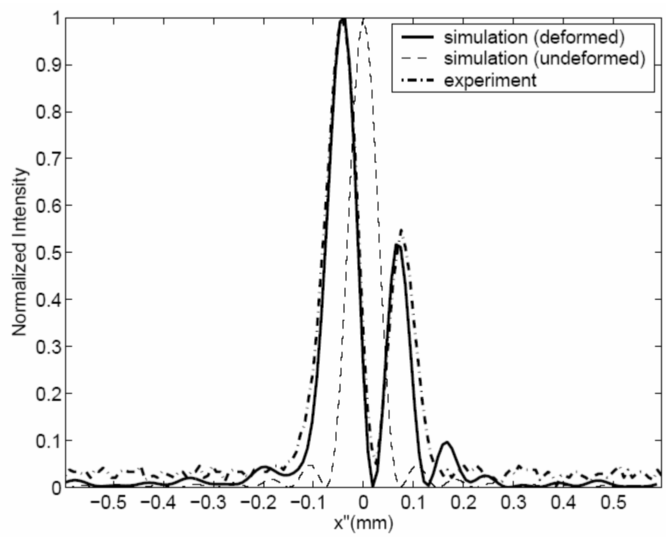
Diffraction from deformed 3D holograms



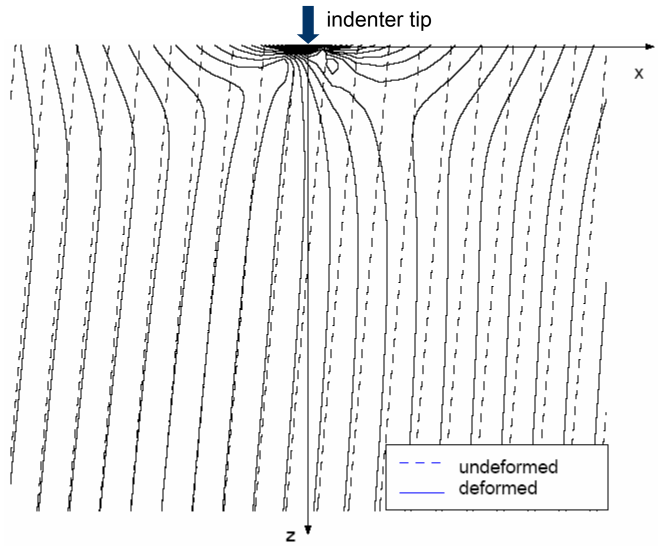
Experimental arrangement: transmission geometry



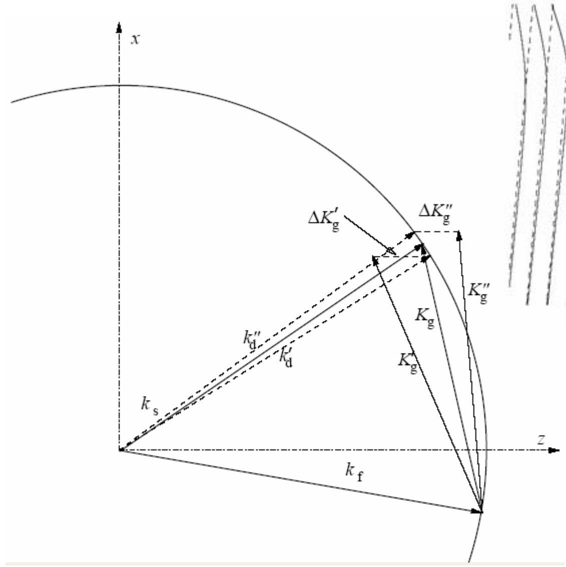
Experiment: diffraction from indented 3D hologram

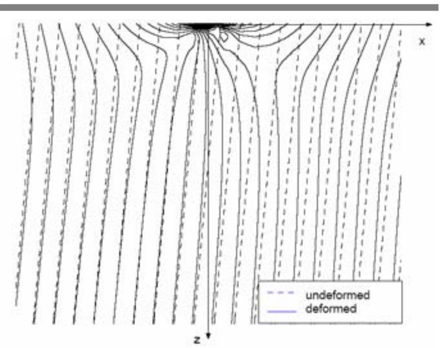


Fringe deformation due to indenter (transmission)

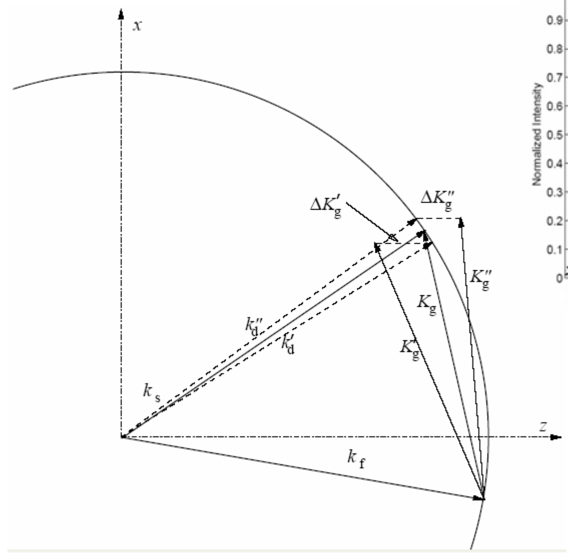


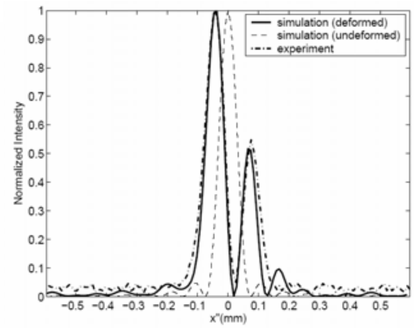
Deformed transmission 3D hologram in *k*-space



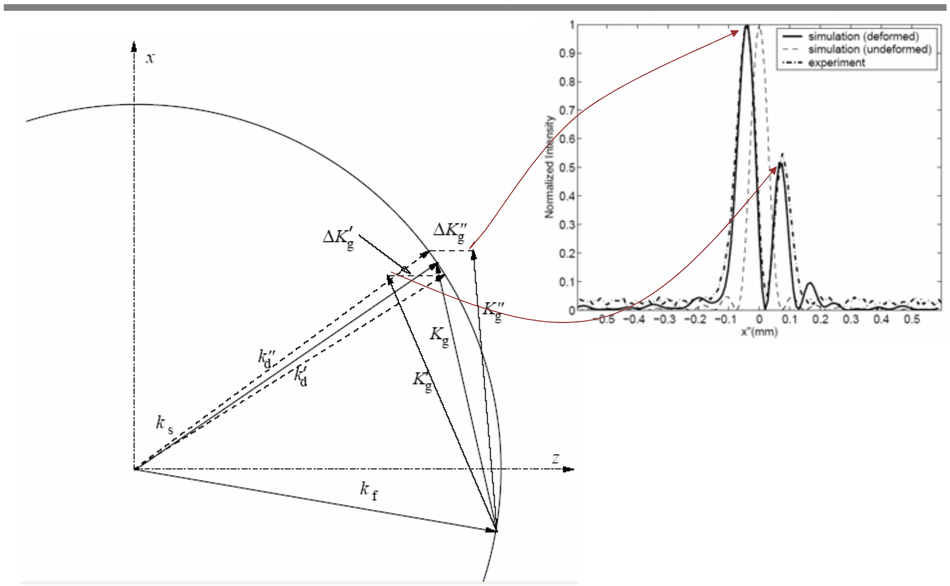


Deformed transmission 3D hologram in *k*-space

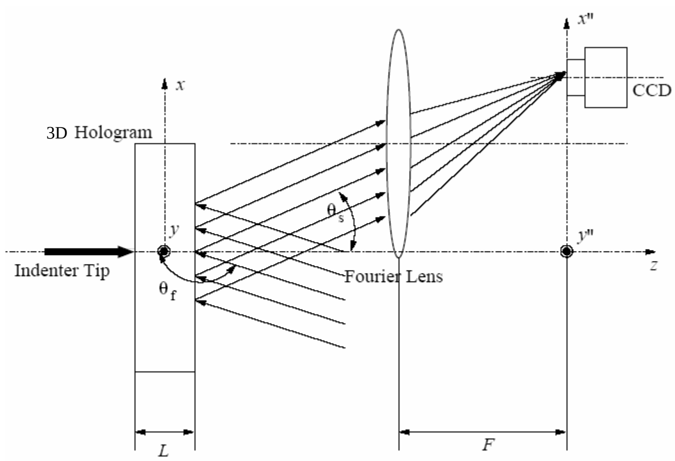




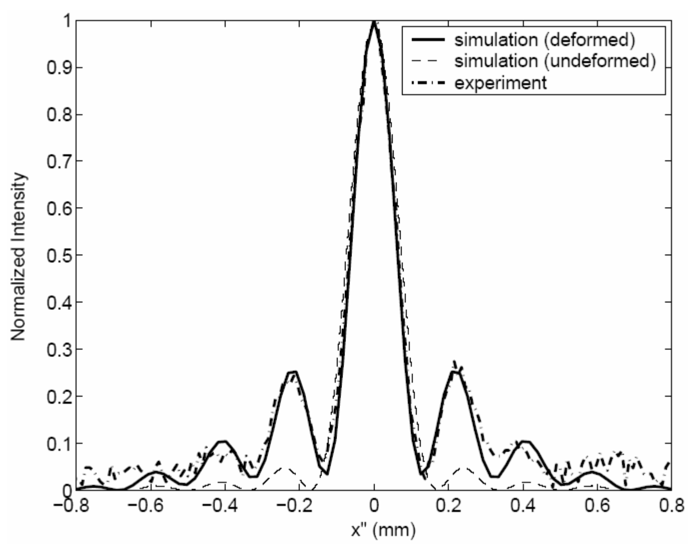
Deformed transmission 3D hologram in *k*-space



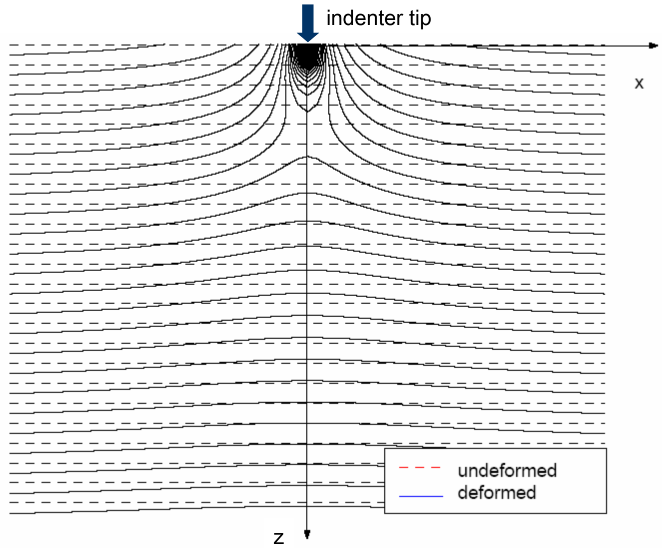
Experimental arrangement: reflection geometry



Experiment: diffraction from indented 3D hologram



Fringe deformation due to indenter (reflection)



Deformed reflection 3D hologram in *k*-space