Today

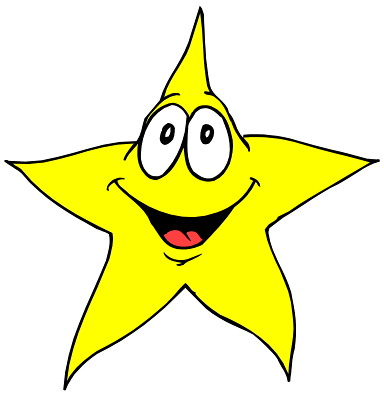
- Defocus

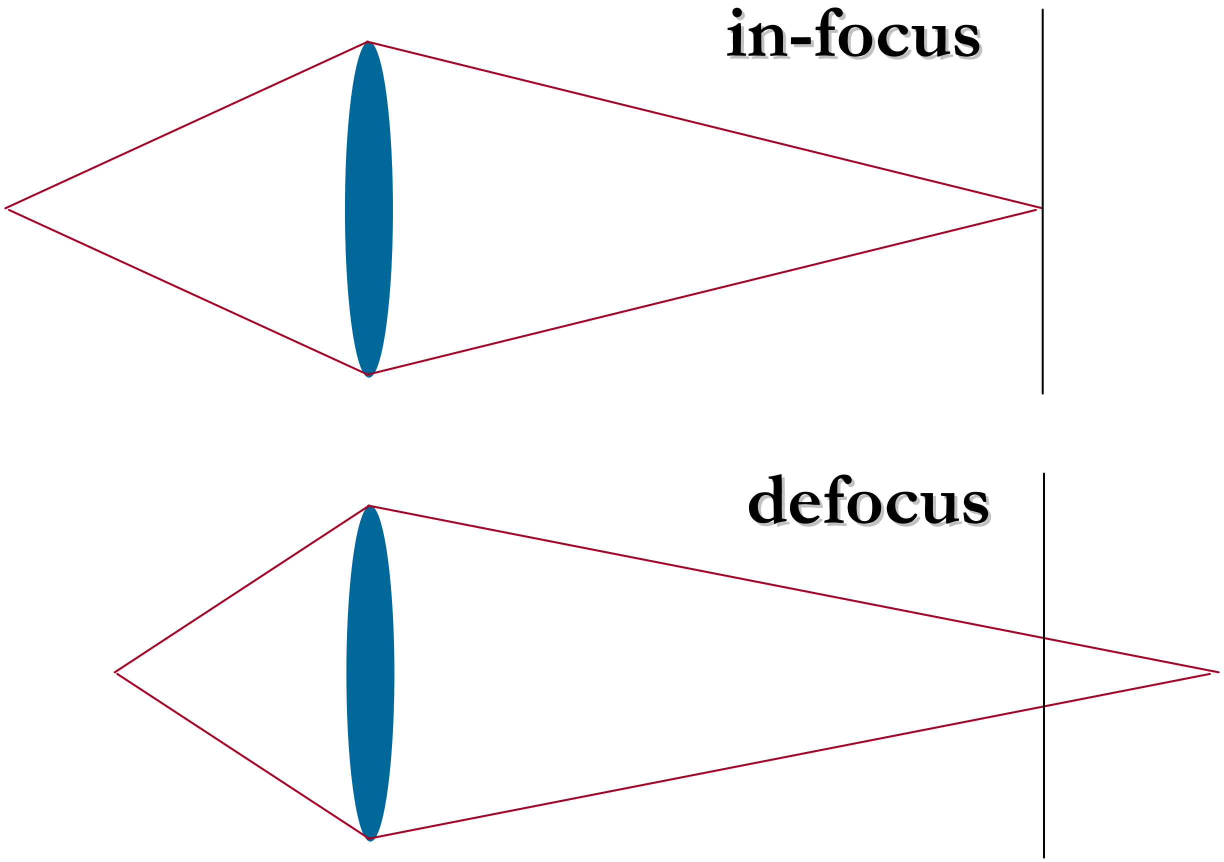
- Deconvolution / inverse filters

Defocus

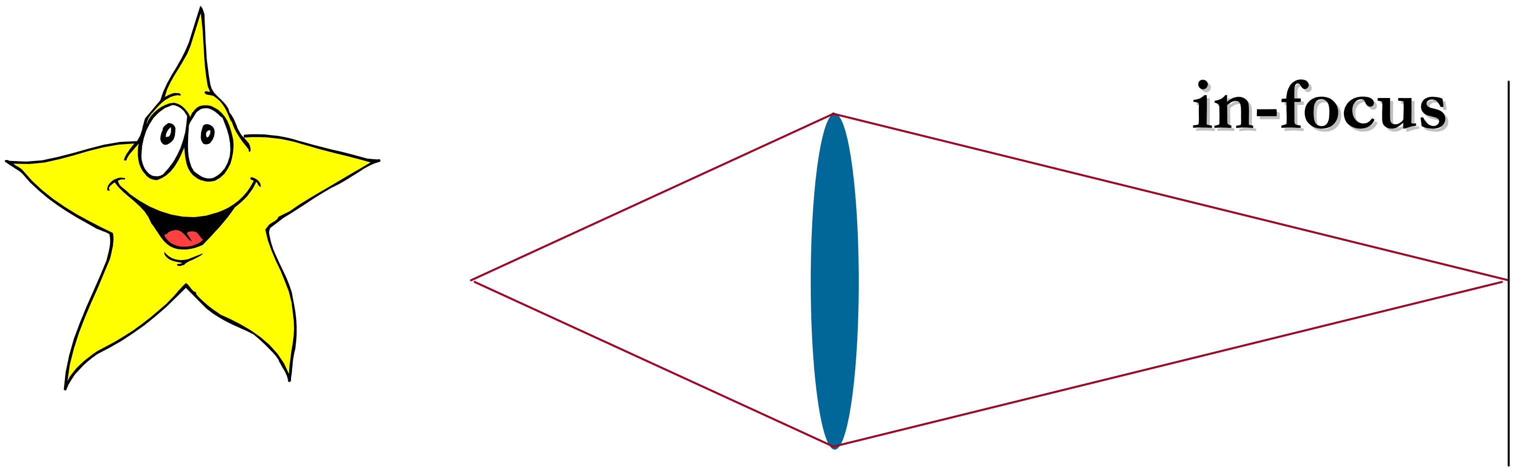
© 20th Century Fox

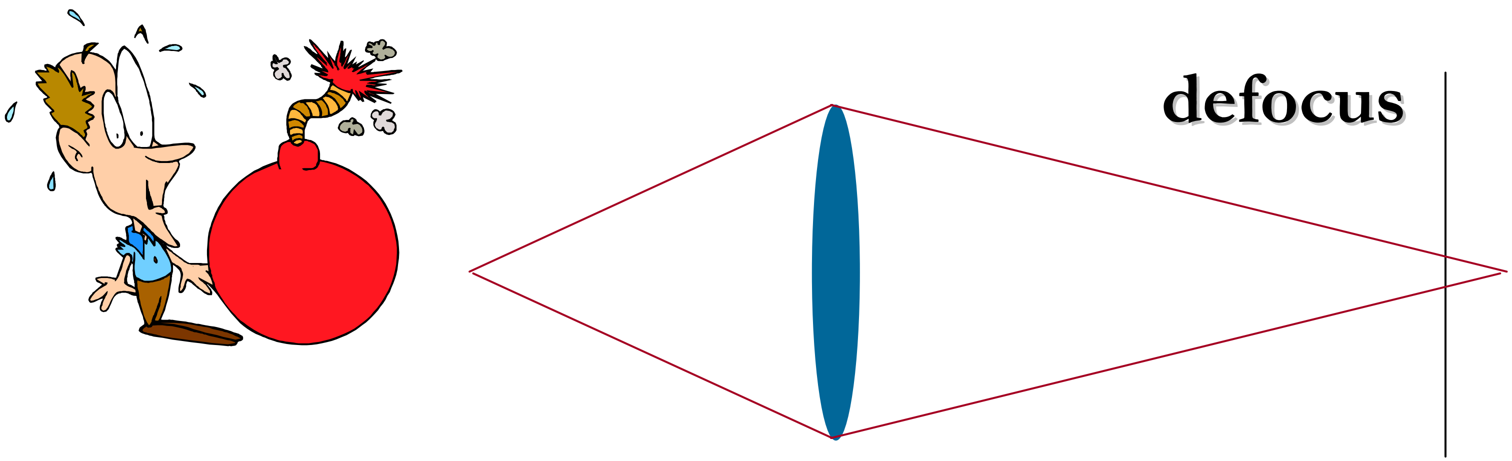
Focus in classical imaging





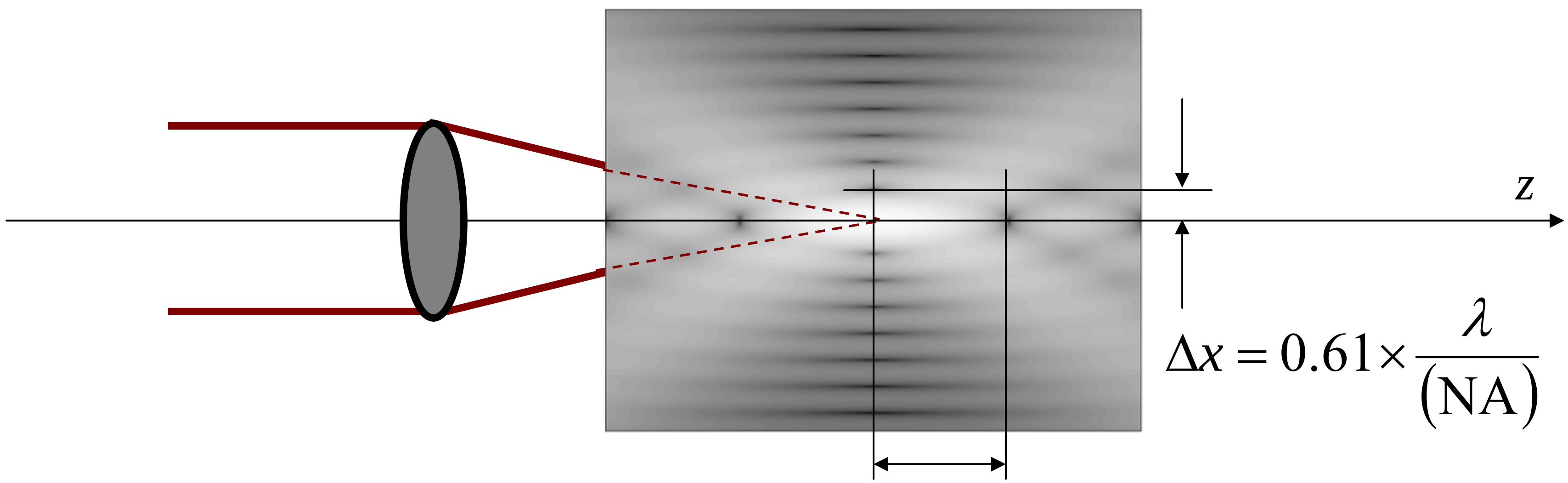
Focus in classical imaging





Intensity distribution near the focus of an ideal lens

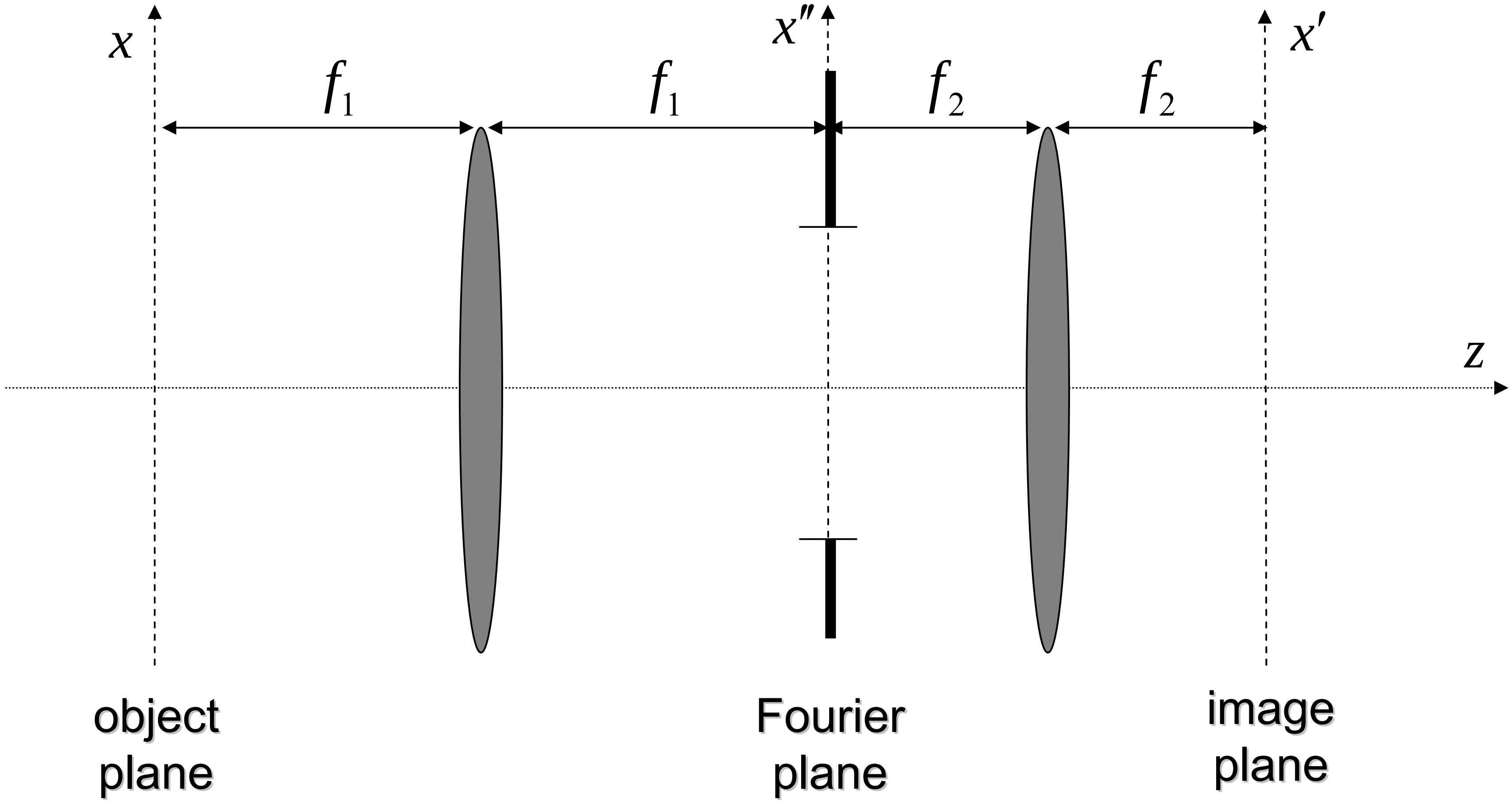
(rotationally symmetric wrt z axis)



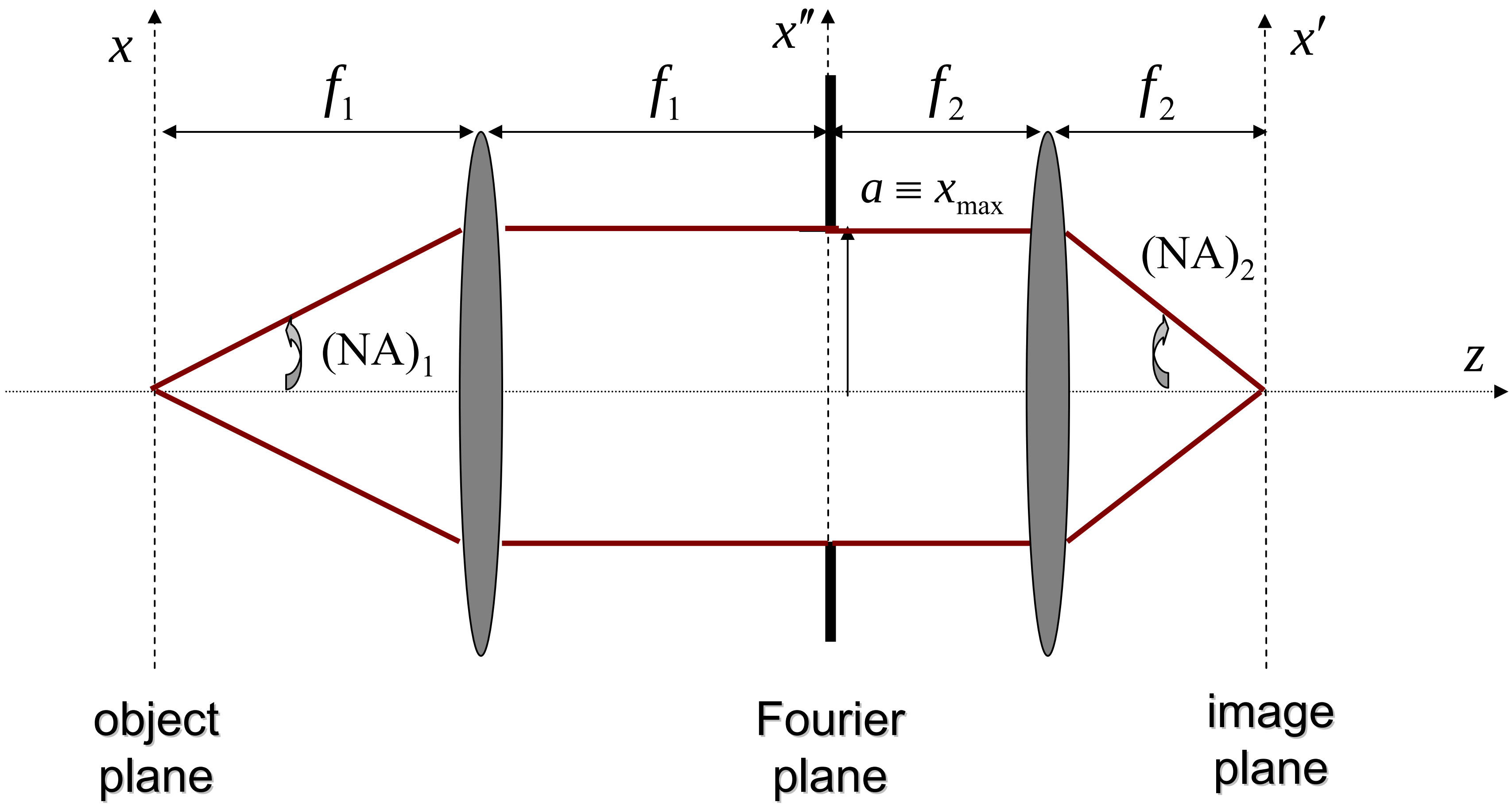
$$\Delta x = 0.61 \times \frac{\lambda}{(\mathrm{NA})}$$

$$\Delta z = \frac{\lambda}{2(\mathrm{NA})^2}$$

Back to the basics: 4F system



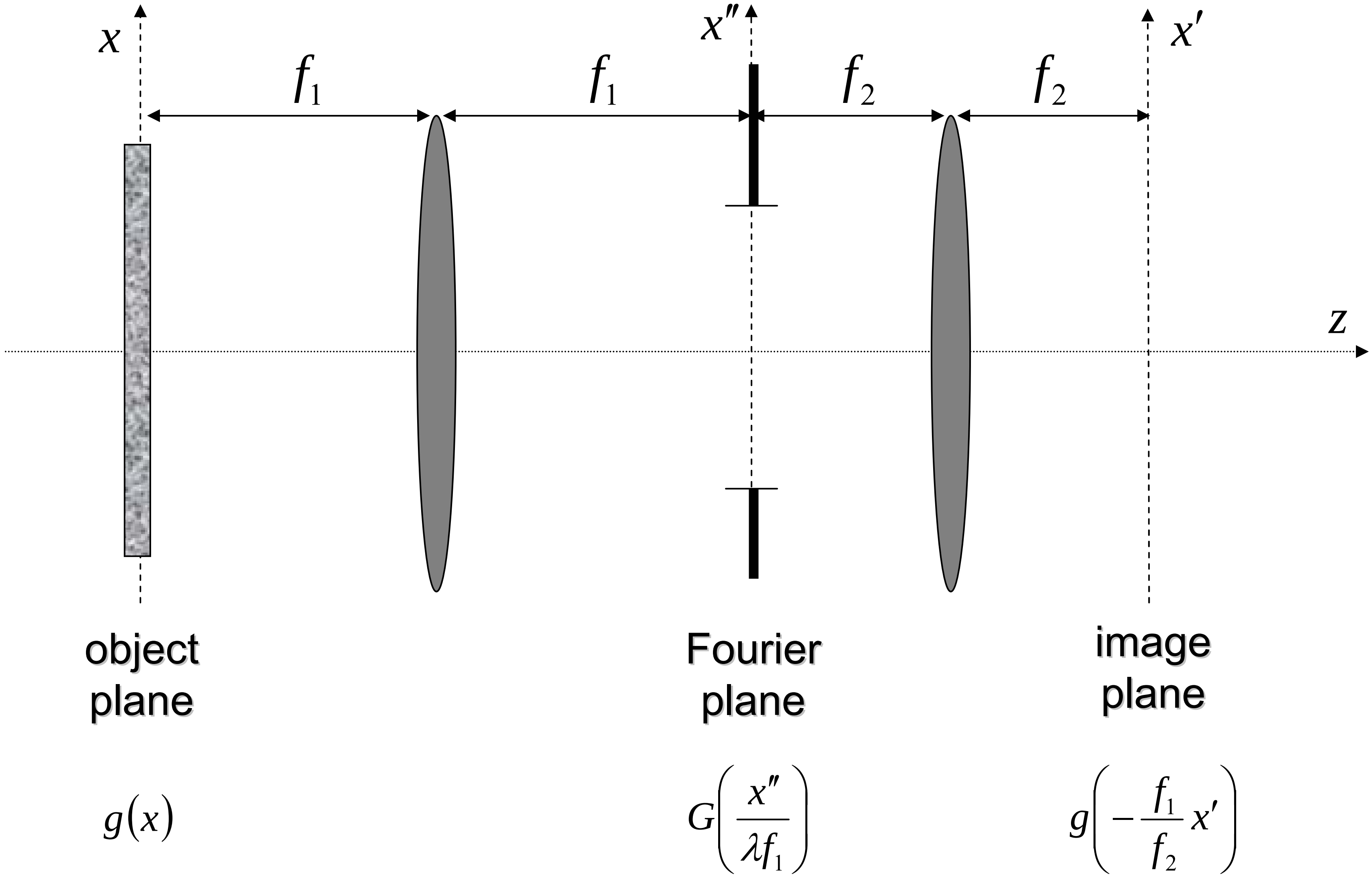
Back to the basics: 4F system



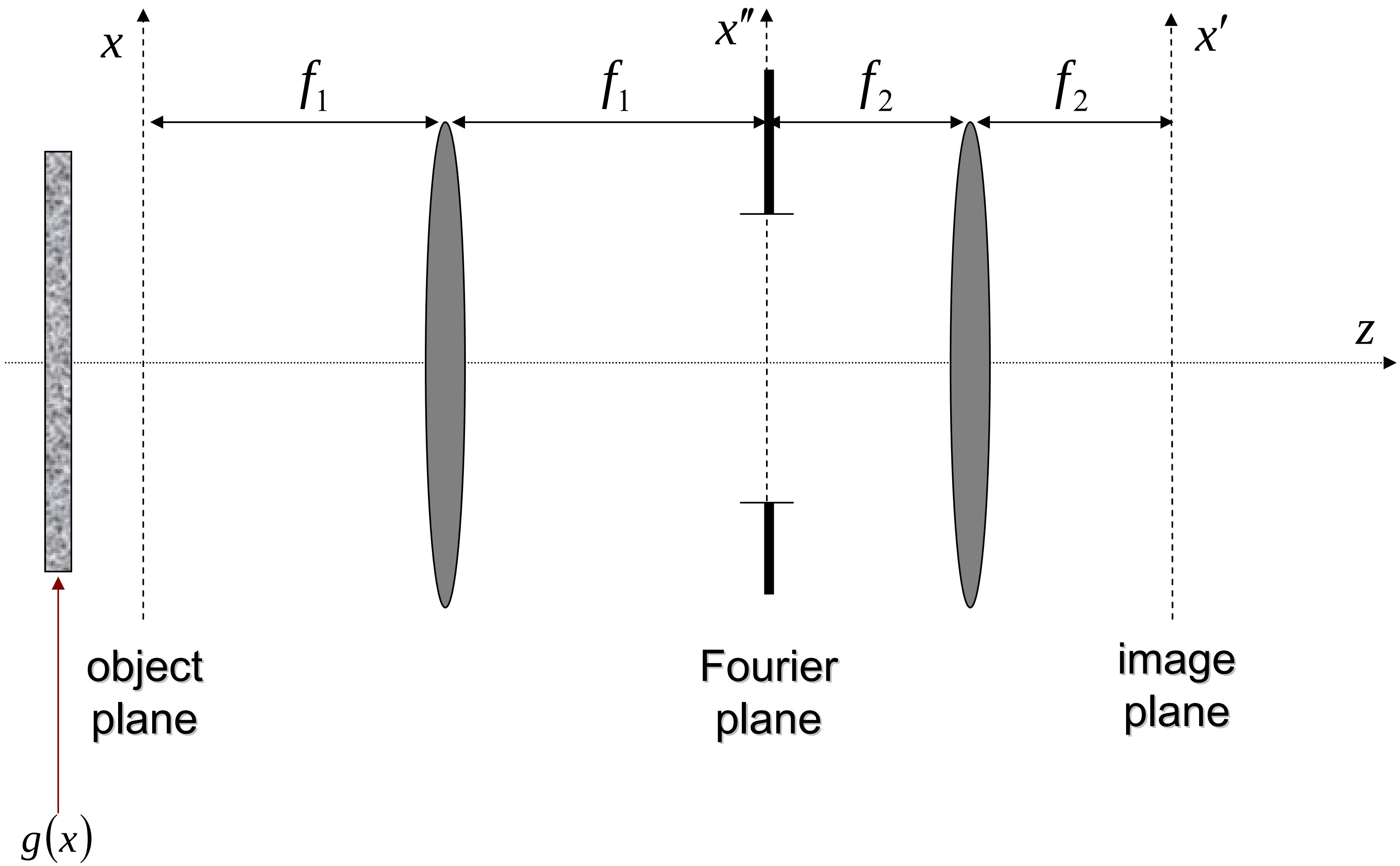
$$(\mathrm{NA})_1 = \frac{x_{max}}{f_1}$$

$$(\mathrm{NA})_2 = \frac{x_{max}}{f_2}$$

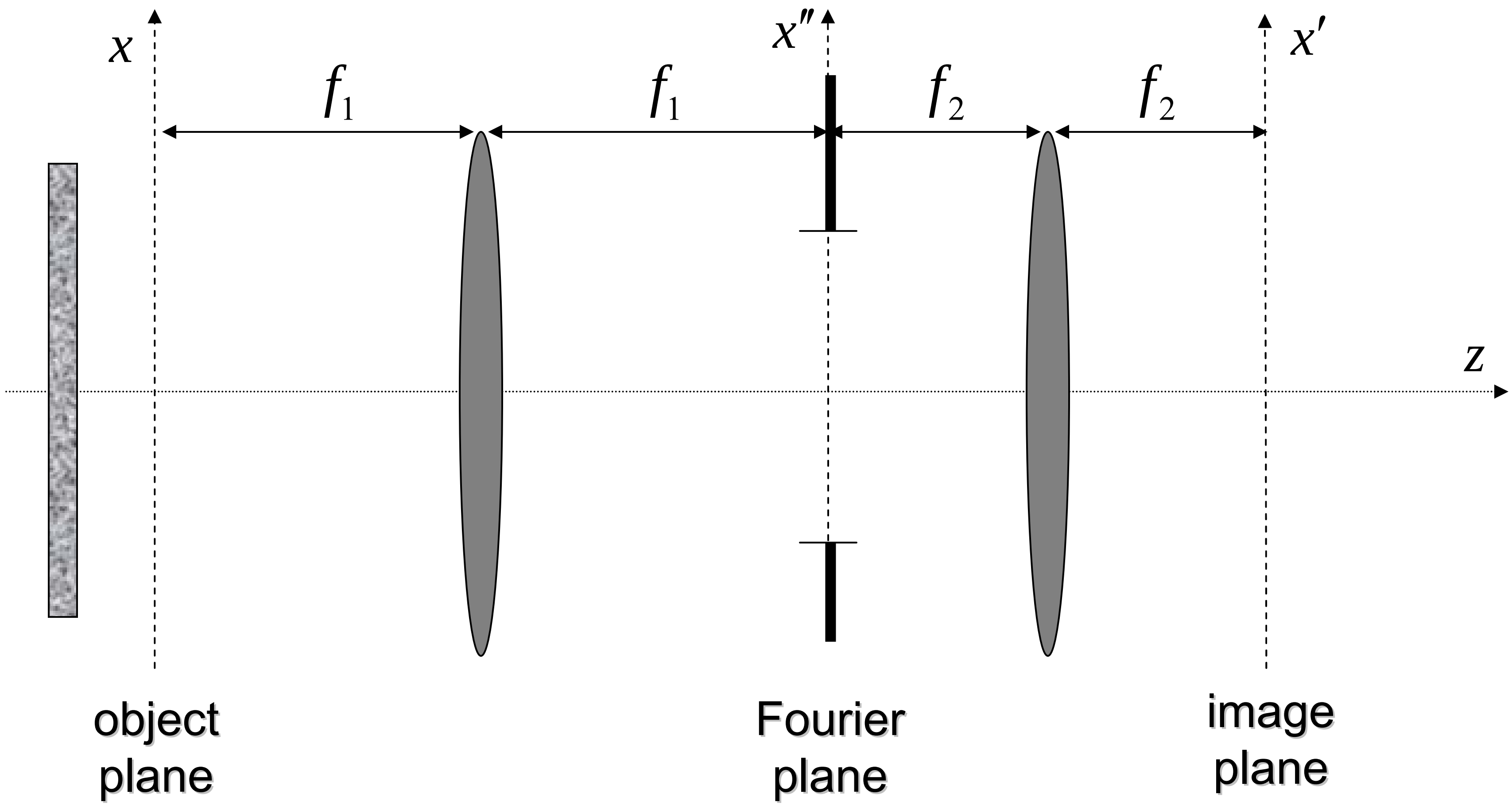
Back to the basics: 4F system



4F system with defocused input

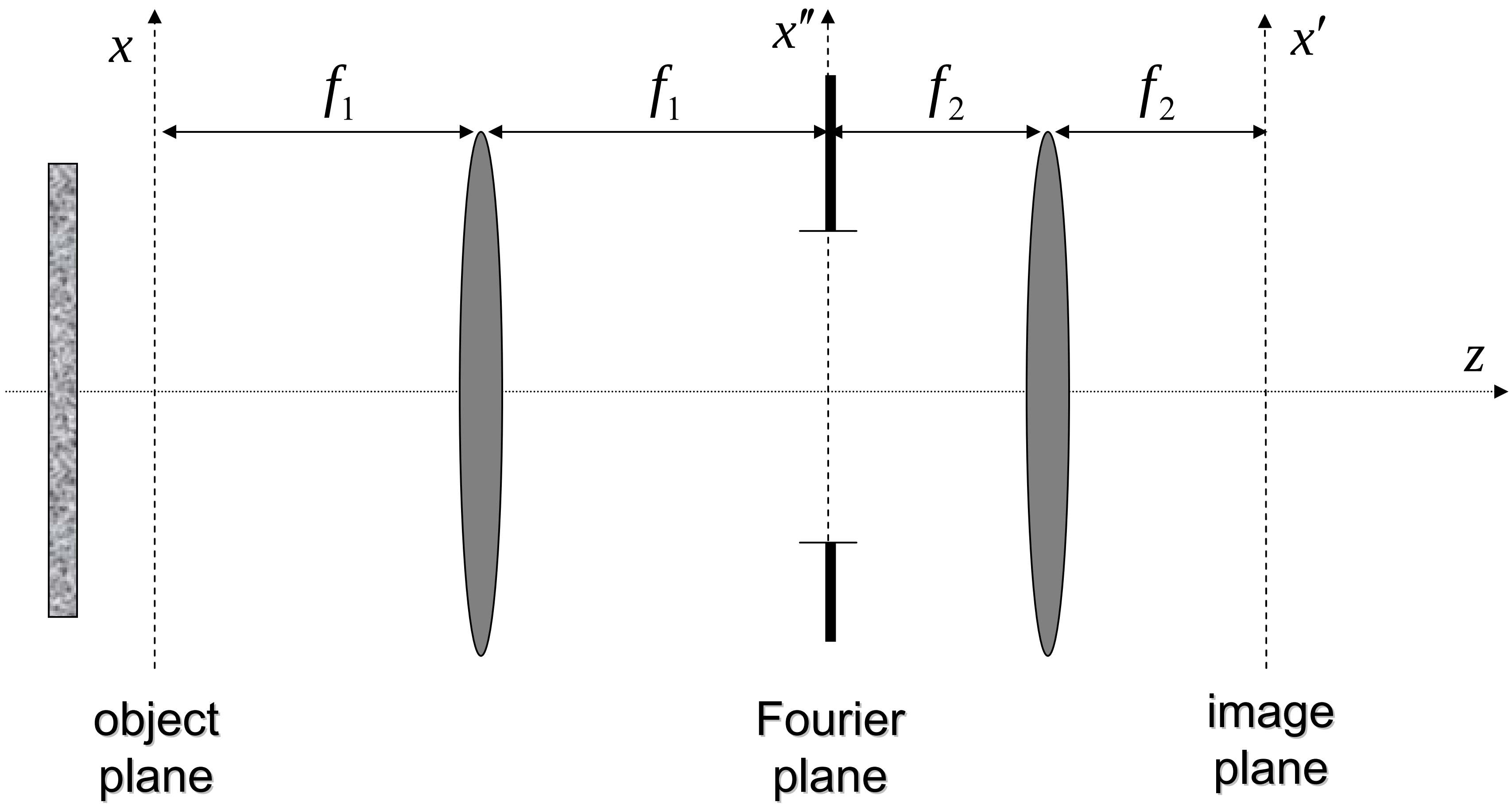


4F system with defocused input



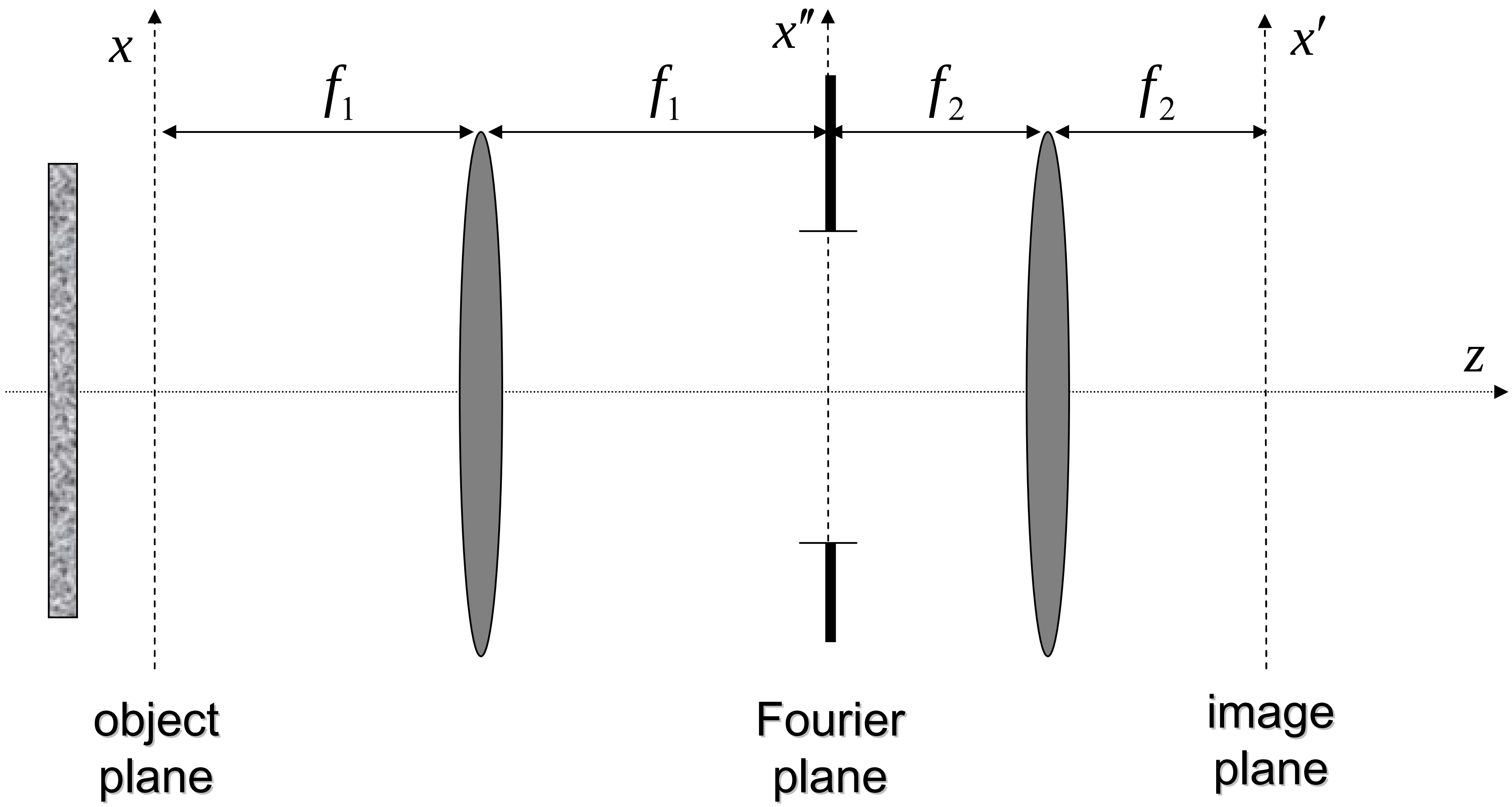
$$g(x) * \exp\left(i\pi \frac{x^2 + y^2}{\lambda \Delta z}\right)$$

4F system with defocused input



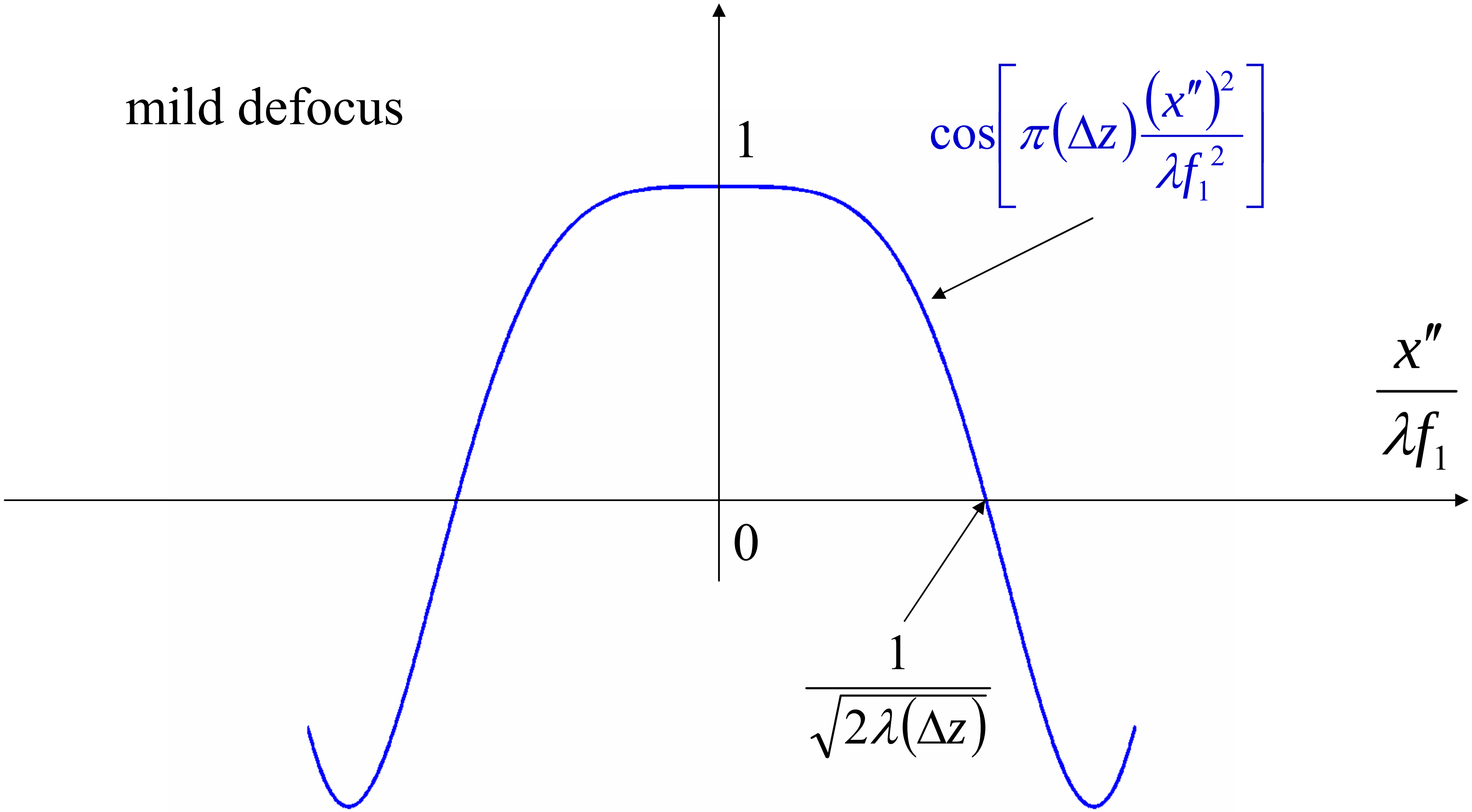
$$g(x) * \exp\left(i\pi \frac{x^2}{\lambda \Delta z}\right) \stackrel{\Im}{\Longrightarrow} G\left(\frac{x''}{\lambda f_1}\right) \cdot \exp\left[i\pi(\lambda\,\Delta z)\left(\frac{x''}{\lambda f_1}\right)^2\right]$$

4F system with defocused input

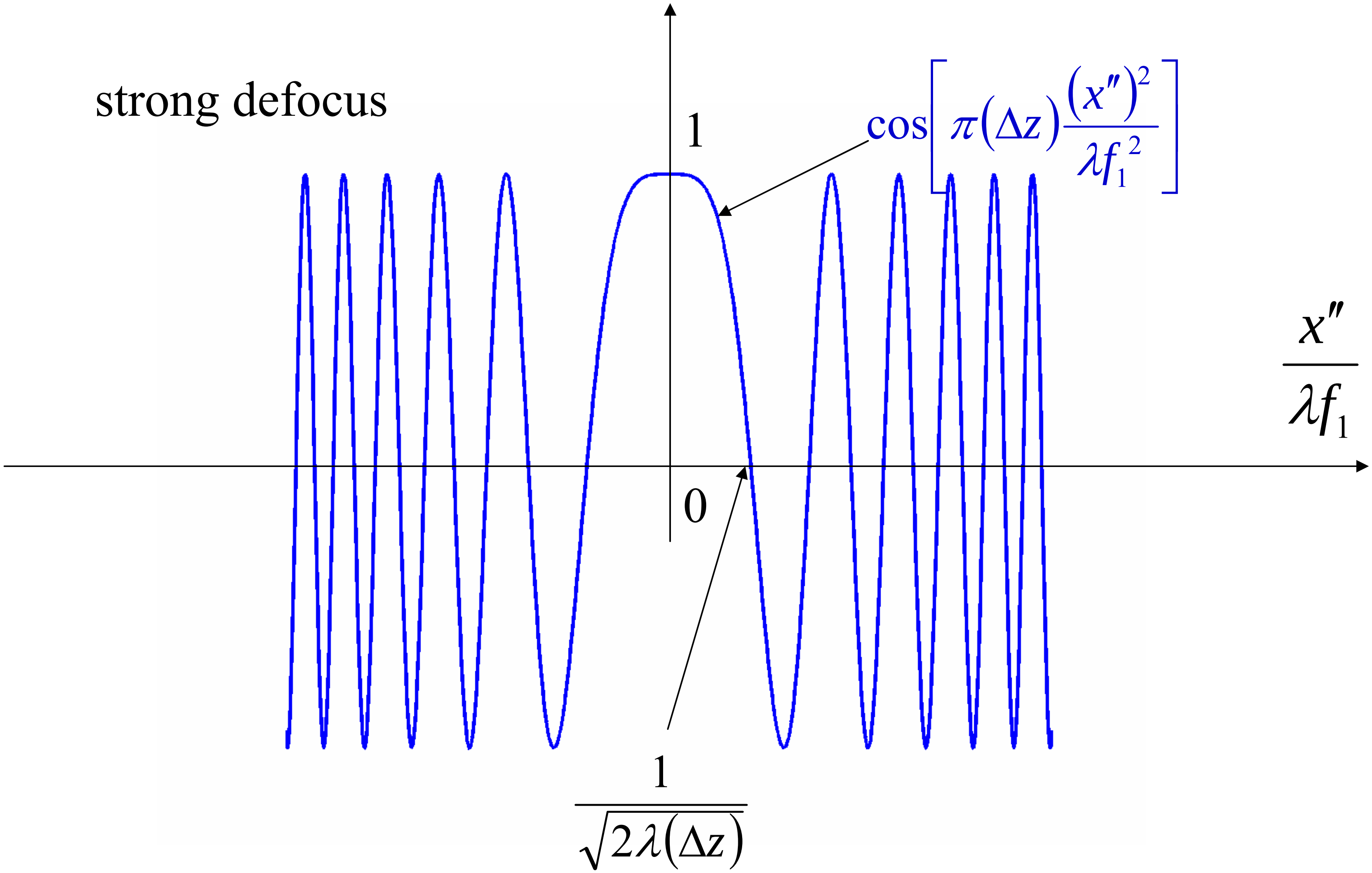


$$g(x) * \exp\left(i\pi \frac{x^2}{\lambda \Delta z}\right) \xrightarrow{\ \Im\ } G\left(\frac{x''}{\lambda f_1}\right) \cdot \exp\left[i\pi(\Delta z)\frac{(x'')^2}{\lambda f_1^{\,2}}\right]$$

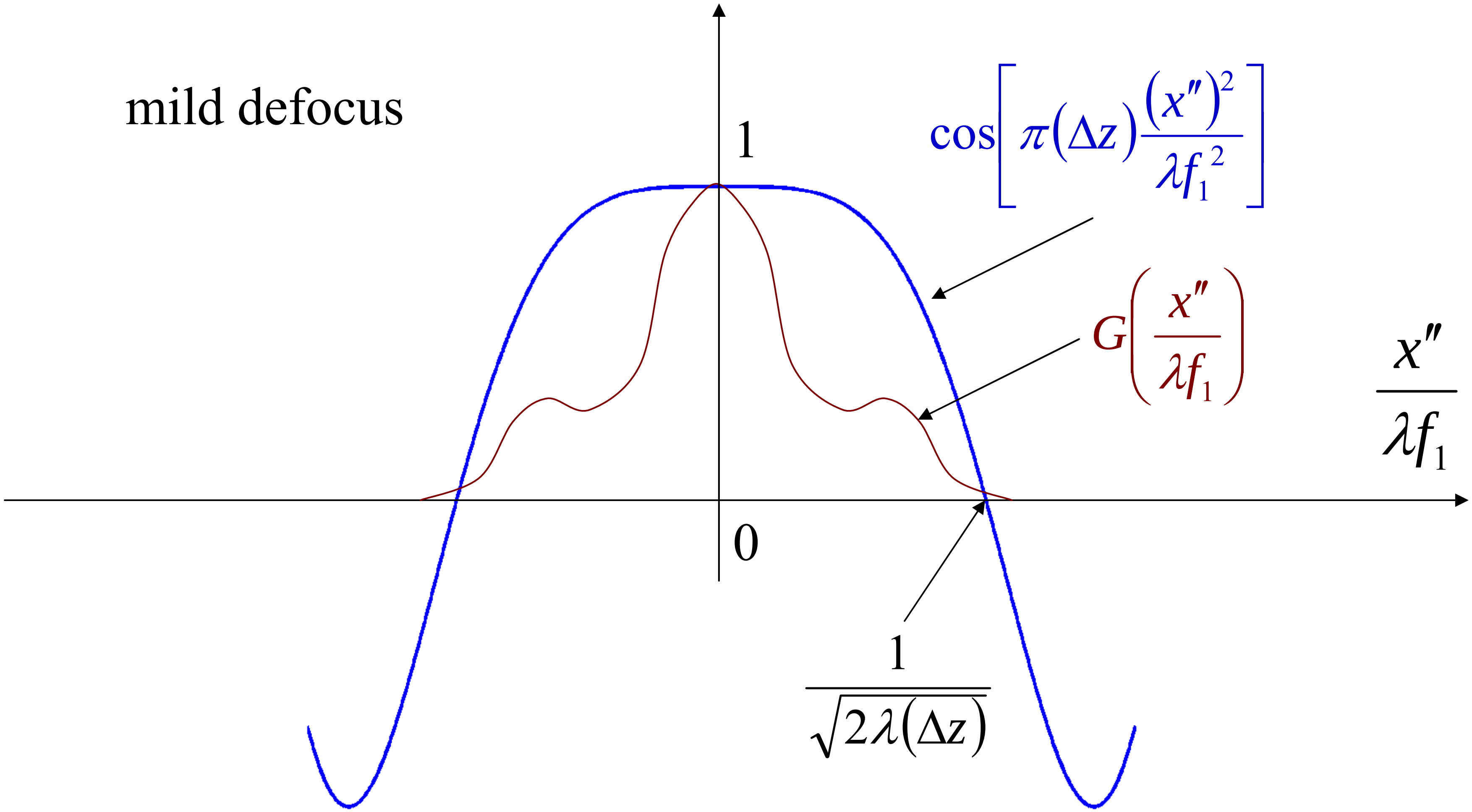
Effect of defocus on the Fourier plane



Effect of defocus on the Fourier plane

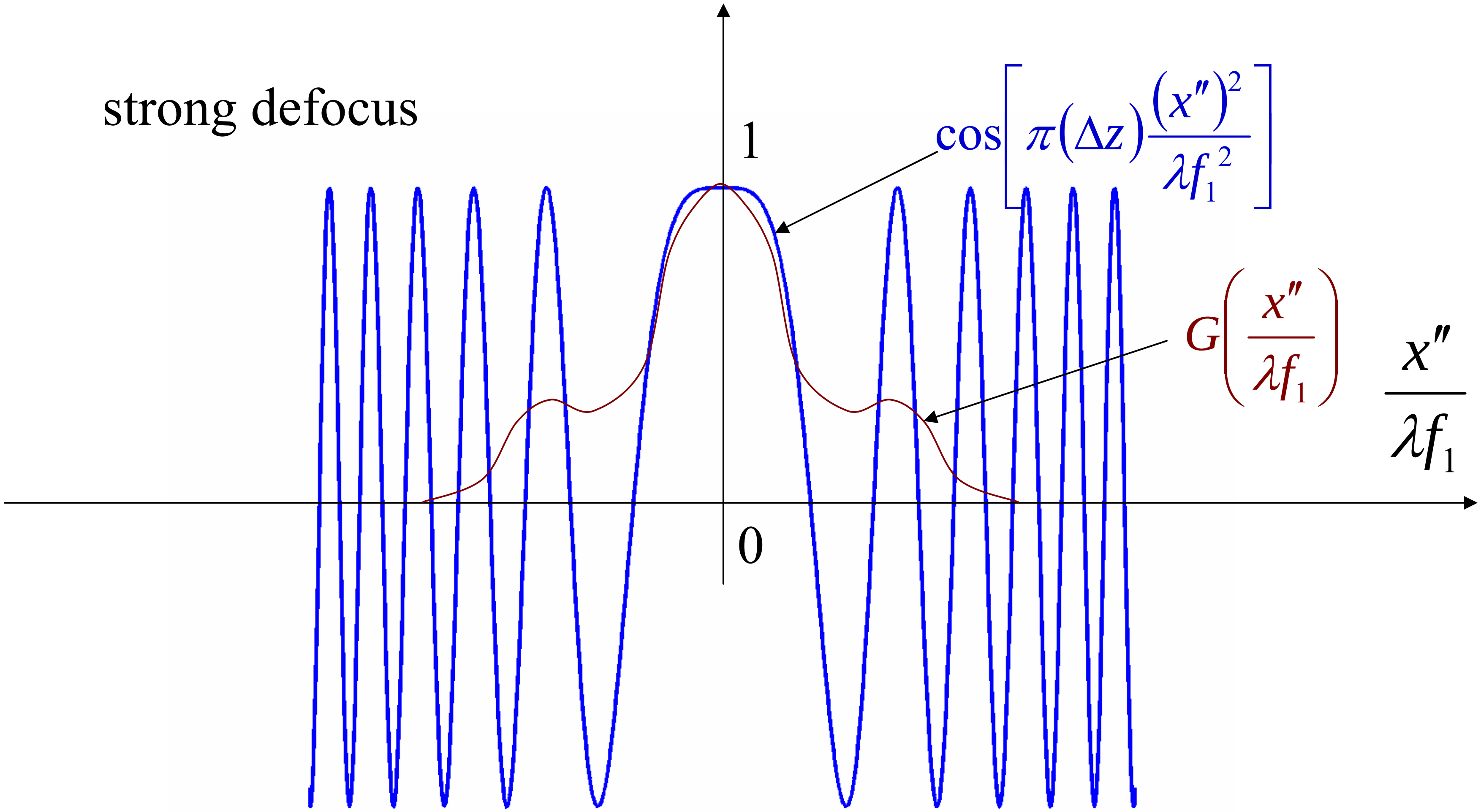


Effect of defocus on the Fourier plane



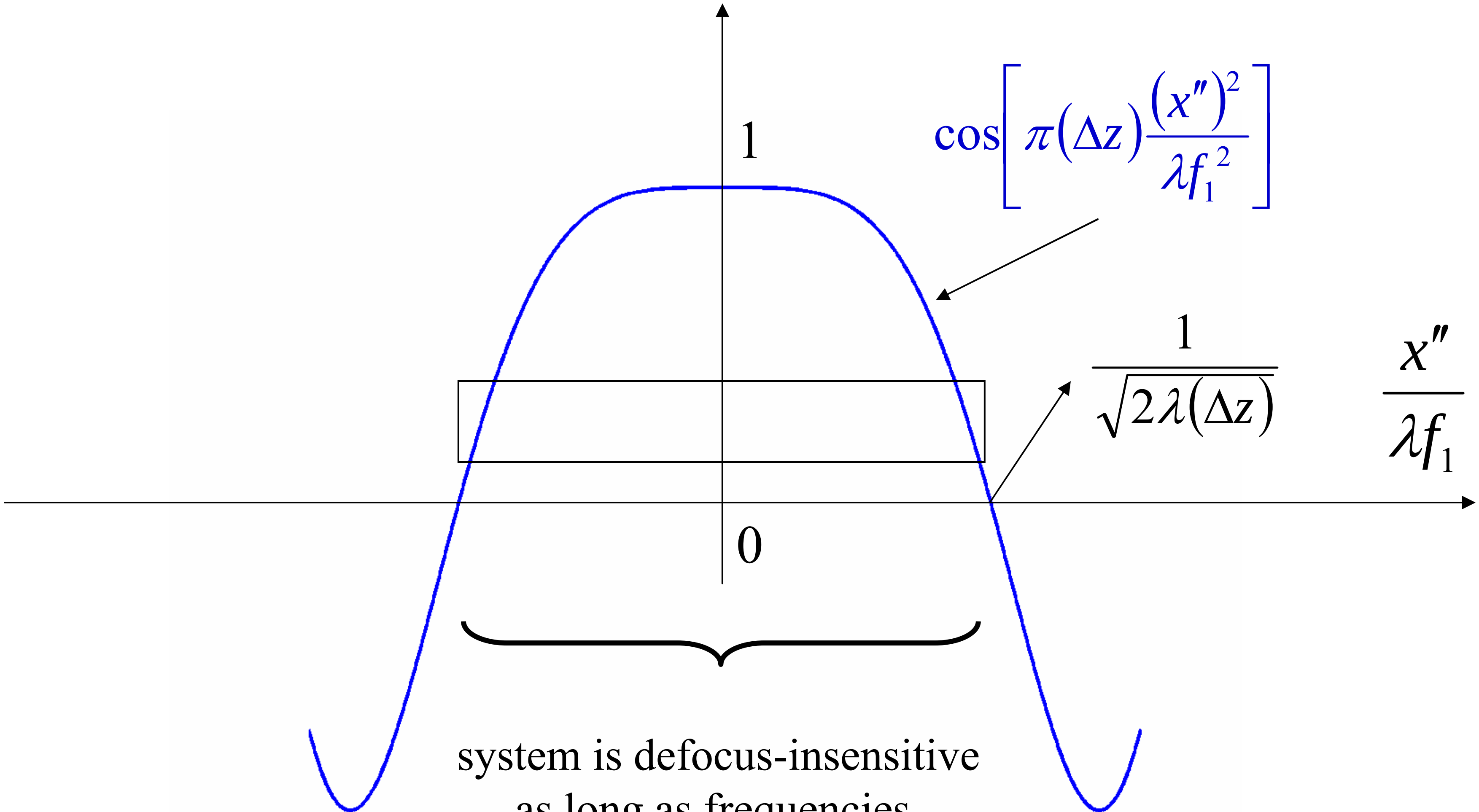
$G\left(\frac{x''}{\lambda f_1}\right) \times \cos\left[\pi(\Delta z)\frac{(x'')^2}{\lambda f_1^{\,2}}\right]$ not too different from $G\left(\frac{x''}{\lambda f_1}\right)$

Effect of defocus on the Fourier plane



$G\left(\frac{x''}{\lambda f_1}\right) \times \cos\left[\pi(\Delta z)\frac{(x'')^2}{\lambda f_1^{\,2}}\right]$ ***very*** different from $G\left(\frac{x''}{\lambda f_1}\right)$

Depth of field

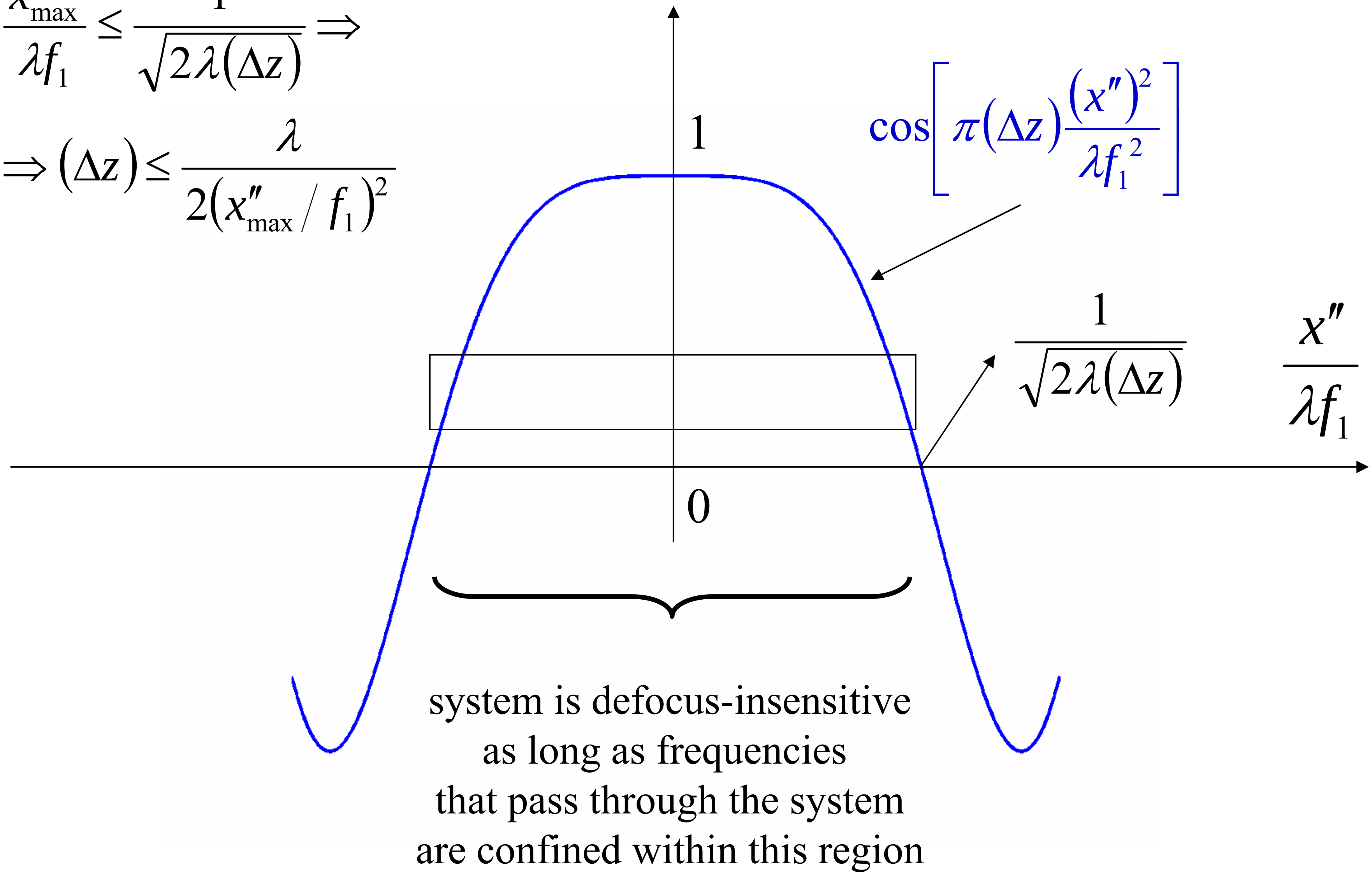


system is defocus-insensitive
as long as frequencies
that pass through the system
are confined within this region

Depth of field

$$\frac{x''_{\max}}{\lambda f_1} \leq \frac{1}{\sqrt{2\lambda(\Delta z)}} \Rightarrow$$

$$\Rightarrow (\Delta z) \leq \frac{\lambda}{2\left(x''_{\max}/f_1\right)^2}$$

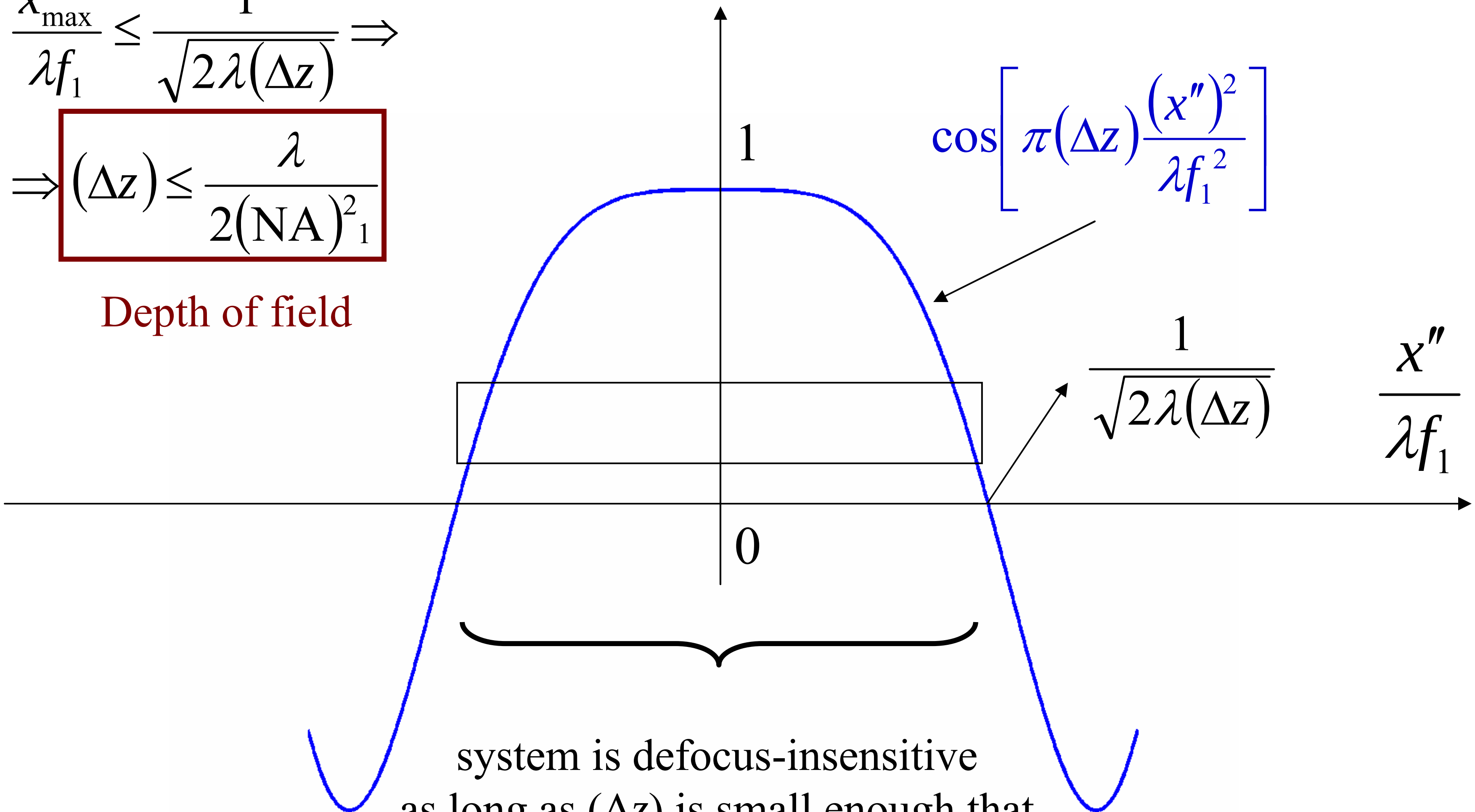


Depth of field

$$\frac{x''_{\max}}{\lambda f_1} \le \frac{1}{\sqrt{2\lambda(\Delta z)}} \Rightarrow$$

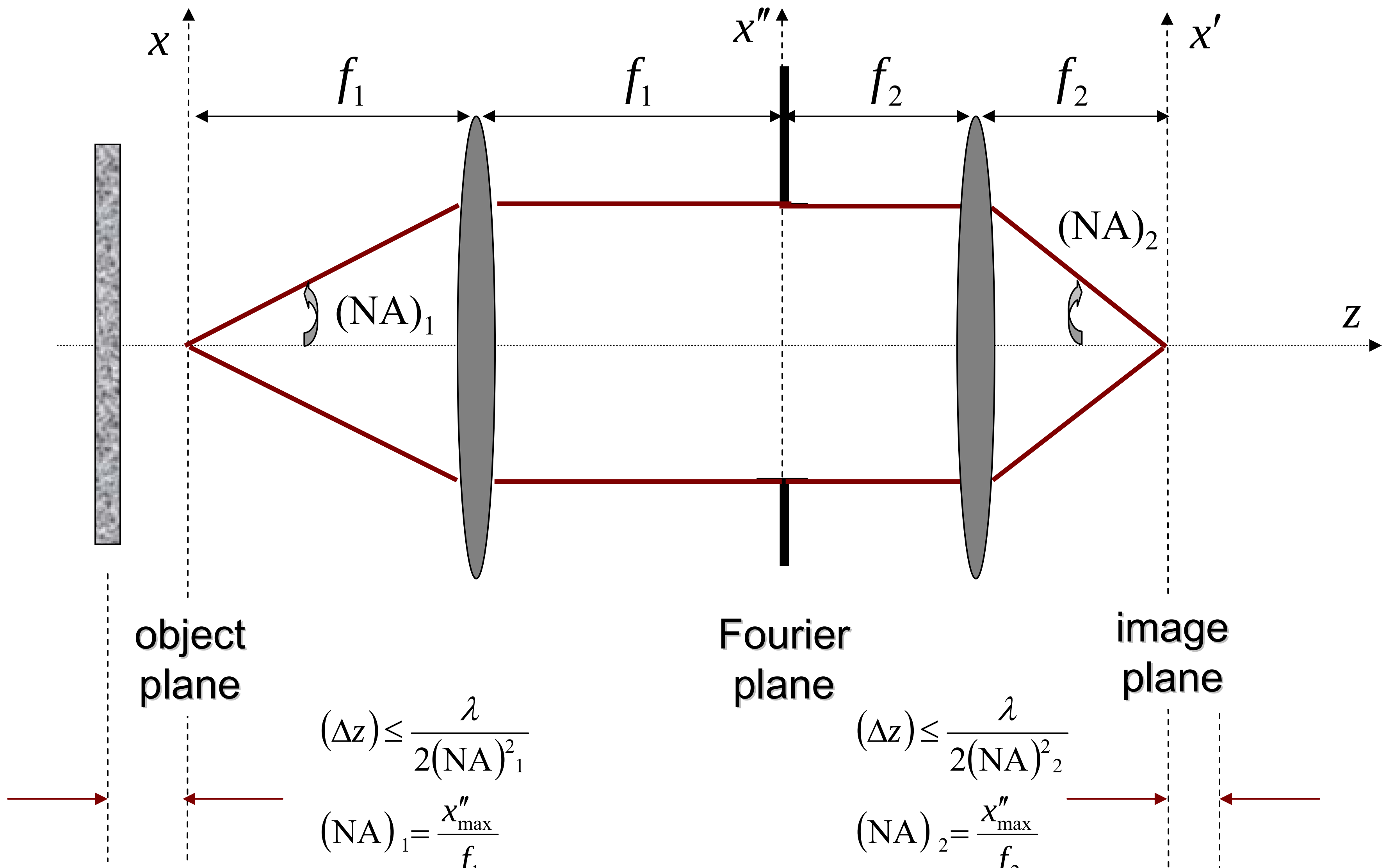
$$\Rightarrow \boxed{(\Delta z) \le \frac{\lambda}{2(\mathrm{NA})^2{}_1}}$$

Depth of field



system is defocus-insensitive
as long as (Δz) is <u>small enough</u> that
frequencies that pass through the system
<u>can be confined</u> within this region

Depth of field & Depth of focus



$$(\Delta z) \le \frac{\lambda}{2(\text{NA})^2{}_1}$$

$$(\text{NA})_1 = \frac{x''_{\max}}{f_1}$$

Depth of field

$$(\Delta z) \le \frac{\lambda}{2(\text{NA})^2{}_2}$$

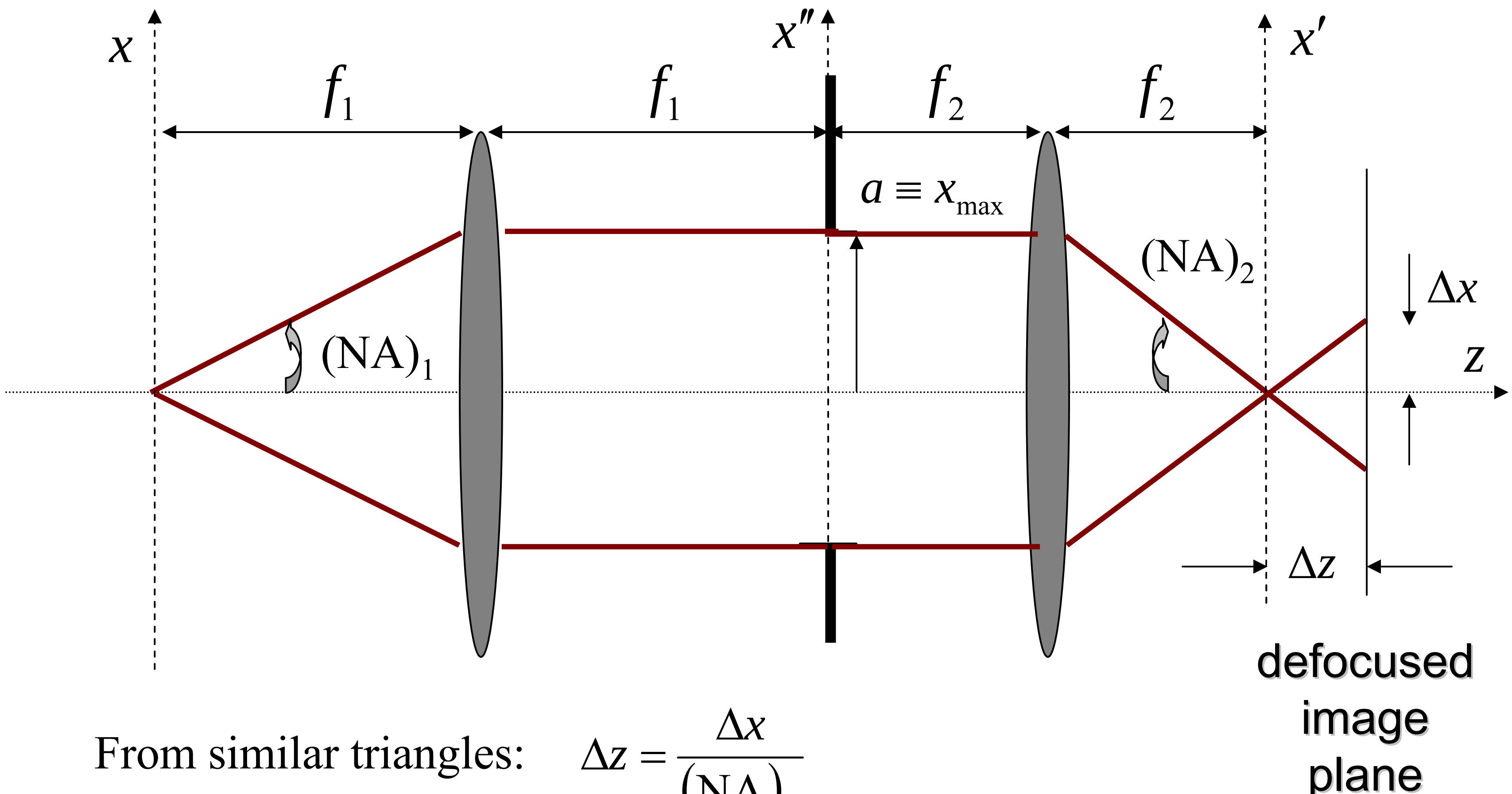
$$(\text{NA})_2 = \frac{x''_{\max}}{f_2}$$

Depth of focus

NA trade – offs

- high NA
  - narrow PSF in the lateral direction (PSF width ~1/NA)
    - sharp lateral features
  - narrow PSF in longitudinal direction (PSF depth $\sim 1/NA^2$)
    - poor depth of field
- low NA
  - broad PSF in the lateral direction (PSF width ~1/NA)
    - blurred lateral features
  - broad PSF in longitudinal direction (PSF depth $\sim 1/NA^2$)
    - good depth of field

Depth of focus: Geometrical Optics viewpoint



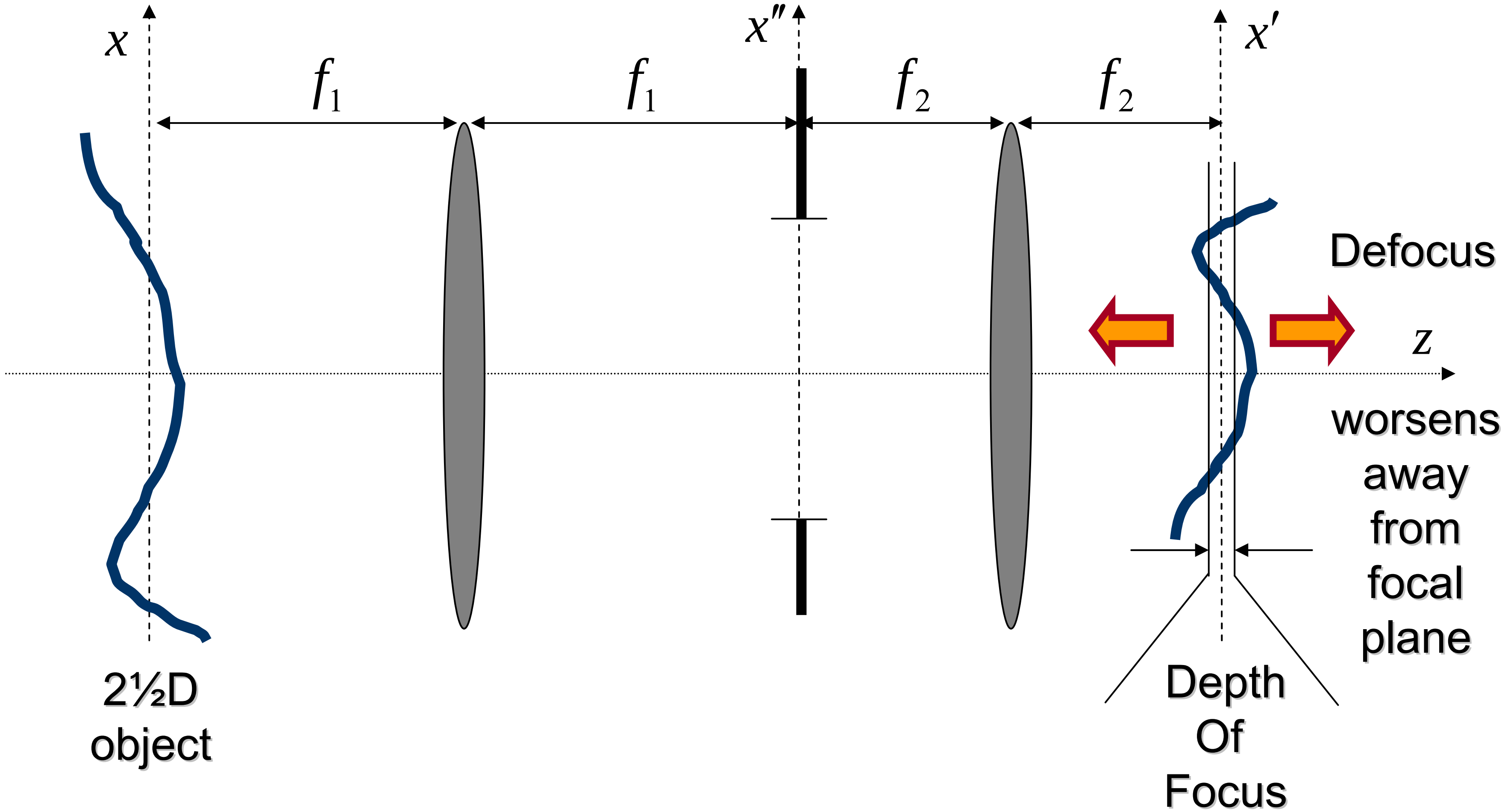
From similar triangles: $\Delta z = \dfrac{\Delta x}{(NA)_2}$

Now require defocused spot ≈ diffraction spot: $\Delta x \approx 0.61 \dfrac{\lambda}{(NA)_2}$

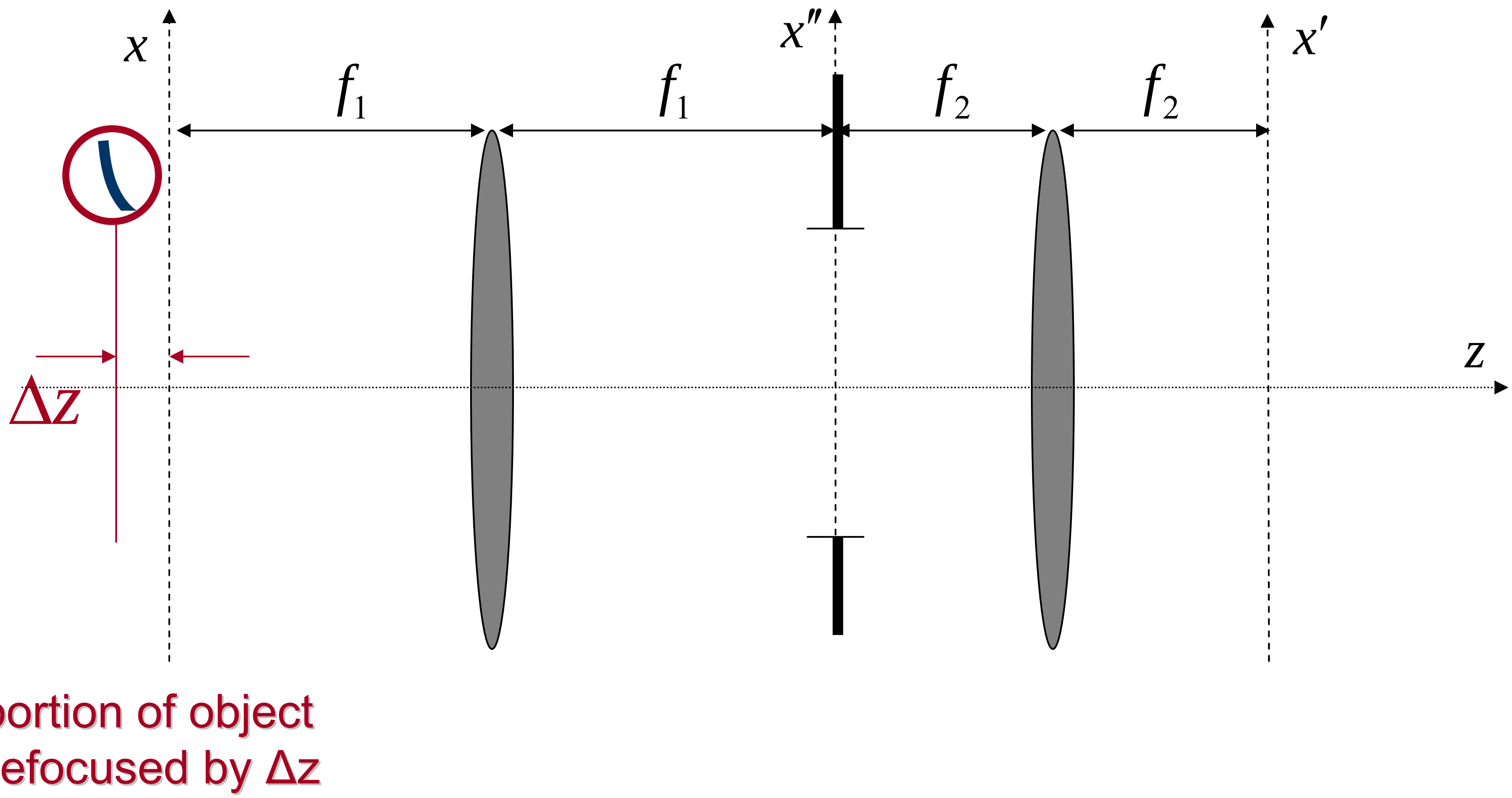
Therefore: $\Delta z \approx 0.61 \dfrac{\lambda}{(NA)_2^2}$

Defocus and Deconvolution
(Inverse filters)

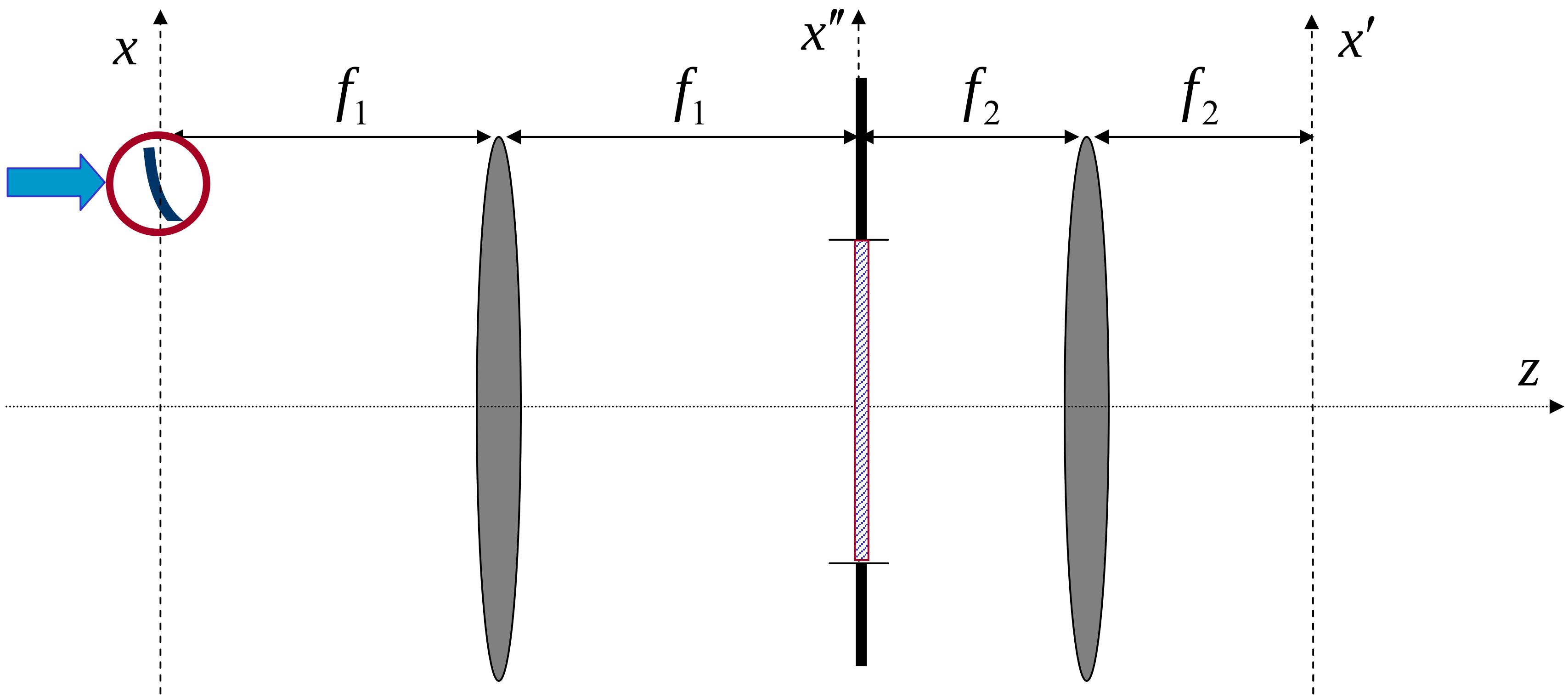
Imaging a 2½D object



Imaging a 2½D object



Imaging a 2½D object



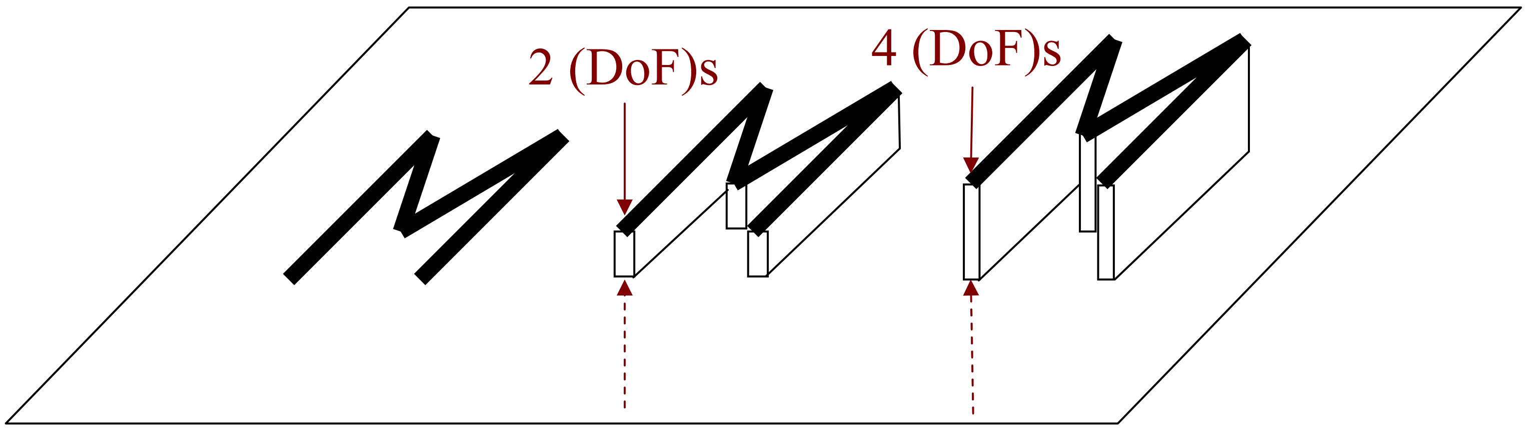
… is equivalent to same portion *in-focus* PLUS …

… fictitious quadratic phase mask on the Fourier plane

$$\exp\left\{-i2\pi\frac{\left(x''^2+y''^2\right)\Delta z}{\lambda f_1^2}\right\}$$

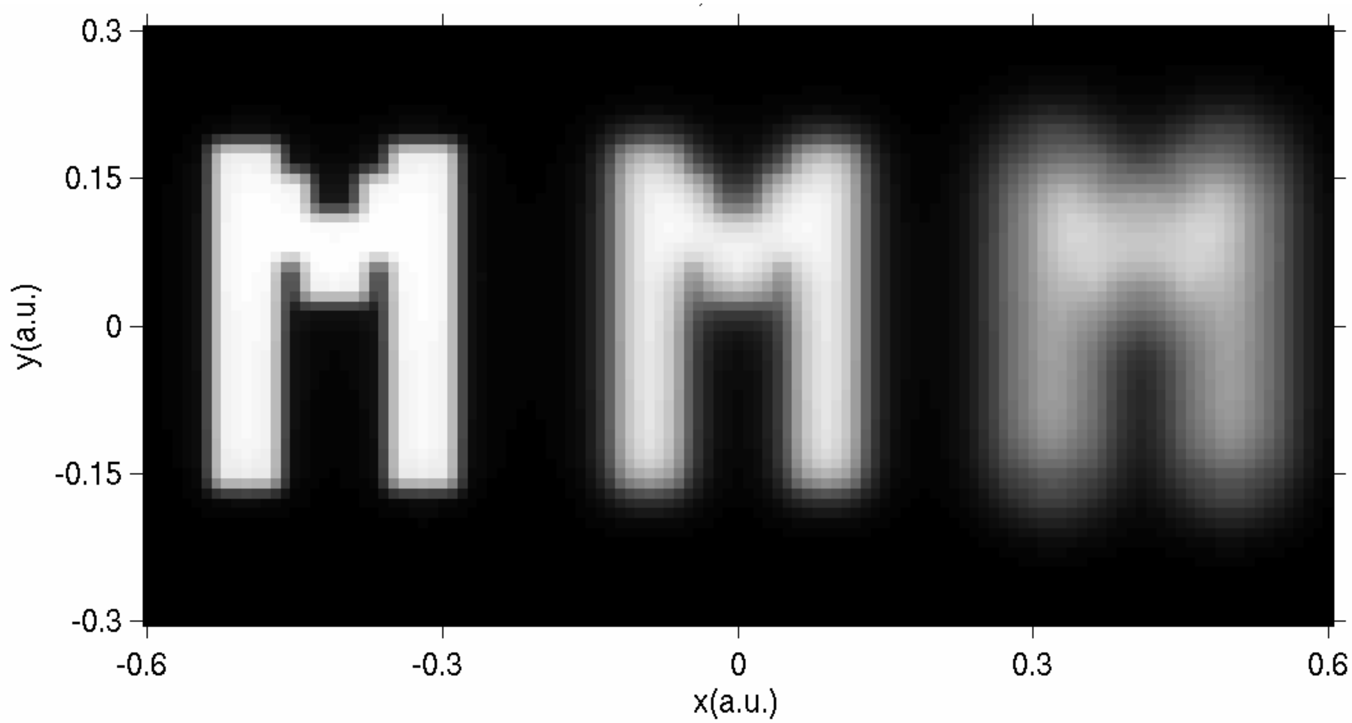
(applied *locally*)

Example



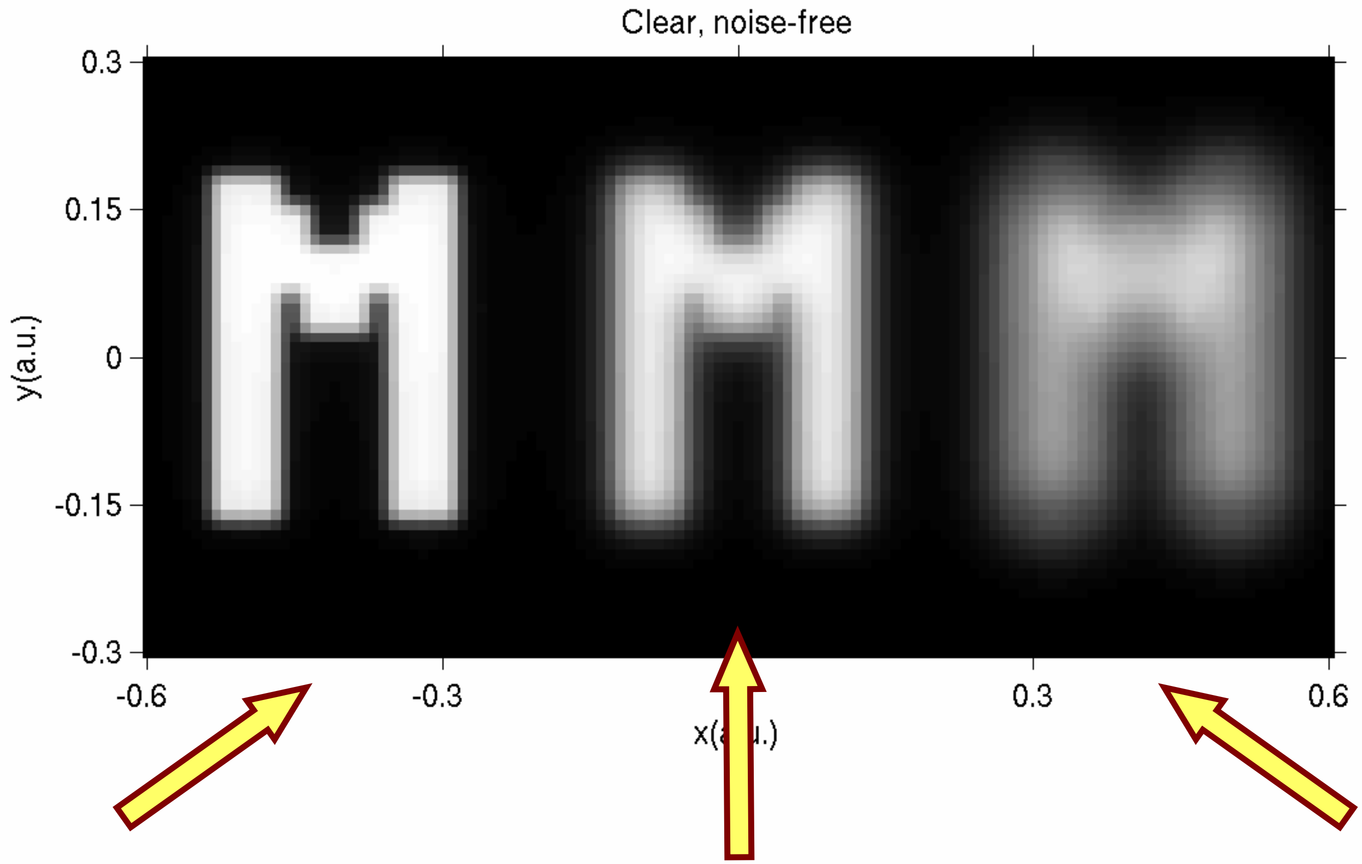
focal plane

Raw image (collected by camera – noise-free)



Distance between planes ≈ 2 Depths of Field
left-most “M” : image blurred by diffraction only
center and right-most “M”s : image blurred by diffraction and defocus

Raw image explanation: *convolution*

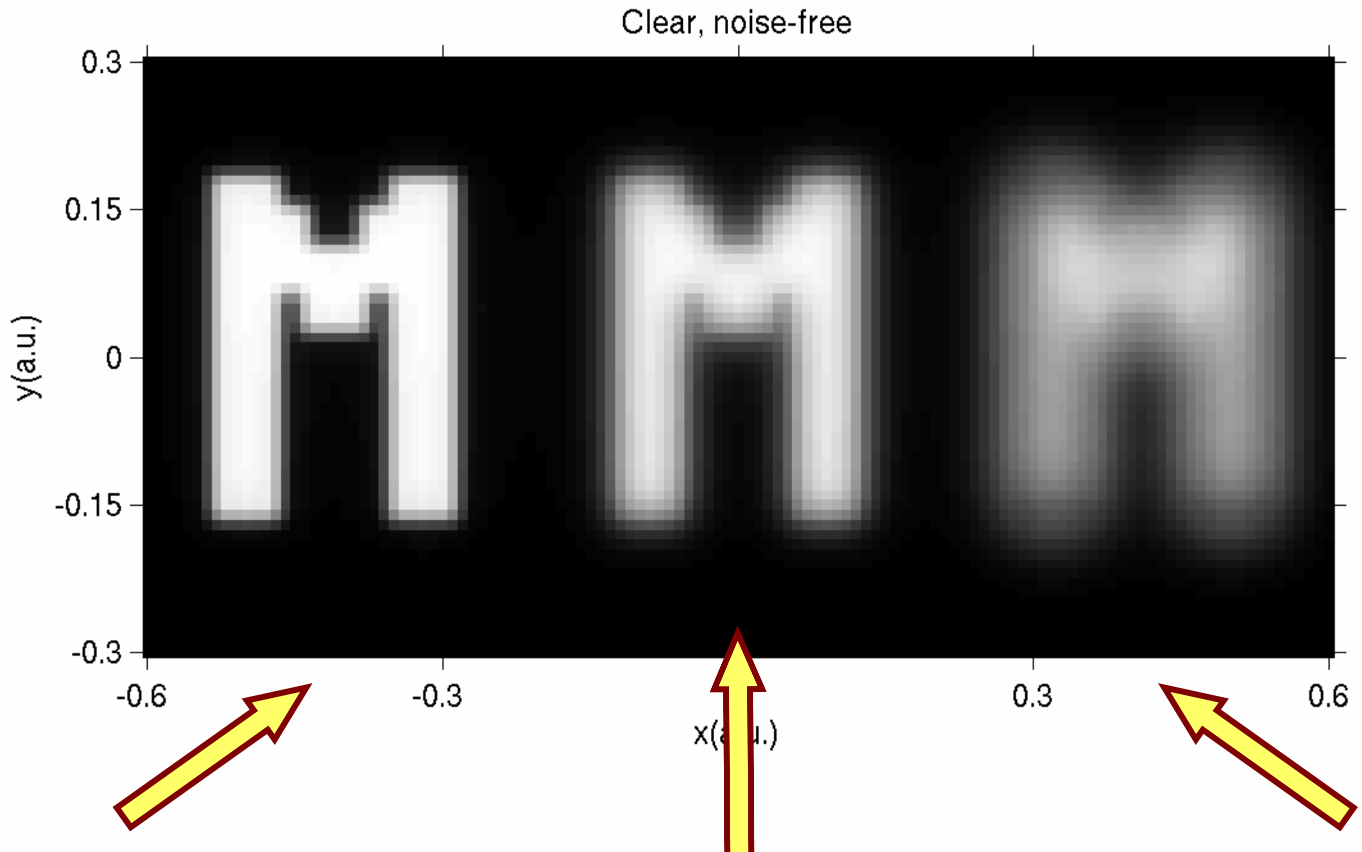


"M" convolved with standard diffraction PSF

"M" convolved with diffraction PSF *and* defocus

"M" convolved with diffraction PSF *and more* defocus

Raw image explanation: *Fourier domain*



in the Fourier domain ...

$$\Im\{"\mathrm{M}"\}\times H_{\text{diffraction}}$$

$$\Im\{"\mathrm{M}"\}\times H_{\text{diffraction}} \times H_{\text{defocus}}(2\text{DoF})$$

$$\Im\{"\mathrm{M}"\}\times H_{\text{diffraction}} \times H_{\text{defocus}}(4\text{DoF})$$

Can diffraction and defocus be "undone" ?

- Effect of optical system (expressed in the Fourier plane):

$$\Im\{\text{"M"}\}\times H_{\text{system}} \qquad \text{where} \qquad H_{\text{system}} = H_{\text{diffraction}} \times H_{\text{defocus}}$$

- To undo the optical effect, multiply by the "inverse transfer function"

$$\left(\Im\{\text{"M"}\}\times H_{\text{system}}\right)\times\frac{1}{H_{\text{system}}} = \Im\{\text{"M"}\} \text{ !!!}$$

Can diffraction and defocus be "undone" ?

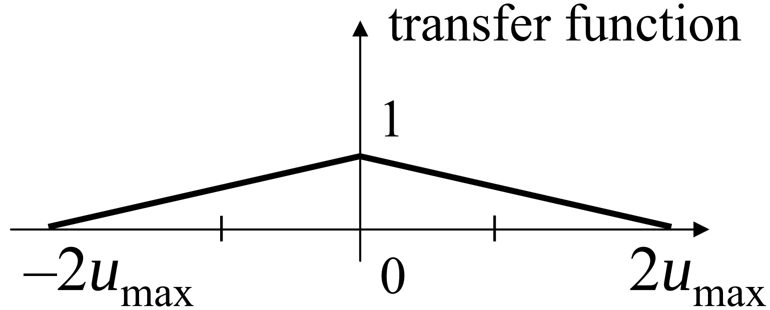
- Effect of optical system (expressed in the Fourier domain):

$$\Im\{\text{"M"}\}\times H_{\text{system}} \qquad \text{where} \qquad H_{\text{system}} = H_{\text{diffraction}} \times H_{\text{defocus}}$$

- To undo the optical effect, multiply by the "inverse transfer function"

$$\left(\Im\{\text{"M"}\}\times H_{\text{system}}\right)\times\frac{1}{H_{\text{system}}} = \Im\{\text{"M"}\} \text{ !!!}$$

- Problems
  - Transfer function goes to zero outsize the system pass-band
  - Inverse transfer function will multiply the FT of the noise as well as the FT of the original signal

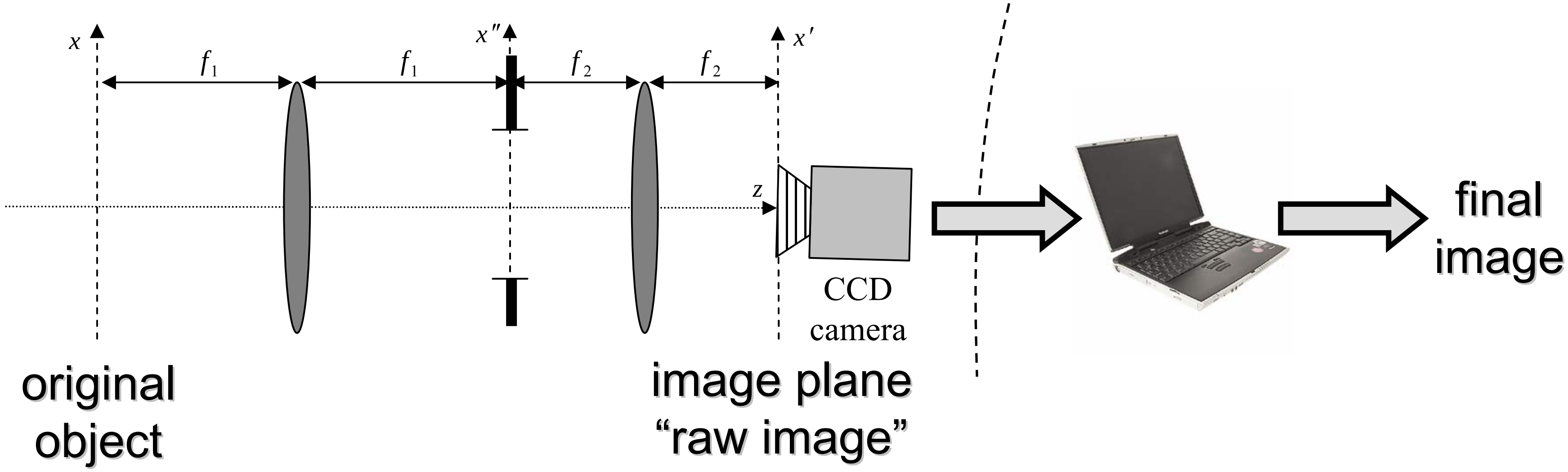


Solution: Tikhonov regularization

$$\Im\left\{\begin{matrix}\text{final}\\\text{image}\end{matrix}\right\} = \left(\Im\left\{\begin{matrix}\text{original}\\\text{object}\end{matrix}\right\} \times H_{\text{system}}\right) \times \frac{H^{*}_{\text{system}}}{\mu + \left|H_{\text{system}}\right|^{2}}$$

"raw image"
(formed by the optics)

post-processing
("inverse filter")



$$\Im^{-1}\left\{\Im\left\{\begin{matrix}\text{original}\\\text{object}\end{matrix}\right\} \times H_{\text{system}}\right\}$$

COMPUTATIONAL
IMAGING

On Tikhonov regularization

$$\Im\left\{\begin{matrix}\text{final}\\ \text{image}\end{matrix}\right\} = \left(\Im\left\{\begin{matrix}\text{original}\\ \text{object}\end{matrix}\right\} \times H_{\text{system}}\right) \times \frac{H^{*}_{\text{system}}}{\mu + \left|H_{\text{system}}\right|^{2}}$$

- μ is the "regularizer" or "regularization parameter"
- choice of μ : depends on the noise and signal energy
- for Gaussian noise *and* image statistics, optimum μ is

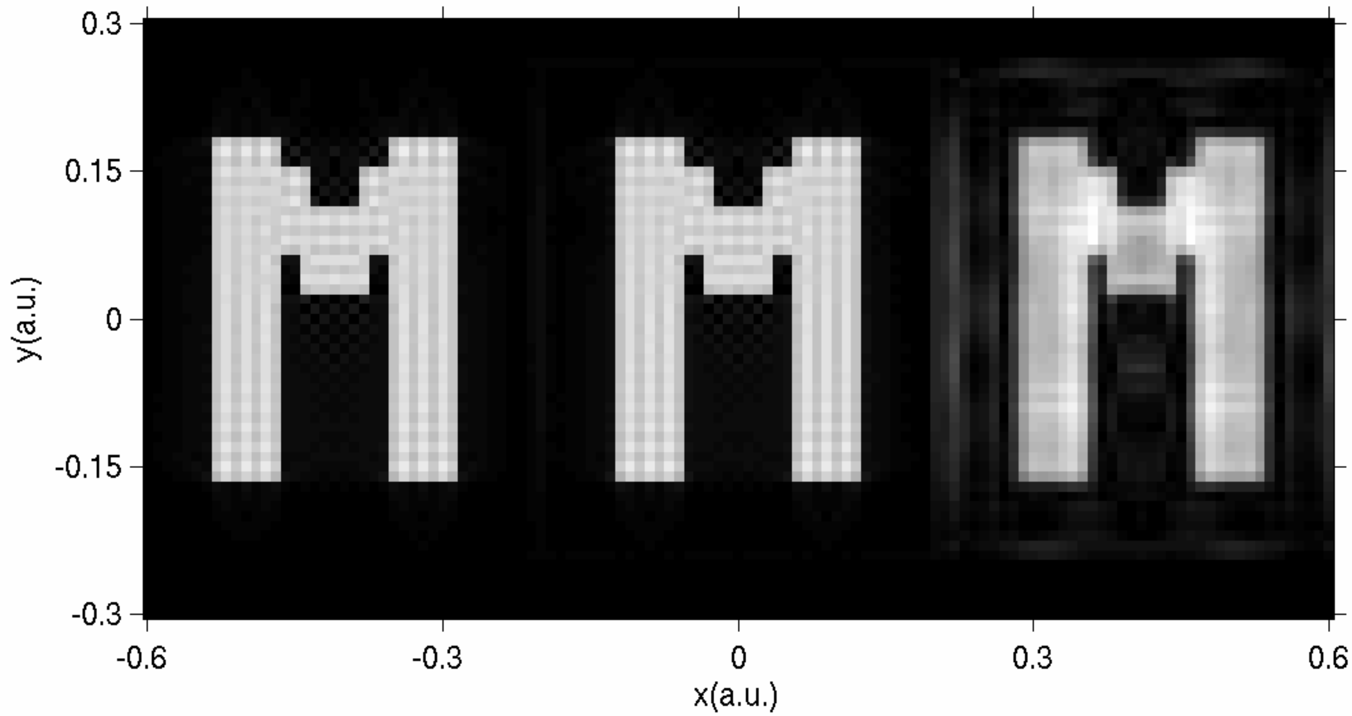
  $$\mu_{\text{optimum}} = \frac{1}{\text{SNR}_{\text{power}}}$$

  "Wiener filter"

- More generally, the optimal inverse filters are nonlinear and/or probabilistic (e.g. maximum likelihood inversion)
- For more details: 2.717

Deconvolution: diffraction *and* defocus

noise free

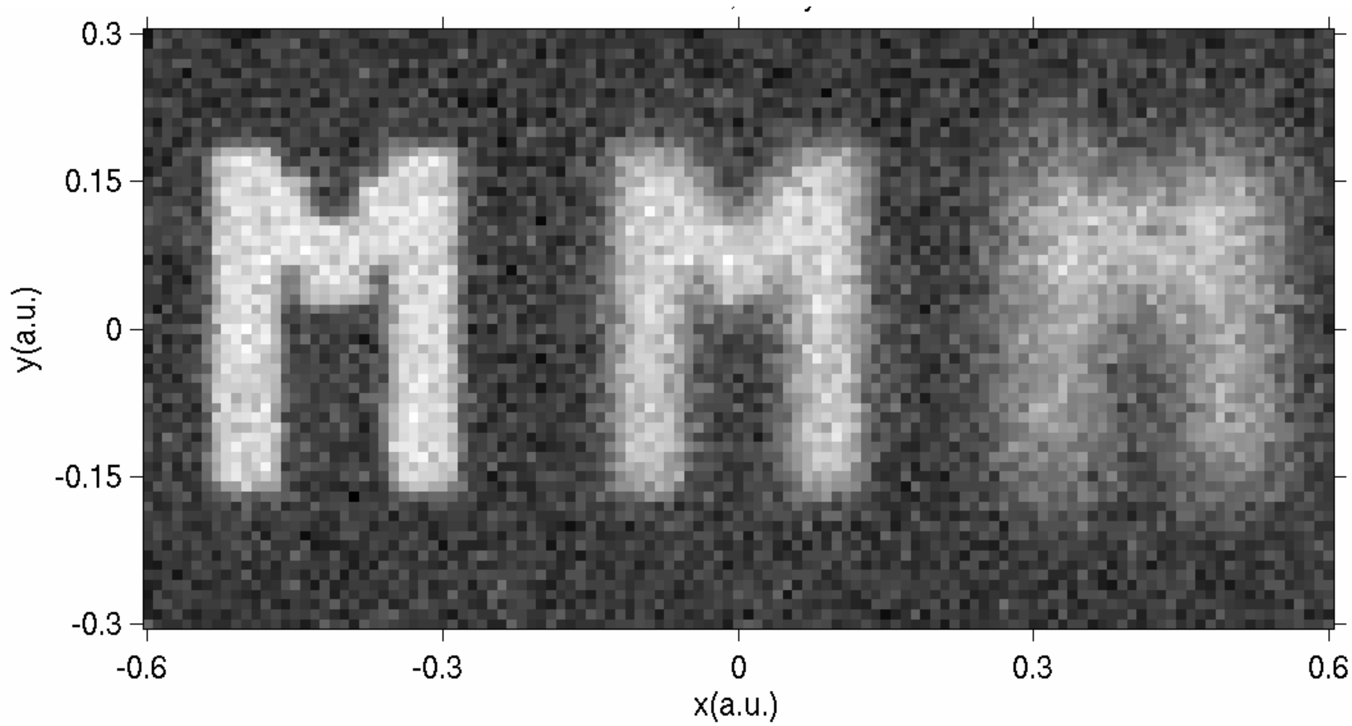


Deconvolution using Tikhonov regularized inverse filter
Utilized *a priori* knowledge of depth of each digit (alternatively, needs depth-from defocus algorithm)

Artifacts due primarily to numerical errors getting amplified by the inverse filter (despite regularization)

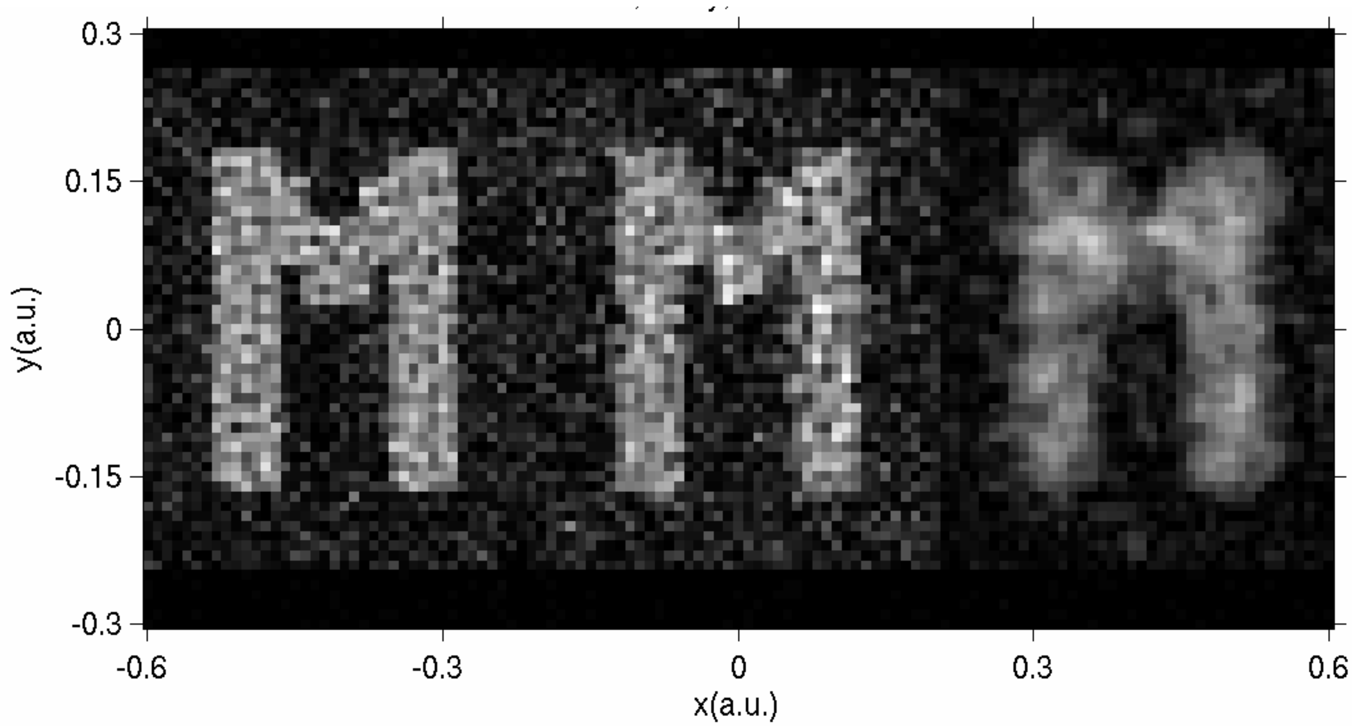
Noisy raw image

SNR=10



Deconvolution in the presence of noise

SNR=10



Deconvolution using Wiener filter (i.e. Tikhonov with μ=1/SNR)
Noise is destructive away from focus (4DOFs)
Utilized *a priori* knowledge of depth of each digit

Artifacts due primarily to noise getting amplified by the inverse filter