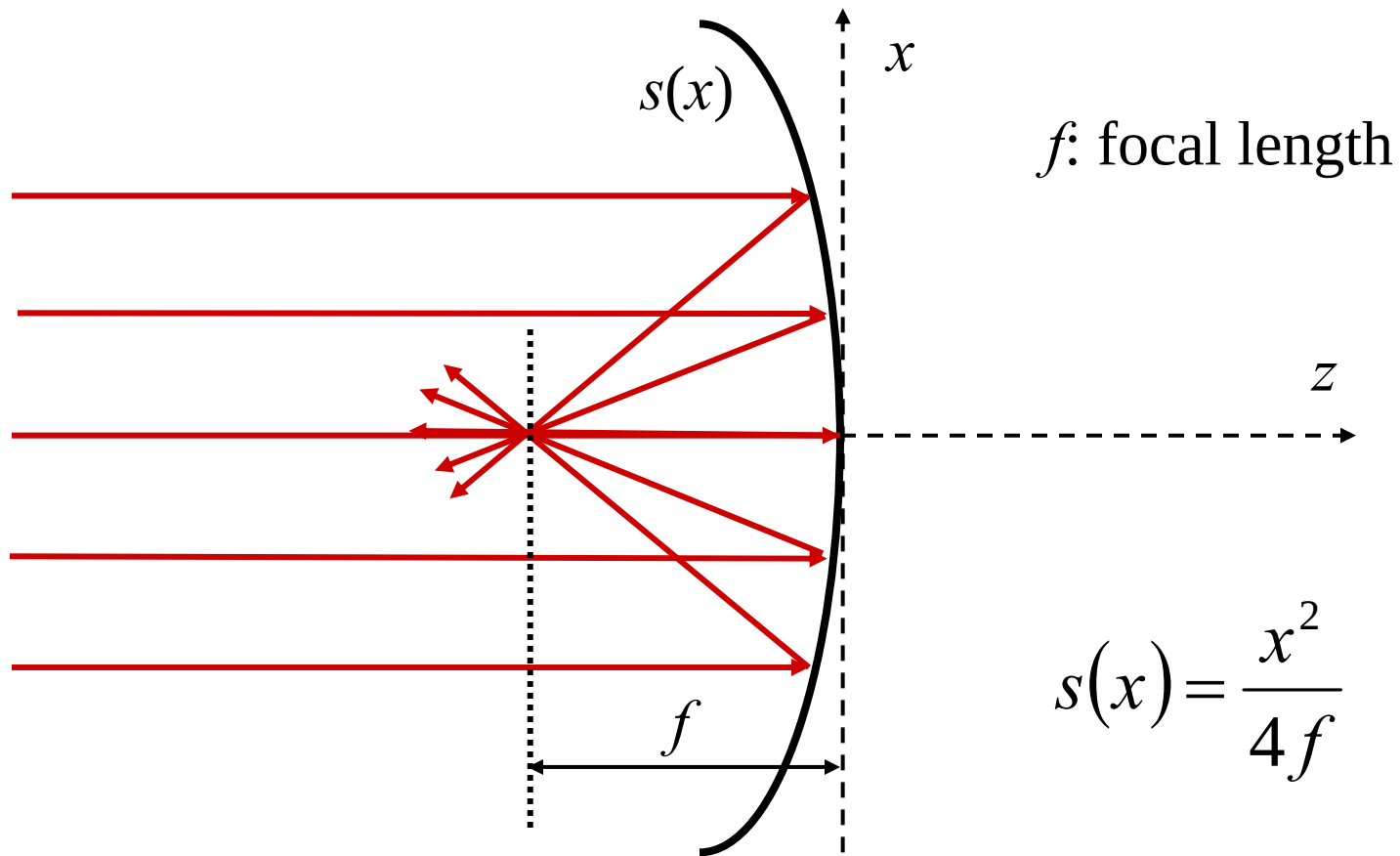


Lenses and Imaging (Part I)

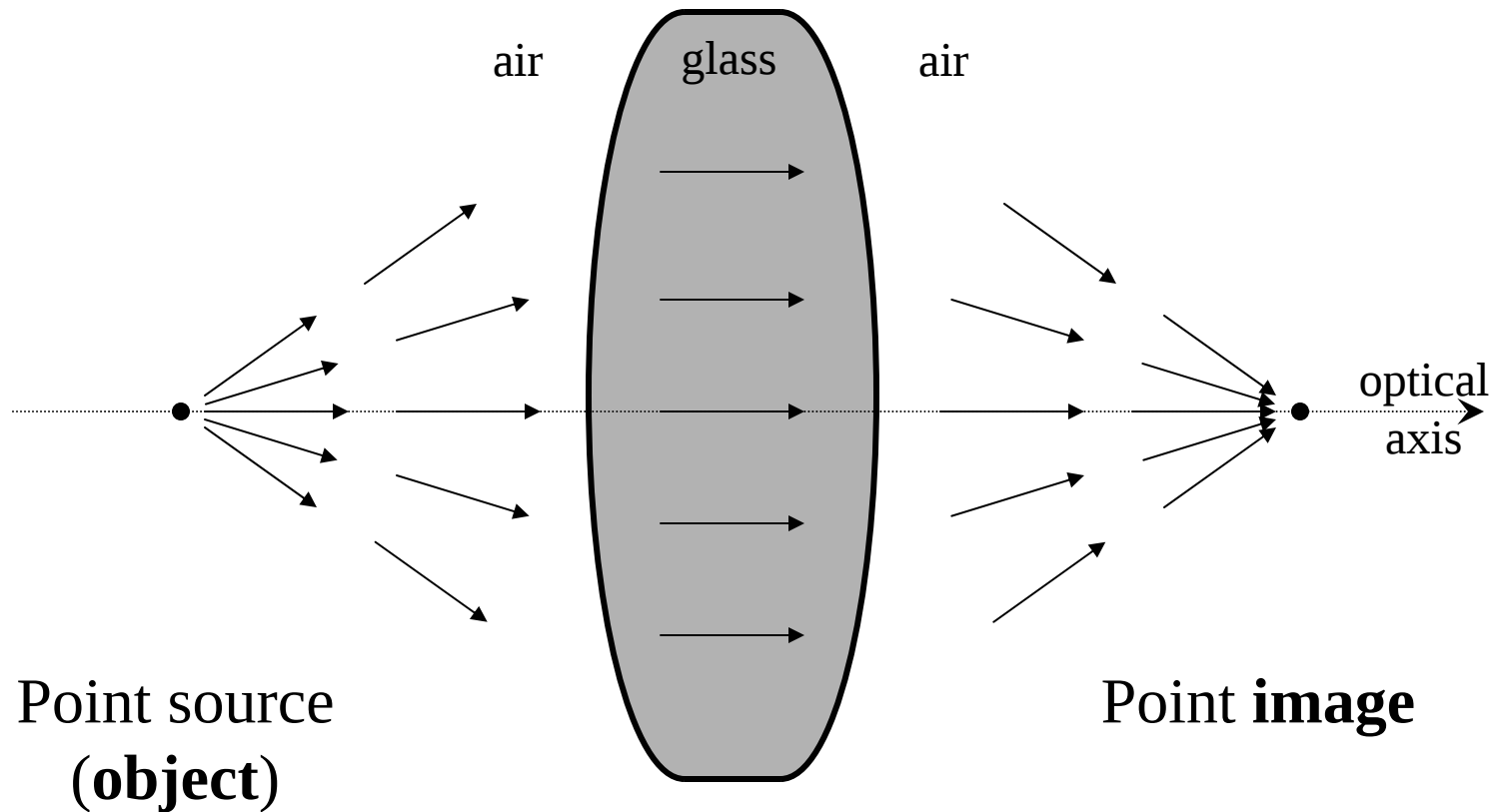
- Review: paraboloid reflector, focusing
- Why is imaging necessary: Huygens' principle
 - Spherical & parallel ray bundles, points at infinity
- Refraction at spherical surfaces (paraxial approximation)
 - Optical power of surfaces: positive, negative surfaces
- Paraxial ray-tracing
 - Matrix formulation of paraxial geometrical optics
- Imaging condition
- Object/image at infinity

Parabloid mirror: perfect focusing

(e.g. satellite dish)



Lens: main instrument of image formation

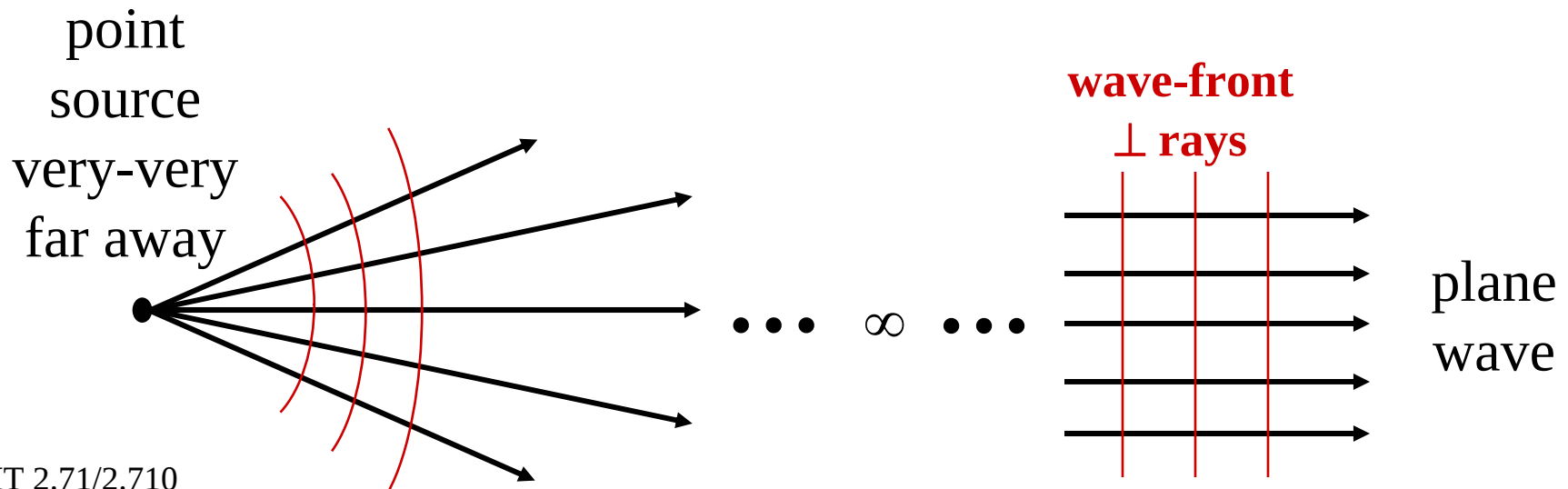
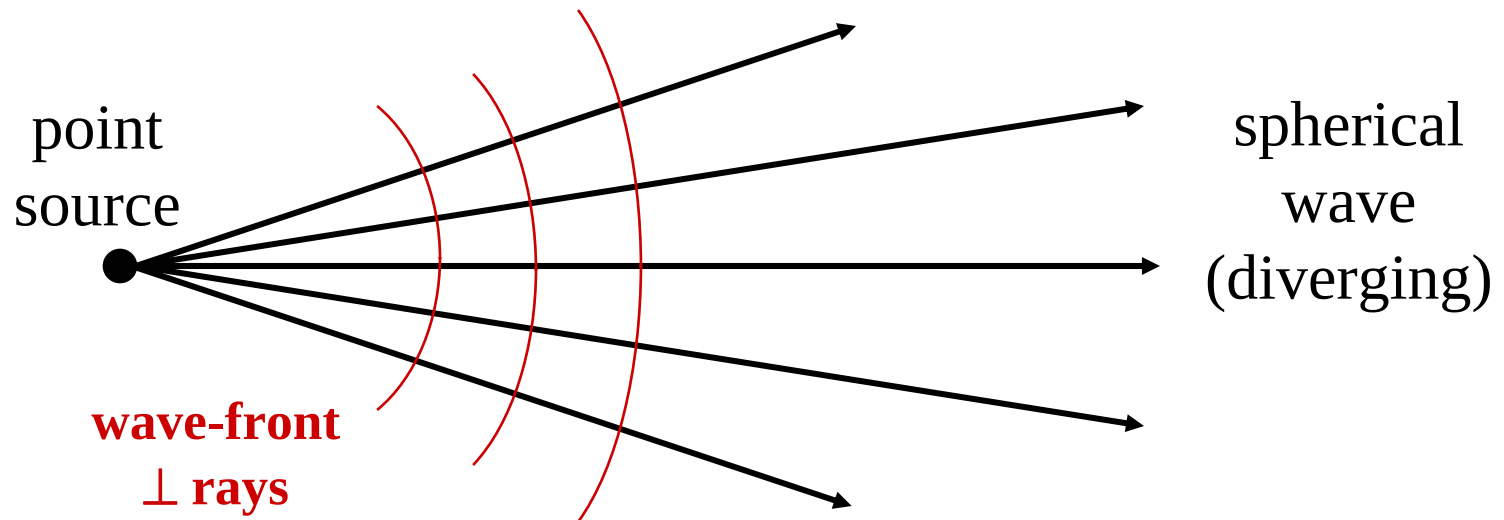


The curved surface makes the rays bend proportionally to their distance from the “optical axis”, according to Snell’s law. Therefore, the divergent wavefront becomes convergent at the right-hand (output) side.

Why are focusing instruments necessary?

- Ray bundles: spherical waves and plane waves
- Point sources and point images
- Huygens principle and why we can see around us
- The role of classical imaging systems

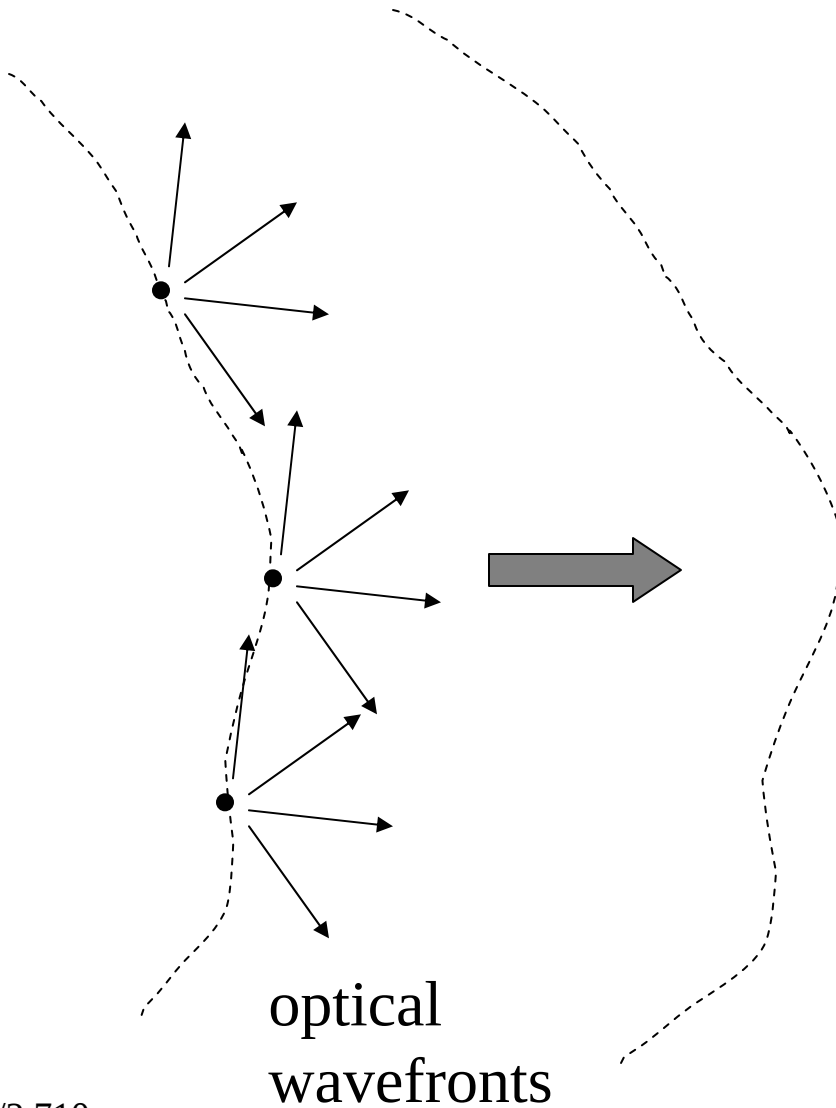
Ray bundles



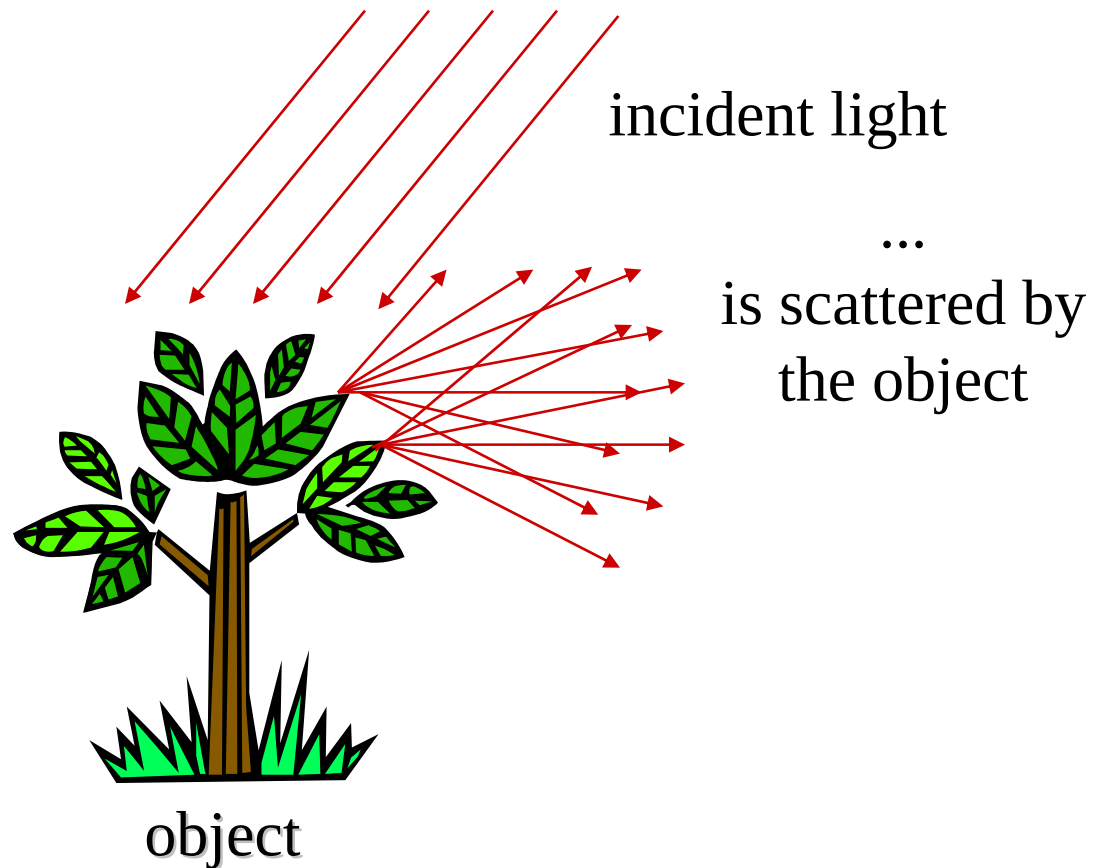
Huygens principle

Each point on the wavefront acts as a secondary light source emitting a spherical wave

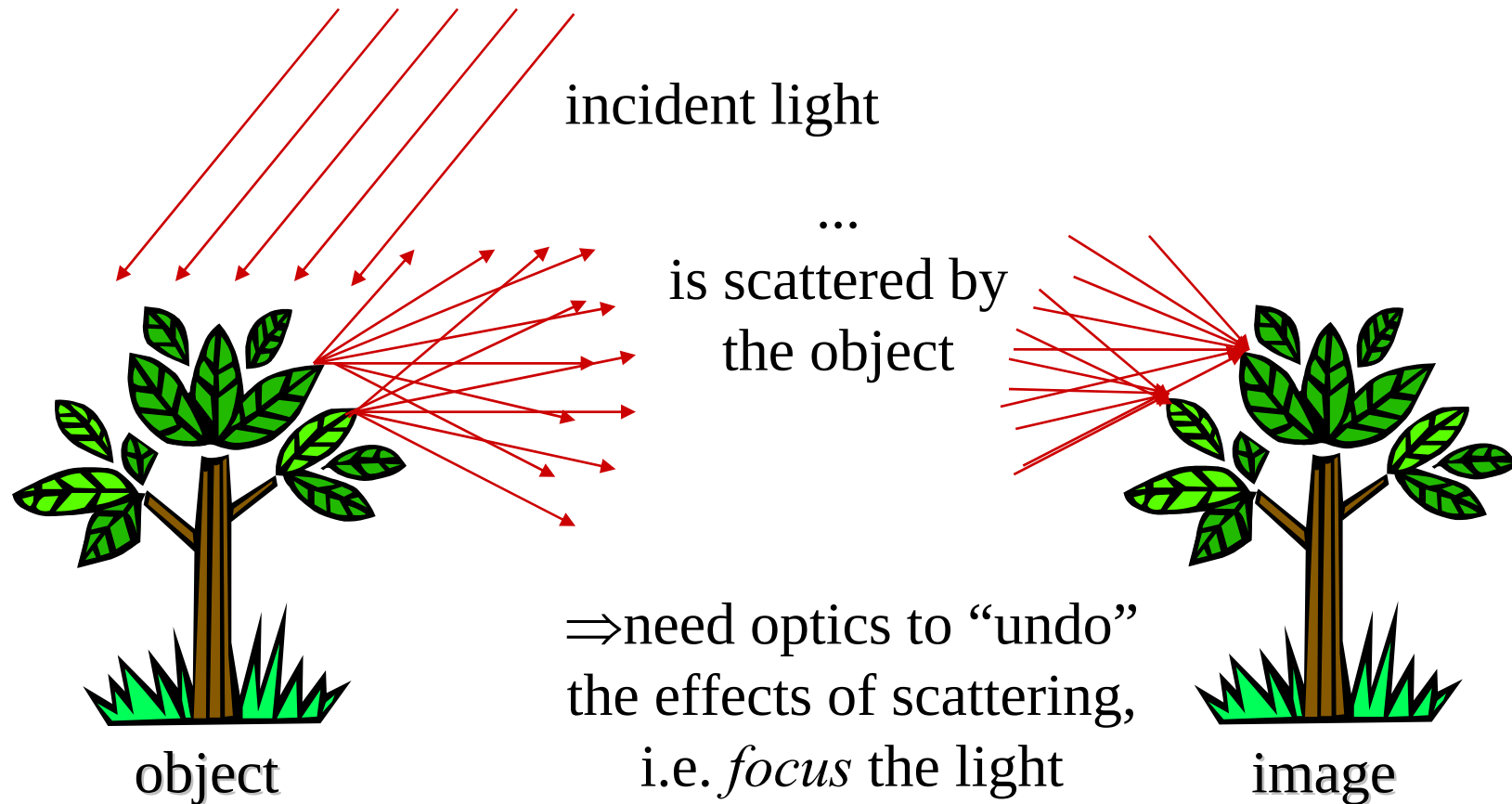
The wavefront after a short propagation distance is the result of superimposing all these spherical wavelets



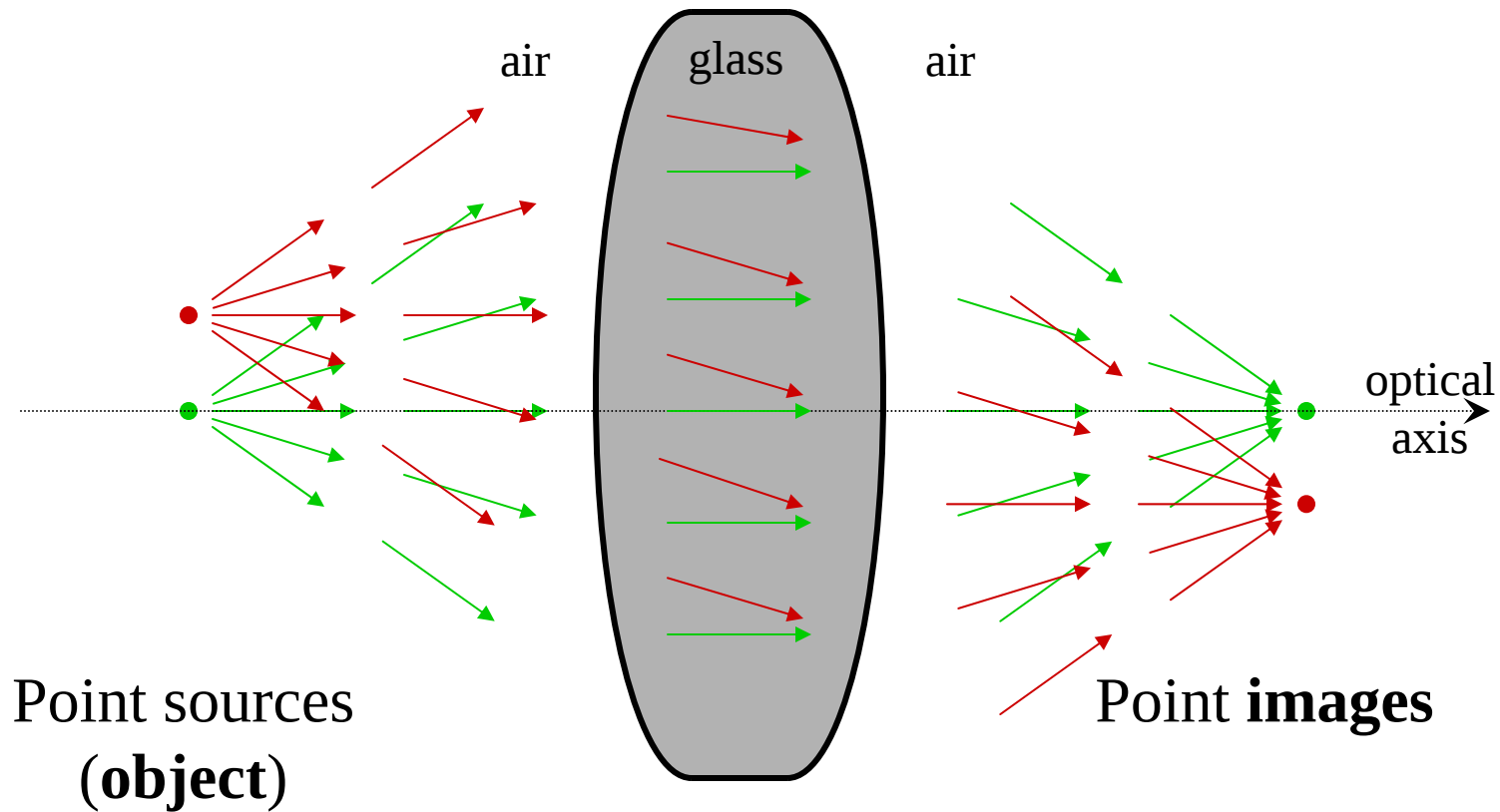
Why are focusing instruments necessary?



Why are focusing instruments necessary?



Ideal lens

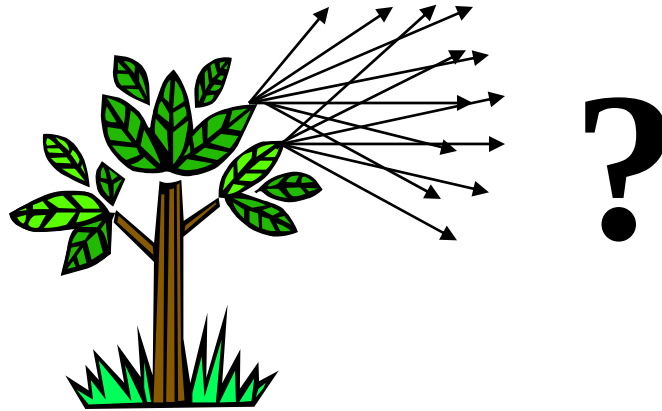


Each point source from the object plane focuses onto a point image at the image plane; NOTE the image inversion

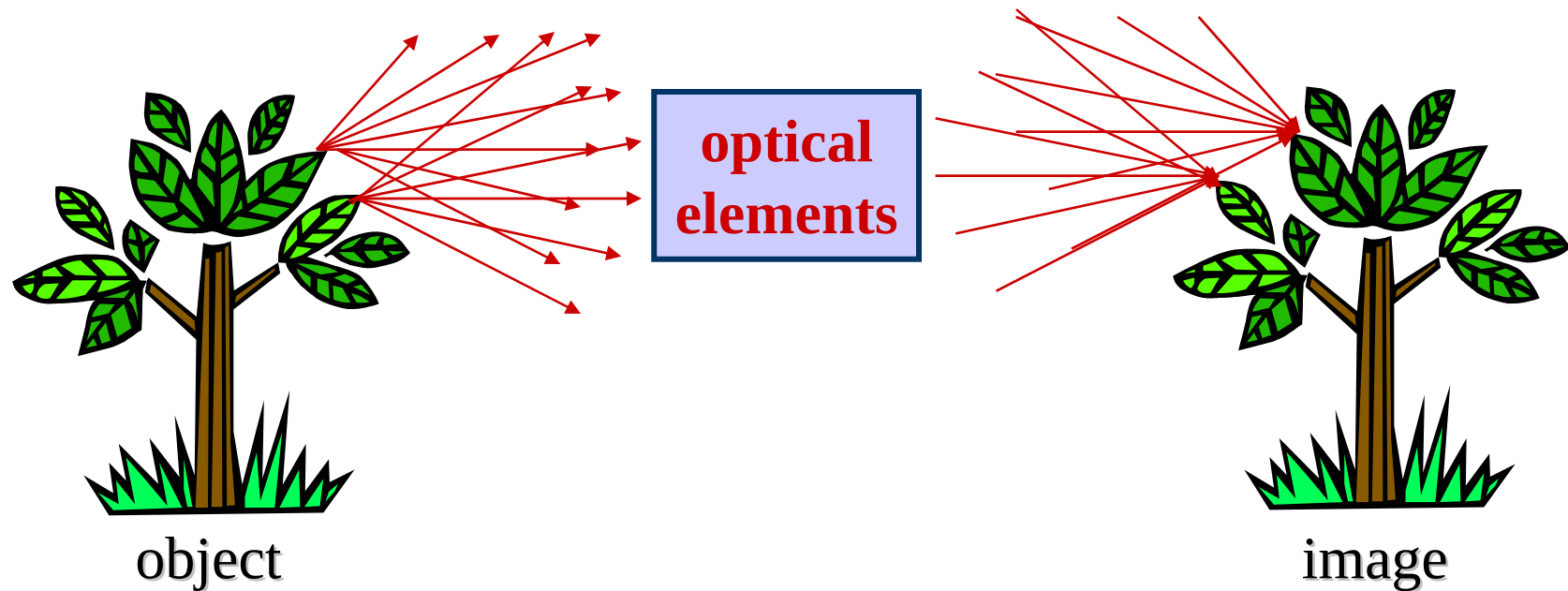
Summary:

Why are imaging systems needed?

- Each point in an object scatters the incident illumination into a spherical wave, according to the Huygens principle.
- A few microns away from the object surface, the rays emanating from all object points become entangled, delocalizing object details.
- To relocalize object details, a method must be found to reassign (“focus”) all the rays that emanated from a single point object into another point in space (the “image.”)
- The latter function is the topic of the discipline of Optical Imaging.



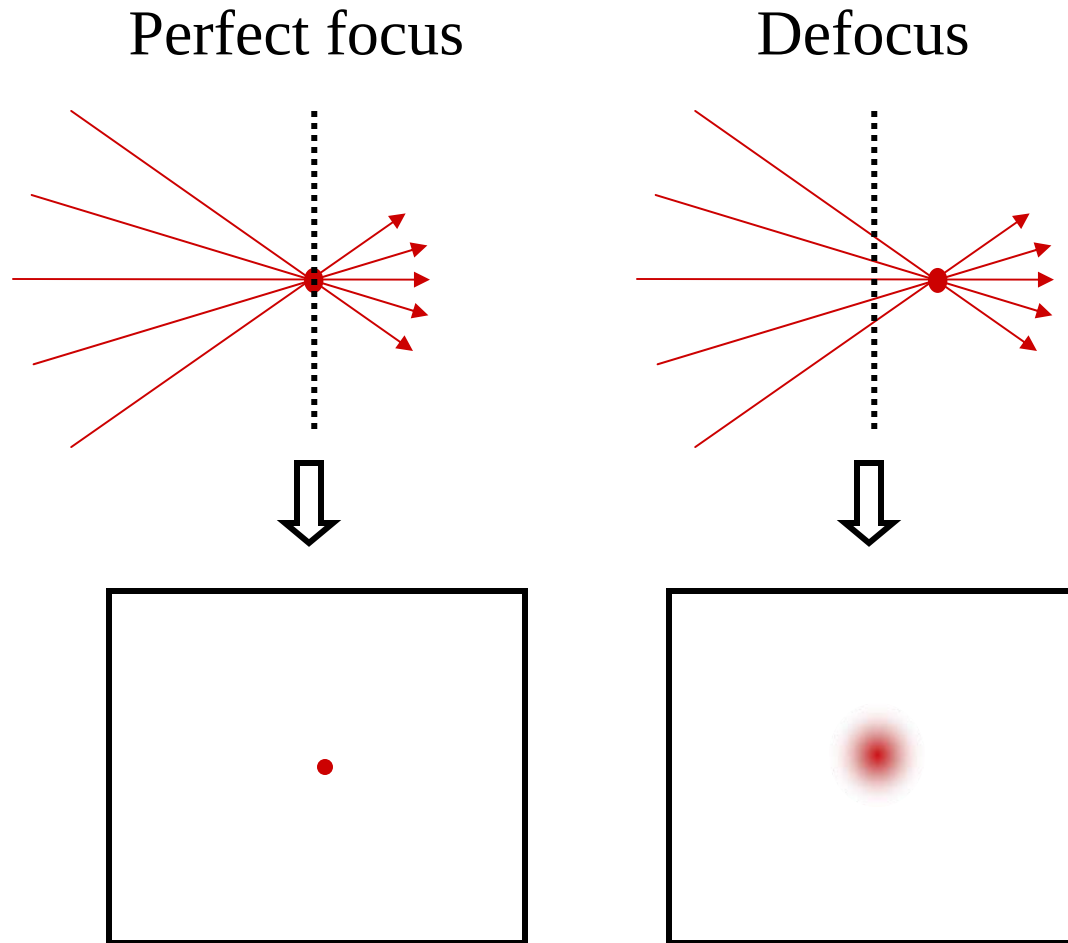
The ideal optical imaging system



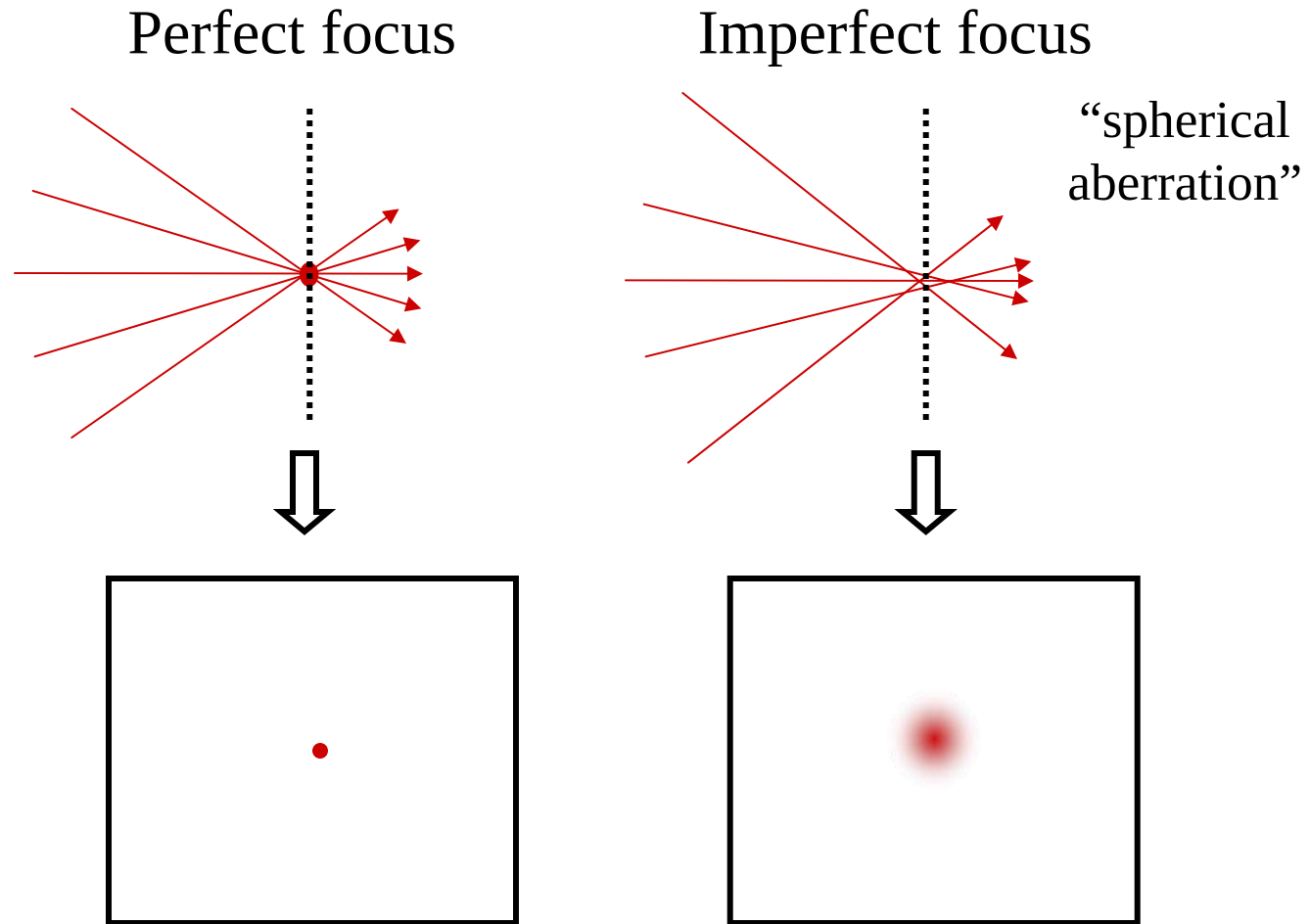
Ideal imaging system:
each point in the object is mapped
onto a single point in the image

Real imaging systems introduce blur ...

Focus, defocus and blur



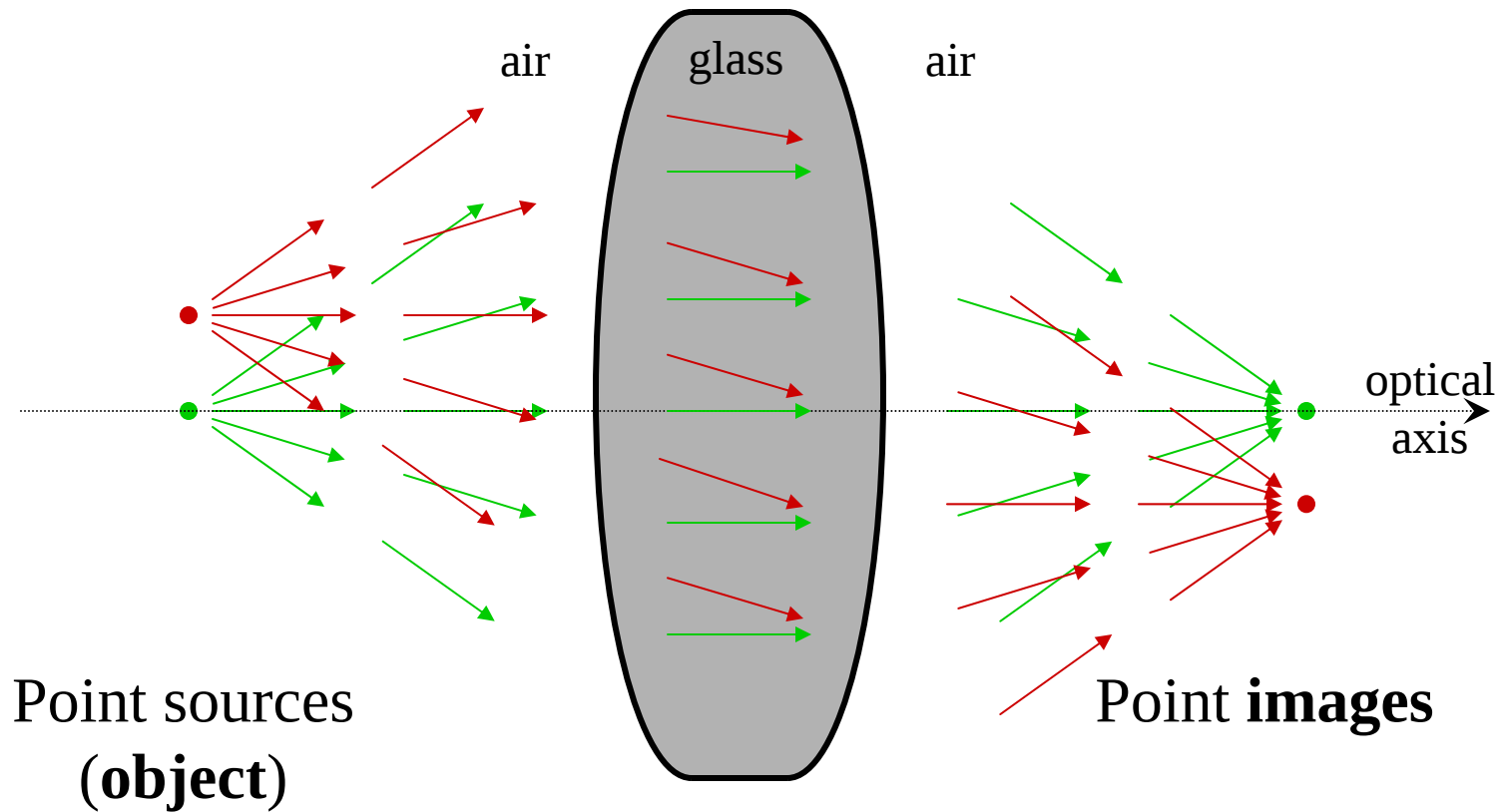
Focus, defocus and blur



Why optical systems do *not* focus perfectly

- Diffraction
- Aberrations
- However, in the paraxial approximation to Geometrical Optics that we are about to embark upon, optical systems do focus perfectly
- To deal with aberrations, we need non-paraxial Geometrical Optics (higher order approximations)
- To deal with diffraction, we need Wave Optics

Ideal lens

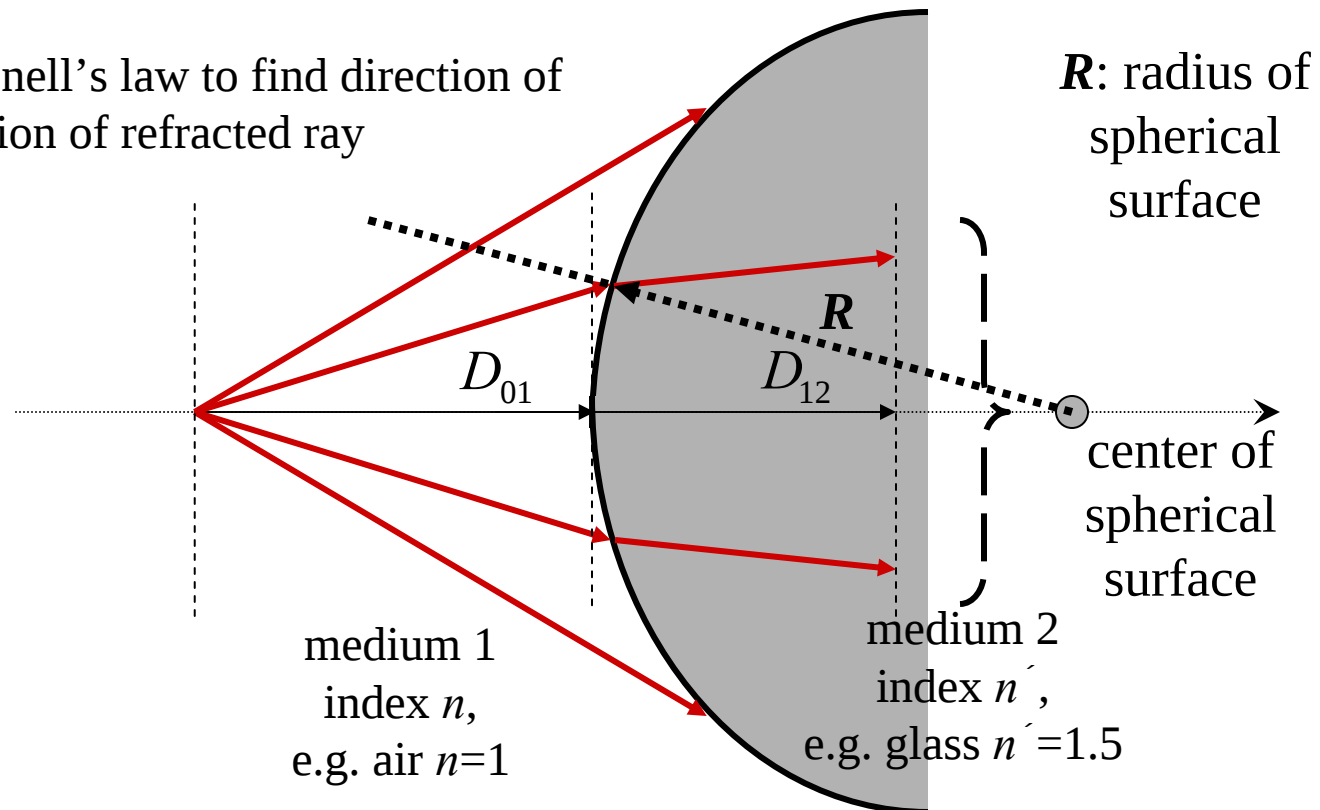


Each point source from the object plane focuses onto a point image at the image plane

Refraction at single spherical surface

for each ray, must calculate

- point of intersection with sphere
- angle between ray and normal to surface
- apply Snell's law to find direction of propagation of refracted ray



Paraxial approximation /1

- In paraxial optics, we make heavy use of the following approximate (1st order Taylor) expressions:

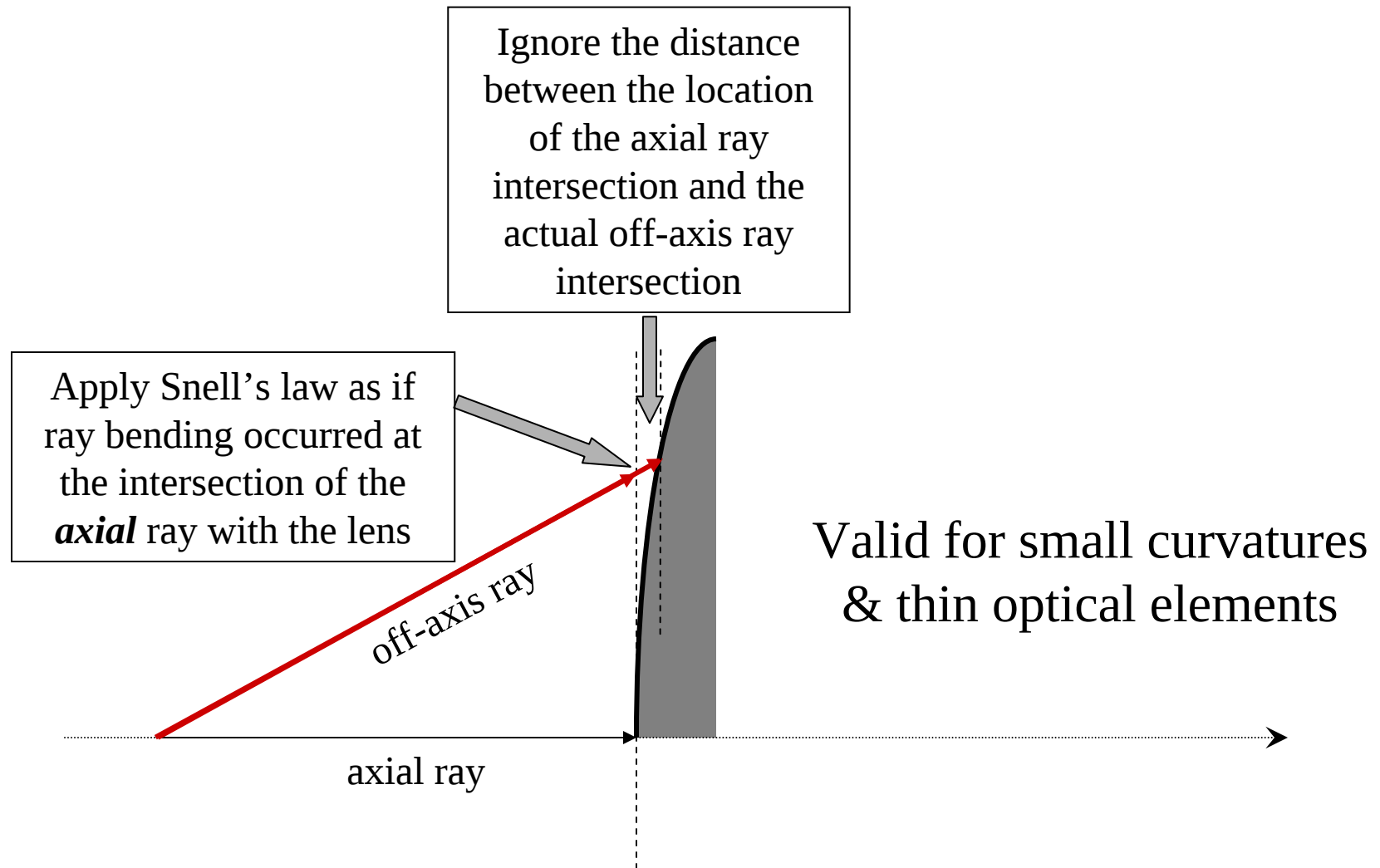
$$\sin \varepsilon \approx \varepsilon \approx \tan \varepsilon \quad \cos \varepsilon \approx 1$$

$$\sqrt{1 + \varepsilon} \approx 1 + \frac{1}{2} \varepsilon$$

where ε is the angle between a ray and the optical axis, and is a small number ($\varepsilon \ll 1$ rad). The range of validity of this approximation typically extends up to ~ 10 - 30 degrees, depending on the desired degree of accuracy. This regime is also known as “Gaussian optics” or “paraxial optics.”

Note the assumption of existence of an optical axis (*i.e.*, perfect alignment!)

Paraxial approximation /2



Refraction at spherical surface

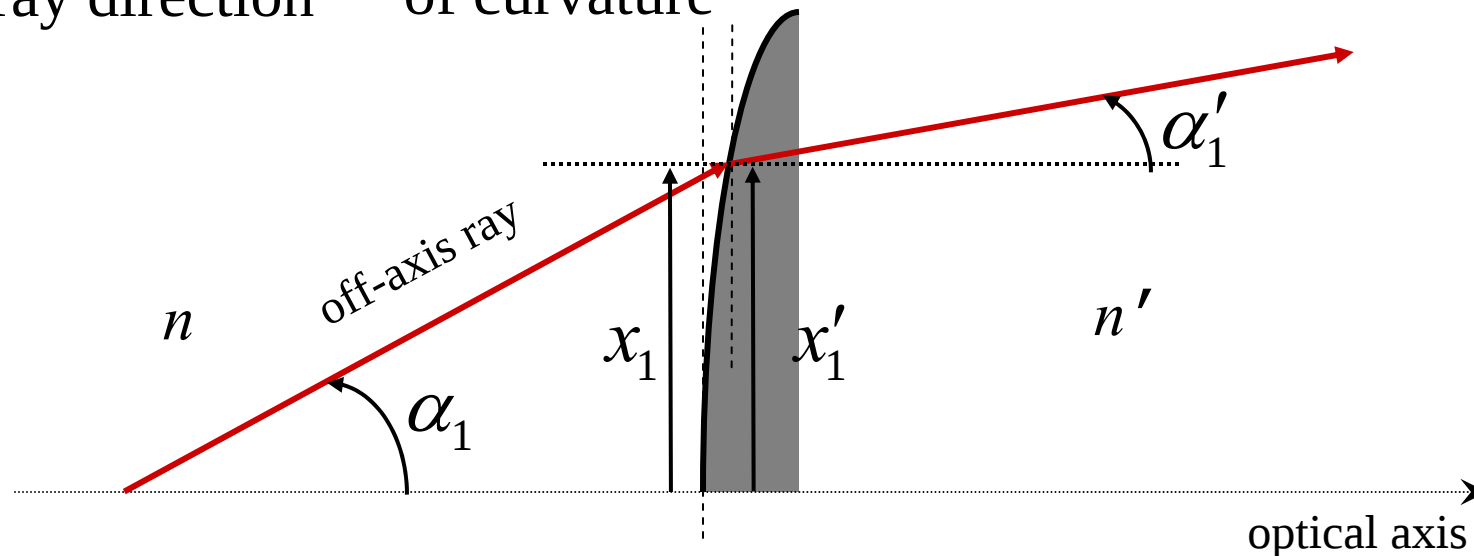
Refraction at positive spherical surface:
$$\begin{cases} x'_1 = x_1 \\ \alpha'_1 = \frac{n}{n'} \alpha_1 - \left[\frac{(n' - n)}{n'R} \right] x_1 \end{cases}$$

x : ray height

R : radius

α : ray direction

of curvature



Refraction at spherical surface

Refraction at positive spherical surface:
$$\begin{cases} x'_1 = x_1 \\ \alpha'_1 = \frac{n}{n'} \alpha_1 - \left[\frac{(n' - n)}{n'R} \right] x_1 \end{cases}$$

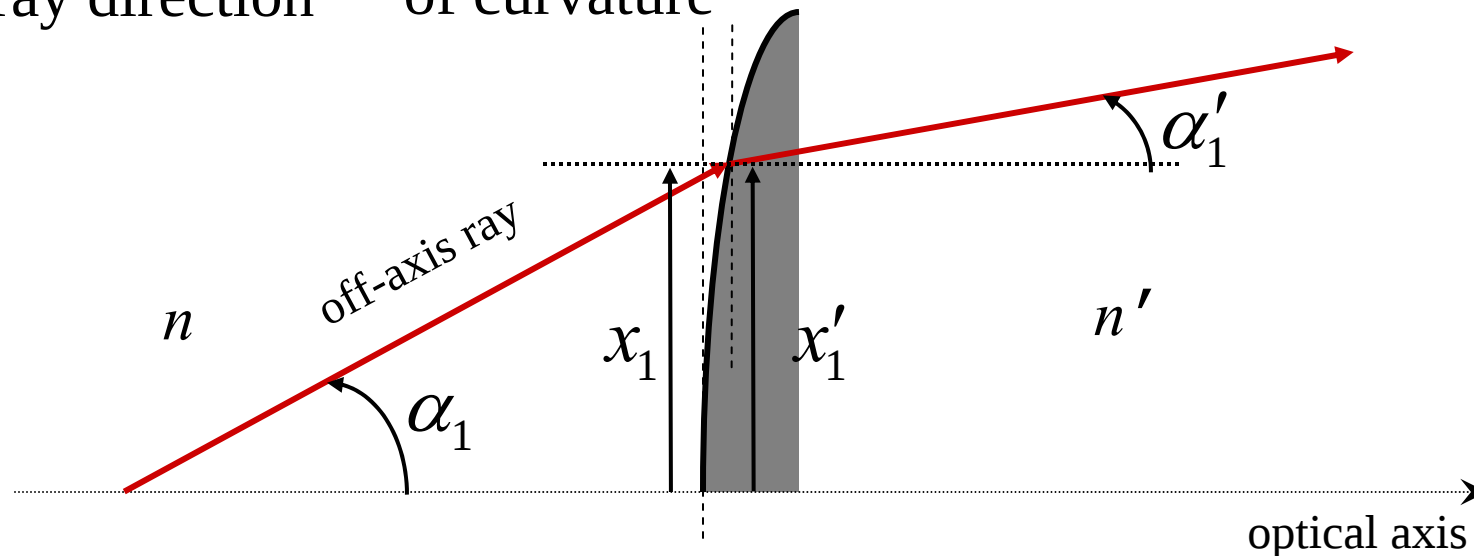
x : ray height

R : radius

α : ray direction

of curvature

Power



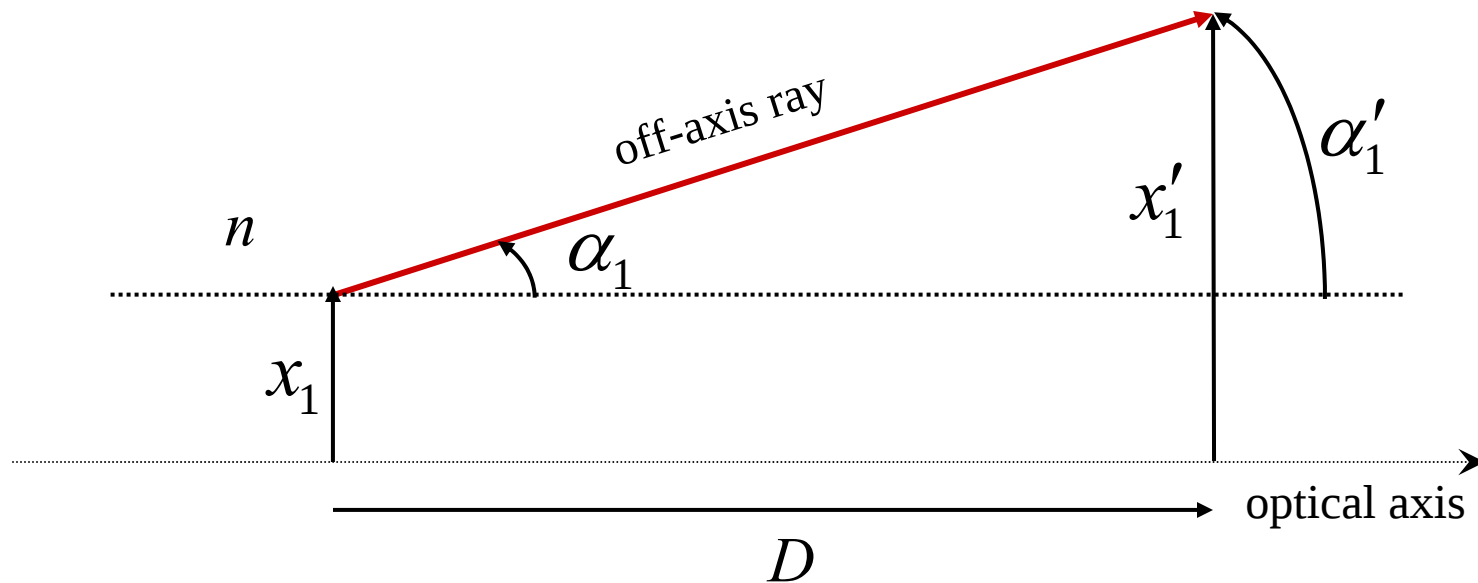
Propagation in uniform space

Propagation through distance D :

$$\begin{cases} x'_1 = x_1 + D\alpha_1 \\ \alpha'_1 = \alpha_1 \end{cases}$$

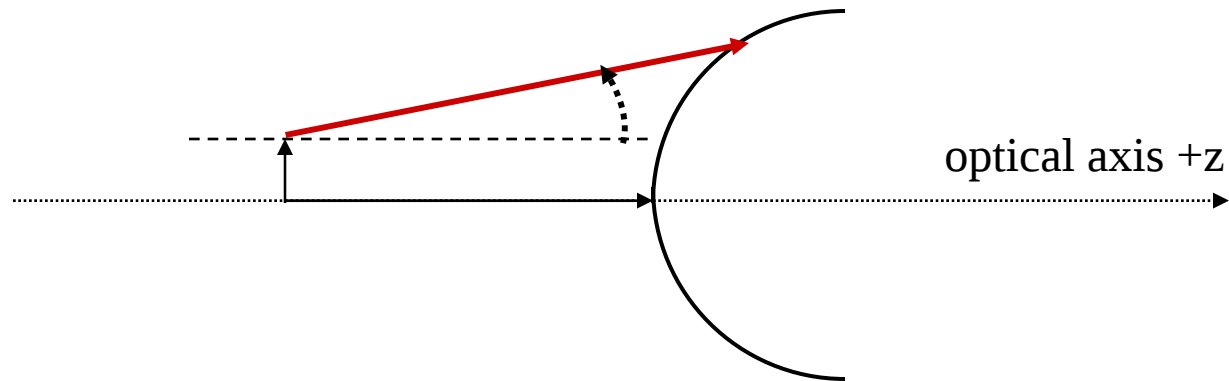
x : ray height

α : ray direction



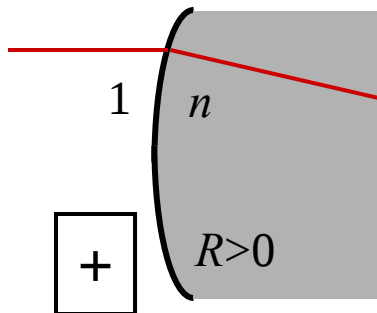
Sign conventions for refraction

- Light travels from left to right
- A radius of curvature is positive if the surface is convex towards the left
- Longitudinal distances are positive if pointing to the right
- Lateral distances are positive if pointing up
- Ray angles are positive if the ray direction is obtained by rotating the $+z$ axis counterclockwise through an acute angle



The power of surfaces

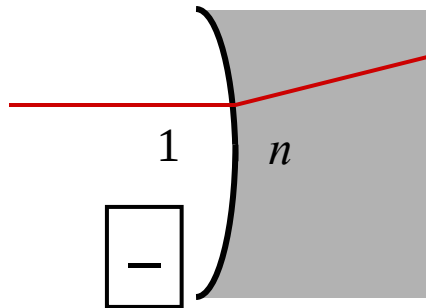
- Positive power bends rays “inwards” (*converging* bundle)



Simple spherical
refractor (positive)

$$P \equiv \frac{(n_{\text{right}} - n_{\text{left}})}{R} = \frac{n - 1}{R} \frac{(+)}{(+)} > 0$$

- Negative power bends rays “outwards” (*diverging* bundle)

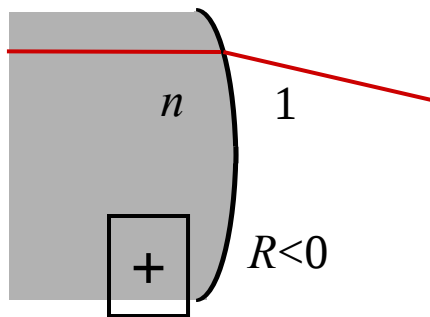


Simple spherical
refractor (negative)

$$P \equiv \frac{(n_{\text{right}} - n_{\text{left}})}{R} = \frac{n - 1}{R} \frac{(+)}{(-)} < 0$$

The power of surfaces

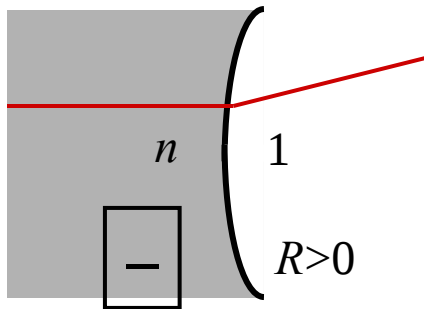
- Positive power bends rays “inwards” (*converging* bundle)



Simple spherical
refractor (positive)

$$P \equiv \frac{(n_{\text{right}} - n_{\text{left}})}{R} = \frac{1 - n \text{ } (-)}{R \text{ } (-)} > 0$$

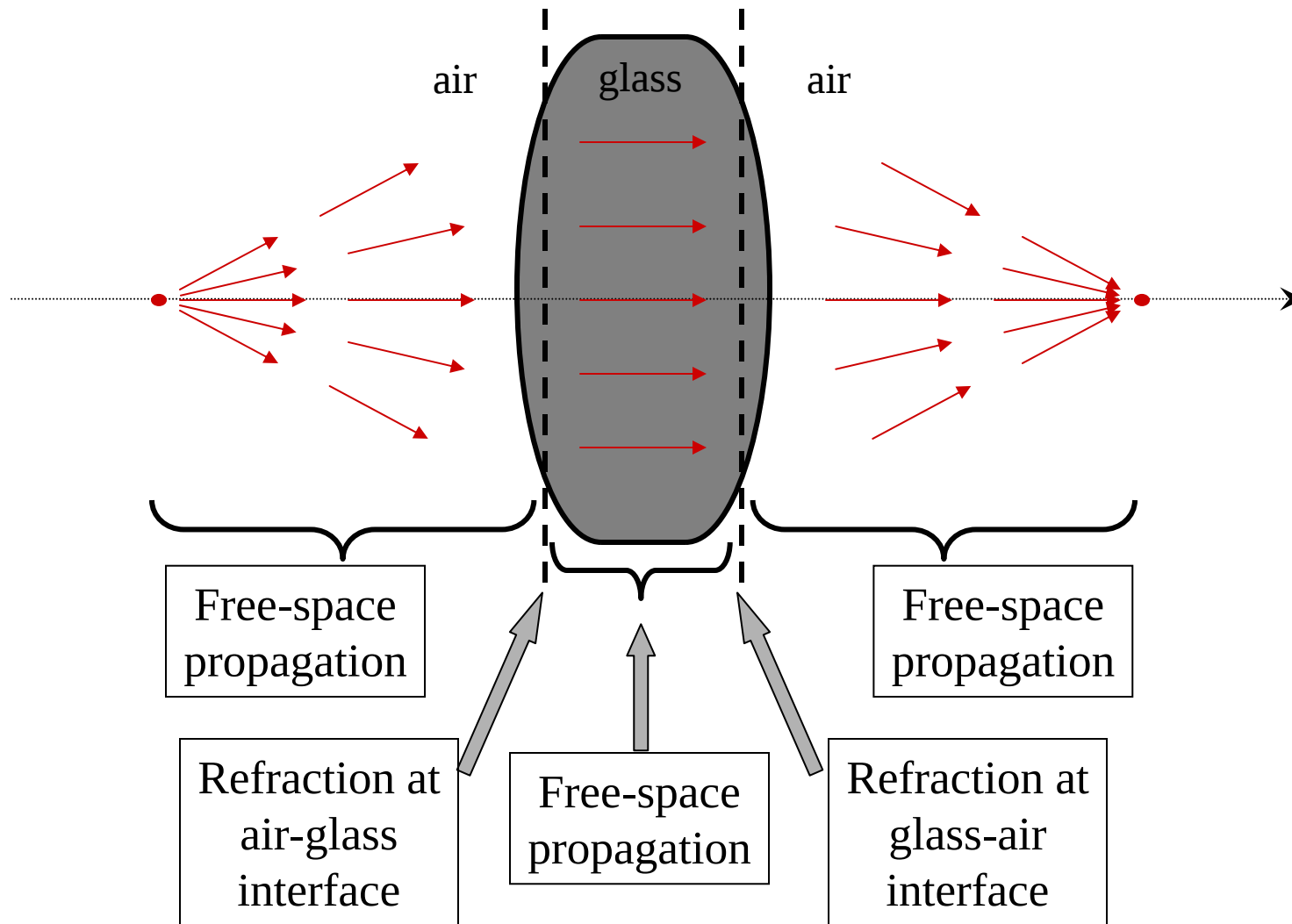
- Negative power bends rays “outwards” (*diverging* bundle)



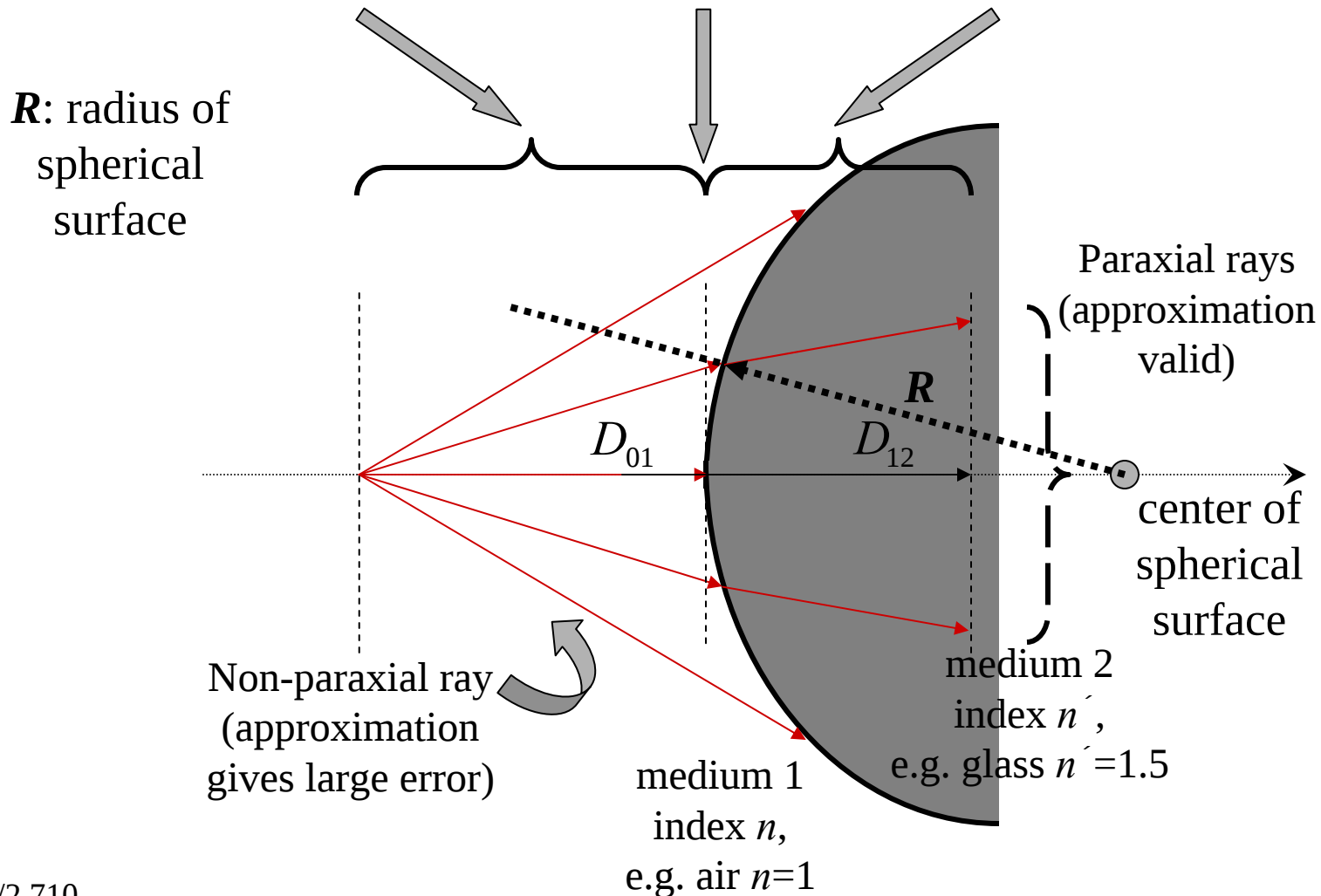
Simple spherical
refractor (negative)

$$P \equiv \frac{(n_{\text{right}} - n_{\text{left}})}{R} = \frac{1 - n \text{ } (-)}{R \text{ } (+)} < 0$$

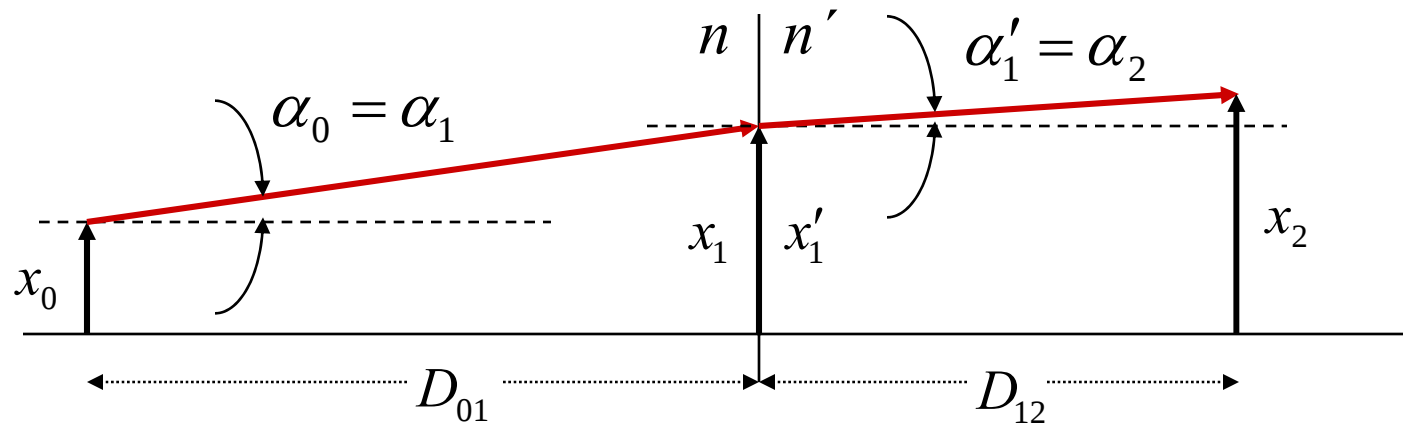
Paraxial ray-tracing



Example: one spherical surface, translation+refraction+translation



Translation+refraction+translation /1

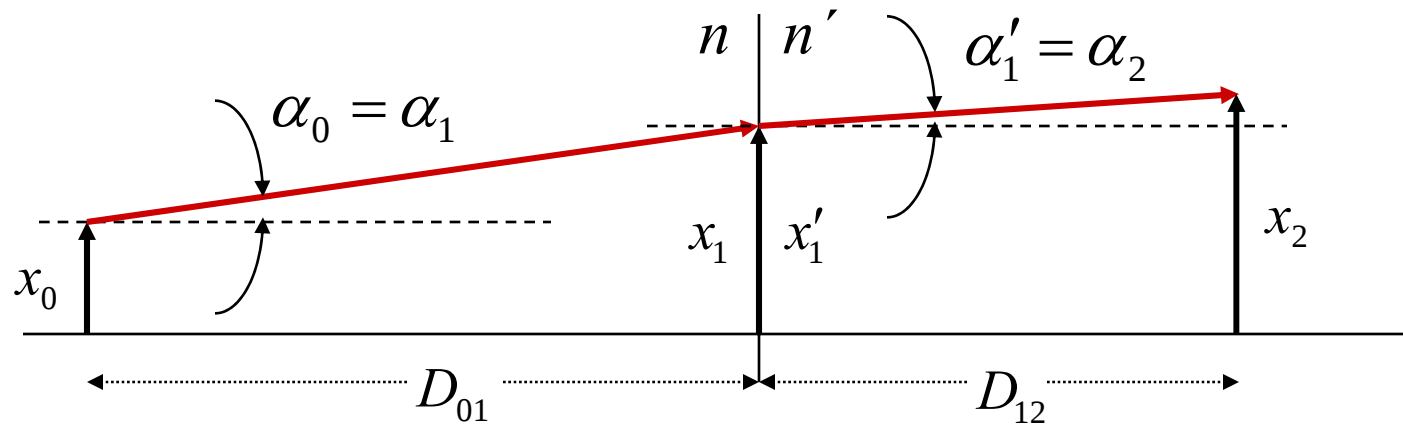


Starting ray: location x_0 direction α_0

Translation through distance D_{01} (+ direction):
$$\begin{cases} x_1 = x_0 + D_{01}\alpha_0 \\ \alpha_1 = \alpha_0 \end{cases}$$

Refraction at positive spherical surface:
$$\begin{cases} x'_1 = x_1 \\ \alpha'_1 = \frac{n}{n'}\alpha_1 - \left[\frac{(n' - n)}{n'R} \right] x_1 \end{cases}$$

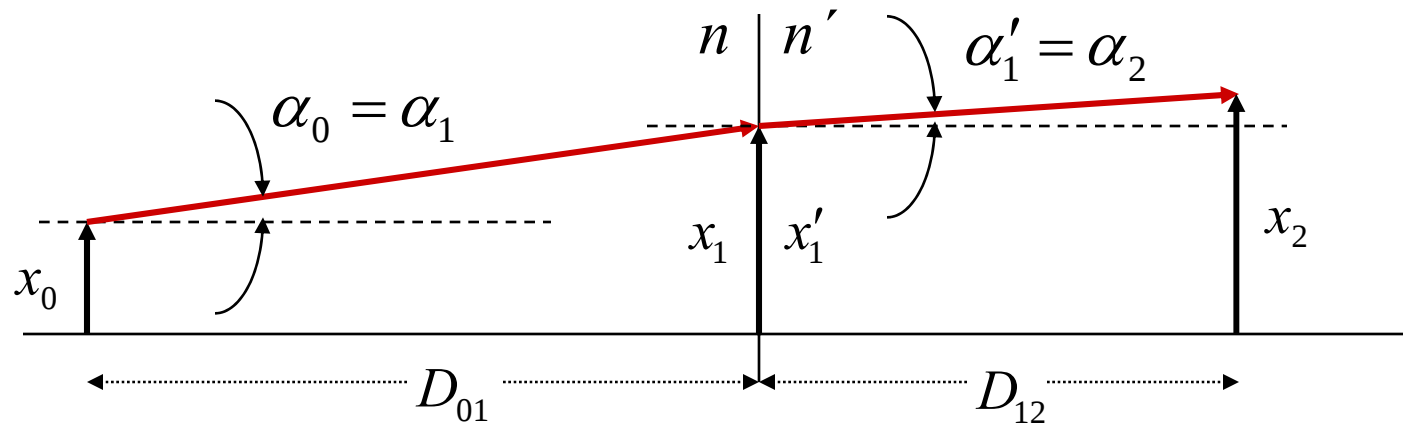
Translation+refraction+translation /2



Translation through distance D_{12} (+ direction):
$$\begin{cases} x_2 = x_1 + D_{12}\alpha'_1 \\ \alpha_2 = \alpha'_1 \end{cases}$$

Put together:

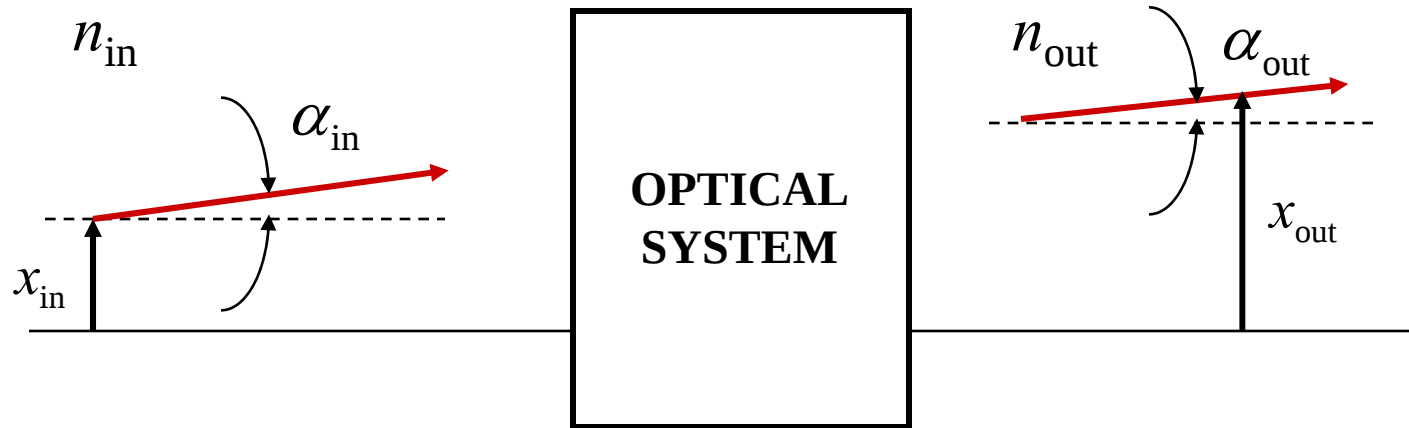
Translation+refraction+translation /3



$$\alpha_2 = \left[\frac{n}{n'} + \frac{(n - n')D_{01}}{n'R} \right] \alpha_0 + \left[\frac{n - n'}{n'R} \right] x_0$$

$$x_2 = \left[D_{01} + \frac{nD_{12}}{n'} + \frac{(n - n')D_{01}D_{12}}{n'R} \right] \alpha_0 + \left[\frac{(n - n')D_{12}}{n'R} + 1 \right] x_0$$

(Paraxial) Ray-tracing in general



For an arbitrary ray entering with

- lateral departure displacement x_{in} wrt the optical axis
- angle of departure α_{in} wrt the optical axis
- in a departure space of refractive index n_{in}

determine the ray's

- lateral arrival displacement x_{out} wrt the optical axis
- angle of arrival α_{out} wrt the optical axis
- in an arrival space of refractive index n_{out}

upon exiting the optical system

Matrix formulation /1

$$x_1 = x_0 + D_{01}\alpha_0$$

$$\alpha_1 = \alpha_0$$

translation by
distance D_{01}

$$x'_1 = x_1$$

$$\alpha'_1 = \frac{n}{n'}\alpha_1 + \left[\frac{(n - n')}{n'R} \right] x_1$$

refraction by
surface with radius
of curvature R

$$x_2 = m_x x_0$$

$$\alpha_2 = -\frac{1}{f'} x_0 + m_\alpha \alpha_0$$

ray-tracing
object-image
transformation

$$\alpha_{\text{out}} = M_{11}\alpha_{\text{in}} + M_{12}x_{\text{in}}$$

$$x_{\text{out}} = M_{21}\alpha_{\text{in}} + M_{22}x_{\text{in}}$$

form common to all

Matrix formulation /2

$$\begin{aligned} \alpha_{\text{out}} &= M_{11}\alpha_{\text{in}} + M_{12}x_{\text{in}} \\ x_{\text{out}} &= M_{21}\alpha_{\text{in}} + M_{22}x_{\text{in}} \end{aligned} \quad \longrightarrow \quad \begin{pmatrix} n_{\text{out}}\alpha_{\text{out}} \\ x_{\text{out}} \end{pmatrix} = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} \begin{pmatrix} n_{\text{in}}\alpha_{\text{in}} \\ x_{\text{in}} \end{pmatrix}$$

Refraction by spherical surface

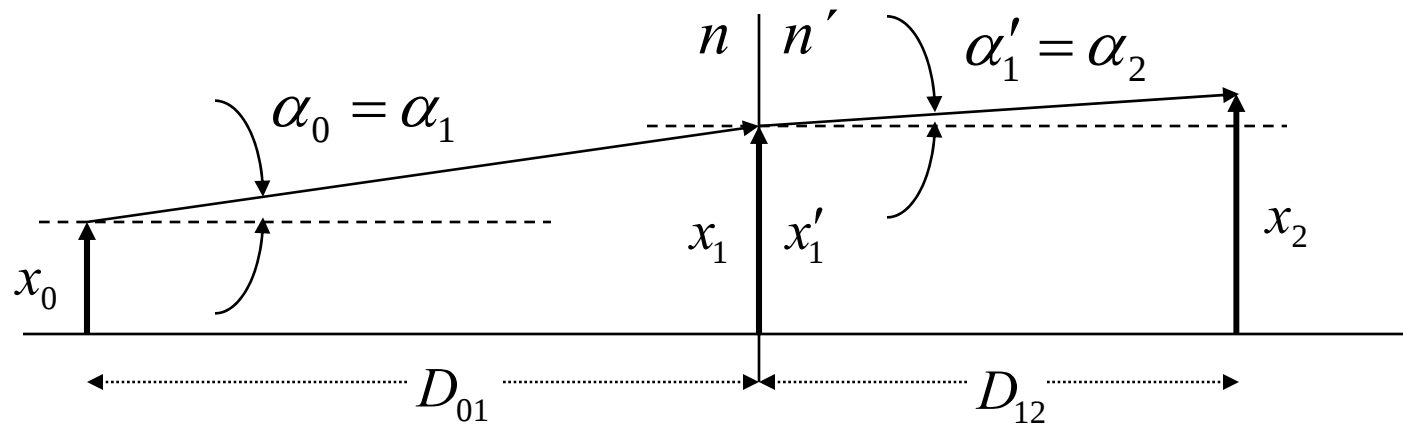
$$\begin{aligned} x'_1 &= x_1 \\ \alpha'_1 &= \frac{n}{n'}\alpha_1 + \left[\frac{(n-n')}{n'R} \right] x_1 \end{aligned} \quad \begin{pmatrix} n'\alpha'_1 \\ x'_1 \end{pmatrix} = \begin{pmatrix} 1 & -\frac{n'-n}{R} \\ 0 & 1 \end{pmatrix} \begin{pmatrix} n\alpha_1 \\ x_1 \end{pmatrix}$$

Power

Translation through uniform medium

$$\begin{aligned} x_1 &= x_0 + D_{01}\alpha_0 \\ \alpha_1 &= \alpha_0 \end{aligned} \quad \begin{pmatrix} n\alpha_1 \\ x_1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ \frac{D_{01}}{n} & 1 \end{pmatrix} \begin{pmatrix} n\alpha_0 \\ x_0 \end{pmatrix}$$

Translation+refraction+translation



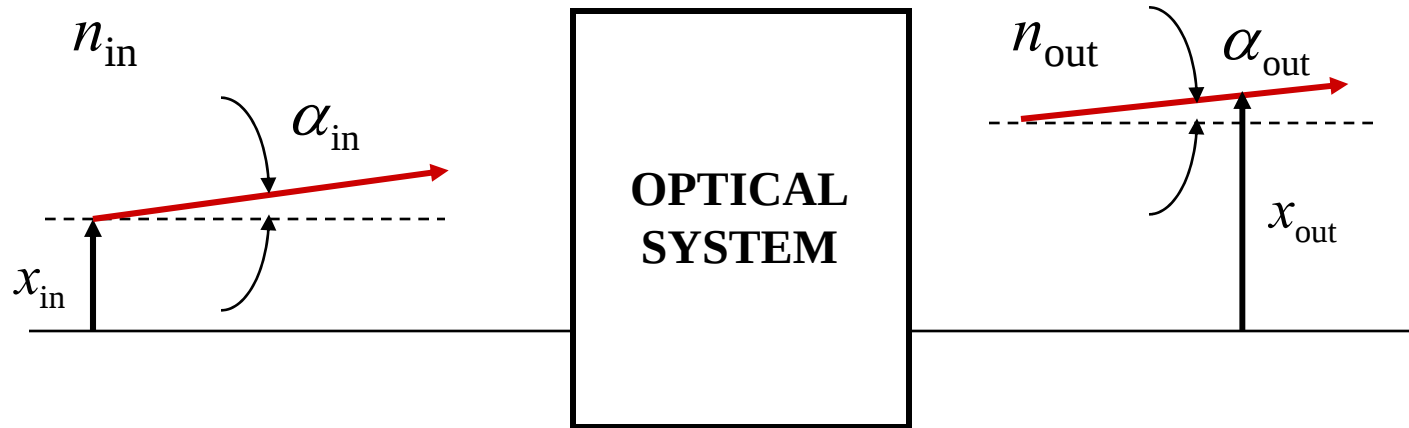
$$\begin{pmatrix} n' \alpha_2 \\ x_2 \end{pmatrix} = \begin{pmatrix} \text{translation} \\ \text{by } D_{12} \end{pmatrix} \begin{pmatrix} \text{refraction} \\ \text{by r.curv. } R \end{pmatrix} \begin{pmatrix} \text{translation} \\ \text{by } D_{01} \end{pmatrix} \begin{pmatrix} n \alpha_0 \\ x_0 \end{pmatrix}$$

result...

$$n' \alpha_2 = \left[n + \frac{(n - n') D_{01}}{R} \right] \alpha_0 + \left[\frac{n - n'}{R} \right] x_0$$

$$x_2 = \left[D_{01} + \frac{n D_{12}}{n'} + \frac{(n - n') D_{01} D_{12}}{n' R} \right] \alpha_0 + \left[\frac{(n - n') D_{12}}{n' R} + 1 \right] x_0$$

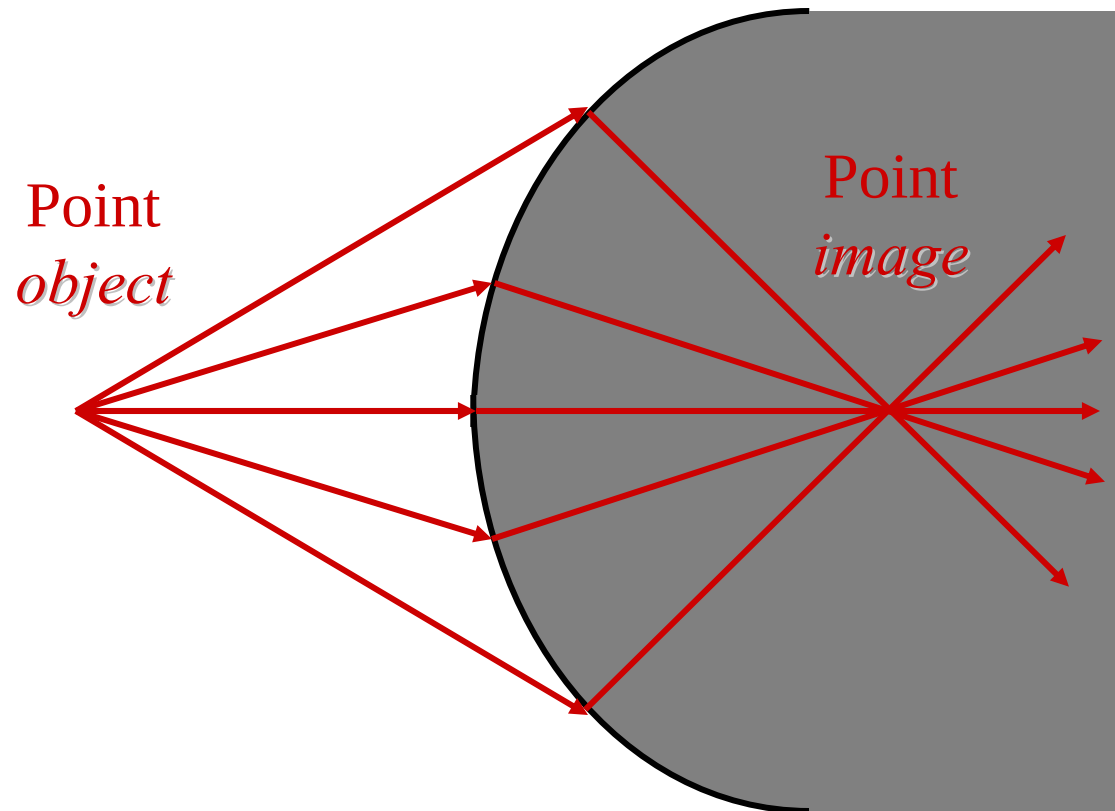
Matrix ray-tracing in general



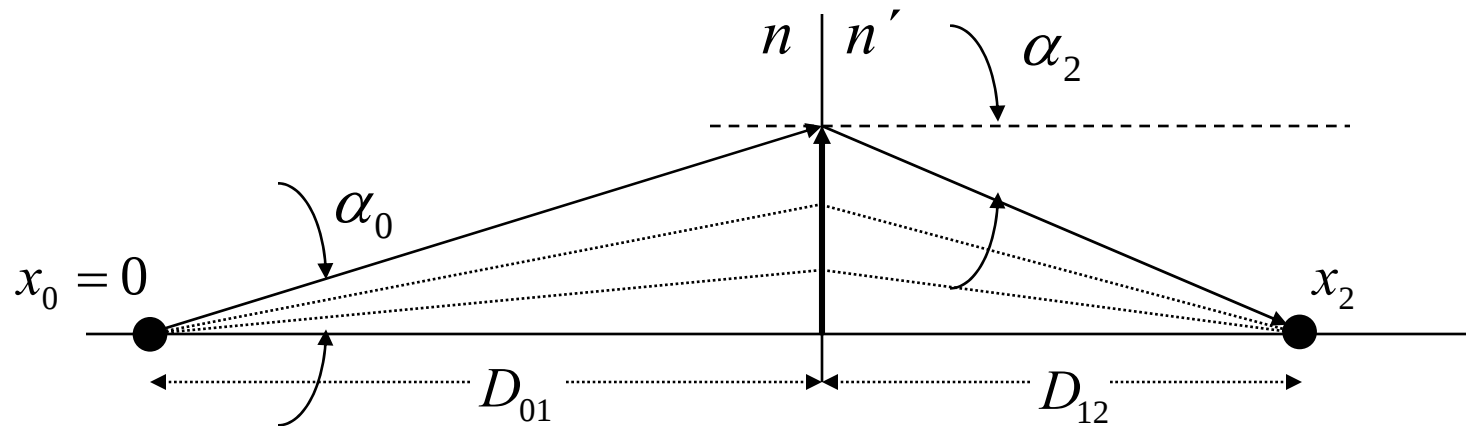
$$\begin{pmatrix} n_{\text{out}} \alpha_{\text{out}} \\ x_{\text{out}} \end{pmatrix} = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} \begin{pmatrix} n_{\text{in}} \alpha_{\text{in}} \\ x_{\text{in}} \end{pmatrix}$$

$\mathbf{M} = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix}$ $\longrightarrow$ is the “system matrix,” found as **the product, in reverse order**, of the matrices of the elements (free space, spherical surfaces) comprising the system

On-axis image formation



On-axis image formation

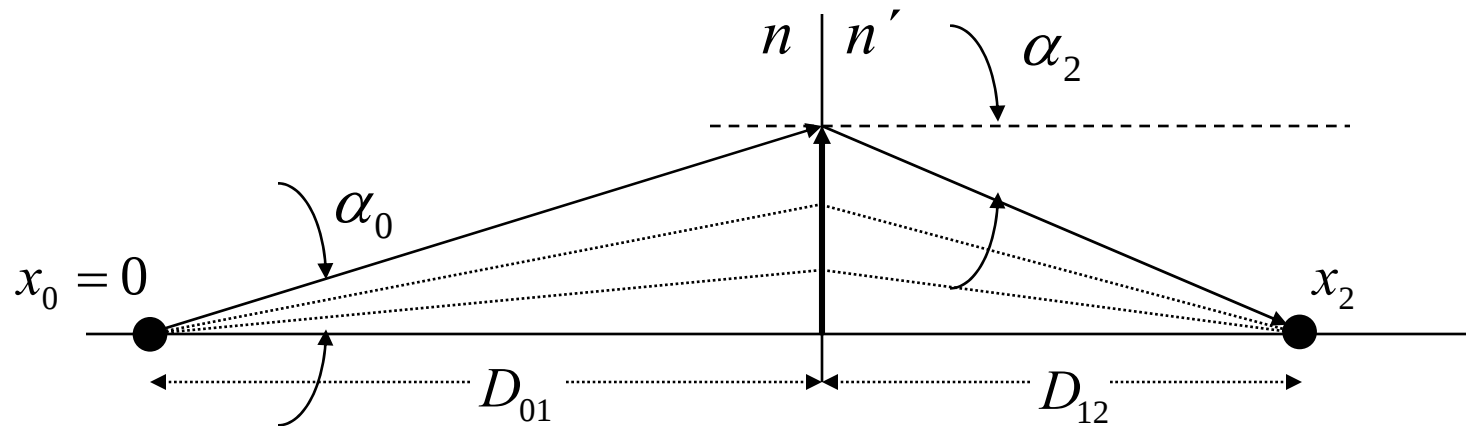


Imaging condition

All rays emanating at x_0 arrive at x_2 irrespective of departure angle α_0 $\Rightarrow \frac{\partial x_2}{\partial \alpha_0} = 0 \Leftrightarrow M_{21} = 0$

$$x_2 = \left[D_{01} + \frac{nD_{12}}{n'} - \frac{(n' - n)D_{01}D_{12}}{n'R} \right] \alpha_0 + [\dots] x_0$$

On-axis image formation



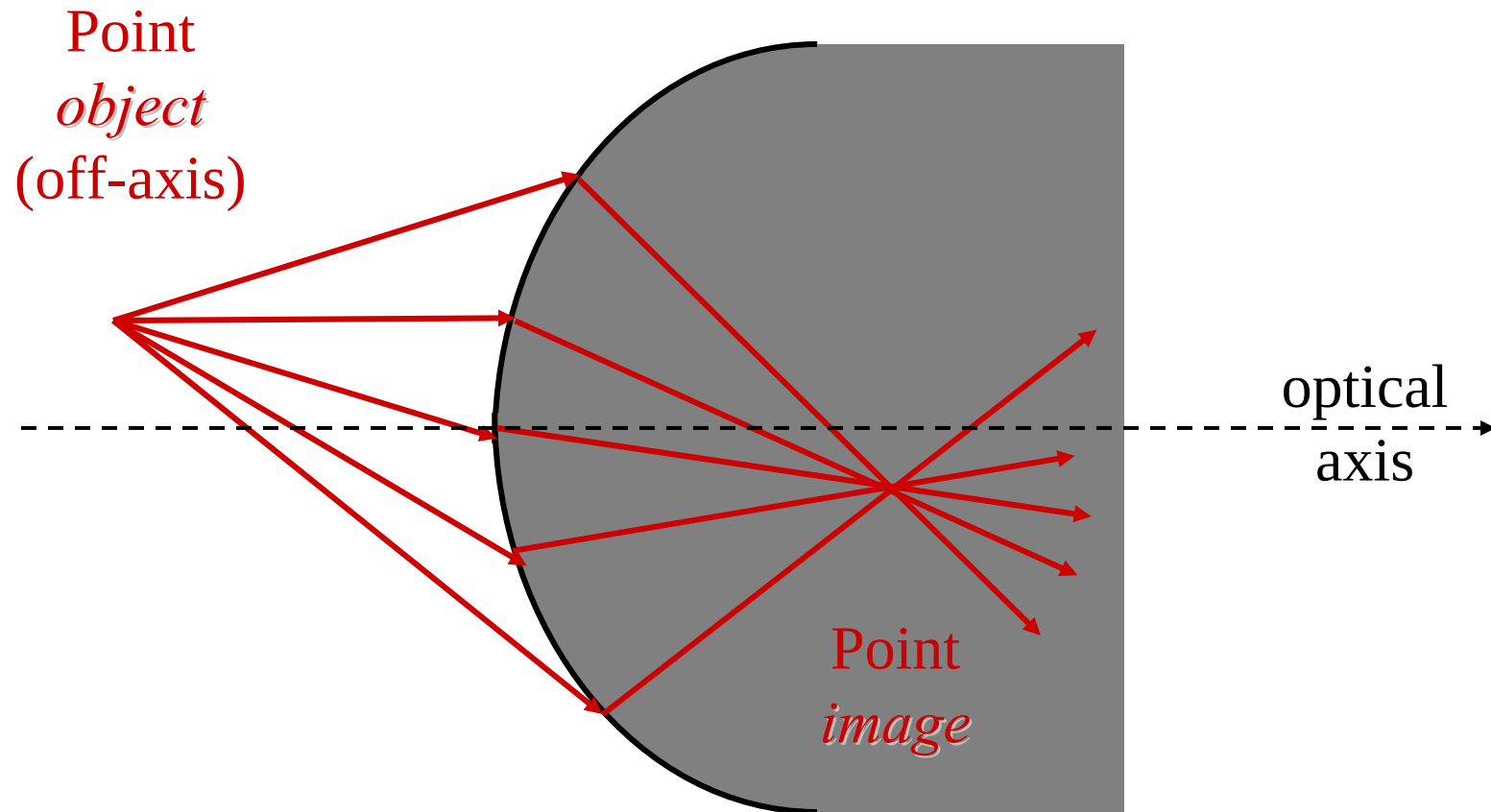
Imaging condition

All rays emanating at x_0 arrive at x_2 irrespective of departure angle α_0 $\Rightarrow \frac{\partial x_2}{\partial \alpha_0} = 0 \Leftrightarrow M_{21} = 0$

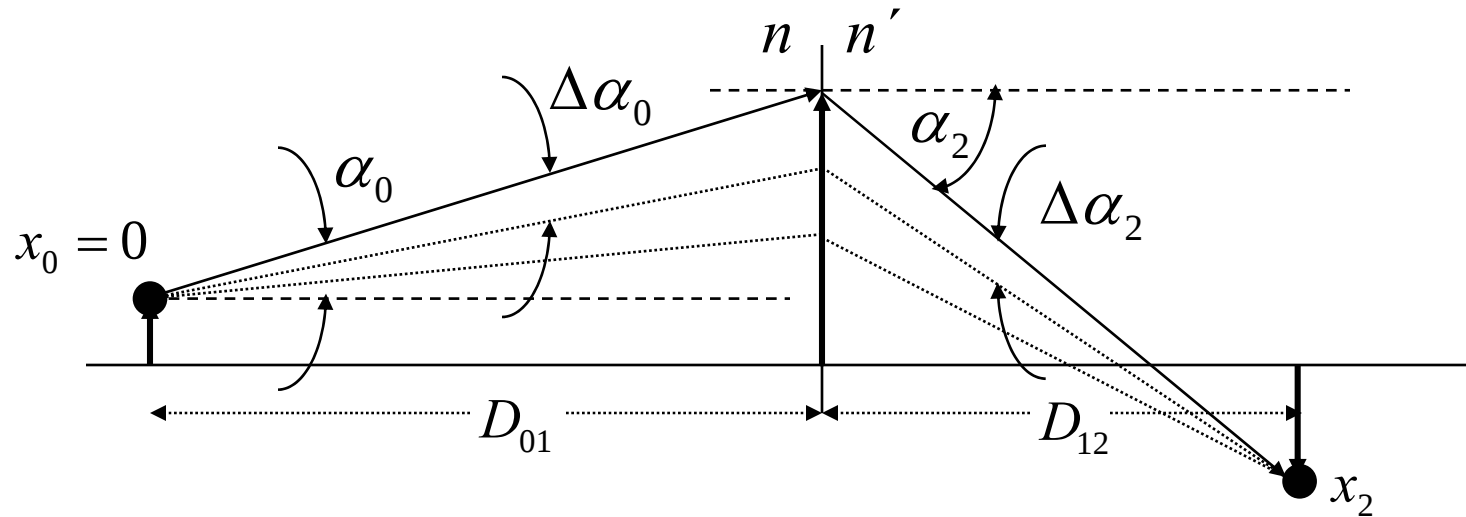
$$\Rightarrow \frac{n'}{D_{12}} + \frac{n}{D_{01}} = \frac{n' - n}{R}$$

“Power” of the spherical surface [units: diopters, $1\text{D} = 1\text{ m}^{-1}$]

Off-axis image formation



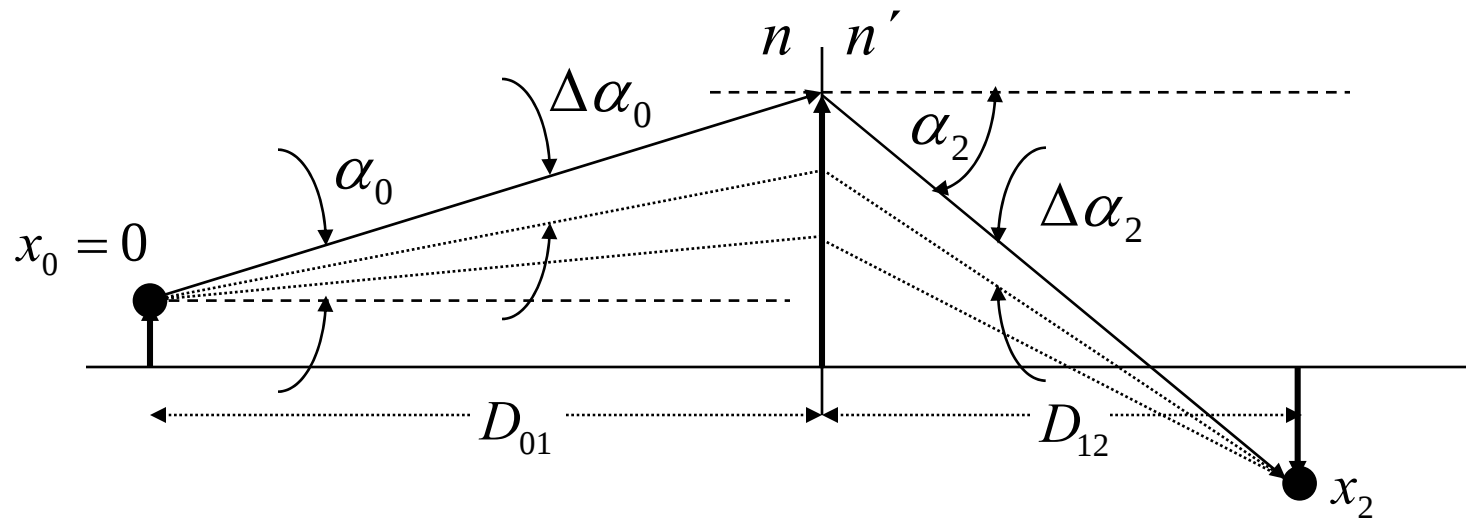
Magnification: lateral (off-axis), angle



Lateral
$$m_x = \frac{x_2}{x_0} = \frac{n - n'}{R} \frac{D_{12}}{n'} + 1 = \dots = -\frac{n}{n'} \frac{D_{12}}{D_{01}}$$

Angle
$$m_\alpha = \frac{\Delta\alpha_2}{\Delta\alpha_0} = -\frac{D_{01}}{D_{12}}$$

Object-image transformation



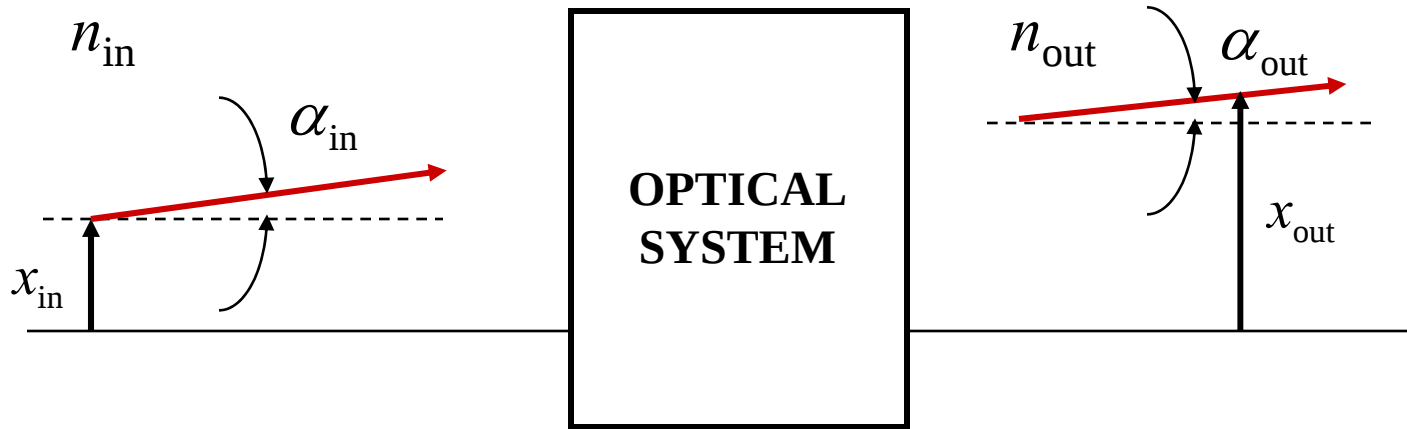
$$\alpha_2 = m_\alpha \alpha_0 - \frac{1}{f'} x_0$$

$$x_2 = m_x x_0$$

Paraxial ray-tracing
transformation between
object and image points

subject to $M_{21} = 0$ (imaging condition)

Matrix ray-tracing: summary



$$\begin{pmatrix} n_{\text{out}} \alpha_{\text{out}} \\ x_{\text{out}} \end{pmatrix} = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} \begin{pmatrix} n_{\text{in}} \alpha_{\text{in}} \\ x_{\text{in}} \end{pmatrix}$$

- Power M_{12}
- Imaging condition $M_{21} = 0$
- Lateral magnification $M_{21}|_{M_{21}=0}$
- Angular magnification $M_{11}|_{M_{21}=0}$

Image of point object at infinity

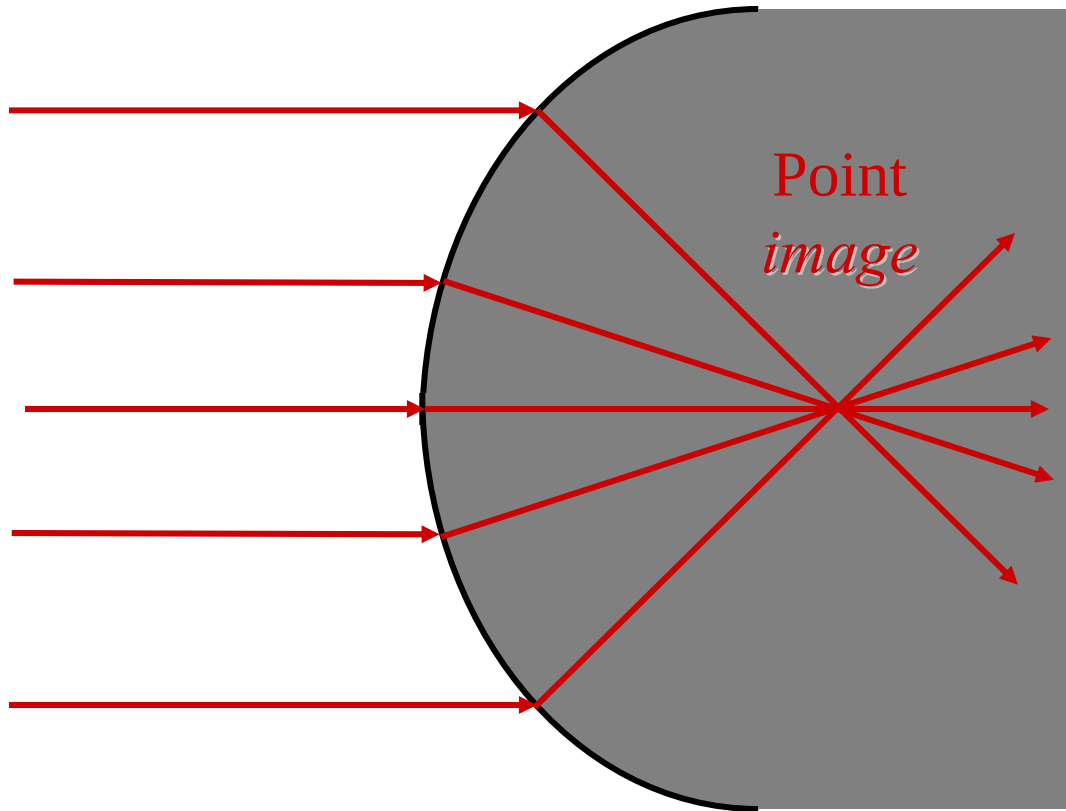
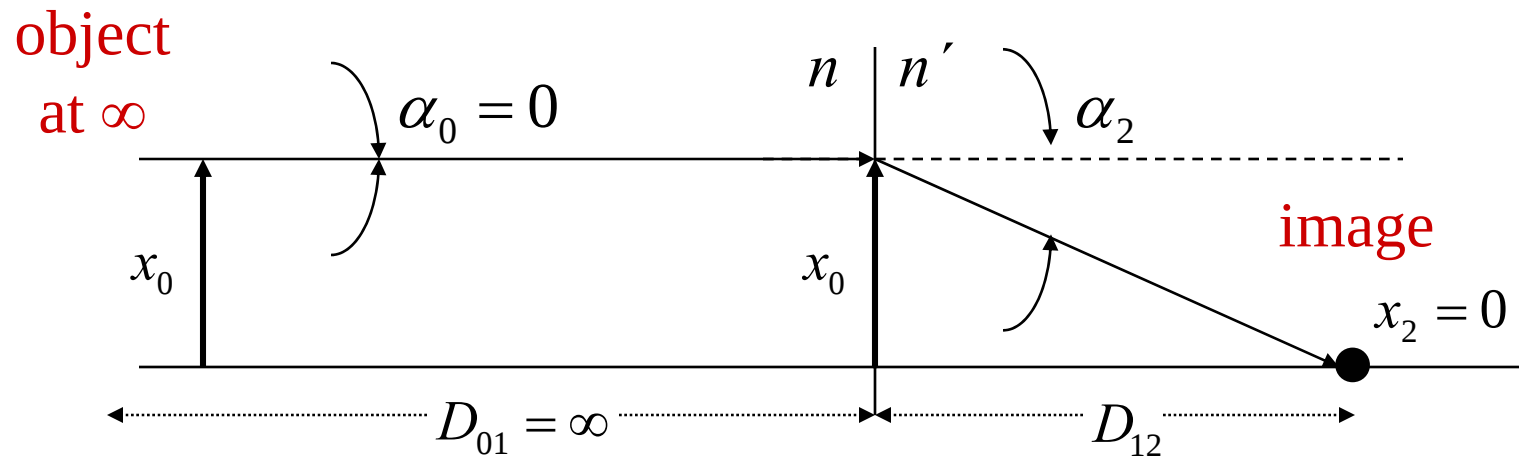


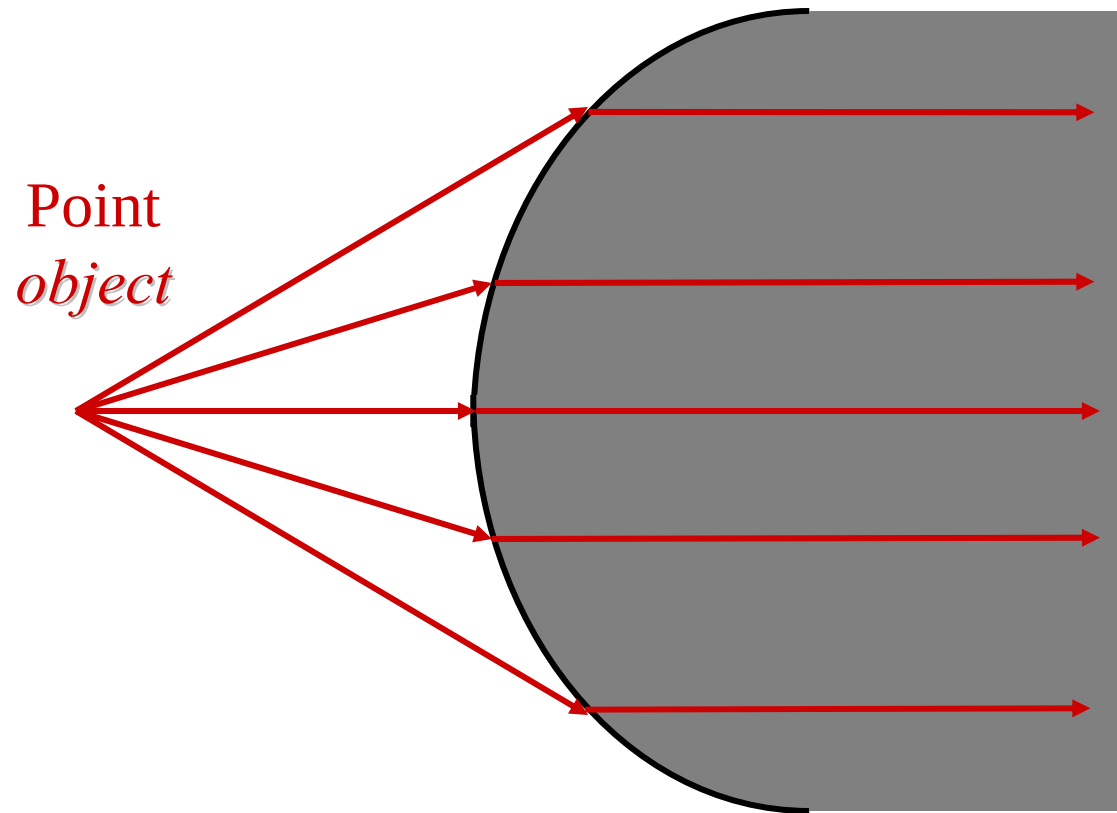
Image of point object at infinity



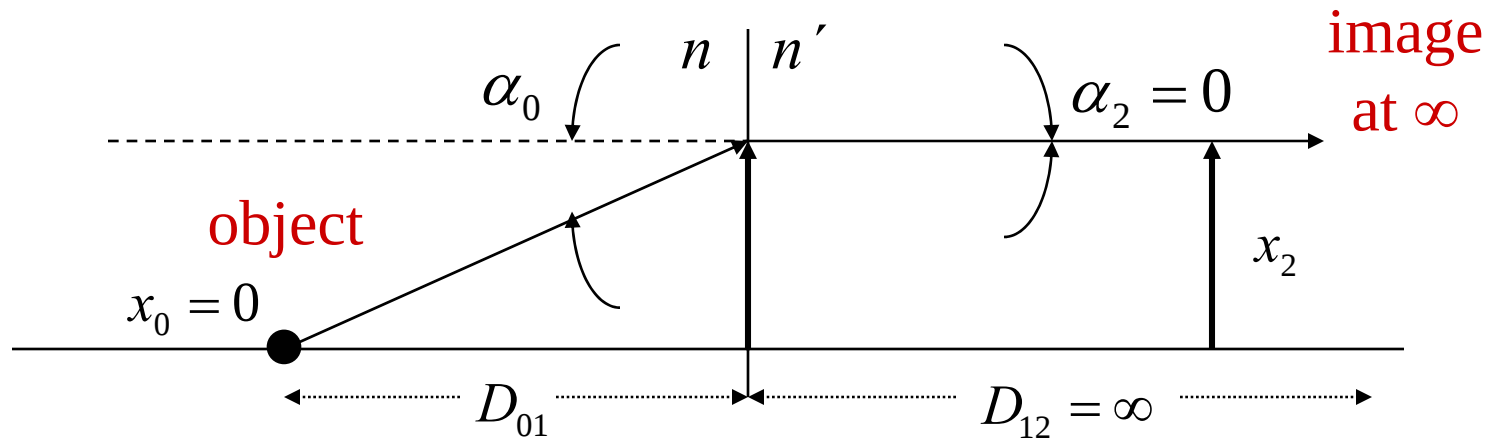
$$\frac{n'}{D_{12}} = \frac{n' - n}{R} \Rightarrow D_{12} = \frac{n'R}{n' - n} \equiv f' : \text{image focal length}$$

Note: $f' = n' \times \frac{R}{n' - n} \equiv \text{ambient refractive index at space of point image} \times \frac{R}{n' - n} \text{ 1/Power}$

Point object imaged at infinity



Point object imaged at infinity



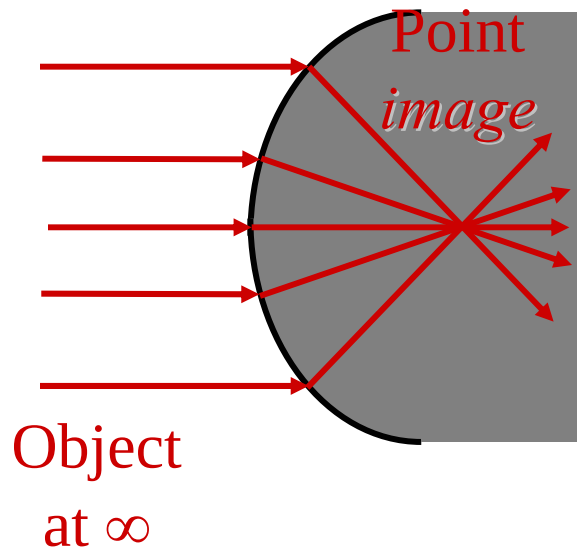
$$\frac{n}{D_{01}} = \frac{n' - n}{R} \Rightarrow D_{01} = \frac{n R}{n' - n} \equiv f : \text{object focal length}$$

Note: $f = n \times \frac{R}{n' - n} \equiv$ n $\times$ $\frac{R}{n' - n}$

ambient refractive index
at space of point object

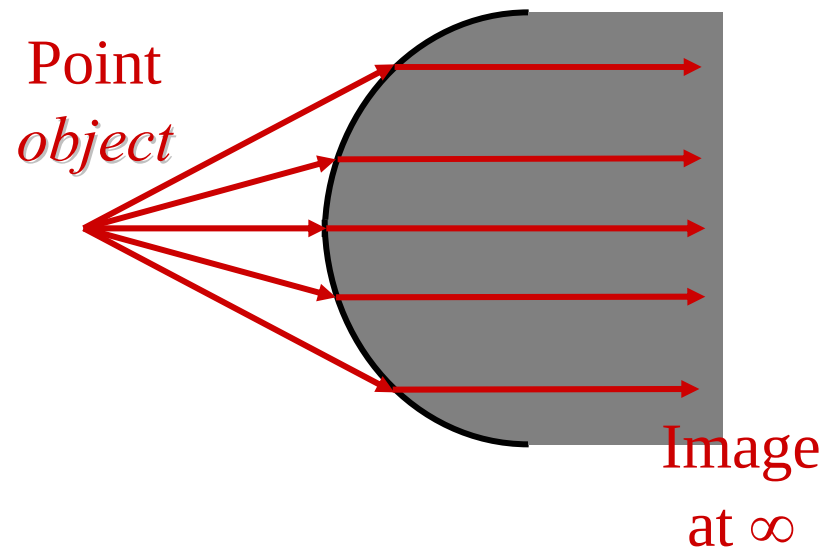
1/Power

Image / object focal lengths



$$f' = n' \times \frac{R}{n' - n}$$

$$= n' \times \frac{1}{\text{Power}}$$



$$f = n \times \frac{R}{n' - n}$$

$$= n \times \frac{1}{\text{Power}}$$