Lenses and Imaging (Part II)

- Review from Part I
- Thin lens
  - matrix ray-tracing
  - imaging condition
  - real and virtual images
- Thick lens
- Generalized optical systems:
  - principal planes
  - effective, front and back focal lengths
- Reflective optics

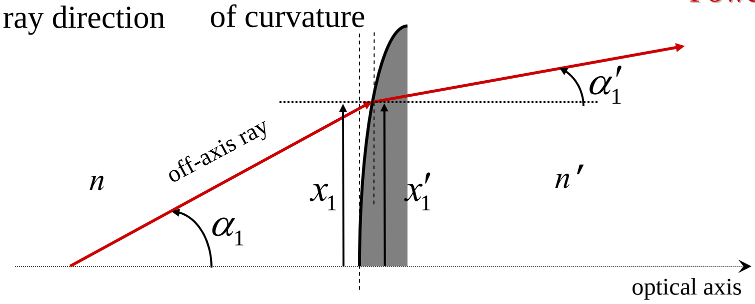
Refraction at spherical surface

Refraction at positive spherical surface:
$$\begin{cases} x_1' = x_1 \\ \alpha_1' = \dfrac{n}{n'}\alpha_1 - \left[\dfrac{(n'-n)}{n'R}\right]x_1 \end{cases}$$

Power

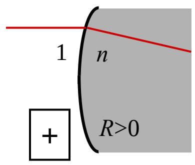
x: ray height
α: ray direction

R: radius
of curvature



The power of surfaces

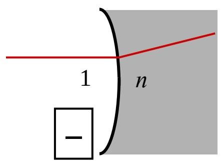
- Positive power bends rays “inwards” (*converging* bundle)



Simple spherical refractor (positive)

$$P \equiv \frac{\left(n_{\text{right}} - n_{\text{left}}\right)}{R} = \frac{n-1}{R} \frac{(+)}{(+)} > 0$$

- Negative power bends rays “outwards” (*diverging* bundle)

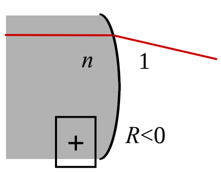


Simple spherical refractor (negative)

$$P \equiv \frac{\left(n_{\text{right}} - n_{\text{left}}\right)}{R} = \frac{n-1}{R} \frac{(+)}{(-)} < 0$$

The power of surfaces

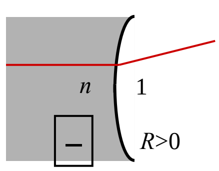
- Positive power bends rays “inwards” (*converging* bundle)



Simple spherical refractor (positive)

$$P \equiv \frac{\left(n_{\text{right}} - n_{\text{left}}\right)}{R} = \frac{1-n}{R} \frac{(-)}{(-)} > 0$$

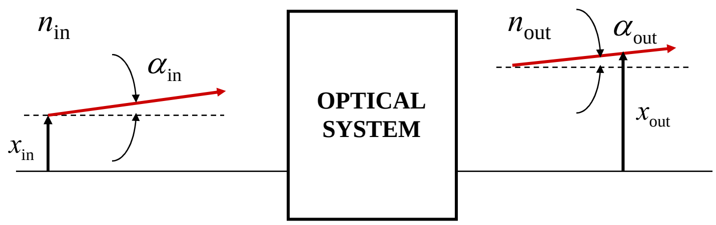
- Negative power bends rays “outwards” (*diverging* bundle)



Simple spherical refractor (negative)

$$P \equiv \frac{\left(n_{\text{right}} - n_{\text{left}}\right)}{R} = \frac{1-n}{R} \frac{(-)}{(+)} < 0$$

(Paraxial) Ray-tracing in general



For an arbitrary ray entering with

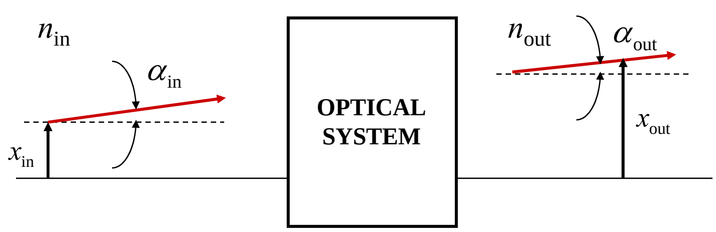
- lateral departure displacement x_{in} wrt the optical axis
- angle of departure α_{in} wrt the optical axis
- in a departure space of refractive index n_{in}

determine the ray's

- lateral arrival displacement x_{out} wrt the optical axis
- angle of arrival α_{out} wrt the optical axis
- in an arrival space of refractive index n_{out}

upon exiting the optical system

Matrix ray-tracing in general

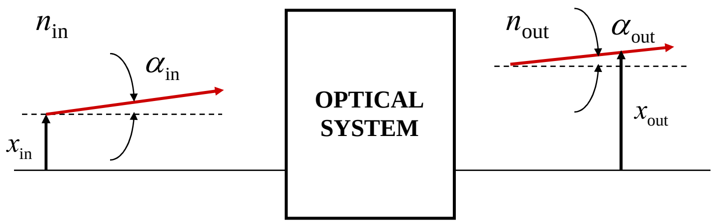


$$\begin{pmatrix} n_{out}\alpha_{out} \\ x_{out} \end{pmatrix} = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} \begin{pmatrix} n_{in}\alpha_{in} \\ x_{in} \end{pmatrix}$$

$$\mathbf{M} = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix}$$

is the "system matrix," found as **the product, in reverse order,** of the matrices of the elements (free space, spherical surfaces) comprising the system

Matrix ray-tracing



$$\begin{pmatrix} n_{\text{out}}\alpha_{\text{out}} \\ x_{\text{out}} \end{pmatrix} = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} \begin{pmatrix} n_{\text{in}}\alpha_{\text{in}} \\ x_{\text{in}} \end{pmatrix}$$

- Power M_{12}
- Imaging condition $M_{21} = 0$
- Lateral magnification $M_{21}\big|_{M_{21}=0}$
- Angular magnification $M_{11}\big|_{M_{21}=0}$

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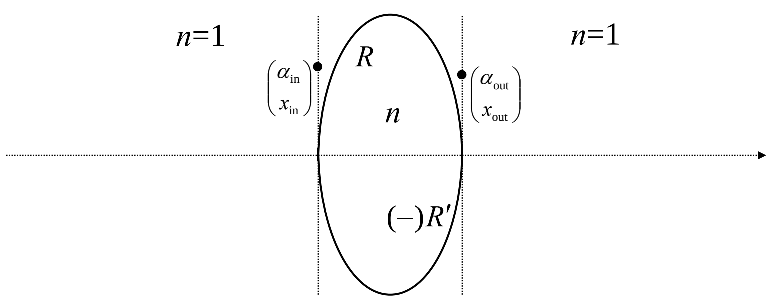
Thin lens in air



$$\begin{pmatrix} \alpha_{\text{out}} \\ x_{\text{out}} \end{pmatrix} = \begin{pmatrix} ? & ? \\ ? & ? \end{pmatrix} \begin{pmatrix} \alpha_{\text{in}} \\ x_{\text{in}} \end{pmatrix}$$

Objective: specify input-output relationship

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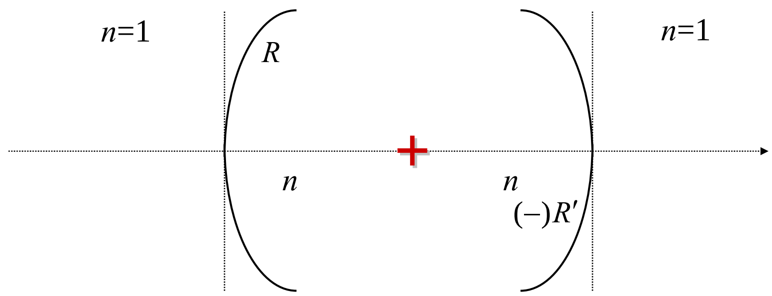
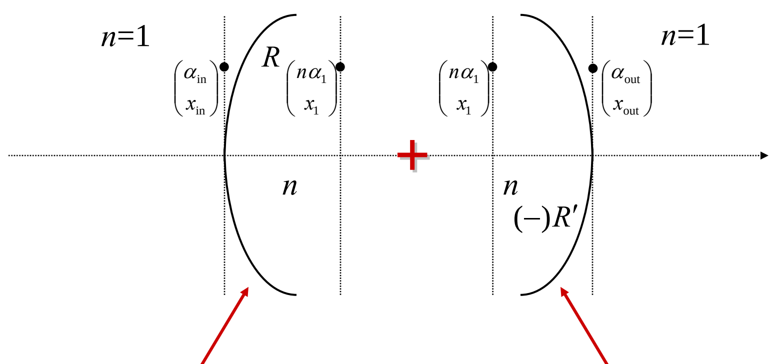
Thin lens in air



Model: refraction from first (positive) surface
+ refraction from second (negative) surface

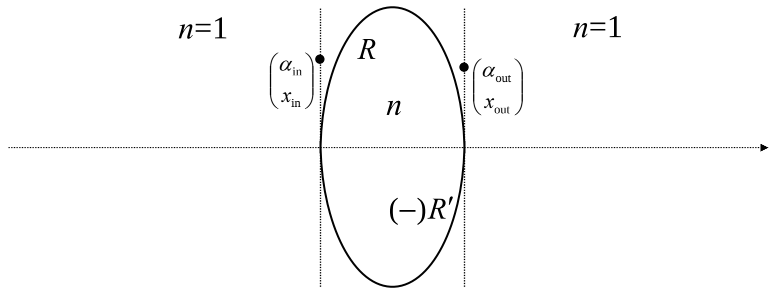
Ignore space in-between (*thin* lens approx.)

Thin lens in air



$$\begin{pmatrix} n\alpha_1 \\ x_1 \end{pmatrix} = \begin{pmatrix} 1 & -\dfrac{n-1}{R} \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \alpha_{\text{in}} \\ x_{\text{in}} \end{pmatrix} \qquad \begin{pmatrix} \alpha_{\text{out}} \\ x_{\text{out}} \end{pmatrix} = \begin{pmatrix} 1 & -\dfrac{1-n}{R'} \\ 0 & 1 \end{pmatrix} \begin{pmatrix} n\alpha_1 \\ x_1 \end{pmatrix}$$

Thin lens in air



$$\begin{pmatrix}\alpha_{\text{out}}\\ x_{\text{out}}\end{pmatrix} = \begin{pmatrix}1 & -\dfrac{1-n}{R'}\\ 0 & 1\end{pmatrix}\begin{pmatrix}1 & -\dfrac{n-1}{R}\\ 0 & 1\end{pmatrix}\begin{pmatrix}\alpha_{\text{in}}\\ x_{\text{in}}\end{pmatrix}$$

$$\begin{pmatrix}\alpha_{\text{out}}\\ x_{\text{out}}\end{pmatrix} = \begin{pmatrix}1 & -(n-1)\left(\dfrac{1}{R}-\dfrac{1}{R'}\right)\\ 0 & 1\end{pmatrix}\begin{pmatrix}\alpha_{\text{in}}\\ x_{\text{in}}\end{pmatrix}$$

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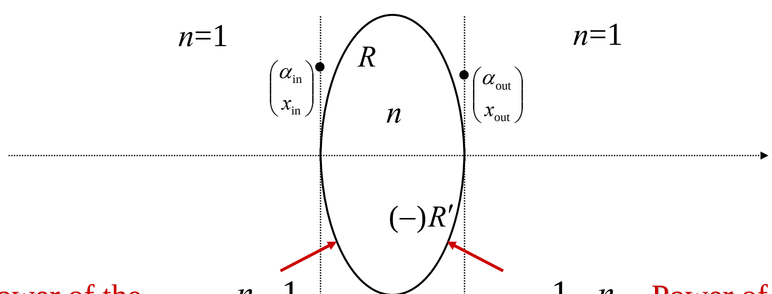
Thin lens in air



Power of the first surface $\quad P = \dfrac{n-1}{R}$

$P' = \dfrac{1-n}{R'}$ $\quad$ Power of the second surface

$$\begin{pmatrix}\alpha_{\text{out}}\\ x_{\text{out}}\end{pmatrix} = \begin{pmatrix}1 & -(n-1)\left(\dfrac{1}{R}-\dfrac{1}{R'}\right)\\ 0 & 1\end{pmatrix}\begin{pmatrix}\alpha_{\text{in}}\\ x_{\text{in}}\end{pmatrix}$$

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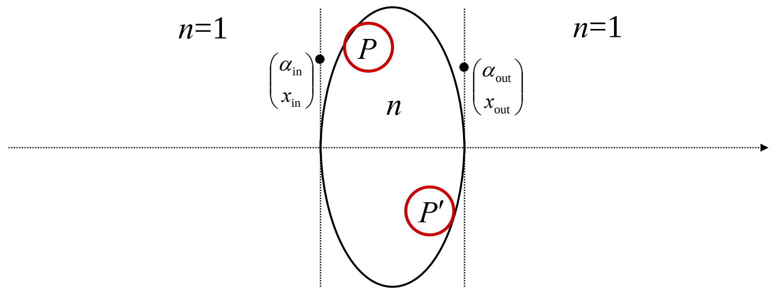
Thin lens in air

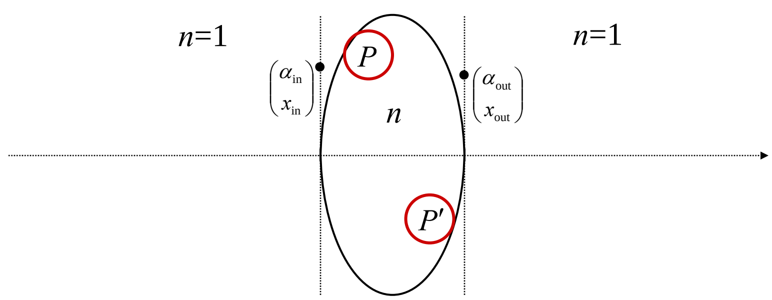


$$\begin{pmatrix}\alpha_{out}\\ x_{out}\end{pmatrix} = \begin{pmatrix}1 & -P_{\text{thin lens}}\\ 0 & 1\end{pmatrix}\begin{pmatrix}\alpha_{in}\\ x_{in}\end{pmatrix}$$

$$P_{\text{thin lens}} = P + P' = \frac{n-1}{R} + \frac{1-n}{R'} = \boxed{(n-1)\left(\frac{1}{R} - \frac{1}{R'}\right)}$$

Lens-maker's formula

Thin lens in air



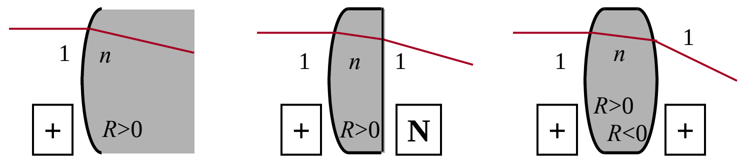
$$\begin{pmatrix}\alpha_{out}\\ x_{out}\end{pmatrix} = \begin{pmatrix}1 & -P_{\text{thin lens}}\\ 0 & 1\end{pmatrix}\begin{pmatrix}\alpha_{in}\\ x_{in}\end{pmatrix} \Rightarrow$$

$$\alpha_{out} = \alpha_{in} - P_{\text{thin lens}}\, x_{in}$$
$$x_{out} = x_{in}$$

Ray bending is proportional to the distance from the axis

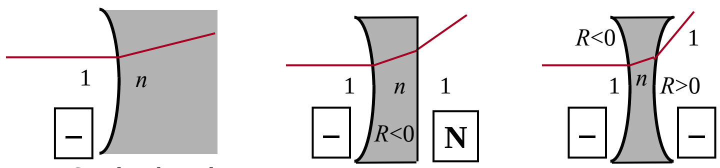
The power of lenses

- Positive power bends rays “inwards” (*converging* bundle)



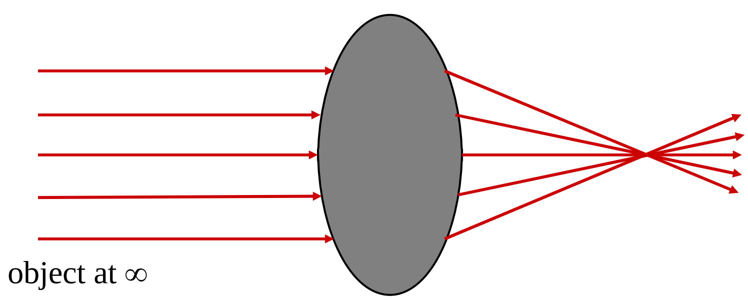
Simple spherical refractor (positive) | Plano-convex lens | Bi-convex lens

- Negative power bends rays “outwards” (*diverging* bundle)



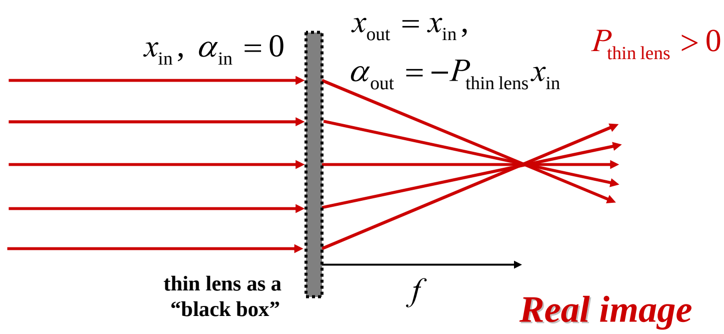
Simple spherical refractor (negative) | Plano-concave lens | Bi-concave lens

Positive thin lens in air



Ray bending is proportional to the distance from the axis

Positive thin lens in air



$$f = -\frac{x_{\text{in}}}{\alpha_{\text{out}}} \Rightarrow \boxed{f = \frac{1}{P_{\text{thin lens}}}}$$

Focal point = image of an object at ∞

Focal length = distance between lens & focal point

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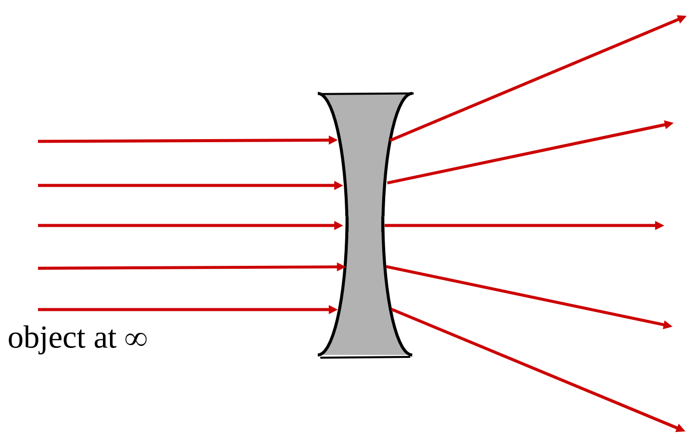
Negative thin lens in air



Ray bending is proportional
to the distance from the axis

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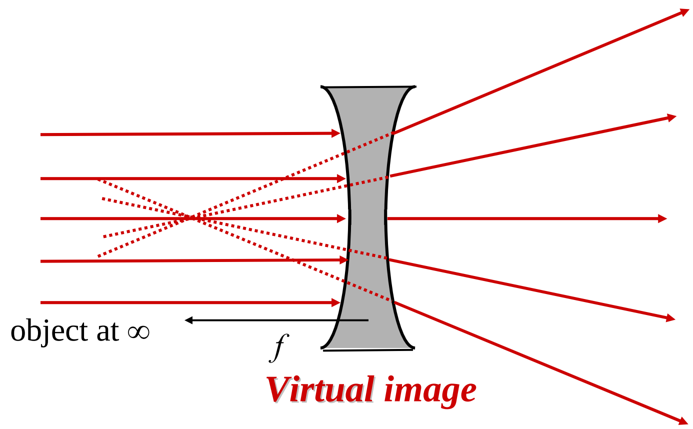
Negative thin lens in air



$$f = \frac{1}{P_{\text{thin lens}}}$$ still applies, now with $P_{\text{thin lens}} < 0 \Rightarrow$

$\Rightarrow \quad f < 0$ (to the *"left"*)

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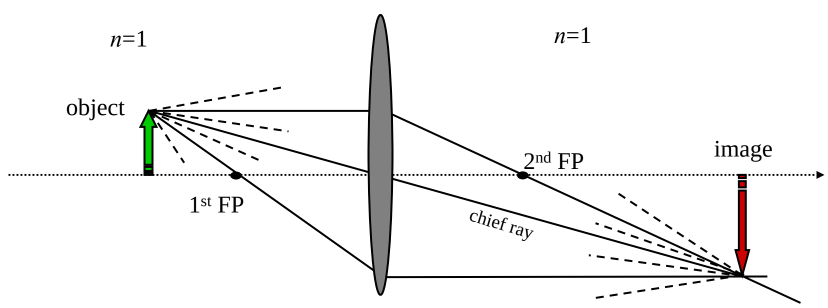
Imaging condition: ray-tracing



- Image point is located at the common intersection of **all** rays which emanate from the corresponding object point
- The two rays passing through the two focal points and the chief ray can be ray-traced directly

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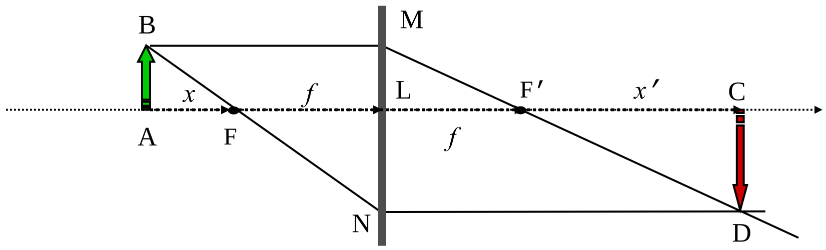
Imaging condition: ray-tracing



• (ABF)~(FLN) and (F′CD)~(MLF′) are pairs of similar triangles

$$\frac{(AB)}{(AF)} = \frac{(LN)}{(FL)} \qquad \frac{(LM)}{(LF')} = \frac{(CD)}{(F'C)} \qquad (AB) = (ML) \qquad (LN) = (CD)$$

$$(AF)\,(F'C) = (FL)\,(LF') \qquad xx' = f^{2}$$

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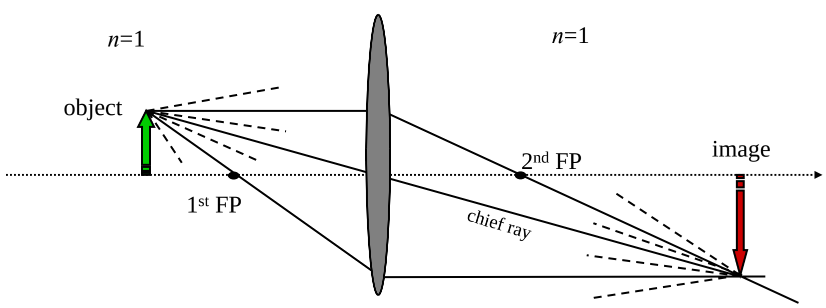
Imaging condition: matrix method



• Location of image point must be independent of ray departure angle at the object

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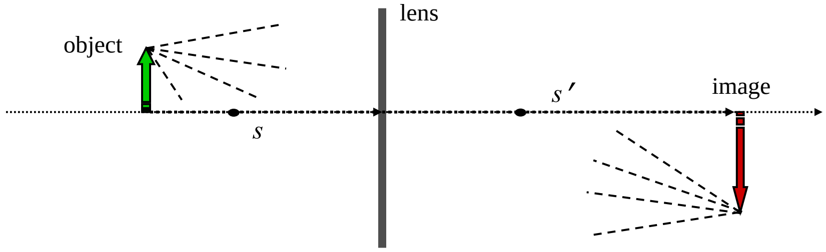
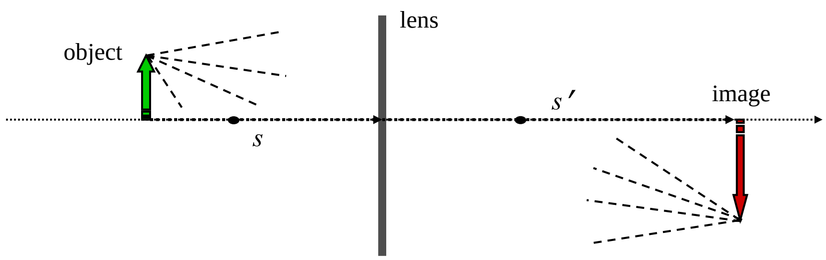
Imaging condition: matrix method



$$\begin{pmatrix}\alpha_{\mathrm{out}}\\ x_{\mathrm{out}}\end{pmatrix} = \begin{pmatrix}1 & 0\\ s' & 1\end{pmatrix}\begin{pmatrix}1 & -\frac{1}{f}\\ 0 & 1\end{pmatrix}\begin{pmatrix}1 & 0\\ s & 1\end{pmatrix}\begin{pmatrix}\alpha_{\mathrm{in}}\\ x_{\mathrm{in}}\end{pmatrix} = \begin{pmatrix}1-\frac{s}{f} & -\frac{1}{f}\\ \underbrace{s+s'-\frac{ss'}{f}}_{=0} & 1-\frac{s'}{f}\end{pmatrix}\begin{pmatrix}\alpha_{\mathrm{in}}\\ x_{\mathrm{in}}\end{pmatrix}$$

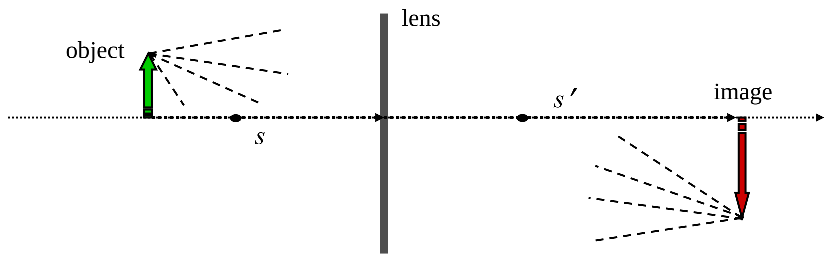
Imaging condition: matrix method



$$s+s'-\frac{ss'}{f}=0 \quad \Leftrightarrow \quad \boxed{\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}}$$

Imaging condition
(*aka* Lens Law)

Imaging condition: matrix method



$$\begin{pmatrix}\alpha_{\text{out}}\\ x_{\text{out}}\end{pmatrix} = \begin{pmatrix}1-\dfrac{s}{f} & -\dfrac{1}{f}\\ 0 & 1-\dfrac{s'}{f}\end{pmatrix}\begin{pmatrix}\alpha_{\text{in}}\\ x_{\text{in}}\end{pmatrix} \Rightarrow x_{\text{out}} = \left(1-\frac{s'}{f}\right)x_{\text{in}}$$

Lateral magnification :

$$M_{\text{T}} = \frac{x_{\text{out}}}{x_{\text{in}}}$$

$$M_{\text{T}} = 1-\frac{s'}{f} = -\frac{s'}{s}$$

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Real & virtual images

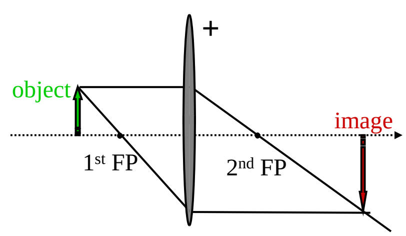

image: real & inverted; M_{T}<0

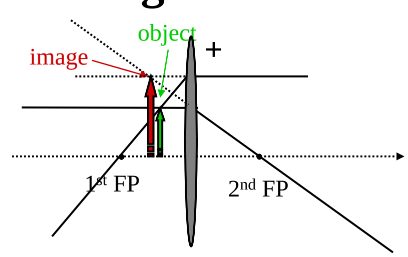

image: virtual & erect; M_{T}>1

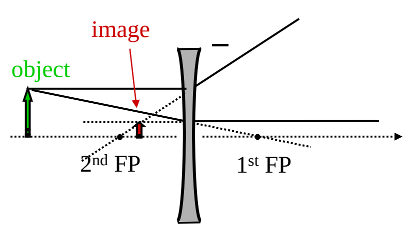

image: virtual & erect; 0<M_{T}<1

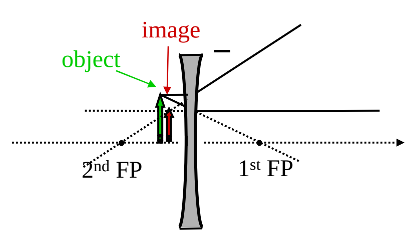

image: virtual & erect; 0<M_{T}<1

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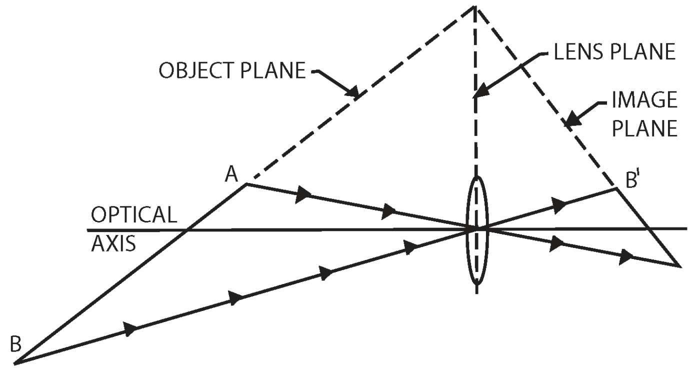
The Scheimpflug condition



The object plane and the image plane
intersect at the plane of the lens.

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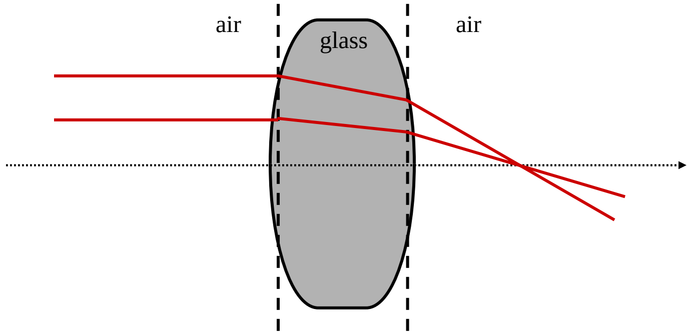
The thick lens



Rays bend in “two steps”

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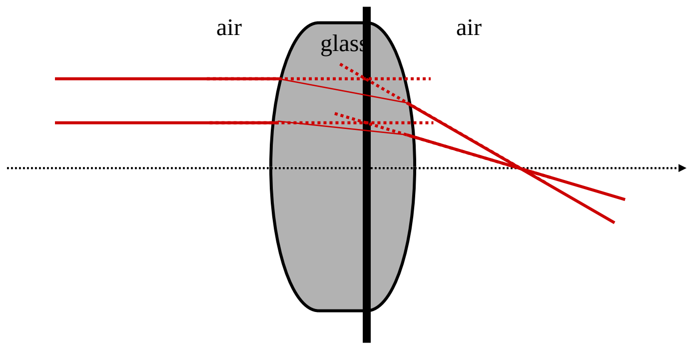
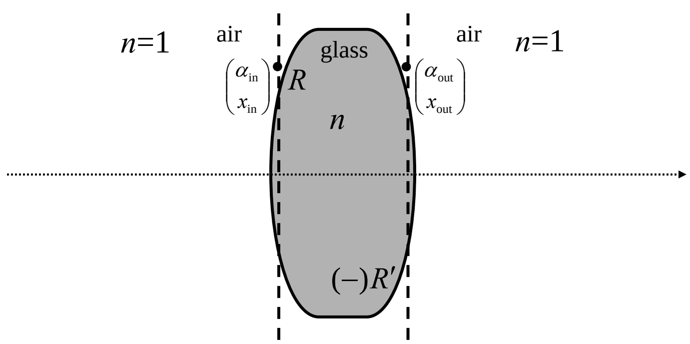
The thick lens



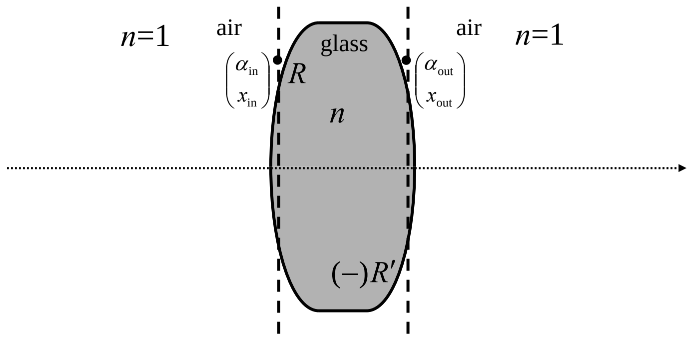
Equivalent to a thin lens placed “somewhere” within the thick element.
The location of this “equivalent thin lens” is the **Principal Plane** of the thick element

The thick lens



$$\begin{pmatrix}\alpha_{out}\\ x_{out}\end{pmatrix} = \begin{pmatrix}1 & -\dfrac{1-n}{R'}\\ 0 & 1\end{pmatrix}\begin{pmatrix}1 & 0\\ \dfrac{d}{n} & 1\end{pmatrix}\begin{pmatrix}1 & -\dfrac{n-1}{R}\\ 0 & 1\end{pmatrix}\begin{pmatrix}\alpha_{in}\\ x_{in}\end{pmatrix}$$

The thick lens



$$\begin{pmatrix}\alpha_{\text{out}}\\ x_{\text{out}}\end{pmatrix}=\begin{pmatrix}1+\dfrac{n-1}{n}\dfrac{d}{R'} & -\left[(n-1)\left(\dfrac{1}{R}-\dfrac{1}{R'}\right)+\dfrac{(n-1)^2 d}{nRR'}\right]\\ \dfrac{d}{n} & 1-\dfrac{n-1}{n}\dfrac{d}{R}\end{pmatrix}\begin{pmatrix}\alpha_{\text{in}}\\ x_{\text{in}}\end{pmatrix}$$

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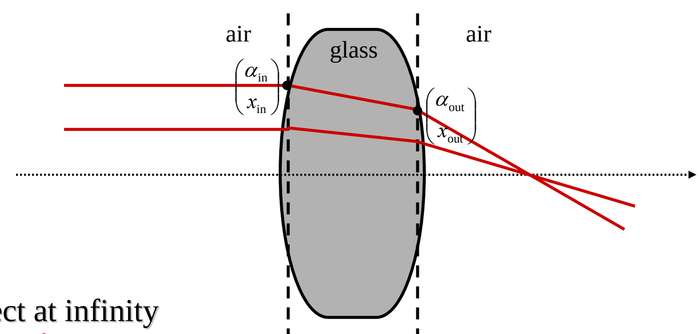
The thick lens: power



Object at infinity

$$\begin{pmatrix}\alpha_{\text{in}}\\ x_{\text{in}}\end{pmatrix}=\begin{pmatrix}0\\ x\end{pmatrix}\Rightarrow\begin{pmatrix}\alpha_{\text{out}}\\ x_{\text{out}}\end{pmatrix}=\begin{pmatrix}-\left[(n-1)\left(\dfrac{1}{R}-\dfrac{1}{R'}\right)+\dfrac{(n-1)^2 d}{nRR'}\right]x\\ \left(1-\dfrac{n-1}{n}\dfrac{d}{R}\right)x\end{pmatrix}\Rightarrow\alpha_{\text{out}}=-Px_{\text{in}}$$

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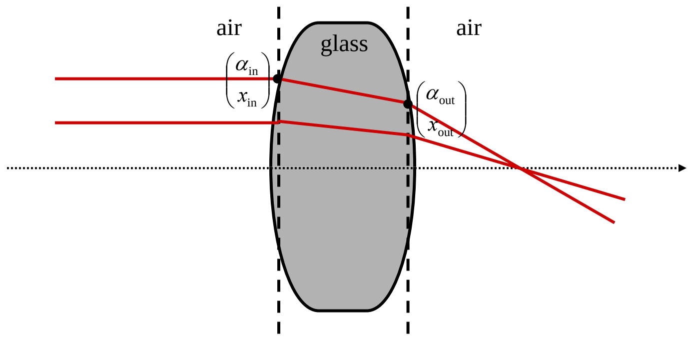
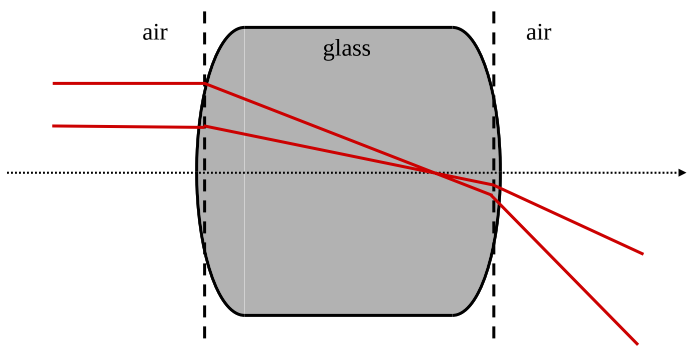
The thick lens: power



$$P=\left[(n-1)\left(\frac{1}{R}-\frac{1}{R'}\right)+\frac{(n-1)^2 d}{nRR'}\right] \qquad \frac{1}{f}=\left[(n-1)\left(\frac{1}{R}-\frac{1}{R'}\right)+\frac{(n-1)^2 d}{nRR'}\right]$$

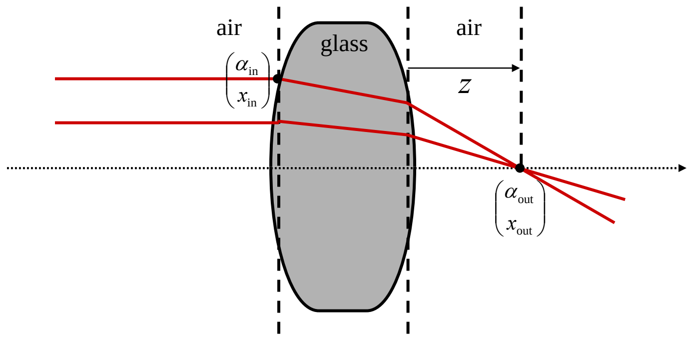
Power $\qquad$ *f*: Effective Focal Length

The *very* thick lens



Funny things happening:
rays diverge upon exiting from the element, *i.e.*
too much positive power leading to a negative element!

The thick lens: back focal length



$$\begin{pmatrix}\alpha_{\text{out}}\\ x_{\text{out}}\end{pmatrix}=\begin{pmatrix}1 & 0\\ z & 1\end{pmatrix}\begin{pmatrix}1+\dfrac{n-1}{n}\dfrac{d}{R'} & -\dfrac{1}{f}\\ \dfrac{d}{n} & 1-\dfrac{n-1}{n}\dfrac{d}{R}\end{pmatrix}\begin{pmatrix}\alpha_{\text{in}}\\ x_{\text{in}}\end{pmatrix}$$

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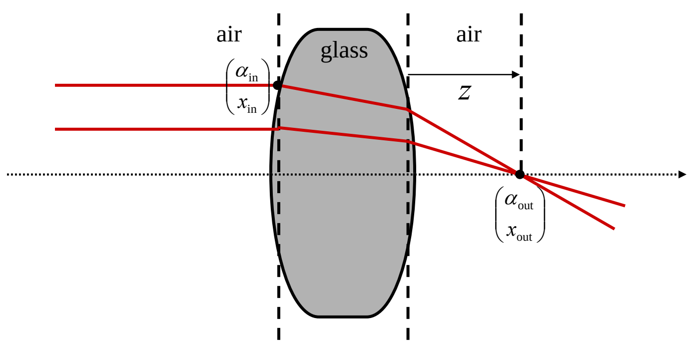
The thick lens: back focal length



$$x_{\text{out}}=0\Rightarrow z=f\left(1-\frac{n-1}{n}\frac{d}{R}\right)$$

z: Back Focal Length

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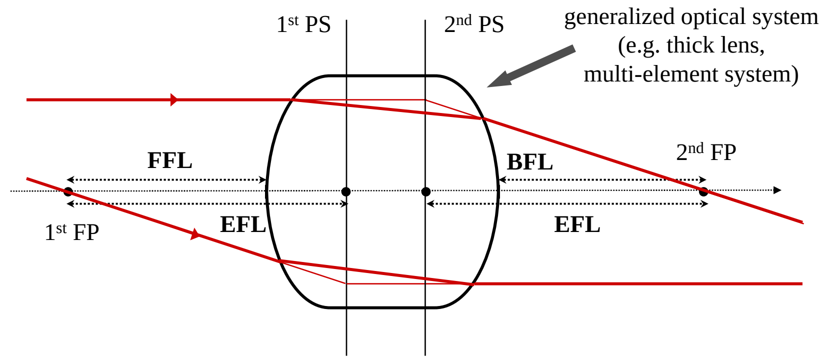
Focal Lengths & Principal Planes



EFL: Effective Focal Length (or simply “focal length”)
FFL: Front Focal Length
BFL: Back Focal Length
FP: Focal Point/Plane **PS**: Principal Surface/Plane

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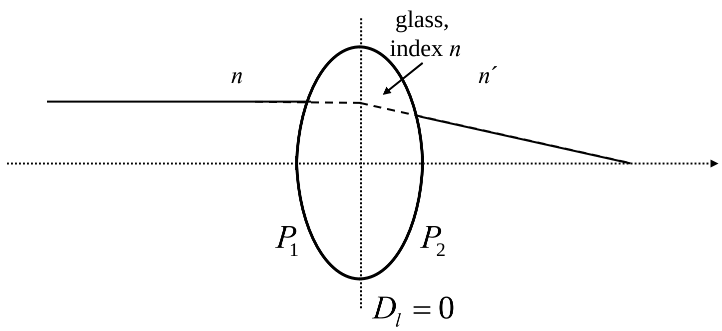
PSs and FLs for thin lenses



$$\frac{1}{(\text{EFL})} \equiv P = P_1 + P_2 \qquad (\text{BFL}) = (\text{EFL}) = (\text{FFL})$$

• The principal planes coincide with the (collocated) glass surfaces
• The rays bend precisely at the thin lens plane (=collocated glass surfaces & PP)

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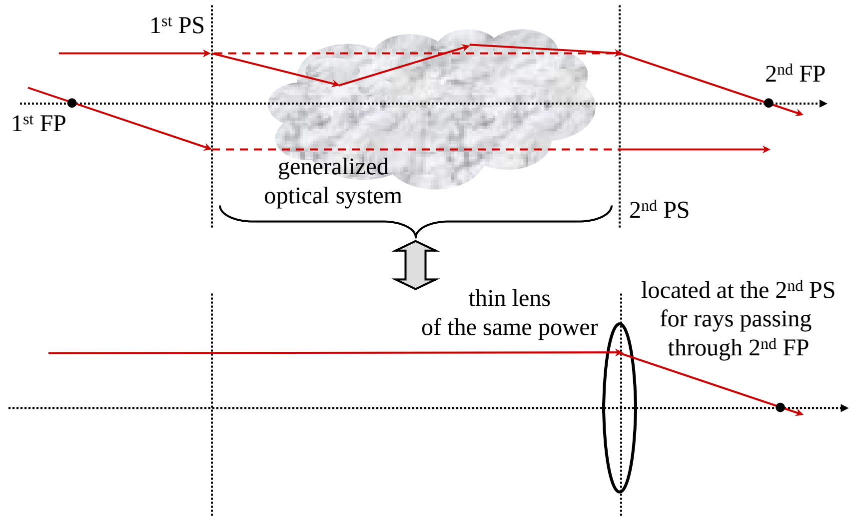
The significance of principal planes /1



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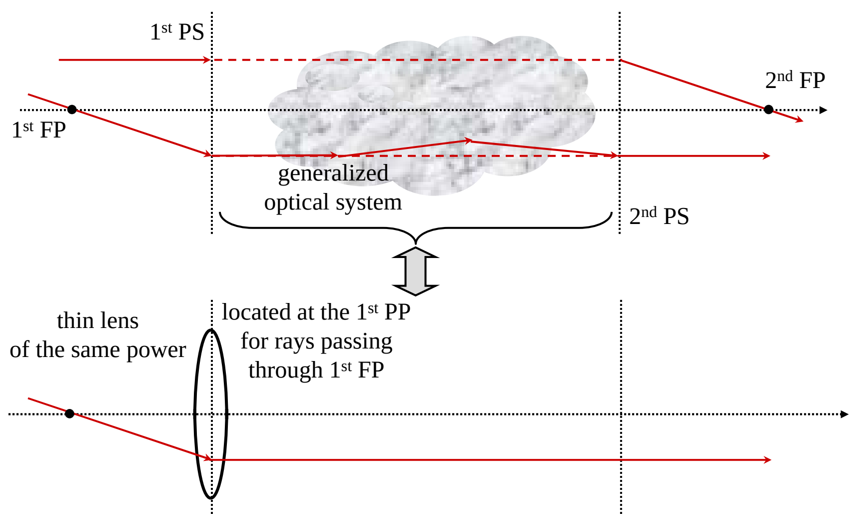
The significance of principal planes /2



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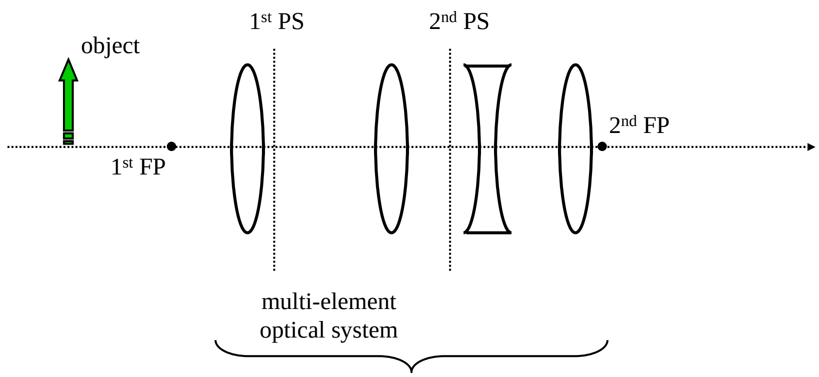
The significance of principal planes /3



image?
magnification?

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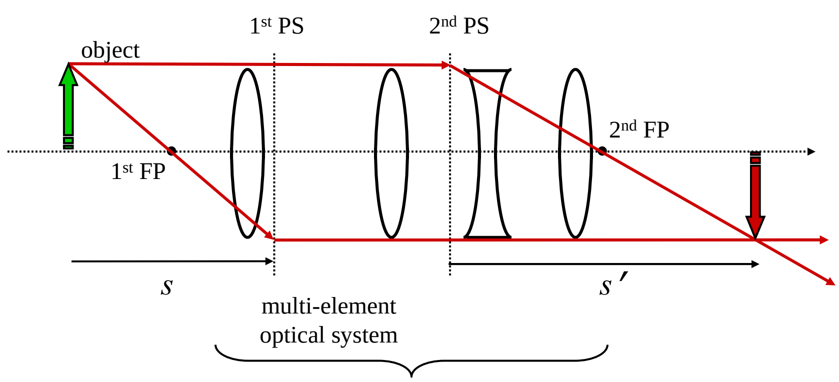
09/21/05 wk3-b-41

The significance of principal planes /3



$$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}, \qquad m_{\text{lateral}}=-\frac{s'}{s}$$ hold, where f = (EFL)

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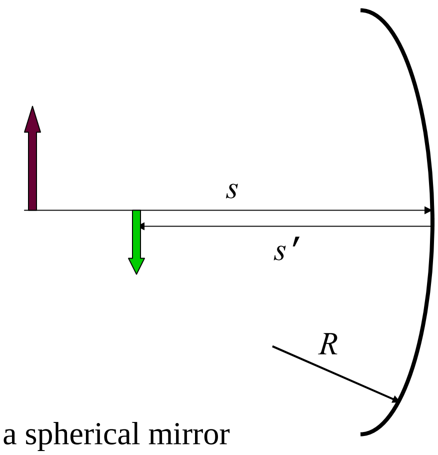
09/21/05 wk3-b-42

Reflective Optics

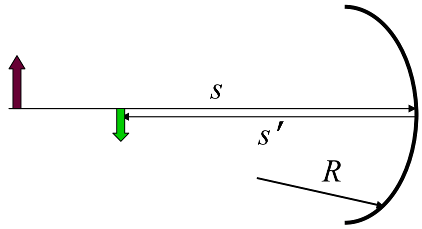


Example:
imaging by a spherical mirror

Sign conventions for reflective optics

- Light travels from left to right *before reflection and from right to left after reflection*
- A radius of curvature is positive if the surface is convex towards the left
- Longitudinal distances *before reflection* are positive if pointing to the right; *longitudinal distances after reflection are positive if pointing to the left*
- Longitudinal distances are positive if pointing up
- Ray angles are positive if the ray direction is obtained by rotating the +z axis counterclockwise through an acute angle

Reflective optics formulae



Imaging condition $$\frac{1}{s}+\frac{1}{s'}=-\frac{2}{R}$$

Focal length $$f=-\frac{R}{2}$$

Magnification $$m_x=-\frac{s'}{s} \qquad m_\alpha=-\frac{s}{s'}$$