

Elementary waves: plane, spherical

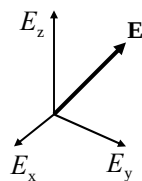
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The EM vector wave equation

$$\nabla^2 \mathbf{E} - \frac{1}{c^2} \frac{\partial^2 \mathbf{E}}{\partial t^2} = 0$$

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

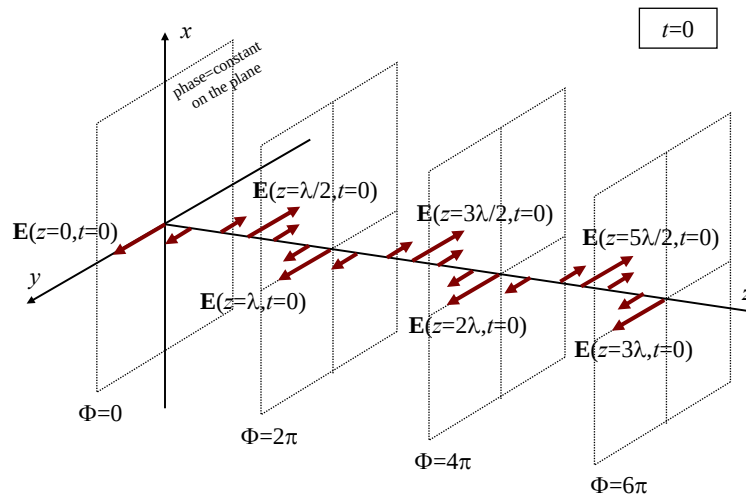
$$\mathbf{E} = E_x \hat{\mathbf{x}} + E_y \hat{\mathbf{y}} + E_z \hat{\mathbf{z}}$$



$$\left. \begin{aligned} \frac{\partial^2 E_x}{\partial x^2} + \frac{\partial^2 E_x}{\partial y^2} + \frac{\partial^2 E_x}{\partial z^2} - \frac{1}{c^2} \frac{\partial^2 E_x}{\partial t^2} &= 0 \\ \frac{\partial^2 E_y}{\partial x^2} + \frac{\partial^2 E_y}{\partial y^2} + \frac{\partial^2 E_y}{\partial z^2} - \frac{1}{c^2} \frac{\partial^2 E_y}{\partial t^2} &= 0 \\ \frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + \frac{\partial^2 E_z}{\partial z^2} - \frac{1}{c^2} \frac{\partial^2 E_z}{\partial t^2} &= 0 \end{aligned} \right\}$$

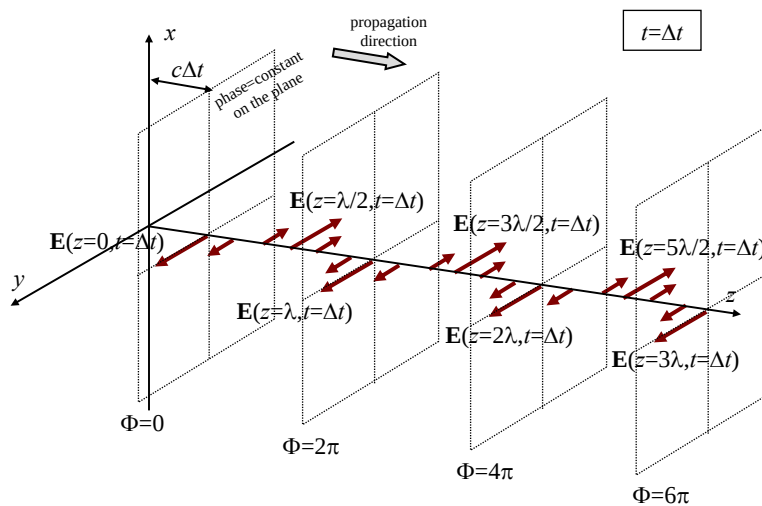
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Harmonic solution in 3D: plane wave



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Plane wave propagating



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Complex representation of 3D waves

$$f(x, y, z, t) = A \cos\left(\frac{2\pi}{\lambda}(x \cos \alpha + y \cos \beta + z \cos \gamma) - \omega t - \phi_0\right)$$

$$f(x, y, z, t) = A \cos(k_x x + k_y y + k_z z - \omega t - \phi_0) \quad k_x = \frac{2\pi}{\lambda} \cos \alpha, \text{ etc.}$$

$$\hat{f}(x, y, z, t) = A e^{i(k_x x + k_y y + k_z z - \omega t - \phi_0)} \quad \text{complex representation}$$

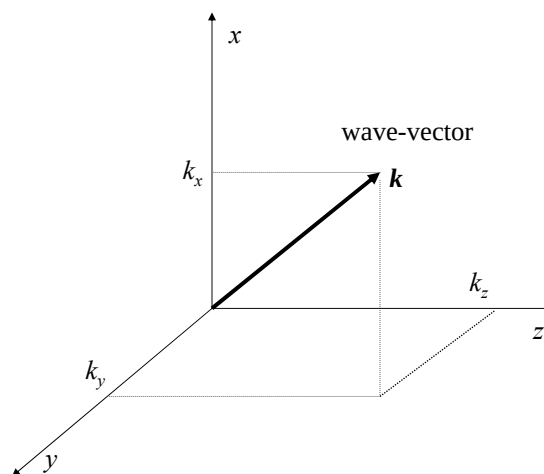
$$A e^{-i\phi(x, y, z)} \quad \text{complex amplitude or "phasor"}$$

$$\text{where } \phi(x, y, z) \equiv k_x x + k_y y + k_z z - \phi_0$$

"Wavefront" : surface $\phi(x, y, z) = \text{const.}$

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Plane wave



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Plane wave

$$a(\mathbf{r}) = A_0 e^{i(\mathbf{k} \cdot \mathbf{r} - \omega t)}$$

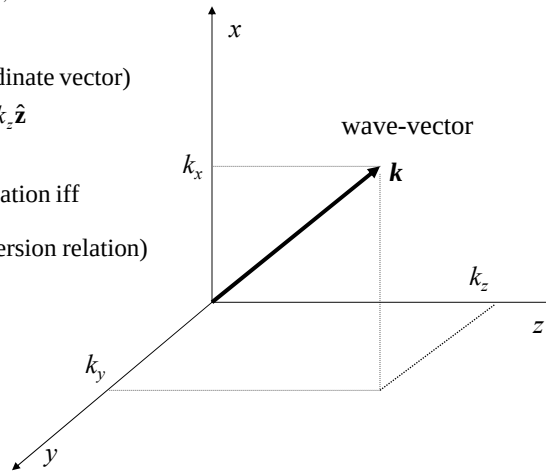
$$\mathbf{r} = x\hat{\mathbf{x}} + y\hat{\mathbf{y}} + z\hat{\mathbf{z}}$$

(Cartesian coordinate vector)

$$\mathbf{k} = k_x\hat{\mathbf{x}} + k_y\hat{\mathbf{y}} + k_z\hat{\mathbf{z}}$$

solves wave equation iff

$$|\mathbf{k}| = \frac{\omega}{c} \quad (\text{dispersion relation})$$



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Plane wave

$$a(\mathbf{r}) = A_0 e^{i(\mathbf{k} \cdot \mathbf{r} - \omega t)}$$

$$\mathbf{r} = x\hat{\mathbf{x}} + y\hat{\mathbf{y}} + z\hat{\mathbf{z}}$$

(Cartesian coordinate vector)

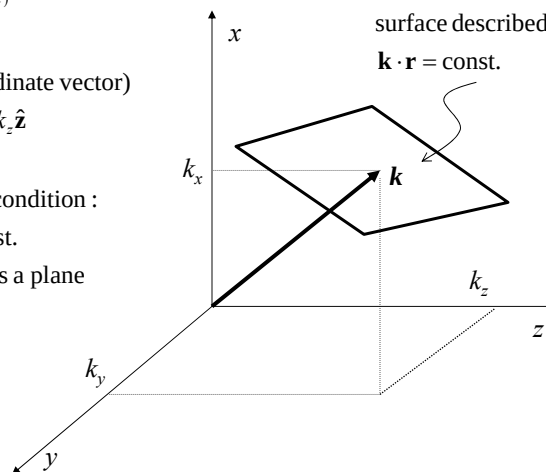
$$\mathbf{k} = k_x\hat{\mathbf{x}} + k_y\hat{\mathbf{y}} + k_z\hat{\mathbf{z}}$$

constant phase condition :

$$\mathbf{k} \cdot \mathbf{r} - \omega t = \text{const.}$$

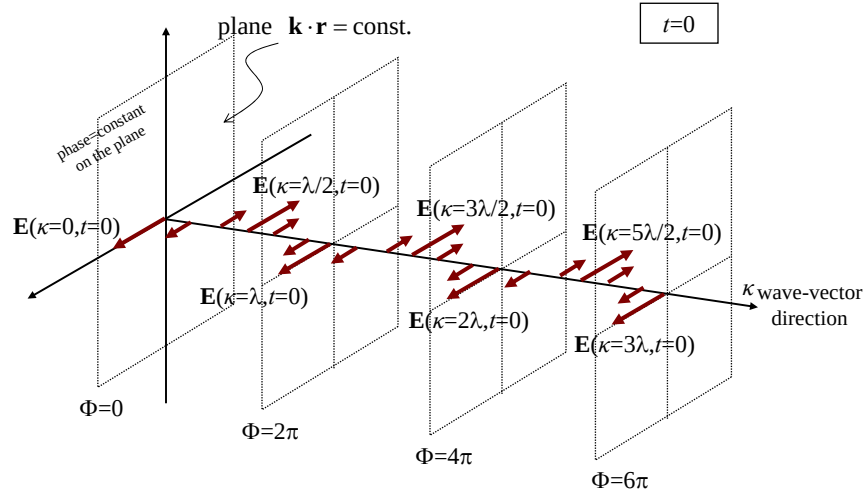
$\Rightarrow$ wave-front is a plane

"wavefront":
surface described by
 $\mathbf{k} \cdot \mathbf{r} = \text{const.}$



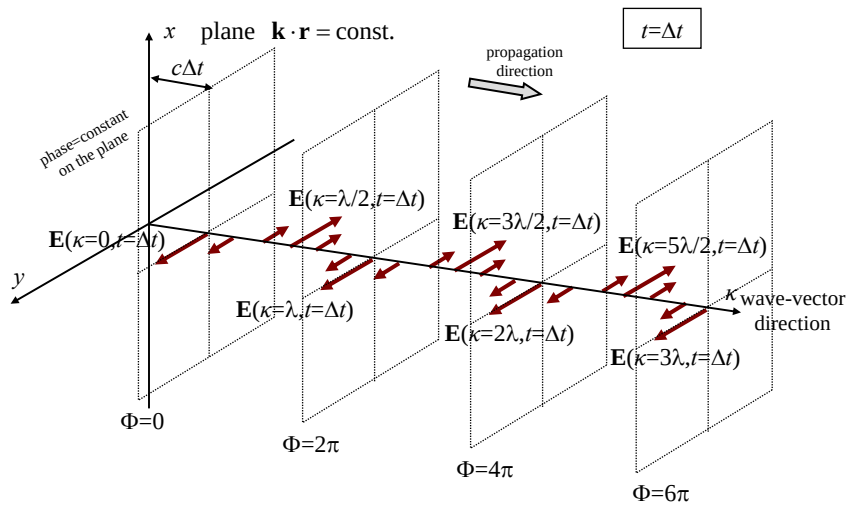
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Plane wave propagating



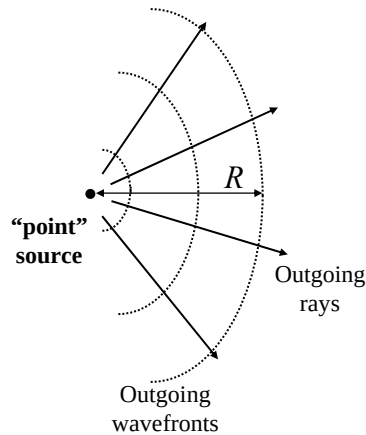
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Plane wave propagating



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Spherical wave



equation of wavefront

$$kR - \omega t = \text{constant}$$



$$a(x, y, z, t) = A \frac{\cos(kR - \omega t + \pi/2)}{R}$$



exponential notation

$$a(x, y, z, t) = A \frac{\exp\{i(kR - \omega t)\}}{iR}$$

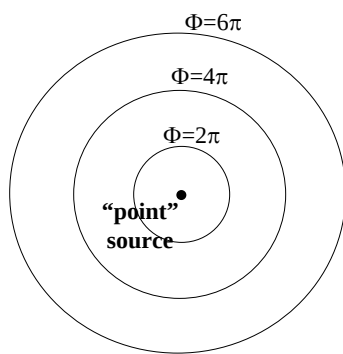


paraxial approximation

$$a(x, y, z) = \frac{A}{iR} \exp\left\{i2\pi \frac{z}{\lambda} + i\pi \frac{x^2 + y^2}{\lambda z}\right\}$$

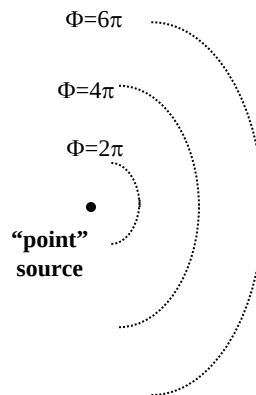
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Spherical wave



spherical wavefronts

exact

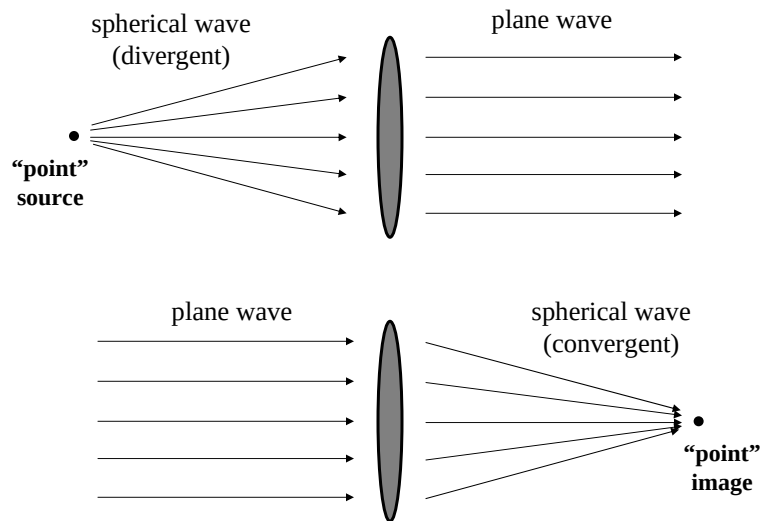


parabolic wavefronts

paraxial approximation/
/Gaussian beams

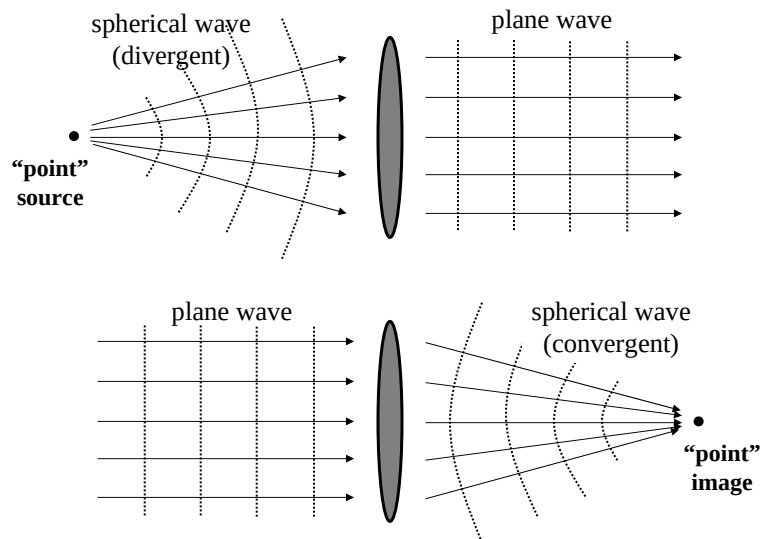
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The role of lenses



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The role of lenses



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Polarization

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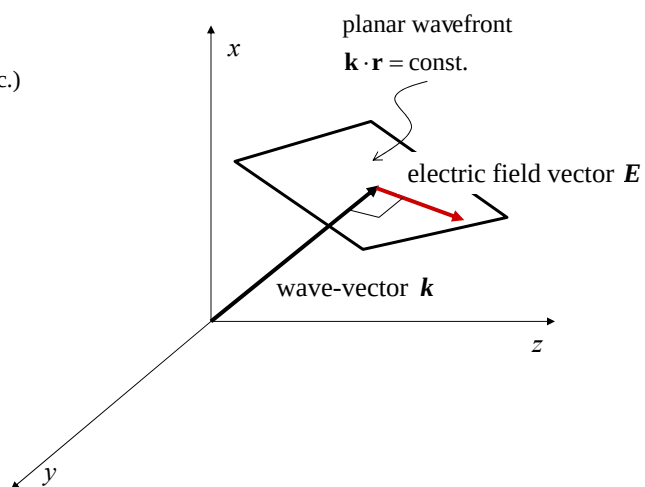
Propagation and polarization

In isotropic media
(e.g. free space,
amorphous glass, etc.)

$$\mathbf{k} \cdot \mathbf{E} = 0$$

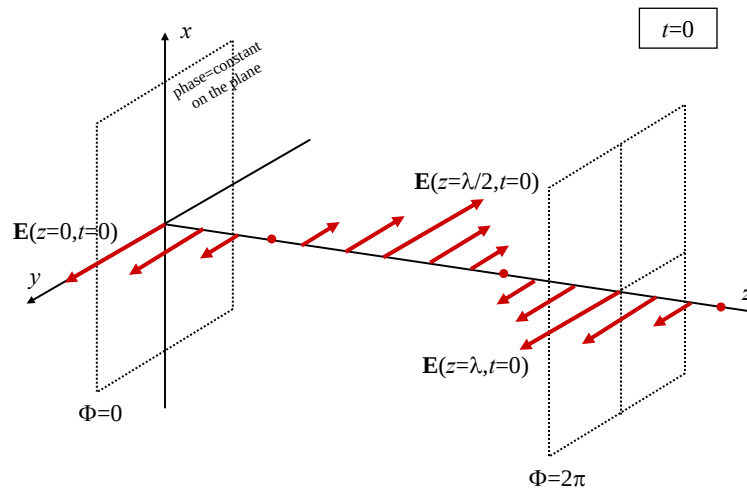
i.e. $\mathbf{k} \perp \mathbf{E}$

More generally,
 $\mathbf{k} \cdot \mathbf{D} = 0$
(reminder: in
anisotropic media,
e.g. crystals, one
could have
 $\mathbf{E}$ not parallel to $\mathbf{D}$)



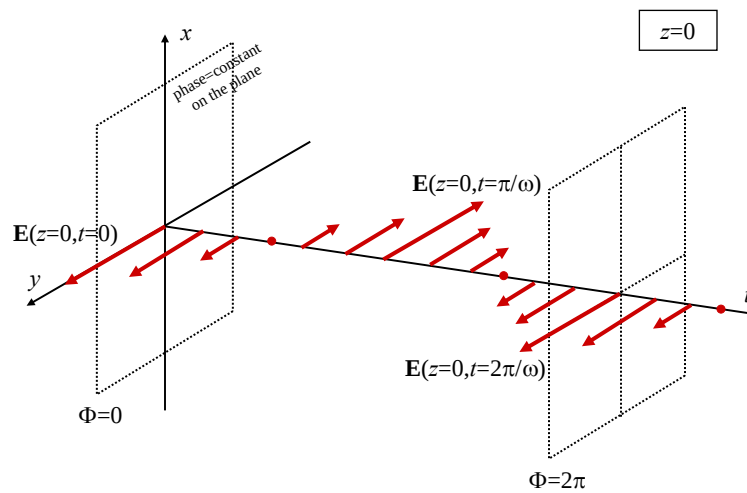
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Linear polarization (frozen time)



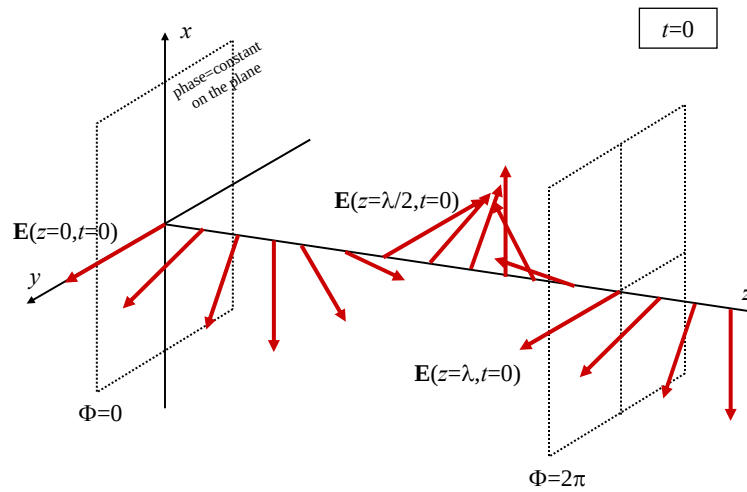
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Linear polarization (fixed space)



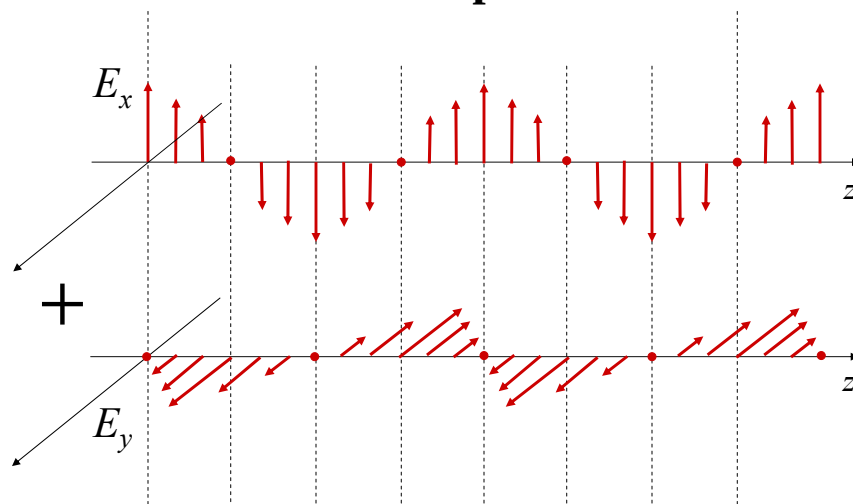
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Circular polarization (frozen time)



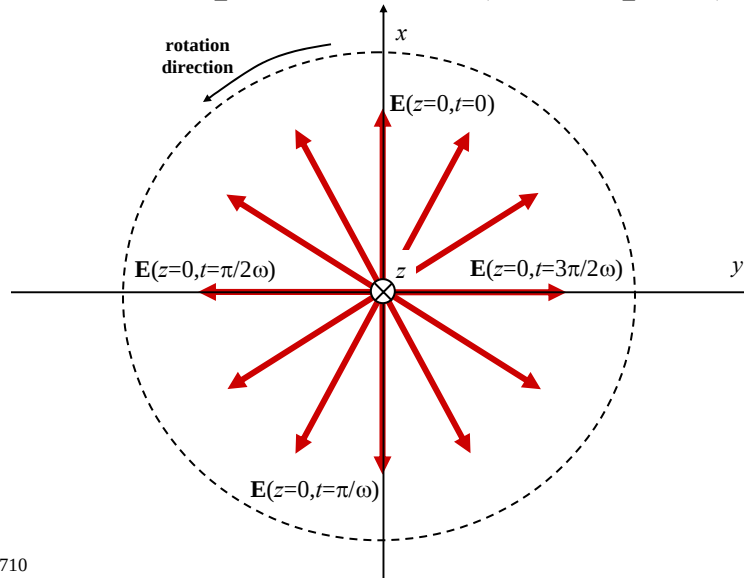
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Circular polarization: linear components



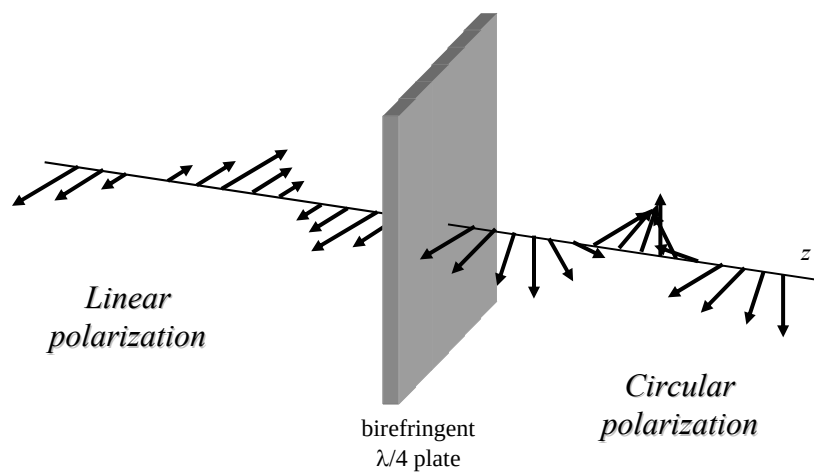
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Circular polarization (fixed space)

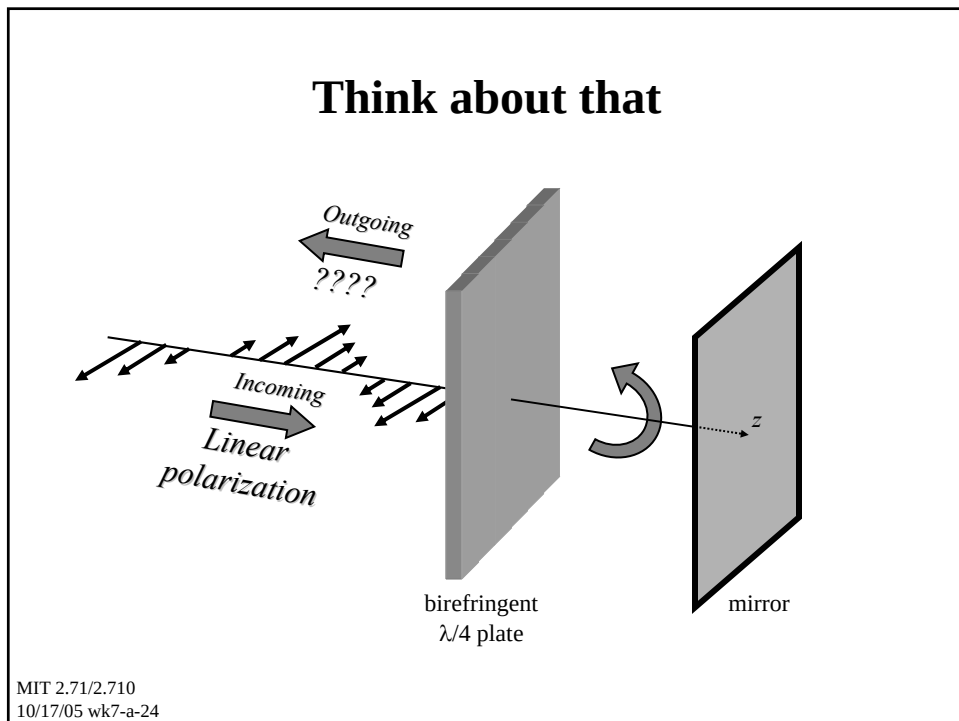
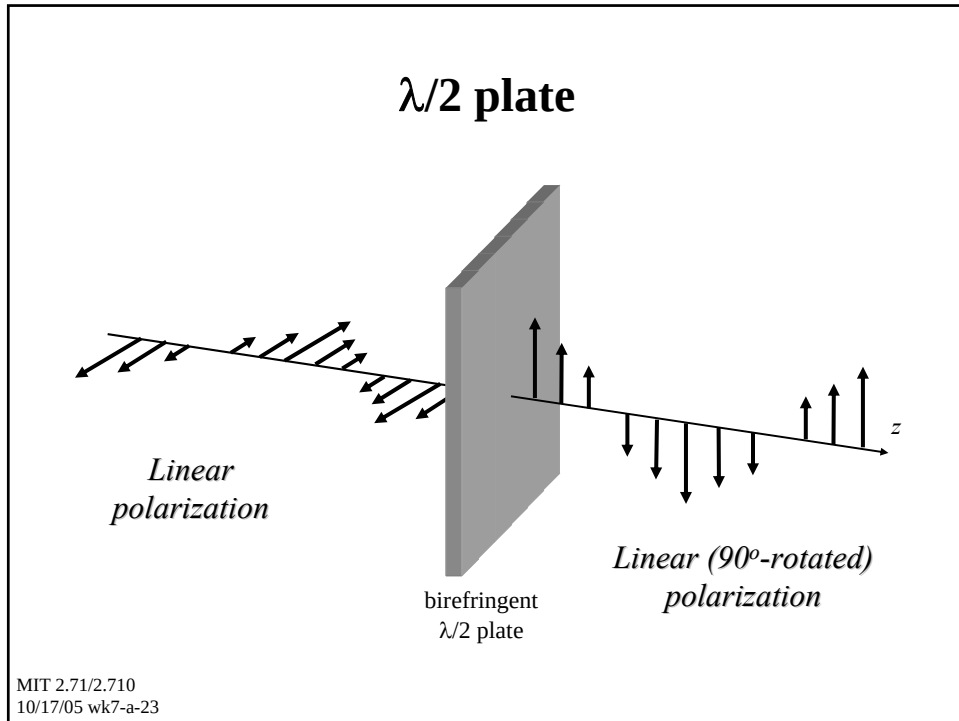


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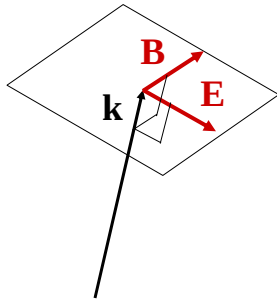
$\lambda/4$ plate



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Relationship between **E** and **B**



$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad \text{where} \quad \mathbf{E} = \hat{\mathbf{x}} E_0 e^{i(\mathbf{k} \cdot \mathbf{r} - \omega t)}$$

$$\Rightarrow \nabla \times \equiv i\mathbf{k} \times \quad \text{and} \quad \frac{\partial}{\partial t} \equiv -i\omega$$

$$\Rightarrow \mathbf{B} = \frac{1}{\omega} \mathbf{k} \times \mathbf{E}$$

Vectors **k**, **E**, **B** form a right-handed triad.

Note: free space or isotropic media only