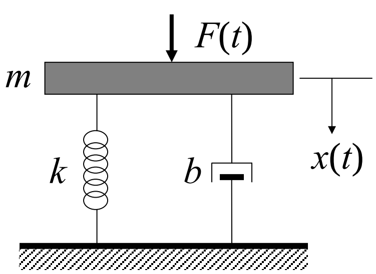
Linear shift invariant systems in the time domain

Linear shift invariant systems

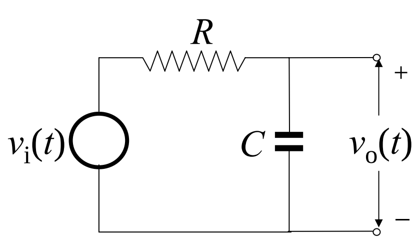


$$m\ddot{x} + b\dot{x} + kx = F(t)$$

$$H(\omega) = \frac{1}{-\omega^2 m - i\omega b + k}$$

$$|H(\omega)|^2 = \frac{1}{m^2}\frac{1}{\left(\omega^2 - \omega_{\mathrm{r}}^2\right)^2 + 4\zeta^2\omega_{\mathrm{r}}^2\omega^2}$$

$$\omega_{\mathrm{r}} = \sqrt{\frac{k}{m}} \qquad \zeta = \frac{b}{2\sqrt{km}}$$



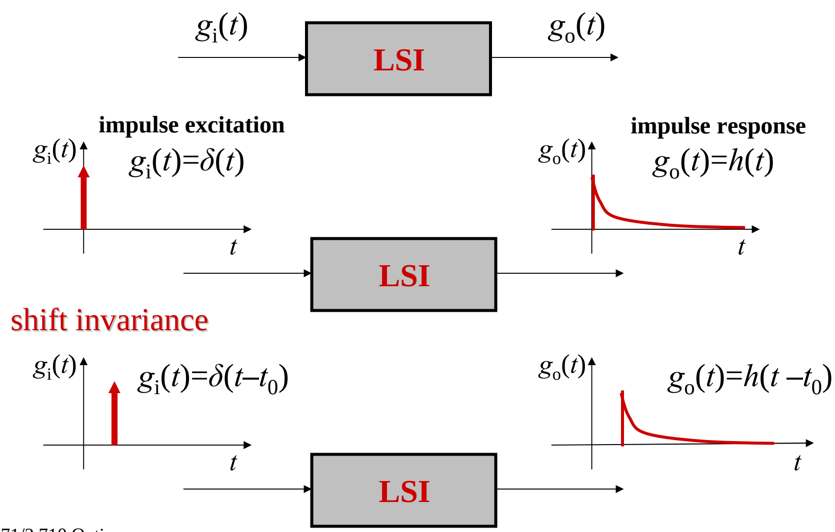
$$RC\dot{v}_{\mathrm{o}} + v_{\mathrm{o}} = v_{\mathrm{i}}(t)$$

$$H(\omega) = \frac{1}{-i\omega\tau + 1}$$

$$|H(\omega)|^2 = \frac{1}{(\omega\tau)^2 + 1}$$

$$\tau = RC$$

Linear shift invariant systems



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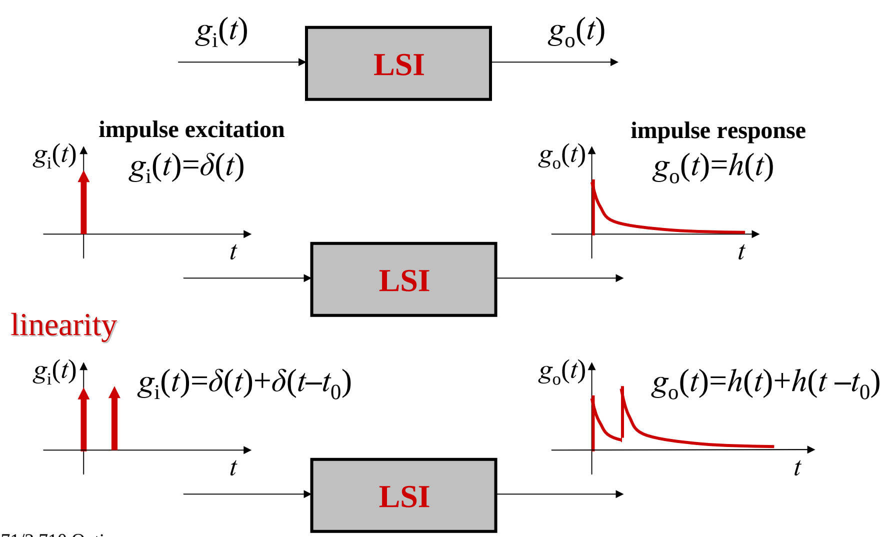
Linear shift invariant systems



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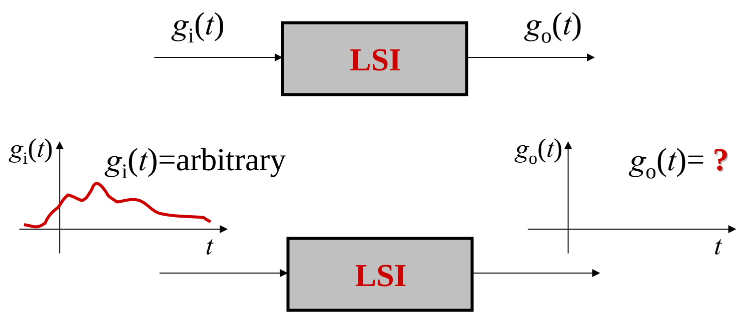
Linear shift invariant systems



$$g_i(t)=\int g_i(t_0)\,\delta(t-t_0)\,\mathrm{d}t_0 \quad \Rightarrow \quad g_o(t)=\int g_i(t_0)\,h(t-t_0)\,\mathrm{d}t_0$$

sifting property of the δ-function — convolution integral

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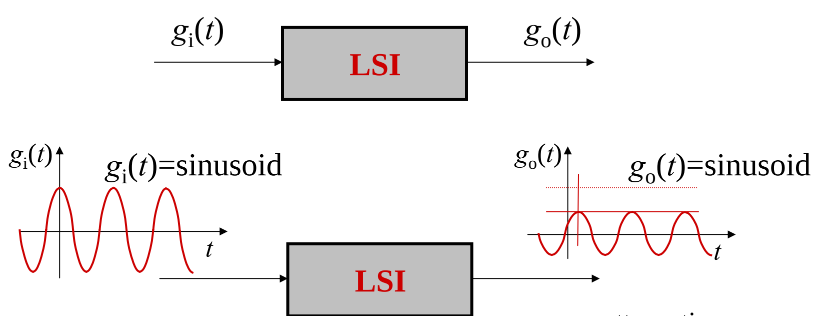
Linear shift invariant systems



attenuation
phase delay

$$g_i(t)=a_0\cos(\omega_0 t)\Rightarrow g_o(t)=a_0\left|H(\omega_0)\right|\cos\left(\omega_0 t-\phi(\omega_0)\right)$$

where $\underbrace{H(\omega)=\left|H(\omega)\right|\exp[i\phi(\omega)]}_{\text{transfer function}}$

same frequency

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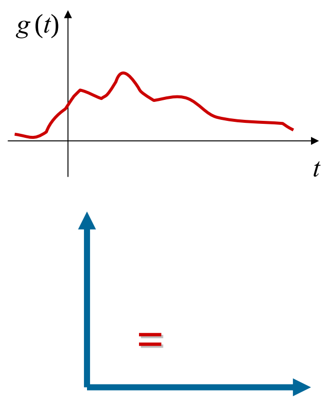
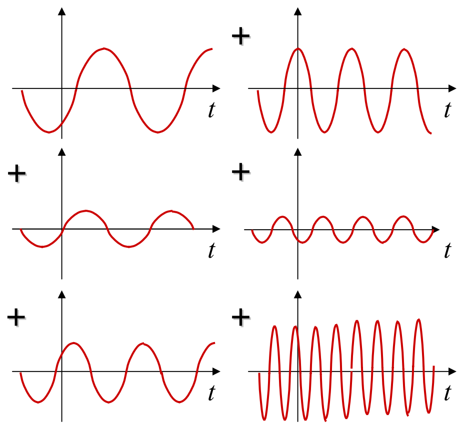
From time to frequency: Fourier analysis



Can I express an arbitrary $g(t)$
as a superposition of sinusoids?



The Fourier integral

(aka **inverse Fourier transform**)

$$g(t) = \int G(\nu)\, \mathrm{e}^{+i2\pi\nu t}\, \mathrm{d}\nu$$

superposition

sinusoids

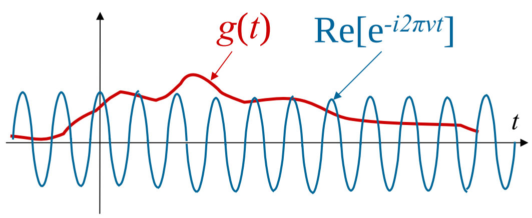
complex weight,
expresses relative amplitude
(magnitude & phase)
of superposed sinusoids

The Fourier transform

The complex weight coefficients $G(\nu)$,
aka **Fourier transform** of $g(t)$
are calculated from the integral

$$G(\nu) = \int g(t)\ \mathrm{e}^{-i2\pi\nu t}\ \mathrm{d}t$$

$$\mathrm{Re}[G(\nu)] = \int$$



$$\mathrm{d}t$$

Fourier transform *pairs*

$$g(t) = \delta(t) \quad \leftrightarrow \quad G(\nu) = 1$$

$$g(t) = \exp(i2\pi\nu_0 t) \quad \leftrightarrow \quad G(\nu) = \delta(\nu - \nu_0)$$

$$g(t) = \cos(2\pi\nu_0 t) \quad \leftrightarrow \quad G(\nu) = \frac{1}{2}[\delta(\nu + \nu_0) + \delta(\nu - \nu_0)]$$

$$g(t) = \sin(2\pi\nu_0 t) \quad \leftrightarrow \quad G(\nu) = \frac{1}{2i}[-\delta(\nu + \nu_0) + \delta(\nu - \nu_0)]$$

$$g(t) = \mathrm{rect}\left(\frac{t}{T}\right) \quad \leftrightarrow \quad G(\nu) = T\,\mathrm{sinc}(T\nu)$$

$$g(t) = \exp\left(-\frac{t^2}{2T^2}\right) \quad \leftrightarrow \quad G(\nu) = T\exp\left(-2T^2\nu^2\right)$$