Wave description of optical imaging systems

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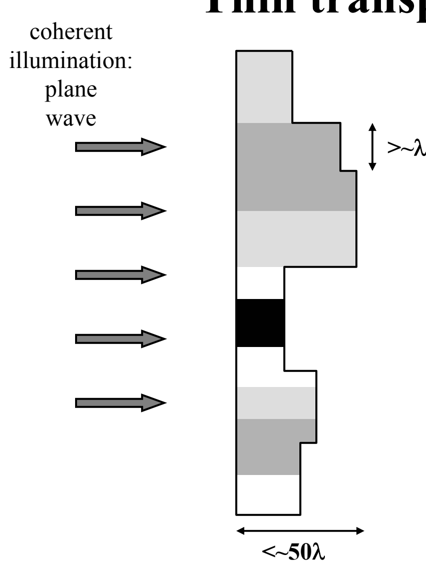
Thin transparencies



Transmission function:

$$g_{\text{in}}(x,y) = t(x,y)\exp\left\{i\phi(x,y)\right\}$$

Field before transparency:

$$a_{-}(x,y,\overset{=0}{\cancel{z}}) = \exp\cancel{\left\{i2\pi\frac{z}{\lambda}\right\}}$$

Field after transparency:

$$a_{+}(x,y,\overset{=0}{\cancel{z}}) = g_{\text{in}}(x,y)\exp\cancel{\left\{i2\pi\frac{z}{\lambda}\right\}}$$

assumptions: transparency at z=0
transparency thickness can be ignored

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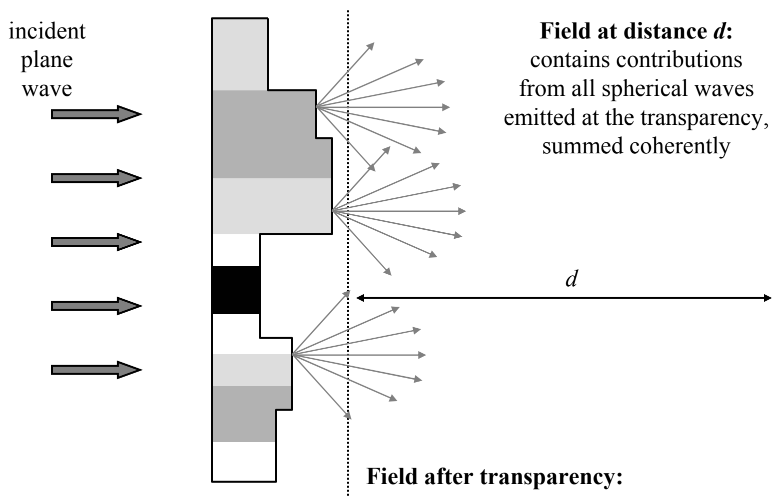
Diffraction: Huygens principle



$$a_{+}(x, y, z) = g_{\text{in}}(x, y)$$

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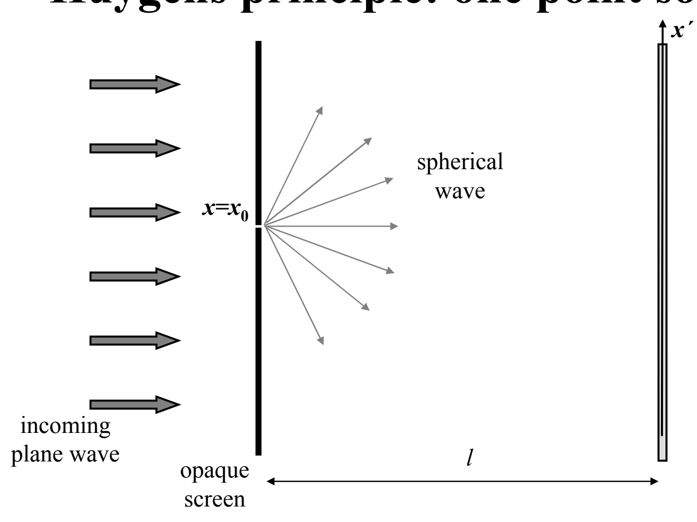
Huygens principle: one point source



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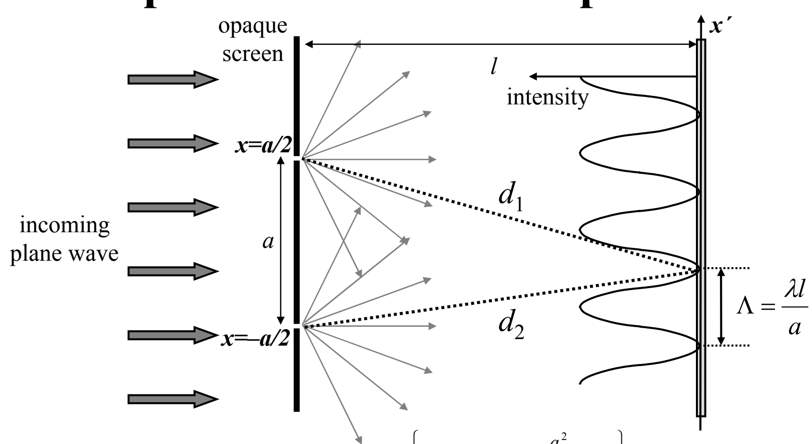
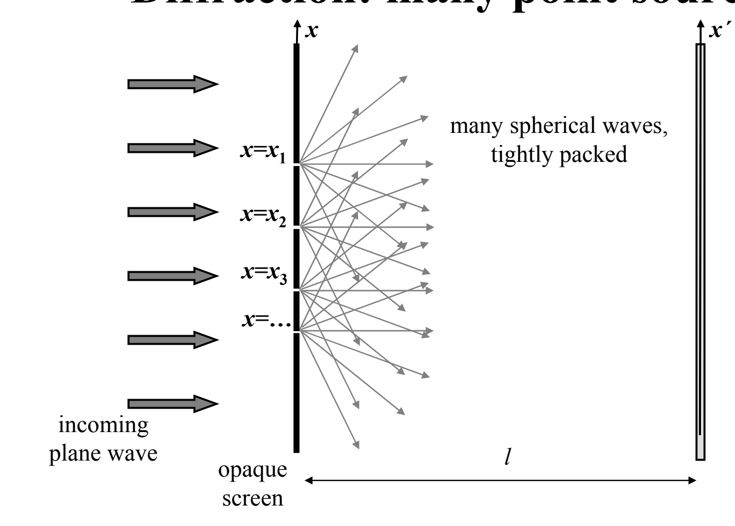
Simple interference: two point sources



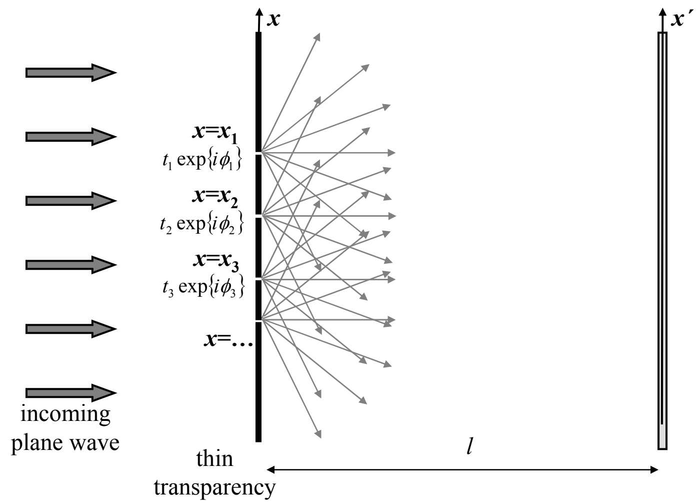
$$\text{Amplitude}: e(x',y') = \frac{2}{i\lambda l}\exp\left\{ i2\pi\frac{l}{\lambda} + i\pi \frac{x'^2 + \frac{a^2}{4} + y'^2}{\lambda l}\right\}\cos\left(\pi\frac{ax'}{\lambda l}\right).$$

$$\text{Intensity}: I(x',y') = \left|e(x',y')\right|^2 = \frac{4}{(\lambda l)^2}\cos^2\left(\pi\frac{ax'}{\lambda l}\right) = \frac{2}{(\lambda l)^2}\left(1+\cos\left\{2\pi\left(\frac{a}{\lambda l}\right)x'\right\}\right).$$

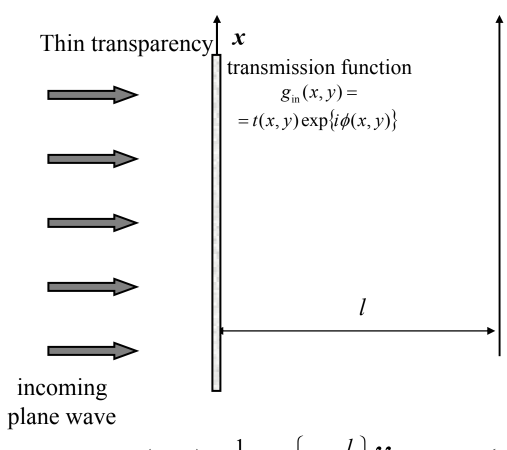
Diffraction: many point sources



Diffraction: many point sources, attenuated & phase-delayed



Diffraction: many point sources attenuated & phase-delayed, math



$$\text{field}(x') = \begin{pmatrix}\text{spherical}\\ \text{wave}\\ \text{from } x_1\end{pmatrix} + \begin{pmatrix}\text{spherical}\\ \text{wave}\\ \text{from } x_2\end{pmatrix} + \begin{pmatrix}\text{spherical}\\ \text{wave}\\ \text{from } x_3\end{pmatrix} + \ldots =$$

$$= \frac{t_1}{i\lambda l}\exp\left\{i\phi_1 + i2\pi\frac{l}{\lambda} + i\pi\frac{(x'-x_1)^2 + y'^2}{\lambda l}\right\} +$$

$$\frac{t_2}{i\lambda l}\exp\left\{i\phi_2 + i2\pi\frac{l}{\lambda} + i\pi\frac{(x'-x_2)^2 + y'^2}{\lambda l}\right\} +$$

$$\frac{t_3}{i\lambda l}\exp\left\{i\phi_3 + i2\pi\frac{l}{\lambda} + i\pi\frac{(x'-x_3)^2 + y'^2}{\lambda l}\right\} \ldots$$

continuous limit

$$\text{field}(x', y') = \frac{1}{i\lambda l}\exp\left\{i2\pi\frac{l}{\lambda}\right\}\iint \underbrace{t(x,y)\exp\{i\phi(x,y)\}}_{g_{\text{in}}(x,y)}\exp\left\{i\pi\frac{(x'-x)^2 + (y'-y)^2}{\lambda l}\right\}\mathrm{d}x\mathrm{d}y.$$

Fresnel diffraction

$$g_{\text{out}}(x',y') = \frac{1}{i\lambda l}\exp\left\{i2\pi\frac{l}{\lambda}\right\}\iint g_{\text{in}}(x,y)\exp\left\{i\pi\frac{(x'-x)^2+(y'-y)^2}{\lambda l}\right\}\mathrm{d}x\mathrm{d}y.$$

The diffracted field is the *convolution* of the transparency with a spherical wave

$$\underbrace{g_{\text{out}}(x',y')}_{\text{amplitude distribution at output plane}} = \underbrace{g_{\text{in}}(x,y)}_{\text{transparency transmission function (complex } te^{i\phi})} * \underbrace{\left[\underbrace{\frac{1}{i\lambda l}\exp\left\{i2\pi\frac{l}{\lambda}\right\}}_{\text{CONSTANT: NOT interesting}}\underbrace{\exp\left\{i\pi\frac{x^2+y^2}{\lambda l}\right\}}_{\text{FUNCTION OF LATERAL COORDINATES: Interesting!!!}}\right]}_{\text{spherical wave @}z=l\text{ (aka Green's function)}}$$

CONSTANT: ***NOT*** interesting

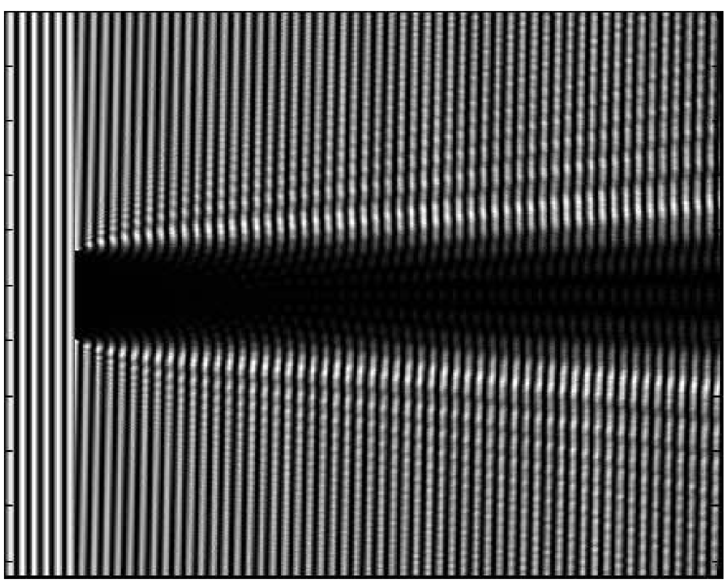
FUNCTION OF LATERAL COORDINATES: ***Interesting!!!***

amplitude distribution at output plane

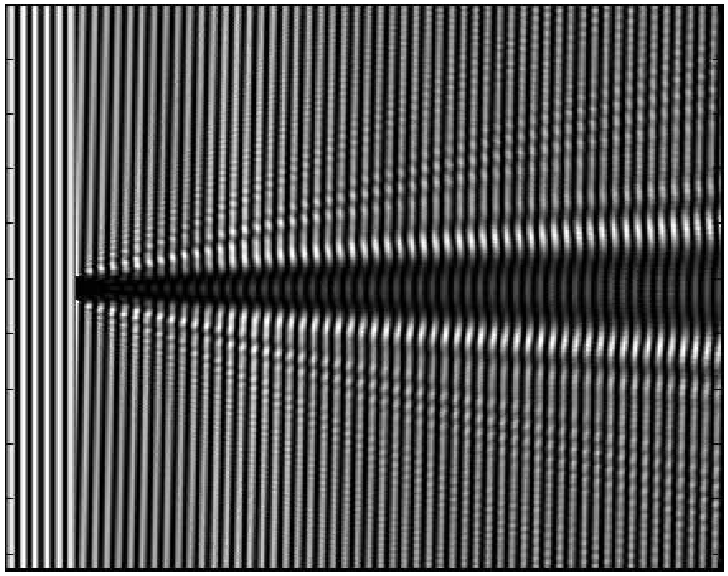
transparency transmission function (complex $te^{i\phi}$)

spherical wave @$z=l$ (aka Green's function)

Diffraction from an obscuration



Diffraction from a small obscuration



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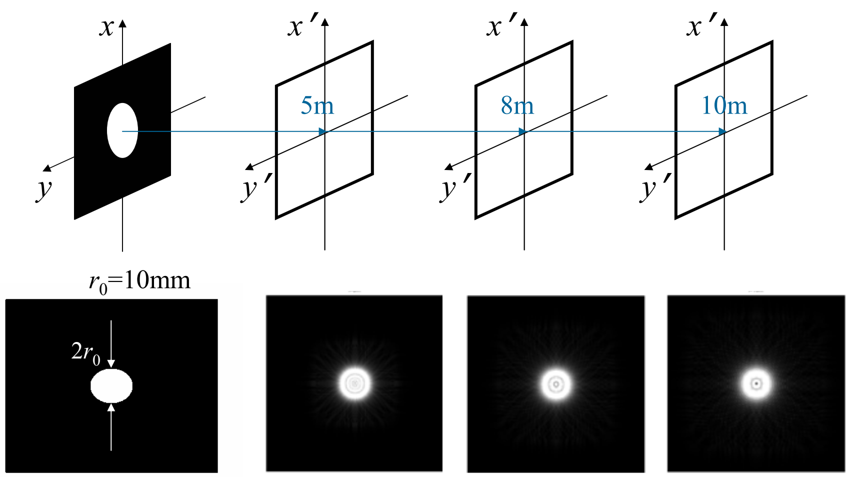
Example: circular aperture



input field $g_{in}(x,y)$ $g_{out}(x,y;5m)$ $g_{out}(x,y;8m)$ $g_{out}(x,y;10m)$

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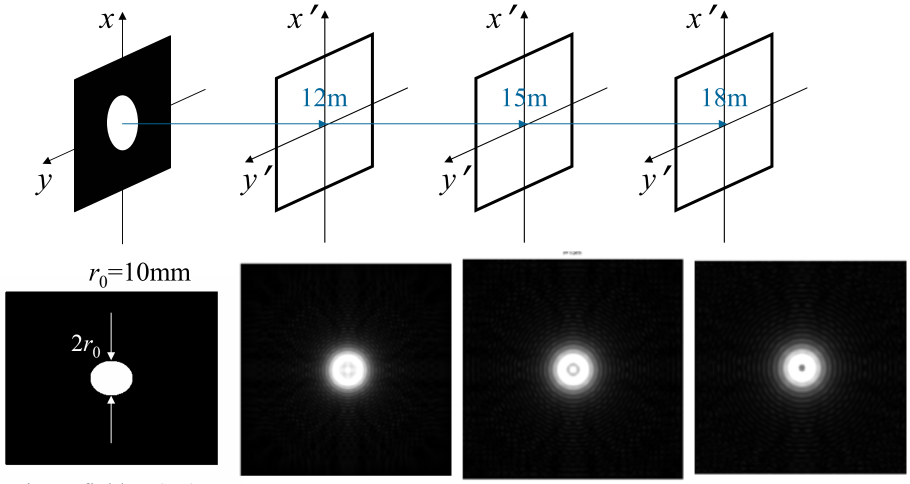
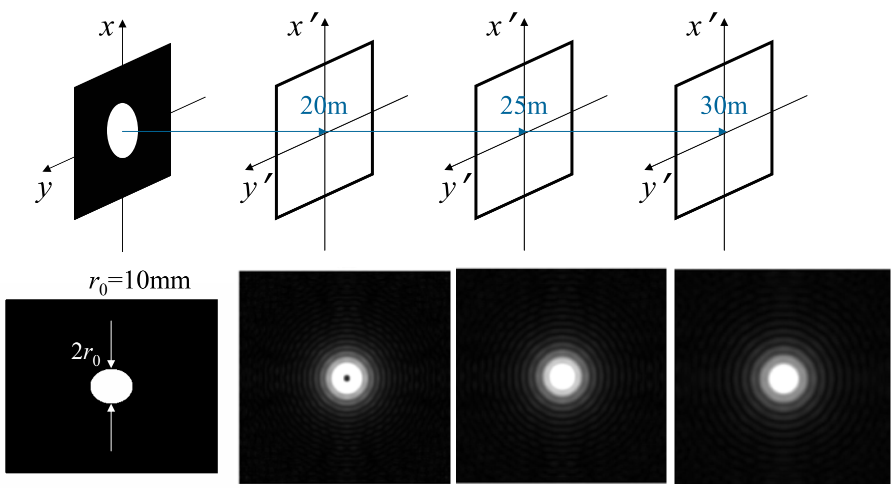
Example: circular aperture



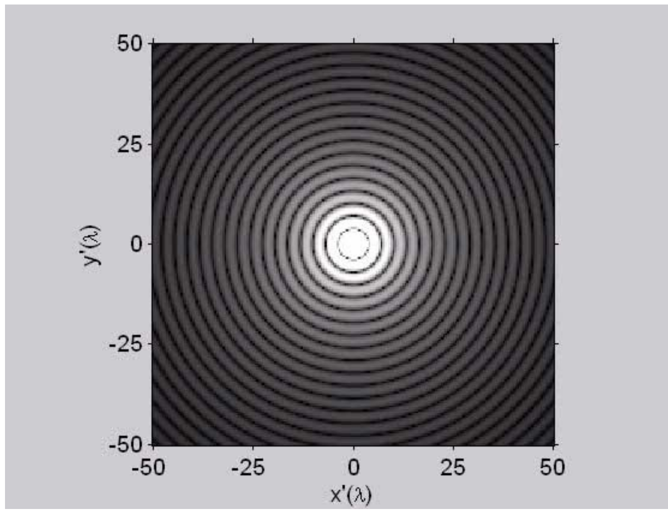
input field $g_{in}(x,y)$ $g_{out}(x,y;12\text{m})$ $g_{out}(x,y;15\text{m})$ $g_{out}(x,y;18\text{m})$

Example: circular aperture



input field $g_{in}(x,y)$ $g_{out}(x,y;20\text{m})$ $g_{out}(x,y;25\text{m})$ $g_{out}(x,y;30\text{m})$

The blinking spot (Poisson spot)



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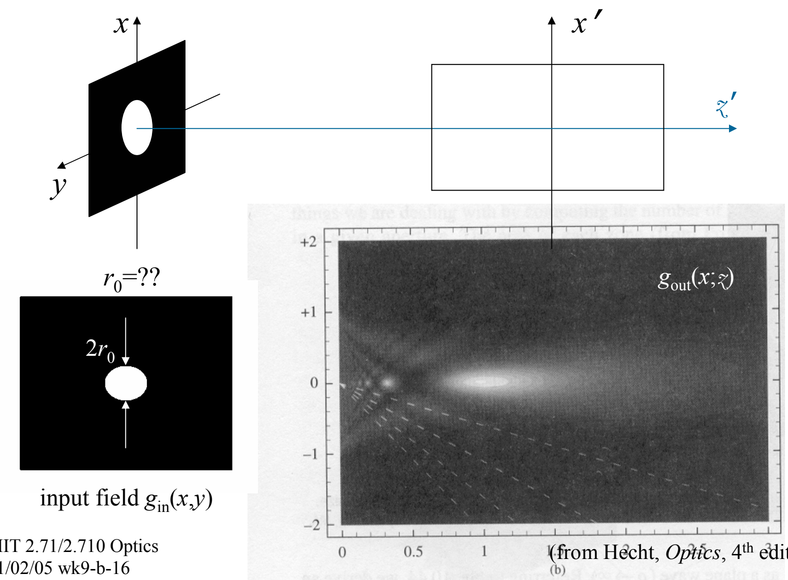
Example: circular aperture



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(from Hecht, *Optics*, 4th edition, page 494)

Fraunhofer diffraction

propagation distance l is "very large"

$$g_{\text{out}}(x',y') = \frac{1}{i\lambda l}\exp\left\{i2\pi\frac{l}{\lambda}\right\}\iint g_{\text{in}}(x,y)\exp\left\{i\pi\frac{(x'-x)^2+(y'-y)^2}{\lambda l}\right\}\mathrm{d}x\mathrm{d}y,$$

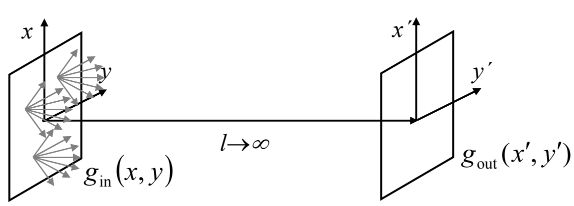
$$g_{\text{out}}(x',y') = \frac{1}{i\lambda l}\exp\left\{i2\pi\frac{l}{\lambda}\right\}\iint g_{\text{in}}(x,y)\exp\left\{i\pi\frac{(x')^2+x^2-2xx'+(y')^2+y^2-2yy'}{\lambda l}\right\}\mathrm{d}x\mathrm{d}y,$$

$$g_{\text{out}}(x',y') = \frac{1}{i\lambda l}\exp\left\{i2\pi\frac{l}{\lambda}+i\pi\frac{(x')^2+(y')^2}{\lambda l}\right\}\iint g_{\text{in}}(x,y)\cancel{\exp\left\{i\pi\frac{x^2+y^2}{\lambda l}\right\}}\exp\left\{-i2\pi\frac{xx'+yy'}{\lambda l}\right\}\mathrm{d}x\mathrm{d}y,$$

$$g_{\text{out}}(x',y') \approx \frac{1}{i\lambda l}\exp\left\{i2\pi\frac{l}{\lambda}+i\pi\frac{(x')^2+(y')^2}{\lambda l}\right\}\iint g_{\text{in}}(x,y)\exp\left\{-i2\pi\frac{xx'+yy'}{\lambda l}\right\}\mathrm{d}x\mathrm{d}y$$

approximation valid if $x^2+y^2 << \lambda l \Leftrightarrow l >> \frac{(x^2+y^2)_{\max}}{\lambda}$

Fraunhofer diffraction

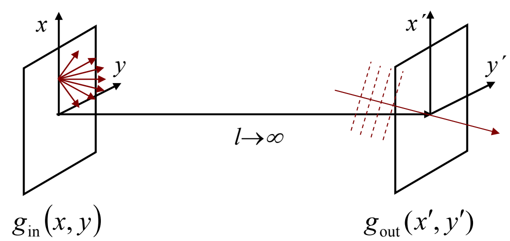


$$g_{\text{out}}(x',y';l) \propto \int g_{\text{in}}(x,y)\exp\left\{-i2\pi\left[x\left(\frac{x'}{\lambda l}\right)+y\left(\frac{y'}{\lambda l}\right)\right]\right\}\mathrm{d}x\mathrm{d}y$$

The **"far-field"** (i.e. the diffraction pattern at a large longitudinal distance l equals the **Fourier transform** of the original transparency calculated at spatial frequencies

$$f_x = \frac{x'}{\lambda l} \qquad f_y = \frac{y'}{\lambda l}$$

Fraunhofer diffraction



spherical wave originating at x $\xrightarrow{l\to\infty}$ plane wave propagating at angle $-x/l$ ⇔ spatial frequency $-x/(\lambda l)$

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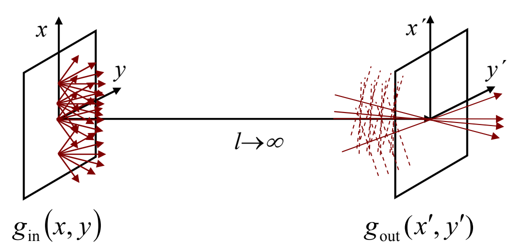
Fraunhofer diffraction



superposition of spherical waves originating at various points along x $\xrightarrow{l\to\infty}$ superposition of plane waves propagating at corresponding angles $-x/l$ ⇔ spatial frequencies $-x/(\lambda l)$

$$g_{\text{out}}(x',y';l) \propto \int g_{\text{in}}(x,y)\exp\left\{-i2\pi\overbrace{\left[x\left(\frac{x'}{\lambda l}\right)+y\left(\frac{y'}{\lambda l}\right)\right]}\right\}\mathrm{d}x\mathrm{d}y$$

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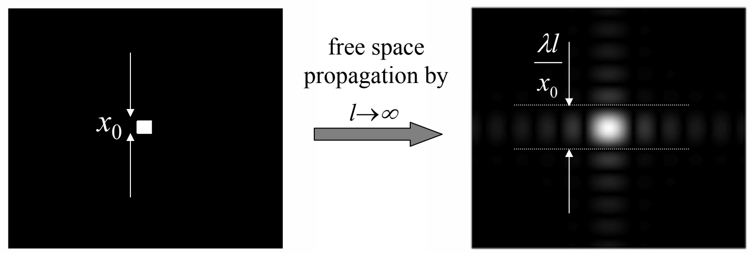
Example: rectangular aperture



input field

far field

$$g_{\text{in}}(x,y)=\text{rect}\left(\frac{x}{x_0}\right)\text{rect}\left(\frac{y}{y_0}\right)$$

$$g_{\text{out}}(x,y)=\frac{e^{i2\pi l/\lambda}}{i\lambda l}\exp\left\{\frac{(x')^2+(y')^2}{\lambda l}\right\}$$
$$\times x_0 y_0\,\text{sinc}\left(\frac{x_0 x'}{\lambda l}\right)\text{sinc}\left(\frac{y_0 y'}{\lambda l}\right)$$

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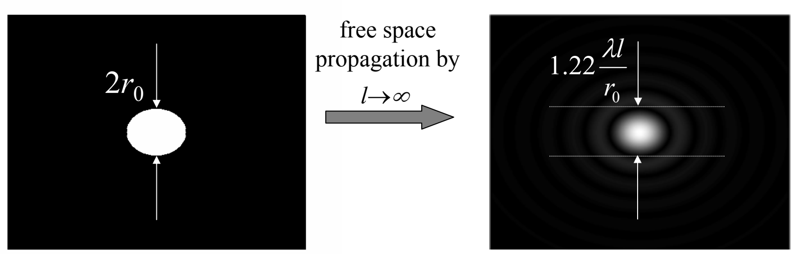
Example: circular aperture



input field

far field

$$g_{\text{in}}(x,y)=\text{circ}\left(\frac{\sqrt{x^2+y^2}}{r_0}\right)$$

$$g_{\text{out}}(x,y)=\frac{e^{i2\pi l/\lambda}}{i\lambda l}\exp\left\{\frac{(x')^2+(y')^2}{\lambda l}\right\}$$
$$\times \pi r_0^2\left[2\frac{J_1\left(\frac{2\pi r_0\sqrt{(x')^2+(y')^2}}{\lambda l}\right)}{\frac{2\pi r_0\sqrt{(x')^2+(y')^2}}{\lambda l}}\right]$$

also known as
Airy pattern, or

$$\text{jinc}\left(\frac{2\pi r_0\sqrt{(x')^2+(y')^2}}{\lambda l}\right)$$

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Gratings

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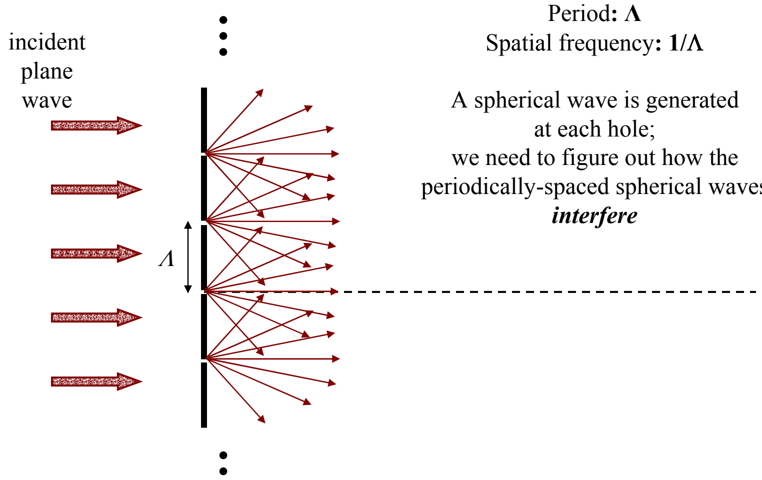
Diffraction from periodic array of holes



Period: **Λ**
Spatial frequency: **1/Λ**

A spherical wave is generated at each hole; we need to figure out how the periodically-spaced spherical waves ***interfere***

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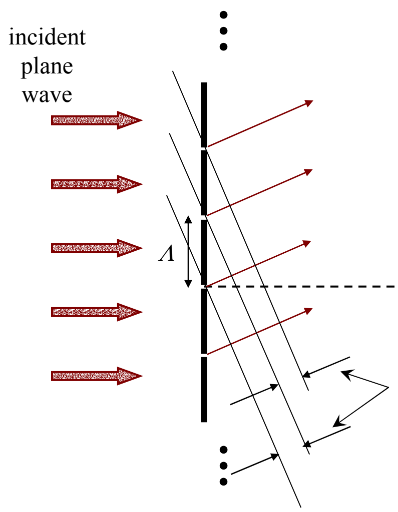
Diffraction from periodic array of holes

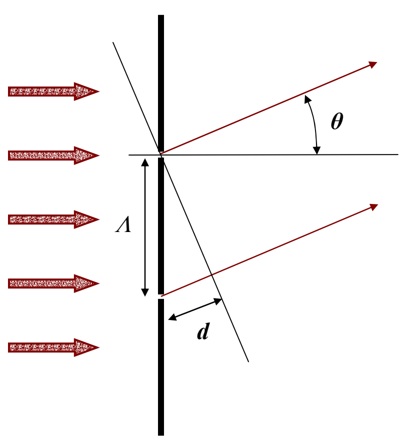


Period: **Λ**
Spatial frequency: **1/Λ**

Interference is ***constructive*** in the direction pointed by the parallel rays if the optical path difference between successive rays equals an integral multiple of **λ** (equivalently, the phase delay equals an integral multiple of **2π**)

Optical path differences

Diffraction from periodic array of holes



Period: **Λ**
Spatial frequency: **1/Λ**

From the geometry we find

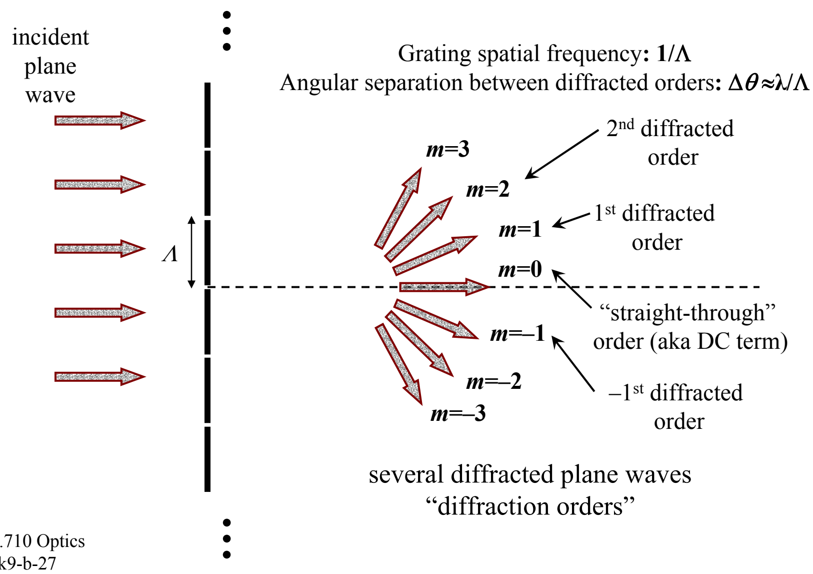
$$d = \Lambda \sin\theta$$

Therefore, interference is constructive iff

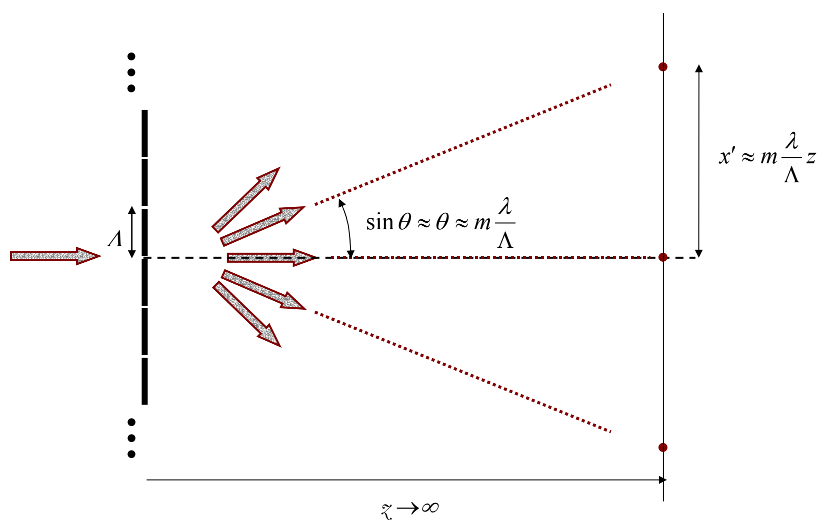
$$\Lambda \sin\theta = m\lambda \Leftrightarrow$$

$$\Leftrightarrow \sin\theta = m\frac{\lambda}{\Lambda}$$

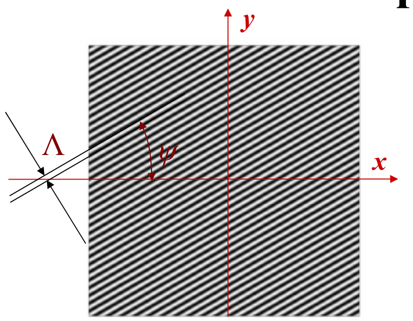
Diffraction from periodic array of holes

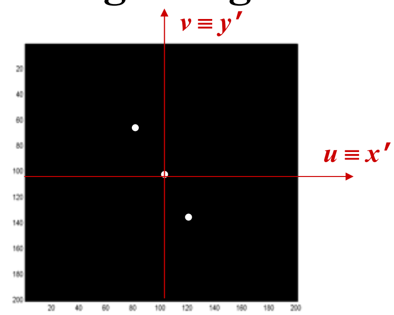


Fraunhofer diffraction from periodic array of holes



Sinusoidal amplitude grating





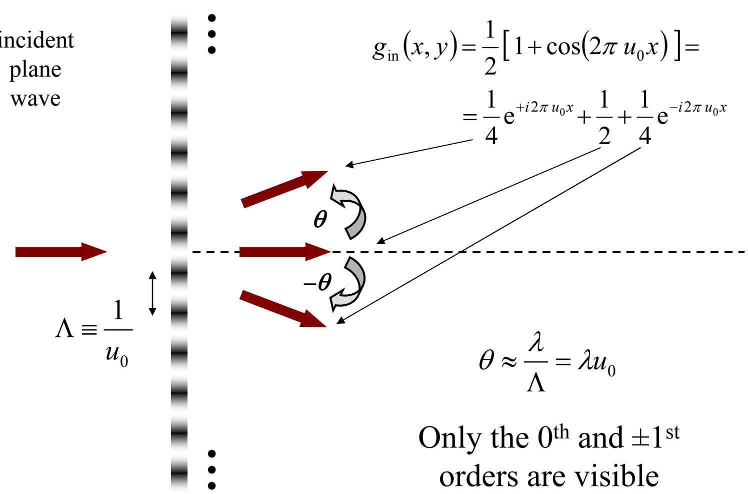
$$g_{\text{in}}(x,y) = \frac{1}{2}\left[1 + \cos\left(2\pi(u_0 x + v_0 y)\right)\right]$$

$$\Lambda = \frac{1}{\sqrt{u_0^2 + v_0^2}}, \quad \tan\psi = \frac{u_0}{v_0}$$

$$g_{\text{out}}(x,y) = \frac{e^{i2\pi z/\lambda}}{i\lambda z} \exp\left\{\frac{(x')^2 + (y')^2}{\lambda z}\right\}$$

$$\times\left[\frac{1}{2}\delta(x')\delta(y') + \frac{1}{4}\delta(x' - \lambda z u_0)\delta(y' - \lambda z v_0) + \frac{1}{4}\delta(x' + \lambda z u_0)\delta(y' + \lambda z v_0)\right]$$

Sinusoidal amplitude grating



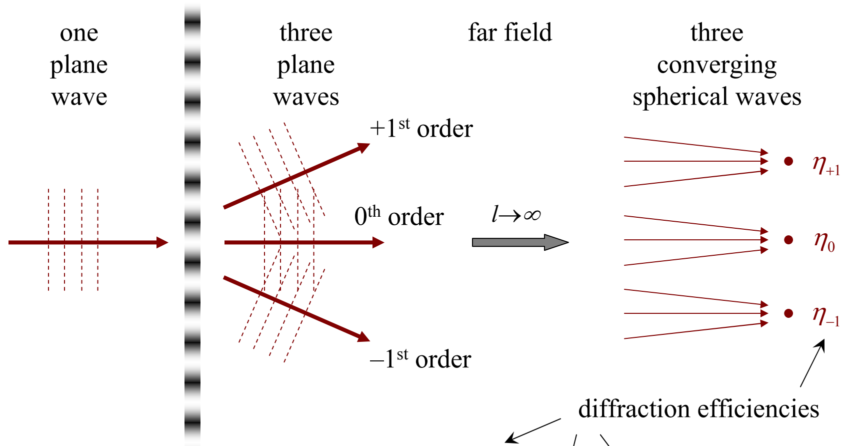
$$g_{\text{in}}(x,y) = \frac{1}{2}\left[1 + \cos(2\pi\, u_0 x)\right] =$$

$$= \frac{1}{4}e^{+i2\pi u_0 x} + \frac{1}{2} + \frac{1}{4}e^{-i2\pi u_0 x}$$

$$\theta \approx \frac{\lambda}{\Lambda} = \lambda u_0$$

Only the 0$^{\text{th}}$ and $\pm 1^{\text{st}}$ orders are visible

Sinusoidal amplitude grating



$$\eta_0 = \frac{1}{4} = 25\%\,, \quad \eta_{+1} = \eta_{-1} = \frac{1}{16} = 6.25\%$$

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Dispersion

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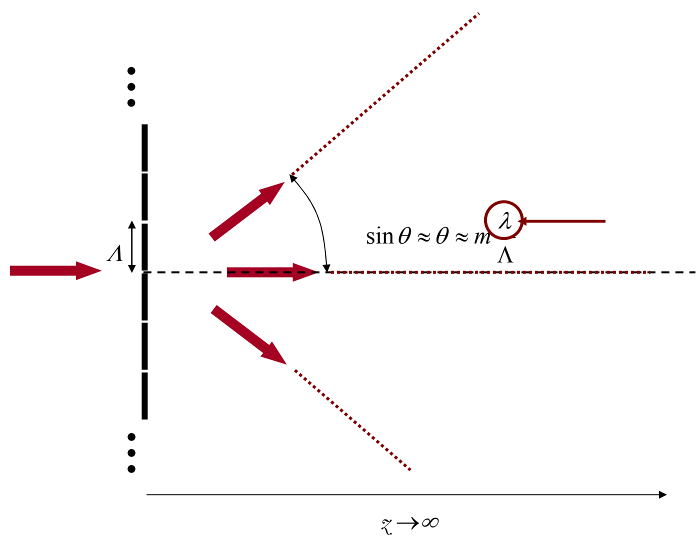
Diffraction from a grating



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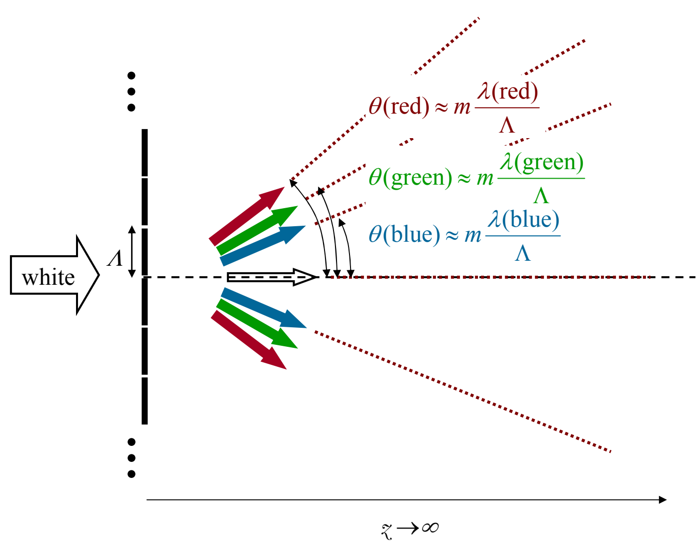
Dispersion from a grating



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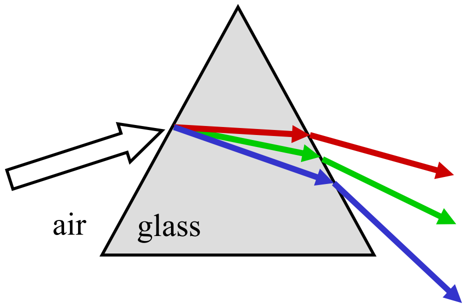
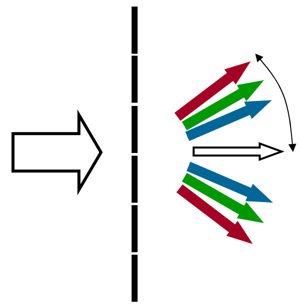
Prism dispersion *vs* grating dispersion





Blue light is refracted at *larger* angle than red:

normal dispersion

Blue light is diffracted at *smaller* angle than red:

anomalous dispersion