2.71 Optics   **Fall '05**
Problem Set #1   Posted Sept. 7, 2005 — Due <u>Wednesday, Sept. 14, 2005</u>

1. **Optics jargon** Below is a list of buzzwords that are currently dominating the headlines of optics research. Pick one, then do a quick Web search and write briefly (5-10 sentences) what it means and why it is considered important.

   a) hyperspectral imaging,

   b) plasmon optics,

   c) long-baseline radio telescope,

   d) photonic bandgap crystals,

   e) negative refractive index,

   f) Bose-Einstein condensate

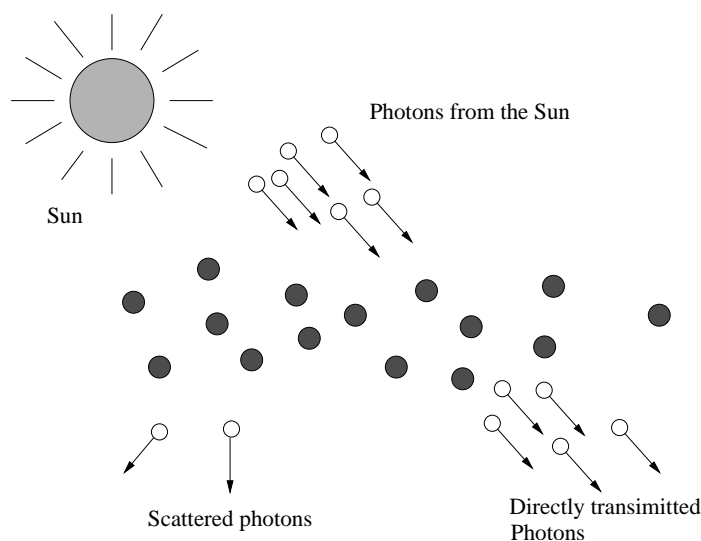   **g)** gravitational lensing.

Figure 2.a

2. **Why is the sky blue?** In 1899, Lord Rayleigh observed that when we look at the sky, we see two kinds of light: "direct" light from the sun, which forms a painfully bright image of the sun on our retina, and "scattered" light which appears as the diffuse blue sky. Rayleigh attributed the blueness of the sky to

the scattering of photons from the sun by particles in the atmosphere, primarily nitrogen. See Figure 2.a above. In this problem, we will walk through Rayleigh's derivation and arrive to the famous law of scattering which explains why blue light is scattered more strongly than other wavelengths.
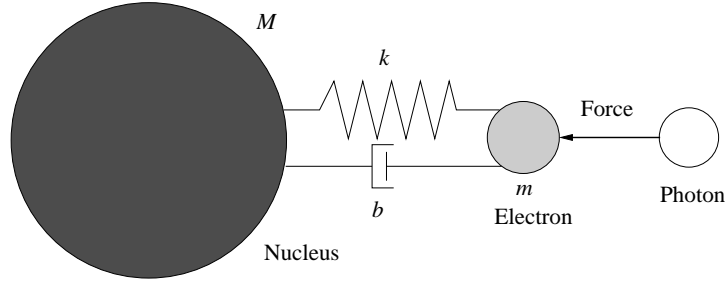

Figure 2.b

Consider the model shown in Figure 2.b, where a photon from the sun is incident on a nitrogen atom. The atom is modeled as a nucleus of mass M and an electron of mass $m \ll M$ bound to the nucleus with a spring of spring constant k and friction coefficient b. The electron is set in motion by the incident electron, whereas the massive nucleus is assumed not to move at all. The force applied by the photon on the electron as function of time t is proportional to $\cos(\omega t)$, where ω is the photon frequency (this is because the force is proportional to the photon's oscillating electric field). The result of this "excitation" of the nitrogen atom by incident light results in the scattering of a new photon, as shown in Figure 2.c. How strong is the scattering? Rayleigh assumed that the scattering strength, sometimes referred to as "scattering cross-section," is proportional to the modulus squared of the acceleration of the electron that results form excitation.
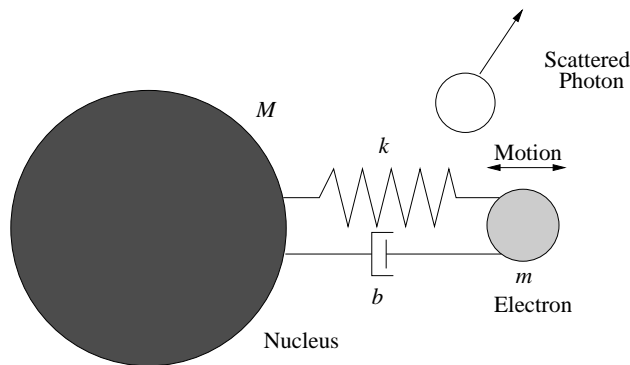

Figure 2.c

a) Derive the transfer function $A(\omega)$ of the electron acceleration with the force by the photon as input.

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b) Identify the resonance frequency and the damping ratio of the system from the transfer function, and sketch the modulus squared $|A(\omega)|^2$ of the transfer function that you obtained.

c) The nitrogen atom has spring constant $k \approx 140\text{N/m}$ and the mass of the electron is $m = 9.11 \times 10^{-31}\text{kg}$. Show that the resonance frequency of this system is higher than all optical frequencies within the visible spectrum of light.

d) Show that $|A(\omega)|^2$ behaves approximately as ω^4 in the visible region; hence higher frequencies (shorter wavelengths) are scattered more strongly than lower frequencies. This is known as " Rayleigh's law of scattering."