MASSACHUSETTS INSTITUTE OF TECHNOLOGY

2.71 Optics **Fall '05**

Problem Set #10 Posted Wednesday, Nov. 30, 2005 — Due Friday, Dec. 9, 2005

1. Consider the optical system shown in Figure 1, where lenses L1, L2 are identical with focal length f and half-aperture a. A thin-transparency object is placed $2f$ to left of L1.

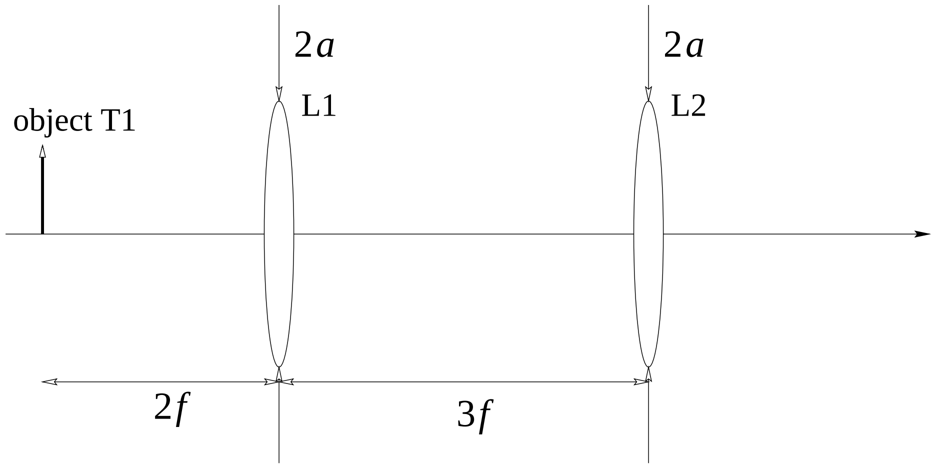

Figure 1

1.a) Where is the image formed? Use geometrical optics, ignoring the lens apertures for the moment.

1.b) If the object T1 is an on-axis point source, describe the Fraunhofer diffraction pattern of the field to the right of L2.

1.c) How are your two previous answers consistent within the approximations of paraxial geometrical and wave optics?

1.d) The point source object T1 is replaced by a clear aperture of full width w and a second thin transparency T2 is placed between the two lenses, at distance f to the left of L2. The system is illuminated coherently with a monochromatic on-axis plane wave at wavelength λ. Write an expression for the field at distance $2f$ to the right of L2 and interpret the expression that you found.

1.e) Derive and approximately sketch, with as much quantitative detail as you can, the intensity observed at distance $2f$ to the right of L2 when T2 is an infinite sinusoidal amplitude grating of period Λ, such that $\Lambda \ll a$

2. You are given an imaging system which consists of two thin transparencies T1, T2 and two thin lenses L1, L2 arranged as shown in Figure 2A. The shapes and dimensions of T1, T2 are shown immediately below in Figure 2B. Transparency T1 is infinitely large in the x dimension. Lenses L1, L2 are identical, with infinitely large apertures and focal lengths $f_1 = f_2 = 10$cm.

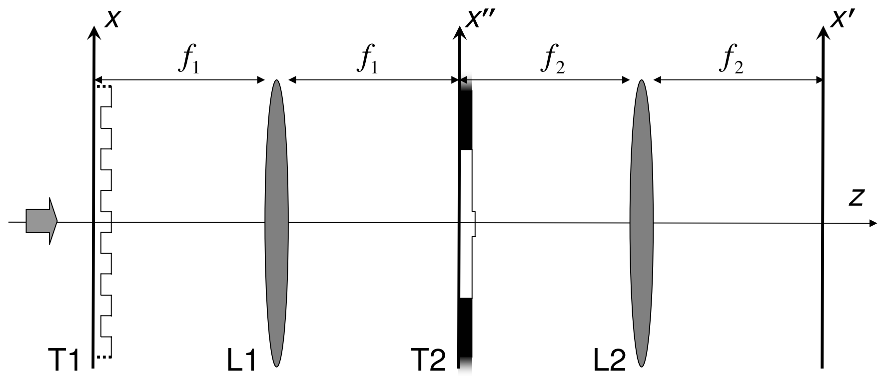

Figure 2A *(not to scale)*

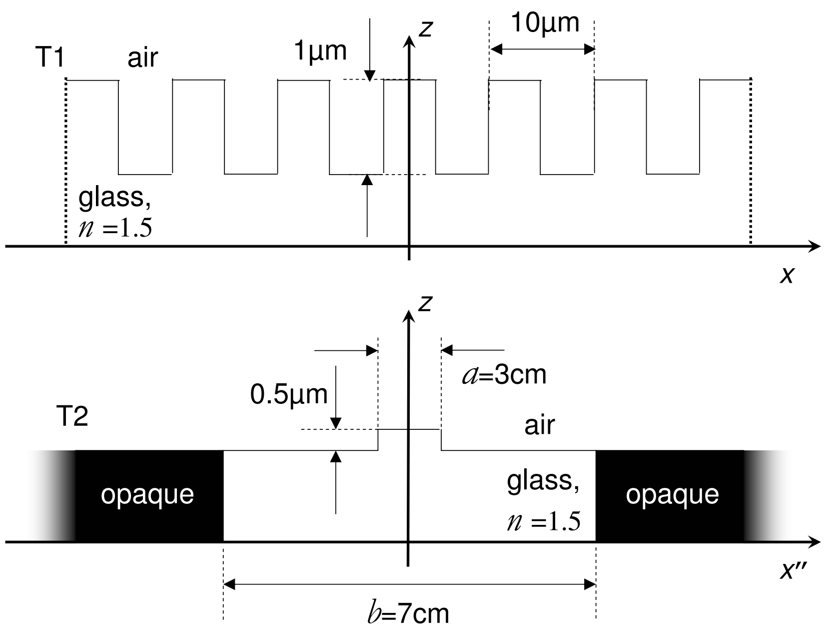

Figure 2B *(not to scale)*

The system is illuminated from the left with monochromatic, spatially coherent light. The illumination is an on–axis plane wave at wavelength $\lambda = 1\mu$m. The observation plane is located one focal distance behind L2.

2.a) What is the intensity immediately after T1?

2.b) What is the optical field immediately before T2?

2.c) What is the intensity measured at the observation plane?

2.d) Comparing your answers (a) and (c), how is T2 helpful in imaging the phase object T1?

(Note: the symbols a, b are defined in Figure B.)

3. Figure 3 below shows the schematic diagram of a simple grating spectrometer. It consists of a sinusoidal amplitude grating of period Λ and lateral size (aperture) a followed by a lens of focal length f and sufficiently large aperture. To analyze this spectrometer, we will assume that it is illuminated from the left in spatially coherent fashion by two plane waves on–axis. One of the plane waves is at wavelength λ and the other is at wavelength $\lambda + \Delta\lambda$, where $|\Delta\lambda| \ll \lambda$. (The two plane waves at different wavelengths are mutually incoherent.) Since the two colors are diffracted by the grating to slightly different angles, the goal of this system is to produce two adjacent but sufficiently well separated bright spots at the output plane, one for each color.

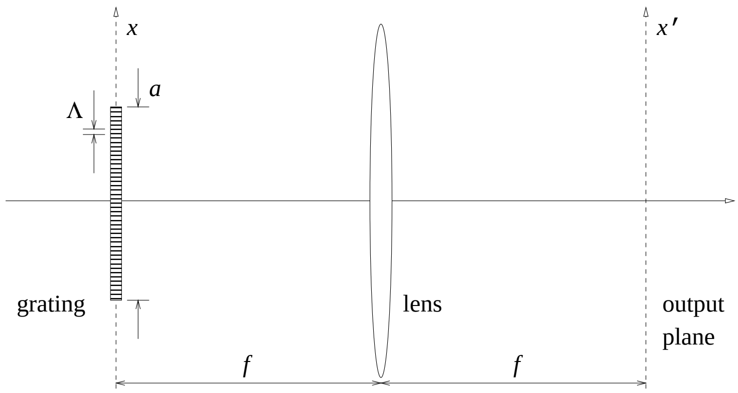

Figure 3

3.a) Estimate the minimum aperture size of the lens so that it does not impair the operation of the spectrometer.

3.b) What is the maximum power efficiency that this spectrometer can achieve?

3.c) Show that a condition for the two color spots to be "sufficiently well" separated is

$$\frac{\lambda}{|\Delta\lambda|} < \frac{a}{\Lambda}.$$

This result is often stated in spectroscopy books as follows: *The resolving power of a grating spectrometer, defined as the ratio of the mean wavelength λ to the spectral resolution $|\Delta\lambda|$, equals the number of periods in the grating.*