

1. Frequencies and wavelengths

- 1.a) MIT's WMBR radio station transmits at frequency 88.1MHz. What is WMBR's wavelength in air?
- 1.b) Microwave antennas, especially those of radio stations, are often of the " $\lambda/2$ " type, i.e. the height of the antenna equals one half of the wavelength. If an antenna is 10m tall, what is the frequency that it is transmitting?
- 1.c) An obsessive flutist keeps playing the A tone (440Hz) on WMBR. What is the wavelength of that sound?
- 1.d) Soft x-rays have a wavelength of 1nm. What is the frequency?
- 1.e) According to quantum mechanics, particles can also be thought of as waves. The "dispersion relation" for these waves, is different than that for light waves that we saw in class; instead, particles obey the "de Broglie" dispersion relationship, which is

$$\lambda = \frac{h}{mv},$$

where λ is the wavelength associated with the particle, m is the particle's rest mass, v is the (non-relativistic) velocity, and h is Planck's constant. Based on this equation, what is the electron's wavelength?

- 1.f) Water waves also obey a different dispersion relation, given by

$$\omega^2 = gk \tanh(kd),$$

where, as usual, $\omega = 2\pi\nu$, $k = 2\pi/\lambda$; g is the acceleration of gravity, and d is the depth of the water. If the depth of a pool is 3m, and the wavelength is 10cm, what is the frequency of the water waves?

- 2. Plane waves and phasor representations** Throughout this problem, by "complete expression" of a wave we mean the space-time representation, e.g. $f(x, y, z, t) = A \cos(kx - \omega t)$ is a plane wave of wave-vector magnitude k and frequency ω propagating in the direction of the $\hat{x}$ coordinate axis. By "phasor expression" we mean the complex representation of the wave, e.g. Ae^{ikx} for the same wave.

- 2.a) Write the complete and phasor expressions for a plane wave $f_1(x, y, z, t)$ propagating at an angle 30° relative to the $\hat{z}$ axis on the xz -plane (i.e., the plane $y = 0$). The wavelength is $\lambda = 1\mu\text{m}$, and the wave speed is $c = 3 \times 10^8 \text{m} \cdot \text{sec}^{-1}$.

- 2.b)** Write the complete and phasor expressions for a plane wave $f_2(x, y, z, t)$ of the same wavelength and wave speed as f_1 but propagating at angle 60° relative to the $\hat{z}$ axis on the yz -plane.
- 2.c)** Use the complete expression to plot $f_1(x, y, z = 0, t = 0)$ and $f_2(x, y, z = 0, t = 0)$ using MATLAB. (*Note:* you will probably need to use `surf` or an equivalent command.)
- 2.d)** The plane $z = 0$ is illuminated by the superposition of the two waves f_1 and f_2 . Plot the waveform received at points A, B, C, D, E with Cartesian coordinates, respectively,

$$(0, 0, 0), \left(\frac{1}{4}, -\frac{1}{4\sqrt{3}}, 0\right), \left(\frac{1}{2}, -\frac{1}{2\sqrt{3}}, 0\right), \left(\frac{3}{4}, -\frac{3}{4\sqrt{3}}, 0\right), \left(1, -\frac{1}{2\sqrt{3}}, 0\right)$$

(all units in microns) as function of time. What do you observe?

3. Superposition of scalar waves

Given two waves

$$E_1(z, t) = E_0 \cos(k_1 z - \omega_1 t),$$

$$E_2(z, t) = E_0 \cos(k_2 z - \omega_2 t),$$

where $\omega_1 = 100\text{Hz}$, $\omega_2 = 101\text{Hz}$ and the wavenumbers k_1, k_2 obey the usual dispersion relation for light waves.

- 3.a)** Show that the superposition can be expressed as a product of two tones, one at frequency lower than both the waves, and one at higher frequency.
- 3.b)** Plot your result. What do you observe?

4. Superposition of polarized waves

Given two waves

$$\mathbf{E}_1(z, t) = \hat{x} E_0 \cos(kz - \omega t),$$

$$\mathbf{E}_2(z, t) = \hat{y} E_0 \cos(kz - \omega t),$$

where E_0 is a constant electric field magnitude, and k, ω are the wavenumber and angular frequency of the waves.

- 4.a)** What is the superposition of the two waves $\mathbf{E}_1, \mathbf{E}_2$? What is the type of its polarization?
- 4.b)** Now suppose the second wave $\mathbf{E}_2$ is delayed by $\pi/2$. What is the polarization of the superposition wave?
- 5. Complex arithmetic** Consider the complex numbers $z_1 = 1 + i2, z_2 = 3 + i4, z_3 = -1 + i2$. Compute the numerical value of the following expressions, and express it in both Cartesian and polar form.

5.a) z_2 (polar form only)

5.b) $z_1 + z_2$

5.c) $\frac{z_2}{z_3}$

5.d) $\left| \frac{z_1}{z_3} \right|$

5.e) $\frac{z_1^2}{z_2 + z_3}$

5.f) $\exp \{z_1\}$

5.g) $\cos \left(\frac{z_1}{z_2} \right)$