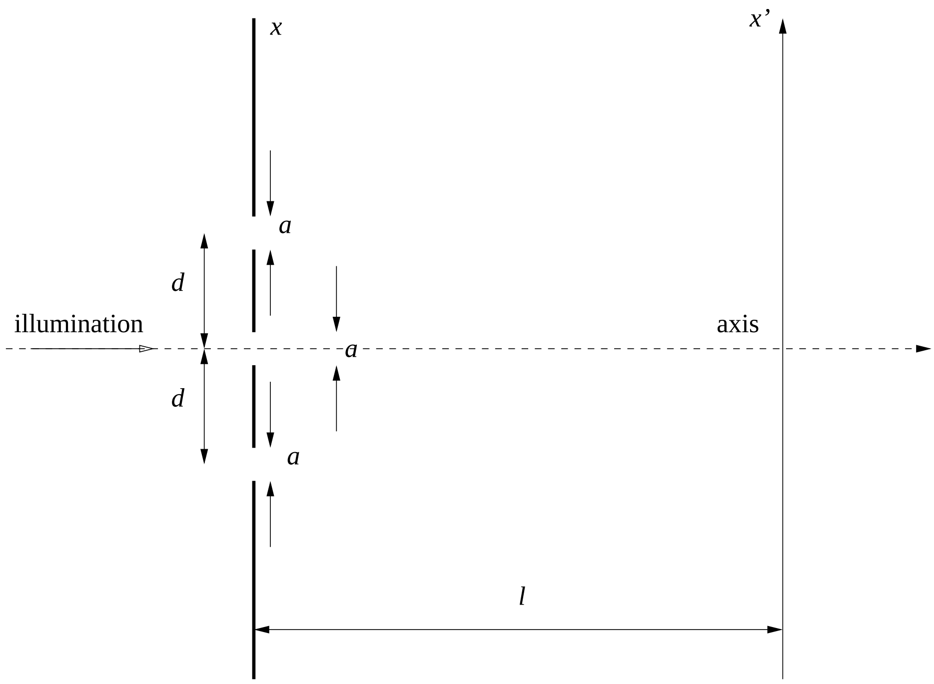
2.71 Optics **Fall '05**

Problem Set #9 Posted Wednesday, Nov. 23, 2005 — Due Wednesday, Nov. 30, 2005

1. (Do this problem in 1 dimension) The diagram below shows an opaque screen with three identical apertures of width a and separation d between adjacent apertures. The system is illuminated by a monochromatic, spatially coherent plane wave incident on-axis. The Fraunhofer diffraction pattern is observed at a distance $l \to \infty$ away from the object. We are given the freedom to place glass plates of arbitrary thickness inside the apertures to manage the relative phase delay between the fields passing from the apertures. (The phase delay must be uniform within each aperture.)

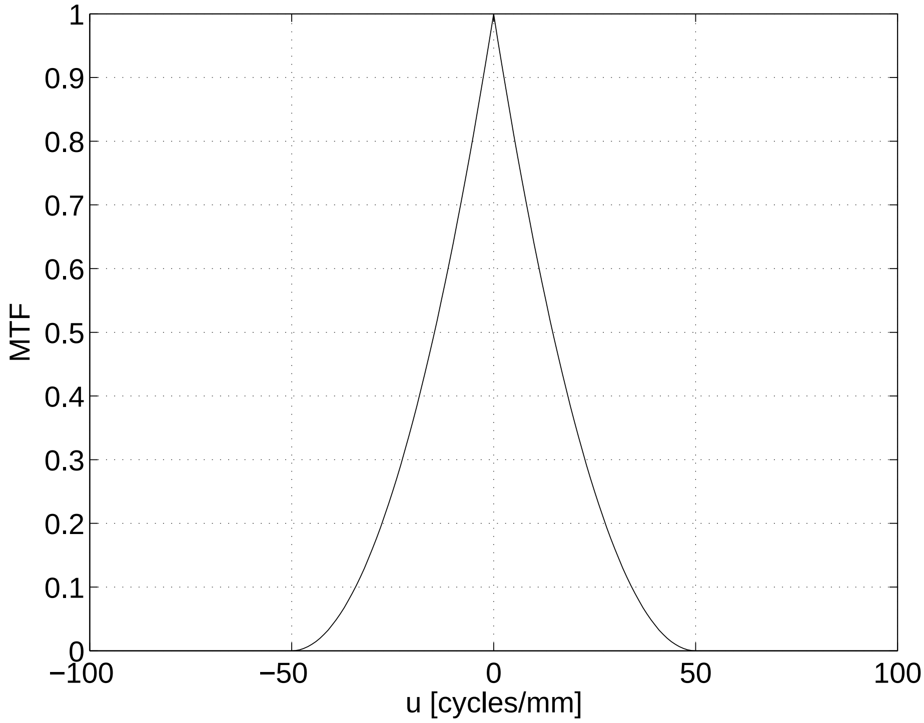
1.a) Devise a set of phase delays between the apertures that result in *destructive* interference on-axis in the Fraunhofer zone.

1.b) Derive and plot the intensity of the entire Fraunhofer diffraction pattern for your choice of phase delays.



2. The modulation transfer function (MTF) of an optical system is given in the figure below. The system is illuminated with quasi-monochromatic, spatially incoherent light. The intensity pattern at the input plane of the system is given by

$$I(x) = \frac{1}{2}\left[1 + \frac{1}{2}\cos\left(2\pi\frac{x}{40\mu\text{m}}\right) + \frac{1}{2}\cos\left(2\pi\frac{3x}{40\mu\text{m}}\right)\right]$$



2.a) What is the contrast of the intensity pattern at the input plane?

2.b) Plot the intensity pattern formed at the output plane, and calculate the image contrast.

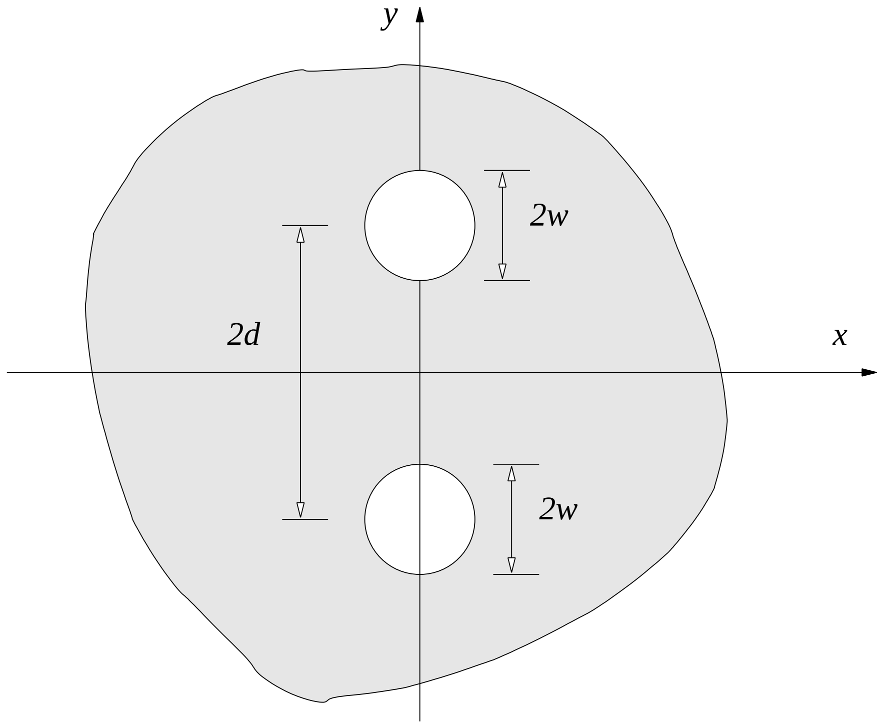
2.c) Can you guess the coherent transfer function and cut-off spatial frequency for this imaging system?

3. An object has intensity transmittance given by

$$t_{\text{I}}(x) = \frac{1}{2}\left[1 + \cos\left(2\pi\frac{x}{\Lambda}\right)\right]$$

and introduces a constant, uniform phase delay across the object plane. This object is placed at a distance $2f$ in front of a positive lens of focal length f, and the image is examined in a plane $2f$ behind the lens. Compare the minimum periods Λ that are visible in the image formed by this system for the cases of coherent and incoherent illumination.

4. Sketch cross-sections of the optical transfer function along the spatial frequency axes u, v for an incoherent imaging system having a pupil function as the two-pinhole combination shown in Figure 3. Assume $w < d$. Do *not* use MATLAB for this calculation. Explain briefly the appearance of your sketches, and be sure to label the various cutoff frequencies and center frequencies.



5. In a diffraction–limited imaging system, the contrast is measured at spatial frequency 25mm^{-1} and it is found to be 68.75%.

5.a) What is the contrast at spatial frequency 50mm^{-1}?

5.b) Is the spatial frequency 50mm^{-1} "visible" through this imaging system under spatially coherent illumination consisting of a plane wave on–axis?

5.c) Does the answer to the previous question change if the on–axis constraint is relaxed, *i.e.* if we allow off–axis plane wave illumination?