MASSACHUSETTS INSTITUTE OF TECHNOLOGY

2.710 Optics **Fall '05**

Problem Set #10 Posted Wednesday, Nov. 30, 2005 — Due Friday, Dec. 9, 2005

1. Consider the optical system shown in Figure 1, where lenses L1, L2 are identical with focal length f and half-aperture a. A thin-transparency object is placed $2f$ to left of L1.

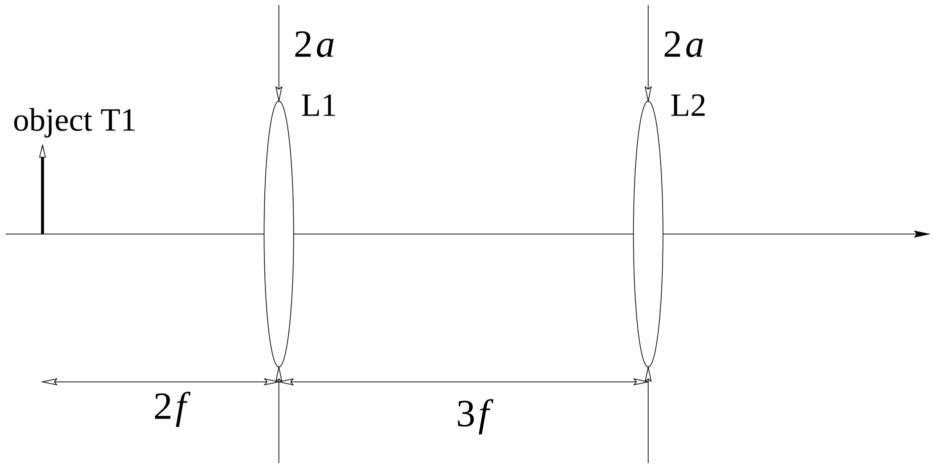

Figure 1

1.a) Where is the image formed? Use geometrical optics, ignoring the lens apertures for the moment.

1.b) If the object T1 is an on-axis point source, describe the Fraunhofer diffraction pattern of the field to the right of L2.

1.c) How are your two previous answers consistent within the approximations of paraxial geometrical and wave optics?

1.d) The point source object T1 is replaced by a clear aperture of full width w and a second thin transparency T2 is placed between the two lenses, at distance f to the left of L2. The system is illuminated coherently with a monochromatic on-axis plane wave at wavelength λ. Write an expression for the field at distance $2f$ to the right of L2 and interpret the expression that you found.

1.e) Derive and approximately sketch, with as much quantitative detail as you can, the intensity observed at distance $2f$ to the right of L2 when T2 is an infinite sinusoidal amplitude grating of period Λ, such that $\Lambda \ll a$

2. You are given an imaging system which consists of two thin transparencies T1, T2 and two thin lenses L1, L2 arranged as shown in Figure 2A. The shapes and dimensions of T1, T2 are shown immediately below in Figure 2B. Transparency T1 is infinitely large in the x dimension. Lenses L1, L2 are identical, with infinitely large apertures and focal lengths $f_1 = f_2 = 10$cm.

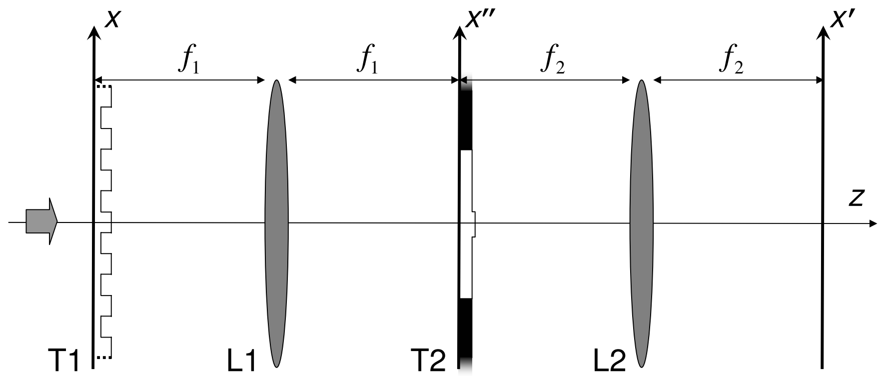

Figure 2A *(not to scale)*

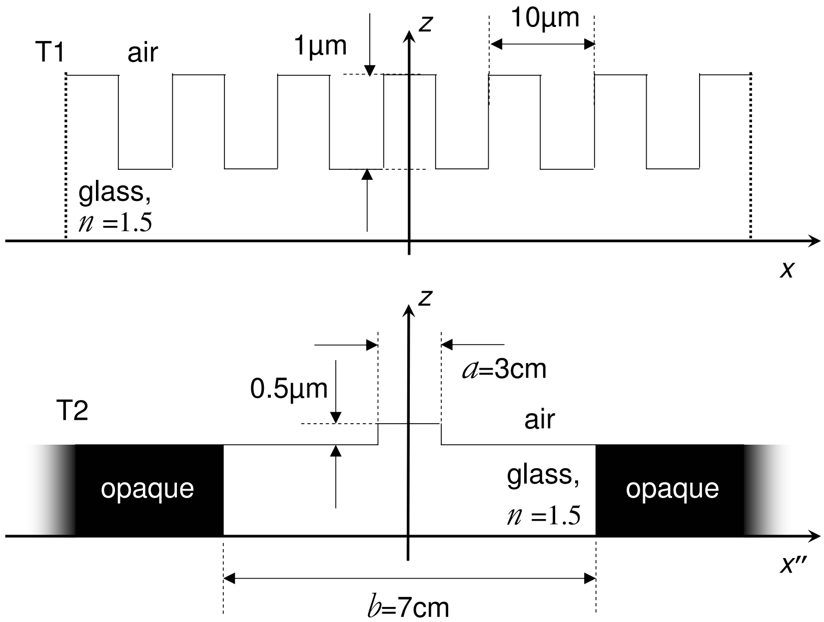

Figure 2B *(not to scale)*

The system is illuminated from the left with monochromatic, spatially coherent light. The illumination is an on–axis plane wave at wavelength $\lambda = 1\mu$m. The observation plane is located one focal distance behind L2.

2.a) What is the intensity immediately after T1?

2.b) What is the optical field immediately before T2?

2.c) What is the intensity measured at the observation plane?

2.d) Comparing your answers (a) and (c), how is T2 helpful in imaging the phase object T1?

2.e) Consider the limit $b \to \infty$. How does then answer (c) change? Is the larger aperture helpful in this case?

2.f) If $a = 0.5$cm and $b \to \infty$, is your answer (d) still valid? If yes, why? If not, what has gone wrong?

(Note: the symbols a, b are defined in Figure B.)

3. An infinite periodic square-wave grating with transmittivity as shown in Figure 3A is placed at the input of the optical system of Figure 3B. Both lenses are positive, $F/1$, and have focal length f. The grating is illuminated with monochromatic, spatially coherent light of wavelength λ and intensity I_0. The spatial period of the grating is $X = 4\lambda$. The element at the Fourier plane of the system is a nonlinear transparency with the intensity transmission function shown in Figure 3C, where the threshold and saturating intensities $I_{\text{thr}} = I_{\text{sat}} = 0.1 I_0$. To calculate the response of this system analytically, we need to make the paraxial approximation; strictly speaking, that is questionable for $F/1$ optics, but we will follow it nevertheless. An additional necessary assumption is discussed in the first question below.

3.a) To answer the second question, we need to neglect the Airy patterns forming at the Fourier plane and pretend they are uniform bright dots. Explain why this assumption is justified and what effects it might have.

3.b) Derive and plot the intensity distribution at the output plane using the above assumption.

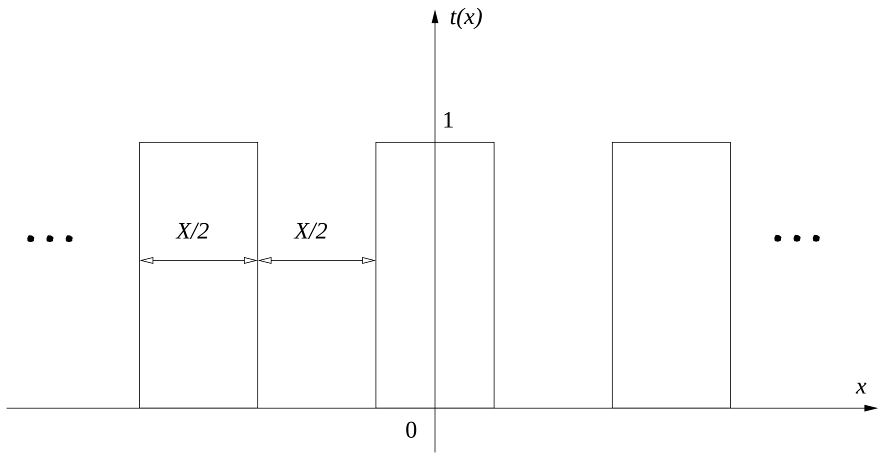

Figure 3A

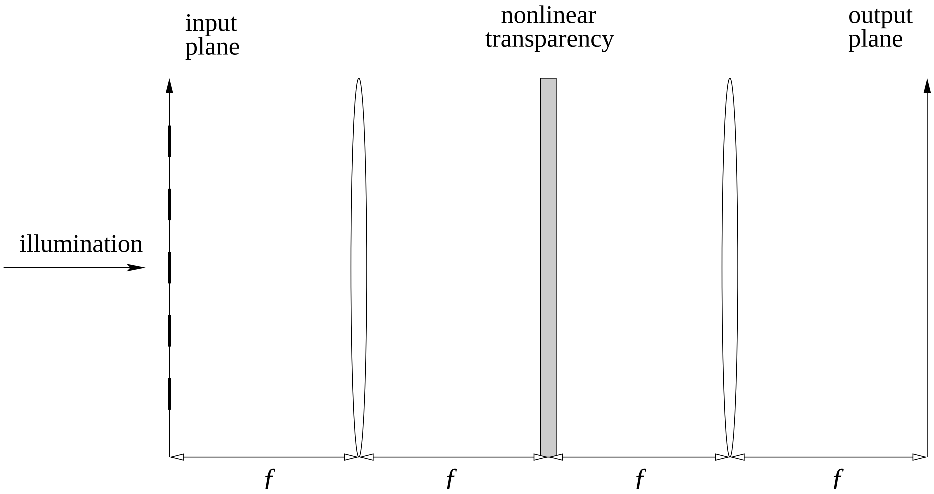

Figure 3B

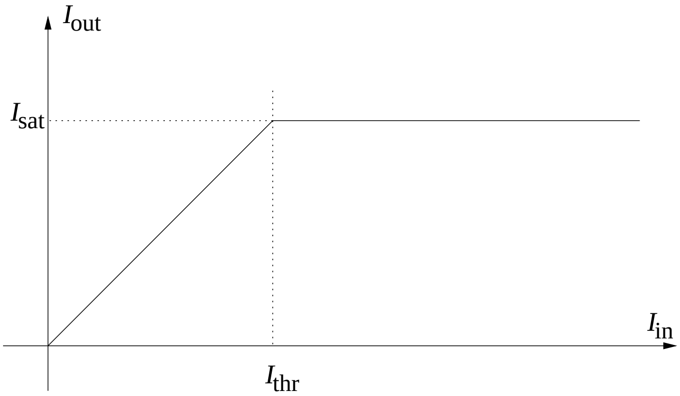

Figure 3C