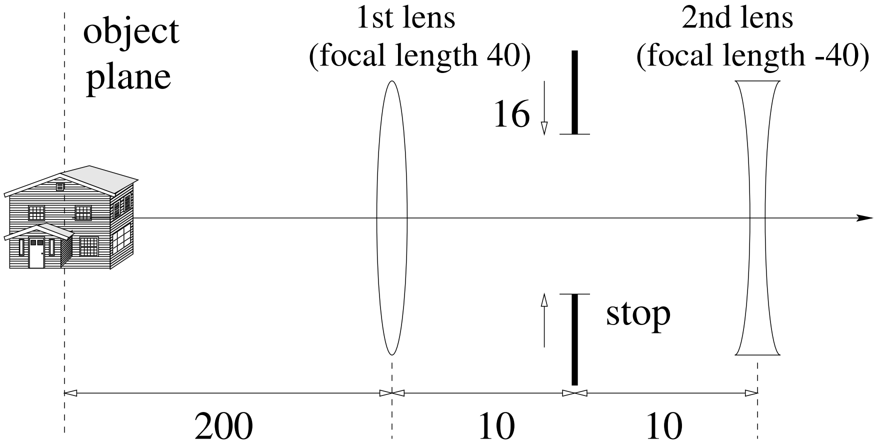
MASSACHUSETTS INSTITUTE OF TECHNOLOGY

2.710 Optics **Fall '05**

Problem Set #4 Posted Sept. 28, 2005 — Due Wednesday, Oct. 5, 2005

1. Consider the two–lens system shown below (all distances in cm). Find the numerical aperture.



2. Consider two identical thin lenses of focal length $f = 5$ inches, spaced one focal length apart (see figure on next page.)

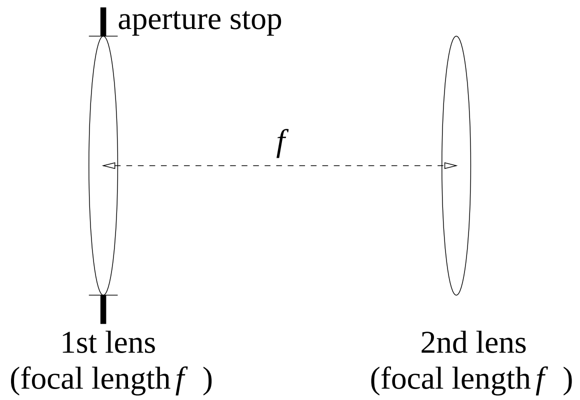
2.a) Calculate the focal length and back focal length of the combination.

2.b) Assuming the stop is located at the first lens and has a diameter of 1 inch, and the second lens is also 1 inch in diameter, calculate and draw the trajectory of the marginal and chief rays.

2.c) What is the field of view of the system?

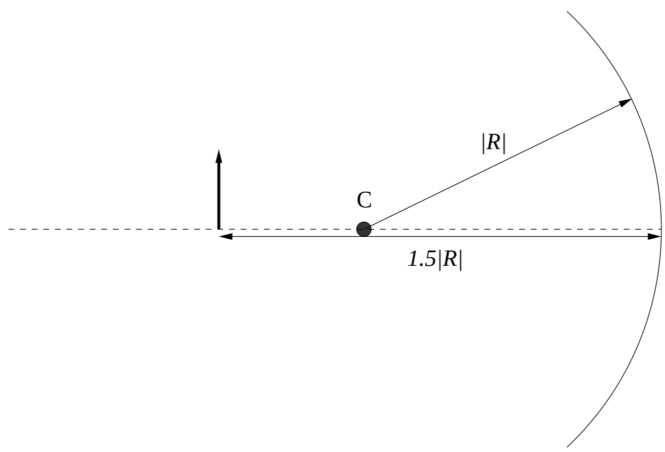
2.d) What is the $f/\#$ of the system?

2.e) How does the performance of the system change if the aperture stop is placed 1 focal length to the front (*i.e.*, to the left) of the first lens?



Optical system for Problem 2.

3. A concave mirror of radius $|R|$ is centered at C. An erect object is located distance $1.5\,|R|$ from the apex of the mirror. Locate the image and calculate the magnification.



4. The system matrix for a thick biconvex lens in air is

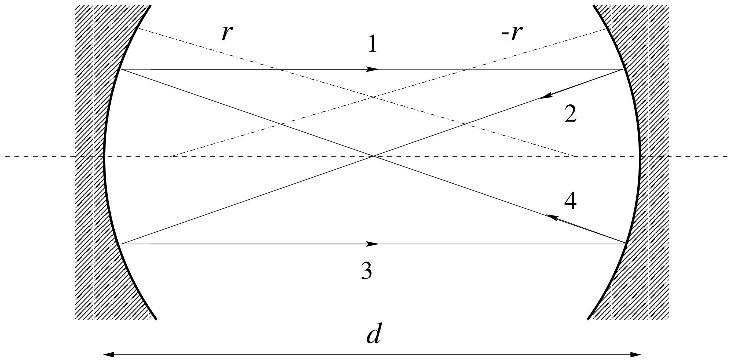
$$\begin{pmatrix} 0.6 & -2.6 \\ 0.2 & 0.8 \end{pmatrix}$$

4.a) Given that the radius of the left–hand side surface is 0.5cm, that the thickness is 0.3cm, and that the index of refraction of the lens material is 1.5, find the radius of the right–hand side surface.

4.b) Locate the principal planes of this thick lens.

4.c) If an object is placed 5cm to the left of the lens, where is the image formed and what is the lateral magnification?

5. Two identical concave mirrors of radius of curvature $|R| = r$ are separated by distance d and are facing each other, as shown in the schematic below. This optical arrangement is referred to as an "optical cavity."



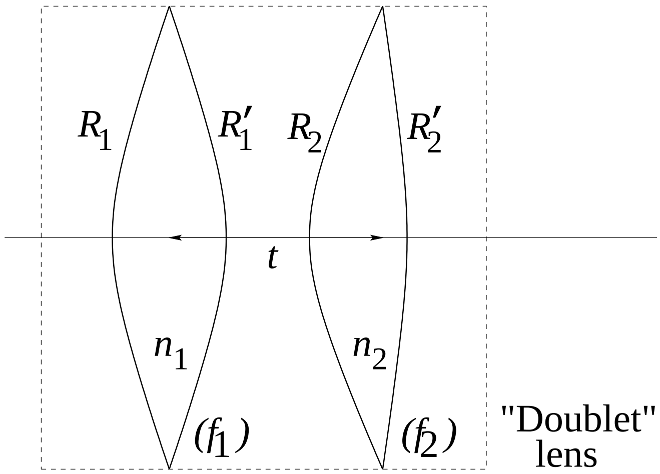
5.a) Show that after traversing the cavity twice (*i.e.*, two round–trips, such that a given ray traces the path 1–2–3–4) the system matrix is

$$\begin{pmatrix} \left(\frac{2d}{r} - 1\right)^2 - \frac{2d}{r} & \frac{4}{r}\left(\frac{d}{r} - 1\right) \\ 2d\left(1 - \frac{d}{r}\right) & 1 - 2\frac{d}{r} \end{pmatrix}$$

5.b) Generalize by deriving an expression for the system matrix after n double passes.

5.c) Interpret the previous expression in terms of the ability of the cavity to retain a light beam confined and show that, in particular, if $d = r$ then the light retraces its original path endlessly as it goes through the cavity. A cavity with $d = r$ is called "confocal," and the self–retracing condition is a partial requirement for "optical resonance" (the other requirement is "constructive interference, that is the optical path length of a round–trip through the cavity should equal an integral number of half–wavelengths; we will learn more about resonance in the Wave Optics part of the class.)

6. Achromatic doublet. When an optical system must operate over a broad range of wavelengths, e.g. over the entire visible range (as opposed to an optical system designed for monochromatic illumination, e.g. from a continuous–wave laser) the dispersion of the optics causes variation of the focal length as function of wavelength. This condition is known as "chromatic aberration." A common method to reduce chromatic aberration is to combine *two* optical elements with different material dispersion curves and optical powers such that the optical power of the compound element equals the desired power and the chromatic aberration of the compound element equals zero. These elements are known as "achromats," or "achromatic doublets." (Note that the term "achromat" as a disability condition refers rather to persons who lack cones in their retina and hence are unable to perceive color.) Consider the achromat shown below, where the individual elements are thin spherical lenses and there is a distance t between them. We seek to minimize the chromatic aberration of the achromat between two wavelengths of interest λ_a and λ_b.



6.a) Let V_j $(j = 1, 2)$ denote Abbe's V-number for the j-th element, and f_j the corresponding focal length. Show that the chromatic dispersion in the optical power of the j-th element is

$$\Delta\left(\frac{1}{f_j}\right) = \frac{1}{V_j f_j}.$$

6.b) Derive the thickness t that is required to eliminate chromatic aberration from the composite element in terms of V_j, f_j $(j = 1, 2.)$ What is then the optical power ϕ of the compound (*i.e.*, the doublet) ? What are t and ϕ if the lenses are identical?

6.c) Practical doublets are often cemented (*i.e.*, "glued" together) in order to minimize the possibility of misalignments and other instabilities. Then $t = 0$ and $R_1{}' = R_2$. Derive the condition that V_j, f_j $(j = 1, 2)$ must satisfy in order to eliminate chromatic aberration in this case. What is then the composite optical power ϕ?

6.d) Design a cemented achromatic doublet for $\phi = 10D$. (You should select the appropriate materials and curvatures for the individual lens components.)

6.e) Plot the focal length of the doublet you designed for all wavelengths between $\lambda_a = 0.2\mu$m and $\lambda_b = 0.7\mu$m and specify the maximum chromatic aberration within this range and the wavelength(s) for which it occurs.