MASSACHUSETTS INSTITUTE OF TECHNOLOGY

2.710 Optics **Fall '05**

Problem Set #5 Posted Wednesday, Oct. 19, 2005 — Due Wednesday, Oct. 26, 2005

1. **Frequencies and wavelengths**

   1.a) MIT's WMBR radio station transmits at frequency 88.1MHz. What is WMBR's wavelength in air?

   1.b) Microwave antennas, especially those of radio stations, are often of the "$\lambda/2$" type, i.e. the height of the antenna equals one half of the wavelength. If an antenna is 10m tall, what is the frequency that it is transmitting?

   1.c) An obsessive flutist keeps playing the A tone (440Hz) on WMBR. What is the wavelength of that sound?

   1.d) Soft x–rays have a wavelength of 1nm. What is the frequency?

   1.e) According to quantum mechanics, particles can also be thought of as waves. The "dispersion relation" for these waves, is different than that for light waves that we saw in class; instead, particles obey the "de Broglie" dispersion relationship, which is

   $$\lambda = \frac{h}{mv},$$

   where λ is the wavelength associated with the particle, m is the particle's rest mass, v is the (non-relativistic) velocity, and h is Planck's constant. Based on this equation, what is the electron's wavelength?

   1.f) Water waves also obey a different dispersion relation, given by

   $$\omega^2 = gk \tanh(kd),$$

   where, as usual, $\omega = 2\pi\nu$, $k = 2\pi/\lambda$; g is the acceleration of gravity, and d is the depth of the water. If the depth of a pool is 3m, and the wavelength is 10cm, what is the frequency of the water waves?

2. **Plane waves and phasor representations** Throughout this problem, by "complete expression" of a wave we mean the space-time representation, e.g. $f(x, y, z, t) = A\cos(kx - \omega t)$ is a plane wave of wave-vector magnitude k and frequency ω propagating in the direction of the $\hat{\boldsymbol{x}}$ coordinate axis. By "phasor expression" we mean the complex representation of the wave, e.g. Ae^{ikx} for the same wave.

   2.a) Write the complete and phasor expressions for a plane wave $f_1(x, y, z, t)$ propagating at an angle 30° relative to the $\hat{\boldsymbol{z}}$ axis on the xz-plane (*i.e.*, the plane $y = 0$). The wavelength is $\lambda = 1\mu$m, and the wave speed is $c = 3 \times 10^8 \text{m} \cdot \text{sec}^{-1}$.

2.b) Write the complete and phasor expressions for a plane wave $f_2(x, y, z, t)$ of the same wavelength and wave speed as f_1 but propagating at angle 60° relative to the $\hat{\boldsymbol{z}}$ axis on the yz-plane.

2.c) Use the complete expression to plot $f_1(x, y, z = 0, t = 0)$ and $f_2(x, y, z = 0, t = 0)$ using MATLAB. (*Note:* you will probably need to use `surf` or an equivalent command.)

2.d) The plane $z = 0$ is illuminated by the superposition of the two waves f_1 and f_2. Plot the waveform received at points A, B, C, D, E with Cartesian coordinates, respectively,

$$(0,0,0), \left(\frac{1}{4}, -\frac{1}{4\sqrt{3}}, 0\right), \left(\frac{1}{2}, -\frac{1}{2\sqrt{3}}, 0\right), \left(\frac{3}{4}, -\frac{3}{4\sqrt{3}}, 0\right), \left(1, -\frac{1}{2\sqrt{3}}, 0\right)$$

(all units in microns) as function of time. What do you observe?

3. Superposition of scalar waves Given two waves

$$\begin{aligned} E_1(z,t) =& E_0 \cos\left(k_1 z - \omega_1 t\right), \\ E_2(z,t) =& E_0 \cos\left(k_2 z - \omega_2 t\right), \end{aligned}$$

where $\omega_1 = 100\text{Hz}$, $\omega_2 = 101\text{Hz}$ and the wavenumbers k_1, k_2 obey the usual dispersion relation for light waves.

3.a) Show that the superposition can be expressed as a product of two tones, one at frequency lower than both the waves, and one at higher frequency.

3.b) Plot your result. What do you observe?

4. Superposition of polarized waves Given two waves

$$\begin{aligned} \boldsymbol{E}_1(z,t) =& \hat{\boldsymbol{x}} E_0 \cos\left(kz - \omega t\right), \\ \boldsymbol{E}_2(z,t) =& \hat{\boldsymbol{y}} E_0 \cos\left(kz - \omega t\right), \end{aligned}$$

where E_0 is a constant electric field magnitude, and k, ω are the wavenumber and angular frequency of the waves.

4.a) What is the superposition of the two waves $\boldsymbol{E}_1$, $\boldsymbol{E}_2$? What is the type of its polarization?

4.b) Now suppose the second wave $\boldsymbol{E}_2$ is delayed by $\pi/2$. What is the polarization of the superposition wave?

4.c) Now suppose that, in addition, the second wave $\boldsymbol{E}_2$ is attenuated to $0.717E_0$. If you were to trace the tip of the superposition electric field vector, as in slide #21 of lecture 7–a, what would the locus inscribed by the tip look like?