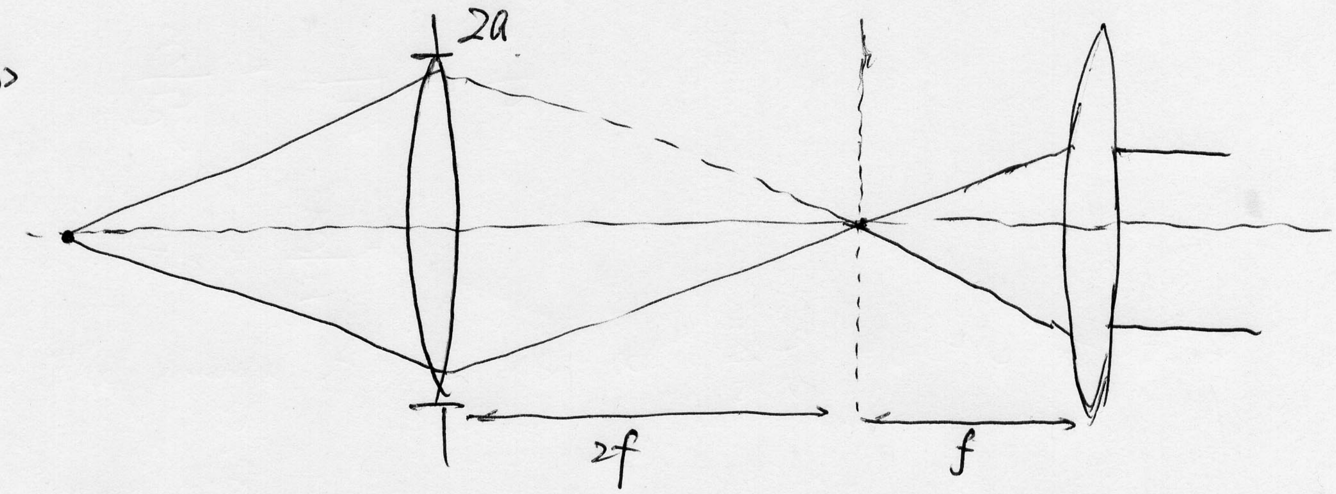
3.

a) Using Lens law cascadedly, the image will be at infinity.

b)



Inorder to obtain the field at 2f to the right of L1, we can think that the system of Lens L1 is illuminated by a point souce at 2f to the left of Lens L1, while the object (tranparency) is the aperture of 2a (the dimension of Lens L1) and the lens is infintely large.

From (5-57) in Goodman, the field at 2f to the right of L1 is

$$U_2(x) = \mathcal{F}\left\{ \mathrm{rect}\left(\frac{x'}{2a}\right)\right\}\Bigg|_{u=\frac{x}{2\lambda f}}$$

$$\left(\frac{z_1}{z_2(z_1-d)} = \frac{2f}{2f(2f-0)} = \frac{1}{2f} \right)$$

So $U_2(x) \propto \mathrm{sinc}\left(\frac{a x}{\lambda f}\right)$

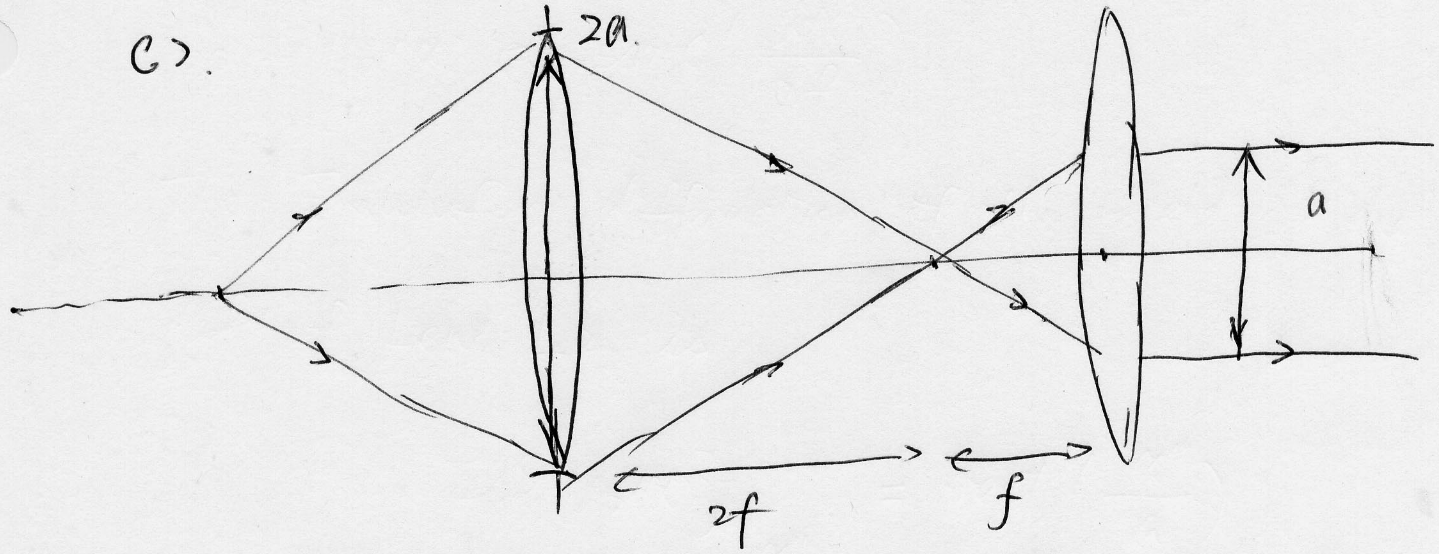
The Fraunhofer diffraction of the field to the left of Lens L2 is

$$\mathcal{F}\{U_2(x)\}\Big|_{\frac{x''}{\lambda f}} = \mathrm{rect}\left(\frac{x''}{a}\right)$$

We can think that we have a transparency with function of $U_2(x)$ at f to the left of Lens L2 and use a plane wave to illuminate it, We can get it's Fraunhofer diffraction at f to the right of Lens L2.

So what we get is a truncated plane wave with width of d.

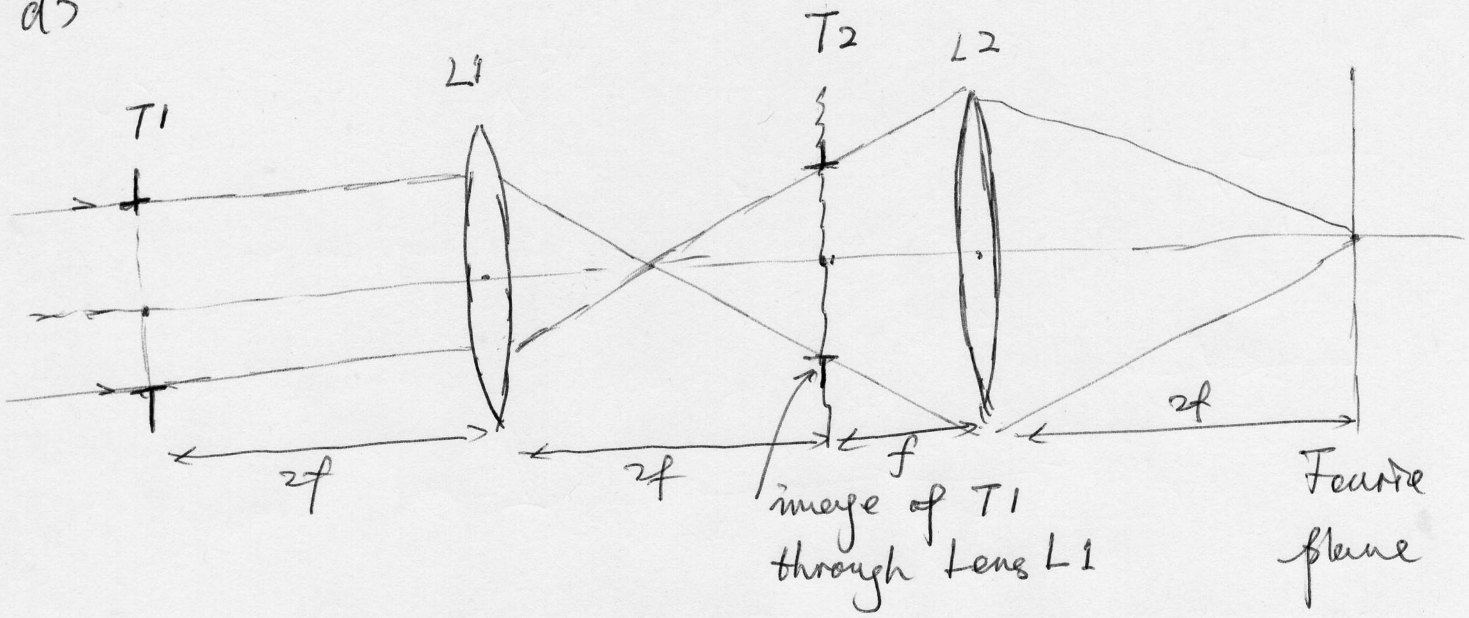
c).



From Geometrical Optics, we know that Lens L1 define the aperture of the system. We can get the width of the output plane wave easily from the plot above:

$$\frac{f}{2f} \cdot 2a = a$$

d)



First, without considering T1 and T2, we can find the Fourier plane (the image of the illumination source which is a plane weave for this case) at 2f to the right of L2.

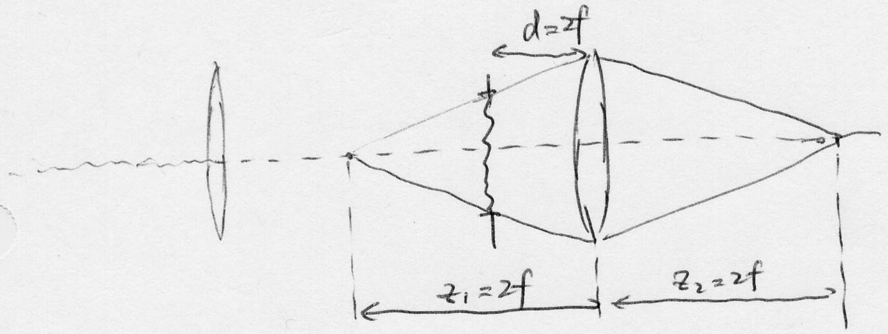
~~Then we can predict we shall see Fourier transform of the ...~~

Then the image of the aperture T1 through Lens L1 is exactly at the same place as the transparency T2. The two objects can be combined by multiplying them together.

Now we can predict that at distance 2f to the right of Lens L2, we will see the Fourier transform of the product of T1 and T2.

If we assumed T2 with the function of $f(x)$, at the distance 2f to the right of L2, the field is

$$U(x') = \mathcal{F}\left\{ \mathrm{rect}\left(\frac{x}{w}\right) \cdot f(x) \right\} \Big|_{u = \frac{x'}{\lambda f}}$$

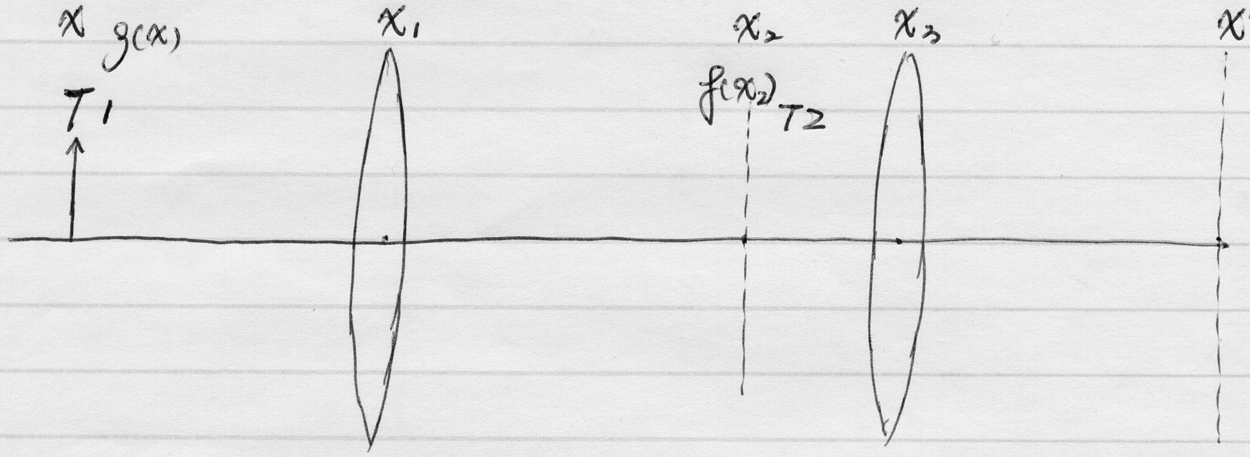


$$\frac{z_1}{z_2(z_1 - d)} = \frac{1}{f}$$

We can also obtain the same results from cascade derivation which is given in the next two pages.

Object T1 : $g(x)$

Transparency T2 : $f(x_2)$



$$\iiiint g(x)\cdot \exp\left\{j\frac{\pi}{\lambda}\cdot\frac{(x_1-x)^2}{2f}\right\} dx \cdot \exp\left\{-j\frac{\pi}{\lambda}\cdot\frac{x_1^2}{f}\right\}$$

$$\exp\left\{j\frac{\pi}{\lambda}\cdot\frac{(x_2-x_1)^2}{2f}\right\}\cdot dx_1\cdot f(x_2)\cdot \exp\left\{j\frac{\pi}{\lambda}\cdot\frac{(x_3-x_2)^2}{f}\right\} dx_2$$

$$\cdot \exp\left\{-j\frac{\pi}{\lambda}\cdot\frac{x_3^2}{f}\right\}\cdot \exp\left\{j\frac{\pi}{\lambda}\cdot\frac{(x'-x_3)^2}{z}\right\} dx_3$$

$$= \iiiint g(x)\cdot f(x_2) \exp\left\{j\frac{\pi}{\lambda}\left[\frac{\cancel{x_1^2}-2xx_1+x^2}{2f} - \frac{\cancel{x_1^2}}{f} + \frac{x_2^2-2x_2x_1+\cancel{x_1^2}}{2f}\right.\right.$$

$$\left.\left.+\frac{\cancel{x_3^2}+x_2^2-2x_3x_2}{f} - \frac{\cancel{x_3^2}}{f} + \frac{x'^2+x_3^2-2x'x_3}{z}\right]\right\} dx\, dx_1\, dx_2\, dx_3$$

$$= \iiiint g(x) f(x_2) \exp\left\{-j\frac{\pi}{\lambda}\frac{(x+x_2)}{f}x_1\right\} \exp\left\{j\frac{\pi}{\lambda}\left[\frac{x^2}{2f}+\frac{x_2^2}{2f}+\frac{x_2^2}{f}\right.\right.$$

$$\left.\left.-\frac{2x_3x_2}{f} + \frac{x'^2+x_3^2-2x'x_3}{z}\right]\right\} dx_1\, dx\, dx_2\, dx_3$$

$$= \iiint g(x)\, f(x_2) \cdot \delta(x+x_2) \cdot \exp\left\{ j\frac{\pi}{\lambda}\left[\frac{x^2}{2f} + \frac{3}{2f}x_2^2 - \frac{2x_3x_2}{f} + \frac{x'^2 + x_3^2 - 2x'x_3}{z} \right]\right\} dx\, dx_2\, dx_3$$

$$= \iint f(x_2)\, g(-x_2)\, \exp\left\{ j\frac{\pi}{\lambda}\left[\frac{2x_2^2}{f} - \frac{2x_3x_2}{f} + \frac{x'^2 + x_3^2 - 2x'x_3}{z} \right]\right\} dx_2\, dx_3$$

$$= \iint f(x_2)\, g(-x_2) \cdot \exp\left\{ j\frac{\pi}{\lambda} \frac{2x_2^2}{f} \right\} \exp\left\{ j\frac{\pi}{\lambda} \cdot \frac{x'^2}{z} \right\} \exp\left\{ j\frac{\pi}{\lambda} \cdot \left[\frac{x_3^2}{z} - \left(\frac{2x_2}{f} + \frac{2x'}{z}\right) x_3 \right]\right\} dx_3\, dx_2$$

$$= \int f(x_2)\, g(-x_2) \cdot \exp\left\{ j\frac{\pi}{\lambda} \cdot \frac{2x_2^2}{f} \right\} \exp\left\{ j\frac{\pi}{\lambda} \frac{x'^2}{z} \right\} \exp\left\{ -j\frac{\pi}{\lambda} \cdot z \cdot \left(\frac{x_2}{f} + \frac{x'}{z} \right)^2 \right\} dx_2$$

$$= \int f(x_2)\, g(-x_2)\, \exp\left\{ j\frac{\pi}{\lambda}\left(\frac{2}{f} - \frac{z}{f^2} \right) x_2^2 \right\} \exp\left\{ -j\frac{2\pi}{\lambda} \cdot \frac{x'}{f} \cdot x_2 \right\} dx_2$$

So if $z = 2f$.

$$= \int f(x_2)\, g(-x_2)\, \exp\left\{ -j\frac{2\pi}{\lambda} \cdot \frac{x'}{f} \cdot x_2 \right\} dx_2$$

a) The transparency can be expressed as

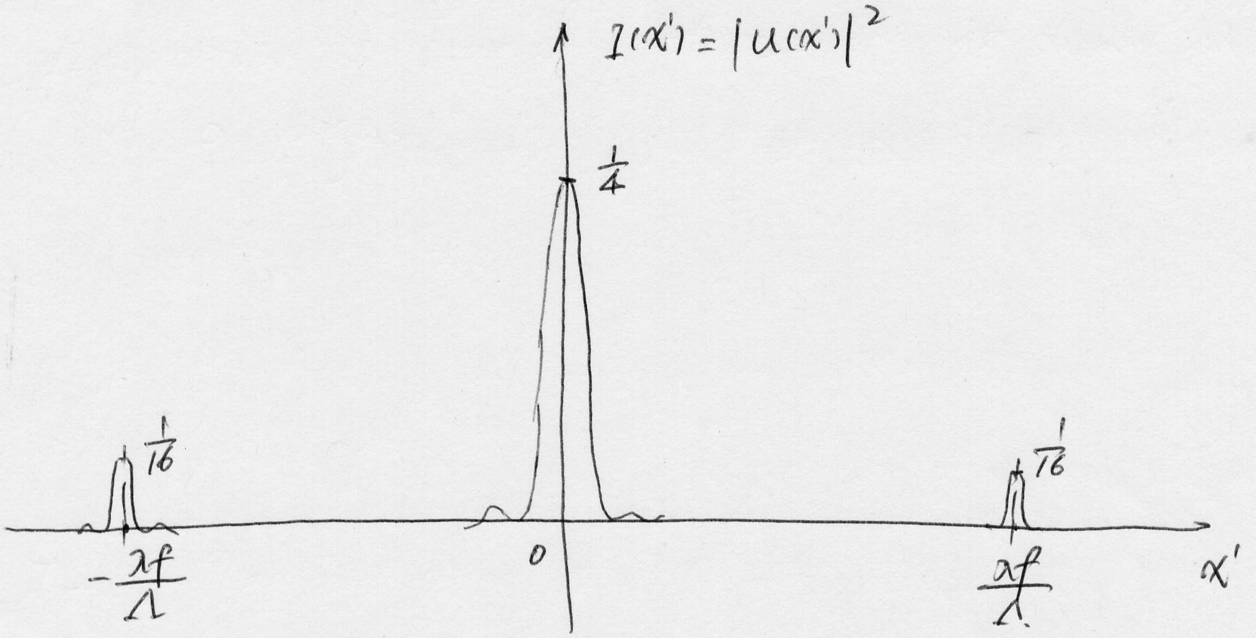
$$f(x) = \frac{1}{2}\left(1 + \cos\left(2\pi \frac{x}{\Lambda}\right)\right)$$

The aperture is $\mathrm{rect}\left(\frac{x}{w}\right)$

$$u(x') = \mathcal{F}\left\{ f(x) \cdot \mathrm{rect}\left(\frac{x}{w}\right)\right\}\Big|_{u=\frac{x'}{\lambda f}}$$

$$= \frac{1}{2}\,\mathrm{sinc}\left(\frac{w x'}{\lambda f}\right) + \frac{1}{4}\,\mathrm{sinc}\left[w\left(\frac{x'}{\lambda f} - \frac{1}{\Lambda}\right)\right]$$

$$+ \frac{1}{4}\,\mathrm{sinc}\left[w\left(\frac{x'}{\lambda f} + \frac{1}{\Lambda}\right)\right]$$



Problem 3

3.a). Assume the grating is

$$t(x) = \frac{1}{2}\left[1 + \cos\left(2\pi \frac{x}{\Lambda}\right)\right] \cdot \mathrm{rect}\left(\frac{x}{a}\right).$$

Then the ± 1 diffraction orders diffracts at the angle of.

$$\sin\theta = \pm \frac{\lambda}{\Lambda}$$

The ensure that the lens does not impair the spectrometer. the lens size must be large enough to accept these diffractions.

so. $$R \geq f \cdot \sin\theta + \frac{a}{2}$$
$$= \frac{\lambda f}{\Lambda} + \frac{a}{2}.$$

3.b). At the output plane, filed is.

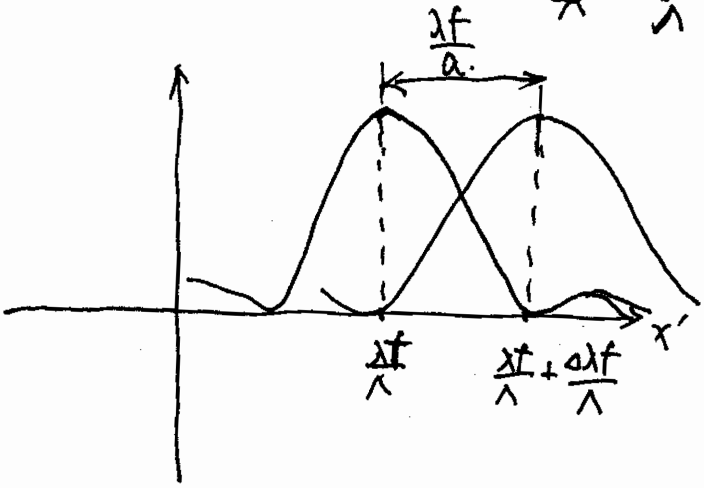
$$t_{out}(x') = \left[\frac{1}{2}\delta(x') + \frac{1}{4}\delta\left(x' - \frac{\lambda f}{\Lambda}\right) + \frac{1}{4}\delta\left(x' + \frac{\lambda f}{\Lambda}\right)\right] * \mathrm{sinc}\left(a\frac{x'}{\lambda f}\right)$$

obviously, the location of 0-th order diffraction is <u>not</u> depend on the wavelength "λ", thus can not be used to distinguish to plane waves.

So. if both "+1" and "−1" orders are collected, the maximal efficiency is 50%. if only one of than is used. it is 25%.

3.c) Let's take a ~~closer~~ closer look of the +1 diffraction order.

There are two "sinc" functions corresponding two wavelengths located at $x' = \frac{\lambda f}{\Lambda}$ and $x' = \frac{(\lambda+\Delta\lambda)f}{\Lambda}$



The cretiaria to resolve these two sinc functions is that the spacing between them is larger than the distance from the peak to the first null of the sinc function as shown above.

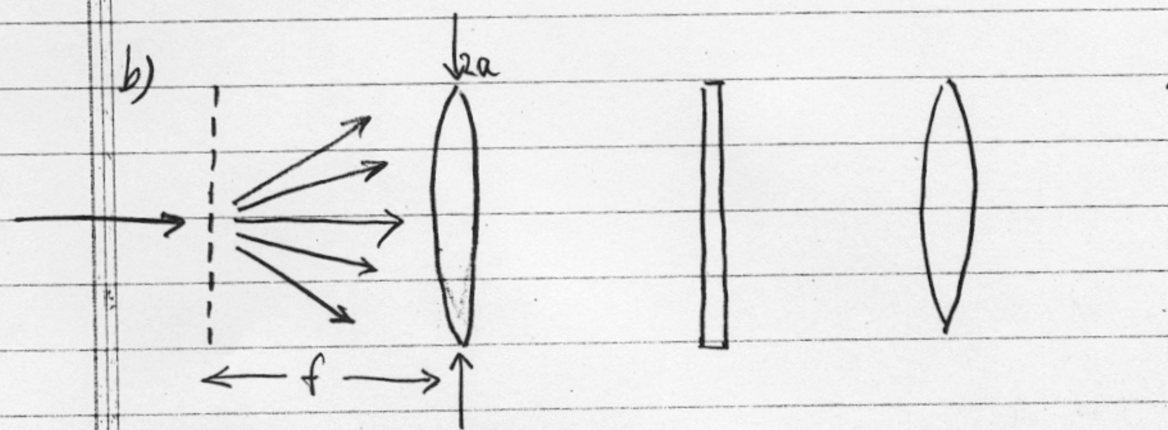
So.

$$\frac{\Delta\lambda f}{\Lambda} \geq \frac{\lambda f}{a}.$$

$$\Rightarrow \frac{\lambda}{\Delta\lambda} \leq \frac{a}{\Lambda}.$$

a) Nonlinearity will be significant only at the peaks of the Airy disks. The system has a low F/# ($\Leftarrow$ high NA), $\Rightarrow$ the Airy disks are very tight and the assumption is justified.

b)



Diffraction angle $\theta = \frac{\lambda}{X} = \frac{1}{4}$

System aperture $= \frac{a}{f} = \frac{1}{2}$ (F/1)

$\Rightarrow$ system admits orders $0, \pm 1, \pm 2$.

0-th order intensity: $\left(\frac{1}{2}\right)^2 I_0 = \frac{1}{4} I_0 \Rightarrow$ transparency transmits $0.1 I_0$

$\pm$1st order intensity: $\left(\frac{1}{2} \frac{\sin\frac{\pi}{2}}{\frac{\pi}{2}}\right)^2 I_0 = 0.101\ I_0 \Rightarrow$

$\Rightarrow$ transparency transmits $0.1 I_0$

$\pm$2nd order intensity: 0

$\Rightarrow$ output $I(x') = 0.1 I_0 \left[1 + 2\cos\left(\frac{2\pi x}{X}\right)\right]$