Problem Set 4

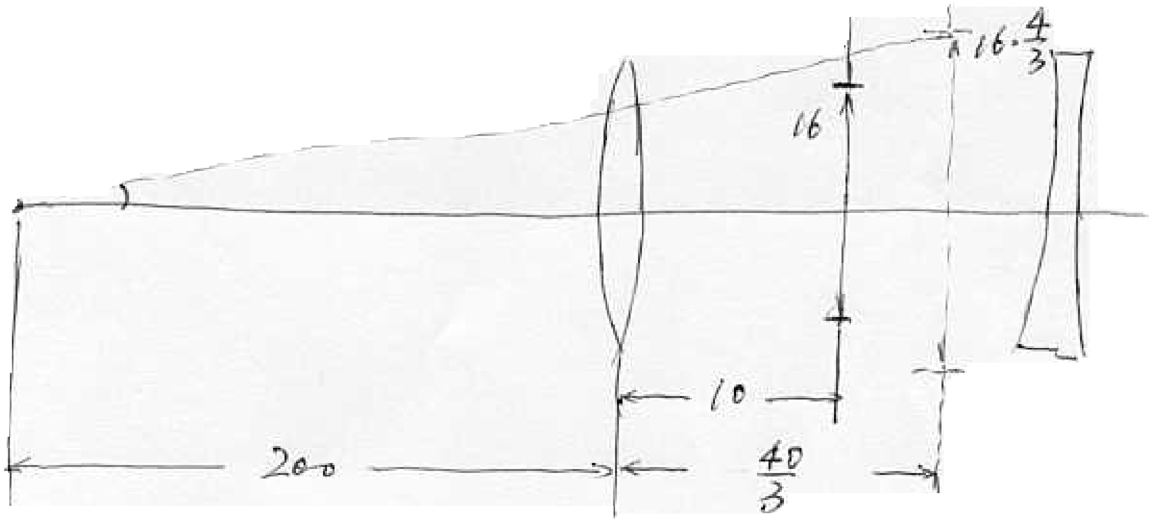
Problem 1

In fact we don't need the 2nd lens in this problem

First image the stop through the first lens

$$\frac{1}{S_o} + \frac{1}{S_i} = \frac{1}{f} \qquad \text{where } S_o = 10 \qquad f = 40$$

$$S_i = -\frac{40}{3} \qquad \text{the magnification is } m_T = \frac{\frac{40}{3}}{10} = \frac{4}{3}$$

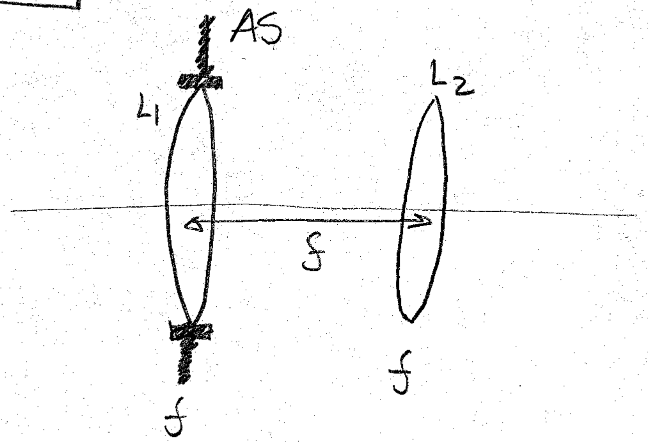


The numerical aperture is

$$NA = \frac{16 \cdot \frac{4}{3} / 2}{200 + \frac{40}{3}} = 1/20$$

PROBLEM SET 4

Problem 2



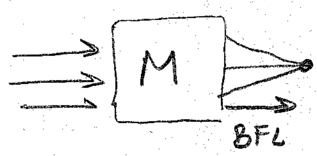
$f = 5''$

a) Calculate the focal length and back focal length of the combination

System Matrix:

$$M = \begin{pmatrix} 1 & -1/f \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ f & 1 \end{pmatrix} \begin{pmatrix} 1 & -1/f \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & -1/f \\ f & 0 \end{pmatrix}$$

$\boxed{\text{Focal Length} = f = 5''}$



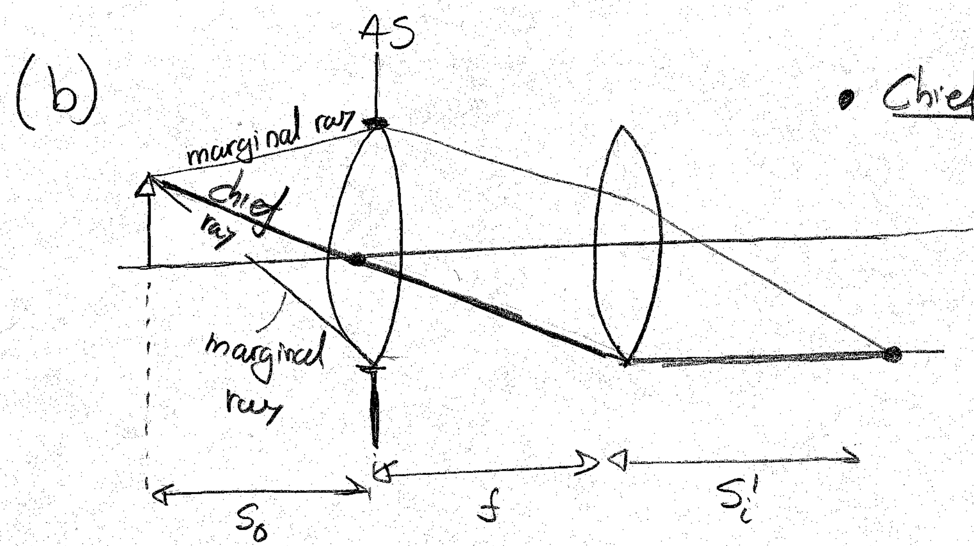
Back Focal Length (BFL):

$$\begin{pmatrix} \alpha_{OUT} \\ X_{OUT} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ BFL & 1 \end{pmatrix} \begin{pmatrix} 0 & -1/f \\ f & 0 \end{pmatrix} \begin{pmatrix} \alpha_{IN} \\ X_{IN} \end{pmatrix} = \begin{pmatrix} 0 & -1/f \\ f & -BFL/f \end{pmatrix} \begin{pmatrix} \overset{0}{\cancel{\alpha_{IN}}} \\ X_{IN} \end{pmatrix}$$

Focus Condition $\rightarrow X_{OUT} = 0$

$$\rightarrow -\frac{BFL}{f} = 0 \Rightarrow \boxed{BFL = 0''}$$

(b)

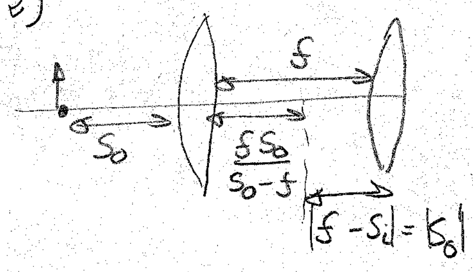


- Chief Ray: Goes through center of aperture stop.
- Marginal Rays: Go through edges of aperture stop.

For a given S_o, we can find S_i':

1) $\frac{1}{S_o} + \frac{1}{S_i} = \frac{1}{f} \rightarrow S_i = \frac{f S_o}{S_o - f} \rightarrow$ Image location of object after the 1st lens.

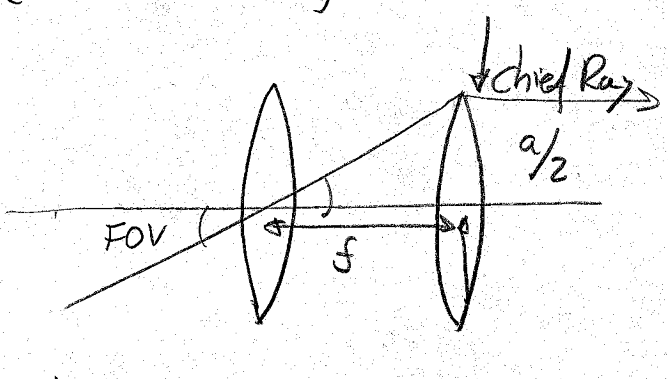
2)



$$|S_o'| = |S_i - f| = \left| \frac{f S_o}{S_o - f} - f \right| = \left| \frac{f^2}{S_o - f} \right|$$

$$\frac{1}{S_o'} + \frac{1}{S_i'} = \frac{1}{f} \rightarrow S_i' = f^2 / S_o$$

(c) Field of view

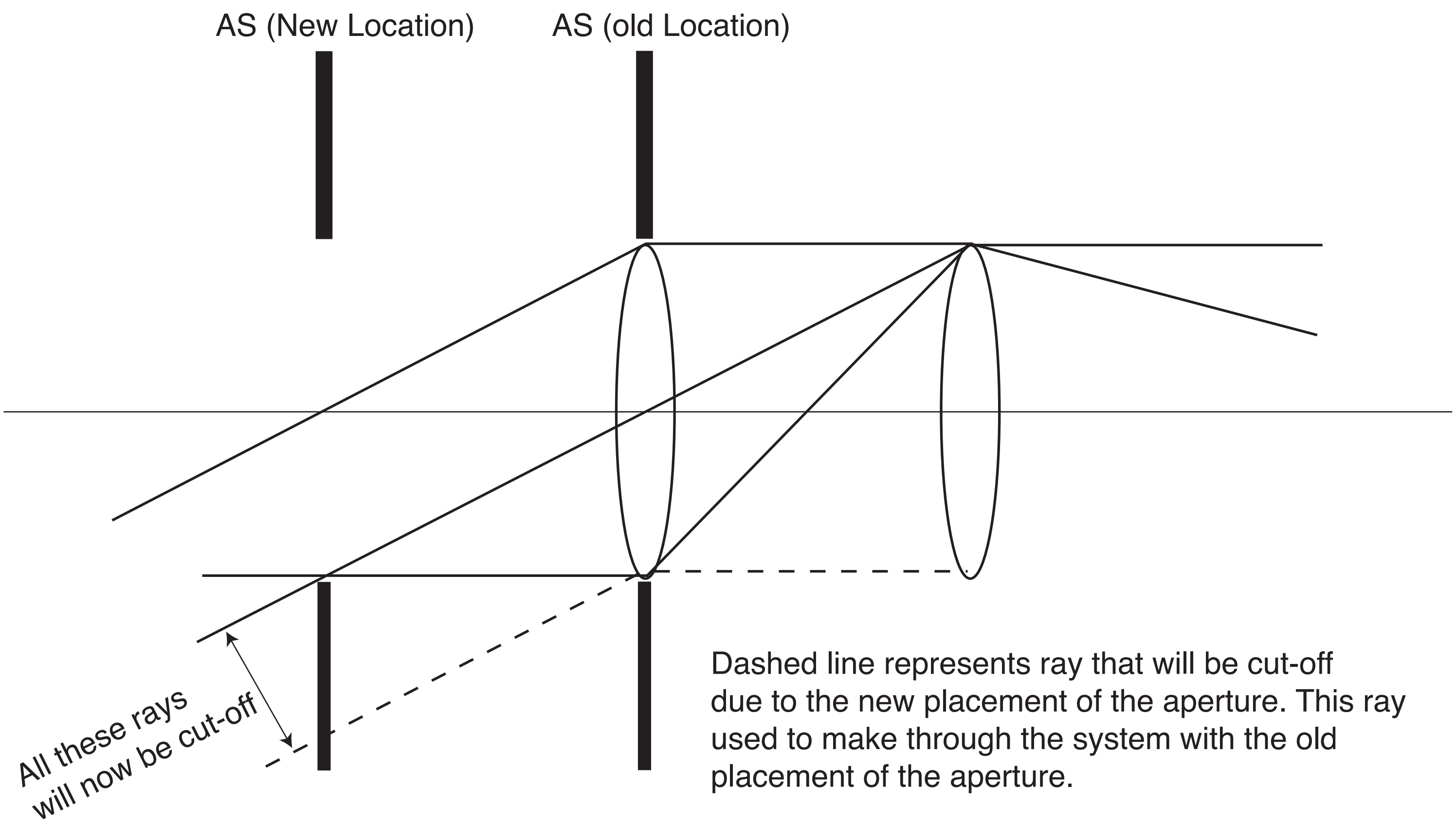


$$FOV = \tan^{-1}\left(\frac{\frac{1}{2}\text{ aperture}}{f} \right) \approx \pm 6^\circ$$

(d) f/# of the System.

$$f/\# = \frac{\text{focal length}}{\text{entrance pupil}} = \frac{5''}{1''} = 5$$

(e)



Hence, the system is VIGNETTING

Problem 3 Curved Mirror

$s = 1.5|R|;$

$s' = ?$

$$\frac{1}{s} + \frac{1}{s'} = -\frac{2}{-|R|}$$

$\Rightarrow$

$$s' = \frac{3}{4}|R|$$

$m_x = -0.5$

$m_\alpha = -2$

Problem #4.

(a) From Eq. in "Hecht" p.p. 250

The power of thick lens:

$$-P = -(n-1)\left[\frac{1}{R_1} - \frac{1}{R_2} + \frac{(n-1)d}{R_1 R_2 n}\right] = -2.6.$$

substitute

$R_1 = 0.5 \quad d = 0.3 \quad n = 1.5$

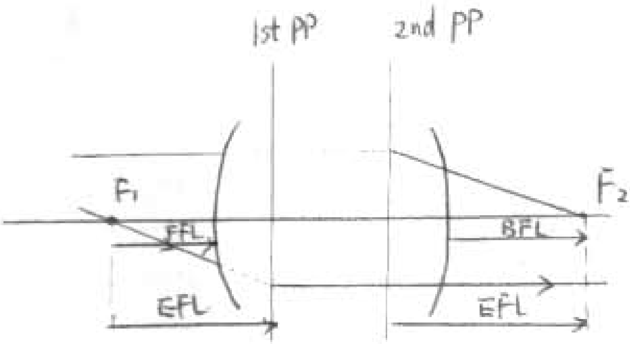
we have

$R_2 = -0.25$ cm.

(b) Use the matrix method.

As $P = \frac{1}{EFL} \Rightarrow EFL = 0.385$ cm.

Then use the matrix method to find BFL, and FFL.



$$\begin{pmatrix} 1 & 0 \\ BFL & 1 \end{pmatrix}\begin{pmatrix} 0.6 & -2.6 \\ 0.2 & 0.8 \end{pmatrix}\begin{pmatrix} 0 \\ h_1 \end{pmatrix} = \begin{pmatrix} 0.6 & -2.6 \\ 0.6BFL + 0.2 & -2.6BFL + 0.8 \end{pmatrix}\begin{pmatrix} 0 \\ h_1 \end{pmatrix}$$

$\Rightarrow -2.6BFL + 0.8 = 0 \Rightarrow BFL = 0.308$ cm.

$$\begin{pmatrix} 0.6 & -2.6 \\ 0.2 & 0.8 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ FFL & 1 \end{pmatrix}\begin{pmatrix} \alpha_1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0.6 - 2.6FFL & -2.6 \\ 0.2 + 0.8FFL & 0.8 \end{pmatrix}\begin{pmatrix} \alpha_1 \\ 0 \end{pmatrix}$$

$\Rightarrow 0.6 - 2.6FFL = 0 \Rightarrow FFL = 0.231$ cm.

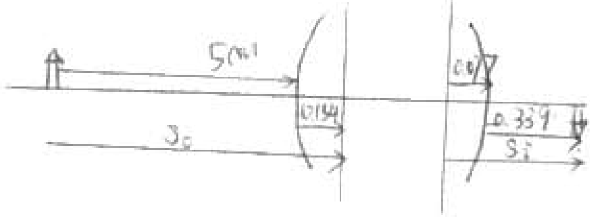
$EFL - BFL = 0.077$ cm $\Rightarrow$ 2nd PP is located 0.077cm to the left of back surface

$EFL - FFL = 0.154$ cm $\Rightarrow$ 1st PP is located 0.154cm to the right of front surface.

(c). object distance.

$$S_o = 5 + 0.154 = 5.154 \text{ cm}.$$

As lens law.



$$\frac{1}{S_o} + \frac{1}{S_i} = \frac{1}{EFL}$$

$$\Rightarrow \frac{1}{5.154} + \frac{1}{S_i} = \frac{1}{0.385}$$

$$S_i = 0.416 \text{ cm}.$$

$$0.416 - 0.077 = 0.339 \text{ cm}.$$

Image is located 0.339cm to the right of back surface.

$$M_L = -\frac{S_i}{S_o} = \frac{0.416}{5.154} = -0.081$$

Problem #5

(a). Matrix for propagation "1" $t_1 = \begin{bmatrix} 1 & 0 \\ d & 1 \end{bmatrix}$

Matrix for reflection at right mirror

$$t_2 = \begin{bmatrix} -1 & -\frac{2}{-r} \\ 0 & 1 \end{bmatrix}$$

Matrix for propagation "2" $t_3 = \begin{bmatrix} 1 & 0 \\ -d & 1 \end{bmatrix}$

Matrix for reflection at left mirror

$$t_4 = \begin{bmatrix} -1 & -\frac{2}{r} \\ 0 & 1 \end{bmatrix}$$

$$T = t_4 t_3 t_2 t_1 = \begin{bmatrix} (\frac{2d}{r}-1)^2 - \frac{2d}{r} & \frac{4}{r}(\frac{d}{r}-1) \\ 2d(1-\frac{d}{r}) & 1-2\frac{d}{r} \end{bmatrix}$$

(b) n double passes equals to.

$$T^{2n}$$

$$= \begin{bmatrix} \left(\frac{2d}{r}-1\right)^2 - \frac{2d}{r} & \frac{4}{r}\left(\frac{d}{r}-1\right) \\ 2d\left(1-\frac{d}{r}\right) & 1-2\frac{d}{r} \end{bmatrix}^{2n}$$

$$= \left(X \Lambda X^{-1}\right)^{2n}$$

$$= X \Lambda^{2n} X^{-1}$$

where

$$X = \begin{bmatrix} -\frac{-d^2+dr-\left(-d(2r-d)(r-d)^2\right)^{\frac{1}{2}}}{r(r-d)d} & -\frac{-d^2+dr+\left(-d(2r-d)(r-d)^2\right)^{\frac{1}{2}}}{r(r-d)d} \\ 1 & 1 \end{bmatrix}$$

$$\Lambda = \begin{bmatrix} \frac{2d^2+r^2-4dr+2\left(-d(2r-d)(r-d)^2\right)^{\frac{1}{2}}}{r^2} & 0 \\ 0 & \frac{2d^2+r^2-4dr-2\left(-d(2r-d)(r-d)^2\right)^{\frac{1}{2}}}{r^2} \end{bmatrix}$$

(c) To confine the beam in the cavity.

must have:

after a round trip 1-2-3-4, the beam goes to the original position, with 0 departure angle.

so we must have.

$$T^2 = X\Lambda^2 X^{\dagger} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

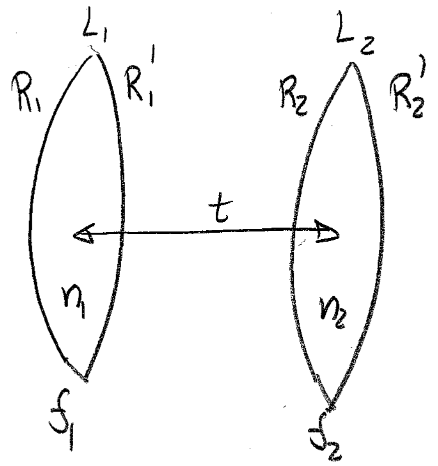
$$\Rightarrow \Lambda^2 = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \Rightarrow r = d$$

Thus, under the condition $r=d$, beam is confined inside the cavity.

and the cavity becomes a resonator.

Problem 6 Achromatic Doublet.

This is the 'official' solution. I thought that my answer would be helpful for you. So I put the solution that I did when I chose this course in the next part.



(a)

Power thin lens $L_1 = \phi_1 = (n_1 - 1)\left(\frac{1}{R_1} - \frac{1}{R_1'}\right)$

Power thin lens $L_2 = \phi_2 = (n_2 - 1)\left(\frac{1}{R_2} - \frac{1}{R_2'}\right)$

$$P_j(\lambda) = (n_j(\lambda) - 1)\left(\frac{1}{R_j} - \frac{1}{R_j'}\right)$$

Total Power: $\frac{1}{f} = P = P_1 + P_2 - P_1 P_2 t$;

Chromatic Dispersion in Optical Power $= \Delta P_j = \Delta\left(\frac{1}{f_j}\right)$

$$\Delta P = \Delta P_1 + \Delta P_2 - t\,(P_2 \Delta P_1 + P_1 \Delta P_2)$$

Change in total power caused by a change in wavelength

$$\Delta P_1 = \Delta n_1\, k_1 \quad \text{where } k_1 = \left(\frac{1}{R_1} - \frac{1}{R_1'}\right)$$

$$= \underbrace{\left(\frac{\Delta n_1}{n_1 - 1}\right)}_{1/V_1} \underbrace{(n_1 - 1)\,k_1}_{P_1} \Rightarrow \Delta\left(\frac{1}{f_1}\right) = \frac{P_1}{V} = \frac{1}{V_1 f_1}$$

$$\Rightarrow \boxed{\Delta\left(\frac{1}{f_j}\right) = \frac{1}{V_j f_j}}$$

(b) t to eliminate chromatic aberration.

Recall, $\Delta P = \Delta P_1 + \Delta P_2 - t\,(P_2 \Delta P_1 + P_1 \Delta P_2)$

Achromatic configuration $\Rightarrow$ $\Delta P = 0$

$$0 = \frac{P_1}{V_1} + \frac{P_2}{V_2} - t\left(P_2 \cdot \frac{P_1}{V_1} + P_1 \frac{P_2}{V_2}\right)$$

$$\Rightarrow \quad t = \frac{P_1 V_2 + P_2 V_1}{P_1 P_2 (V_1 + V_2)} = \frac{f_1 V_1 + f_2 V_2}{(V_1 + V_2)}$$

$$\boxed{t = \frac{f_1 V_1 + f_2 V_2}{V_1 + V_2}}$$

Optical Power of Compound:

$$P = \frac{1}{f_1} + \frac{1}{f_2} - \frac{1}{f_1 f_2} \cdot \frac{f_1 V_1 + f_2 V_2}{V_1 + V_2}$$

$$= \frac{f_1 + f_2}{f_1 f_2} - \frac{1}{f_1 f_2}\left(\frac{f_1 V_1 + f_2 V_2}{V_1 + V_2}\right) = \frac{f_1 V_2 + f_2 V_1}{f_1 f_2 (V_1 + V_2)}$$

$$\Rightarrow \quad \boxed{P = \frac{1}{V_1 + V_2}\left(\frac{V_1}{f_1} + \frac{V_2}{f_2}\right)}$$

For identical lenses:

$V_1 = V_2$ & $P_1 = P_2$. $\Rightarrow$ $t = \dfrac{2fV}{2V} = f$

$$\boxed{\begin{array}{c} t = f \\ \text{Power} = 1/f \end{array}}$$

(c) Cemented Doublets

$t=0 \qquad R_1' = R_2$

Condition to eliminate chromatic aberration:

$P = \frac{1}{f} = P_1 + P_2$; Total Power

$$\Delta P = 0 = \frac{P_1}{V_1} + \frac{P_2}{V_2} \Rightarrow \frac{P_2}{P_1} = -\frac{V_2}{V_1}$$

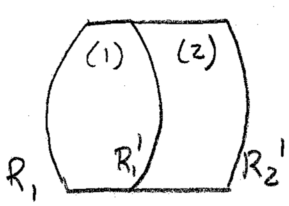
$$\boxed{f_1 V_1 + f_2 V_2 = 0}$$

Composite Power: $\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{f_1} - \frac{V_2}{f_1 V_1} = \frac{1}{f_1}\left(1 - \frac{V_2}{V_1}\right)$

$$\boxed{\frac{1}{f} = P = \frac{1}{f_1}\left(1 - \frac{V_2}{V_1}\right)}$$

(d) $\phi = 10 D$

$$\phi = P = \frac{1}{f_1}\left(1 - \frac{V_2}{V_1}\right) = 10 \qquad -[1]-$$



Pick two materials: (1) BK7 $V_D = 64.17$ $n = 1.517$

(2) SF6 $V_D = 25.43$ $n = 1.805$

From -[1]- we can get f_1:

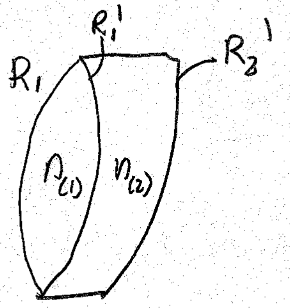
$$f_1 = \frac{1}{10}\left(1 - \frac{25.43}{64.17}\right) = 0.06 \text{ m} \qquad P_1 = \frac{1}{f_1} = 16.56 \text{ D}$$

Now we can get f_2:

$$\phi_2 = P_2 = \frac{1}{f_2} = \frac{-V_2}{V_1 f_1} = \frac{-25.43}{64.17 \cdot 0.06} = -6.60 \text{ D} \rightarrow f_2 = \frac{1}{\phi_2} = -0.15 \text{ m}$$

So, $\begin{cases} f_1 = 0.06\,m & \phi_1 = 16.56\ D \\ f_2 = -0.15\,m & \phi_2 = -6.60\ D \end{cases}$

Now we can get R_1, R_1', R_2'



$$\text{Power} = \frac{1}{f} = P_1 + P_2 = 10\ D$$

Set $R_1 = -R_1'$

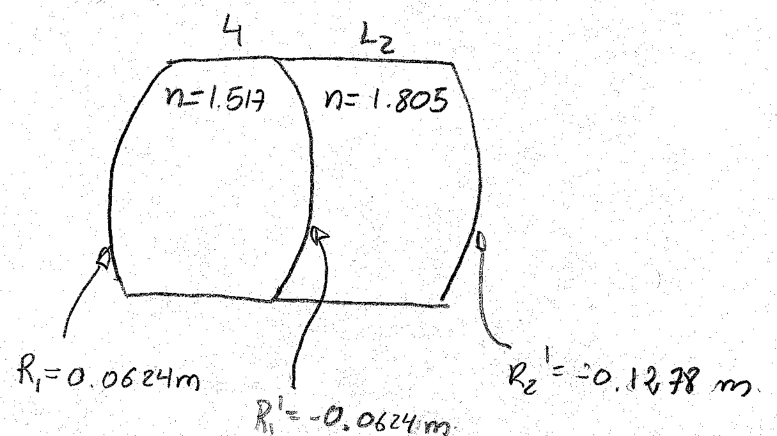
$$P_1 = (n_{(1)} - 1)\left(\frac{2}{R_1}\right) = 16.56\ D$$

$$\Rightarrow R_1 = \frac{(1.517-1)\,2}{16.56} = \underline{0.0624\ m}$$

$$P_2 = (n_{(2)} - 1)\left(+\frac{1}{R_1'} - \frac{1}{R_2'}\right) = -6.60D.$$

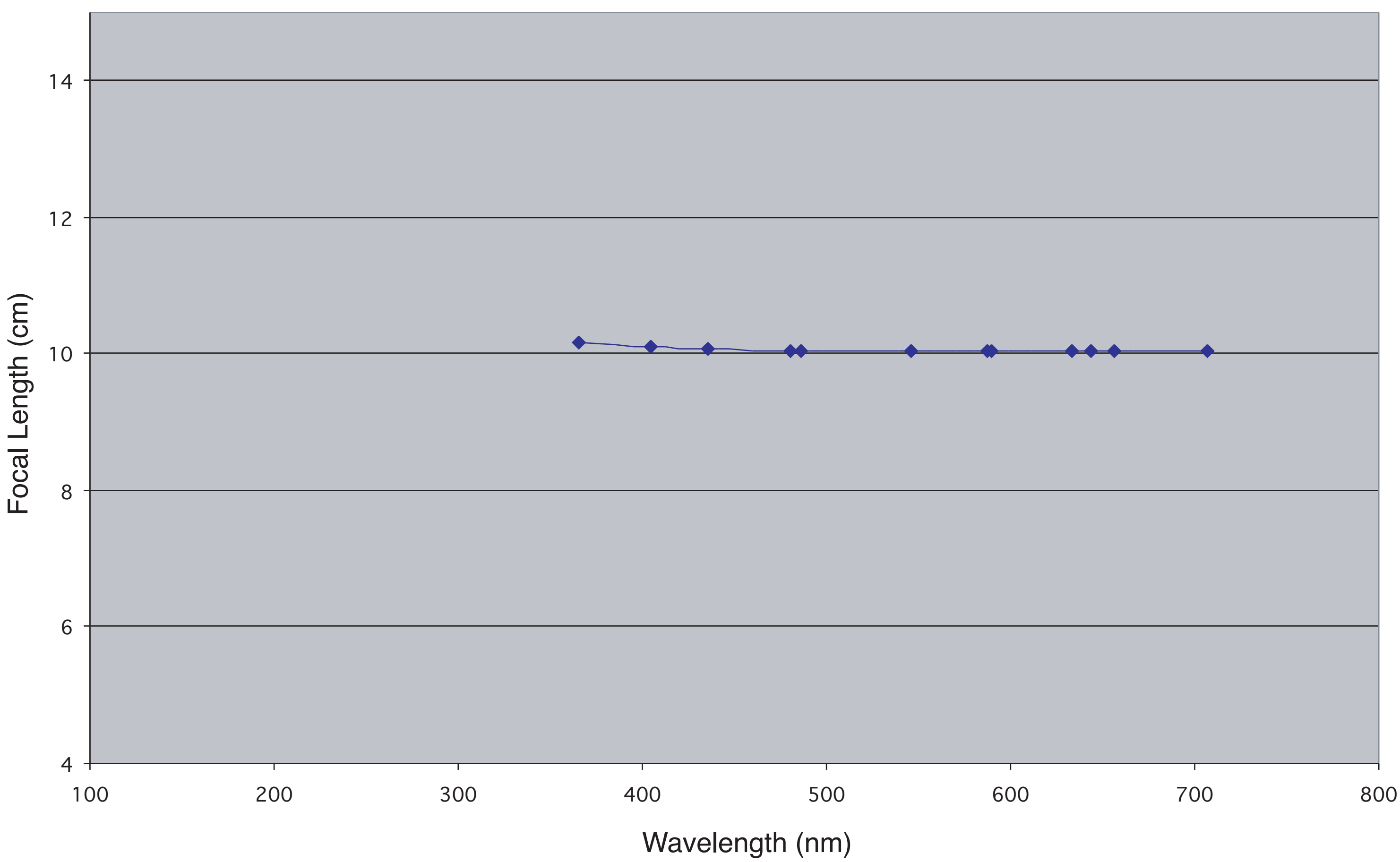
$$= +\frac{1}{R_1'} - \frac{1}{R_2'} = \frac{-6.60}{0.805}\ ; \quad \frac{1}{R_2'} = \frac{1}{R_1'} + \frac{6.6}{0.805} = \frac{-1}{0.0624} + \frac{6.6}{0.805}$$

$$\Rightarrow \underline{R_2' = -0.1278\ m}$$



(e)

Plot of Focal Length vs. Wavelength for Designed Achromat



Materials used BK7 and SF6. Only plotted down to 365nm wavelength, for SF6 is very absorbing after that. The maximum chromatic aberration occurs at the lowest wavelength, 365nm in this case.

2. Solution:

Because the jth element is thin lens. ~~we~~ we can get

$$P_j = \frac{1}{f_j} = (n_j - 1)\left(\frac{1}{R_j} - \frac{1}{R_j'}\right)$$

The chromatic dispersion is

$$\Delta(P_j) = \Delta\left(\frac{1}{f_j}\right) = \Delta n_j\left(\frac{1}{R_j} - \frac{1}{R_j'}\right)$$

$$= \frac{\Delta n_j}{n_j - 1} (n_j - 1)\left(\frac{1}{R_j} - \frac{1}{R_j'}\right)$$

$$= \frac{1}{V_j f_j}$$

The optical power ϕ of the compound

$$\phi = P_1 + P_2 - t P_1 P_2$$

$$\Delta(\phi) = \Delta(P_1) + \Delta(P_2) - t[\Delta(P_1)\cdot P_2 + \Delta(P_2)\cdot P_1]$$

$$= \frac{1}{V_1 f_1} + \frac{1}{V_2 f_2} - t\left[\frac{1}{V_1 f_1}\cdot\frac{1}{f_2} + \frac{1}{V_2 f_2}\cdot\frac{1}{f_1}\right]$$

In order to elimate the chromatic aberration, we must have $\Delta(\phi) = 0$. So we get

$$t = \frac{f_1 V_1 + f_2 V_2}{V_1 + V_2}$$

So we get

$$\phi = \frac{1}{f_1} + \frac{1}{f_2} - t\cdot\frac{1}{f_1}\cdot\frac{1}{f_2} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{f_1 V_1 + f_2 V_2}{f_1 f_2 (V_1 + V_2)}$$

the lenses are indentical $f_1 = f_2 \quad V_1 = V_2$

$$t = \frac{f_1 V_1 + f_2 V_2}{V_1 + V_2} = f_1 = f_2$$ ✓

$$= \frac{1}{f_1} + \frac{1}{f_2} - t \cdot \frac{1}{f_1} \cdot \frac{1}{f_2} = \frac{1}{f_1} = \frac{1}{f_2}$$ ✓

c> We have got

$$\Delta(\phi) = \frac{1}{V_1 f_1} + \frac{1}{V_2 f_2} - t\left[\frac{1}{V_1 f_1} \cdot \frac{1}{f_2} + \frac{1}{V_2 f_2} \cdot \frac{1}{f_1}\right]$$

In glued lens, $t = 0$

$$\Delta(\phi) = \frac{1}{V_1 f_1} + \frac{1}{V_2 f_2}$$

In order to eliminate the chromatic aberration, $\Delta(\phi) = 0$

$$\frac{1}{V_1 f_1} + \frac{1}{V_2 f_2} = 0 \Rightarrow \frac{f_2}{f_1} = -\frac{V_1}{V_2}$$

The composite optical power is

$$\phi = \frac{1}{f_1} + \frac{1}{f_2} = \left(1 - \frac{V_2}{V_1}\right)\frac{1}{f_1}$$ ✓

d> Select BSC-1 for lens 1, DF-1 for lens 2.

$V_{BSC-1} = 63.5 \qquad V_{DF-1} = 38.0$

$n_{BSC-1} = 1.5110 \qquad n_{DF-1} = 1.6050$

$$\phi = \left(1 - \frac{V_2}{V_1}\right) \cdot \frac{1}{f_1} = 10$$

So we get

$$\frac{1}{f_1} = 24.9 \qquad \frac{1}{f_2} = -\frac{V_2}{V_1} f_1 = -14.9$$

fig 4

We design the glued lens shown as Fig 4. The 2nd lens has a flat back surface. $\frac{1}{R_2'} = 0$. So.

$$\frac{1}{f_2} = (n_2 - 1) \cdot \frac{1}{R_2}$$

$$R_2 = (n_2 - 1) f_2 = -0.0406\,m = -4.06\,cm$$

$$R_1' = |R_2| = 4.06\,cm$$

$$\frac{1}{f_1} = (n_1 - 1)\left(\frac{1}{R_1} - \frac{1}{R_1'}\right)$$

$$R_1 = \frac{1}{\frac{1}{f_1(n_1-1)} + \frac{1}{R_1'}} = 0.0136\,m = 1.36\,cm$$

e) $$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$$

$$= (n_1(\lambda) - 1)\left(\frac{1}{R_1} - \frac{1}{R_1'}\right) + (n_2(\lambda) - 1)\frac{1}{R_2}$$

We use formula below to caculate $n(\lambda)$

$$n = n_{A'}B_{A'} + n_C B_C + n_F B_F + n_g B_g$$

$$B_{A'} = -9.74802711 + 12.81327632\lambda^2 + \frac{2.00567317 2}{\lambda^2 - 0.035} + \frac{-0.1314220822}{(\lambda^2 - 0.035)^2}$$

$$B_C = 17.41684396 - 18.92716898\lambda^2 + \frac{-3.95868618 0}{\lambda^2 - 0.035} + \frac{0.2723451558}{(\lambda^2 - 0.035)^2}$$

$$B_F = -9.05109 9716 + 8.12060510 2\lambda^2 + \frac{2.77795372 0}{\lambda^2 - 0.035} + \frac{-0.2296678272}{(\lambda^2 - 0.035)^2}$$

$$B_g = 2.38228285 8 - 2.00671246 2\lambda^2 + \frac{-0.8294271 3}{\lambda^2 - 0.035} + \frac{0.0887447538 3}{(\lambda^2 - 0.035)^2}$$

BSC

$n_{A'}$ 1.5015 n 1 50866 n_F .5700 n_b 1.52540

DF

$n_{A'}$ =1.58703 n 59855 n_F 1 445 n_b 1 63180

The focal length for all wavelengths between .2µm and 0.7µm is shown as Fig 5 The refraction index of BSC is shown in Fig.6. The refraction index of DF-1 is shown in Fig 7. Because the formula used in the wavelengths less that 0.4µm has much error. So we redraw all the figure for the wavelengths between 0.6µm and 0.7µm as Fig 8. Fig 9. Fig1

From Fig 5. 6 7 We see that if the wavelength is less than .4µm the refraction and focal length has much error

From Fig 8, We can find that at the wavelength of .55µm there ~~is~~ the maximum aberration

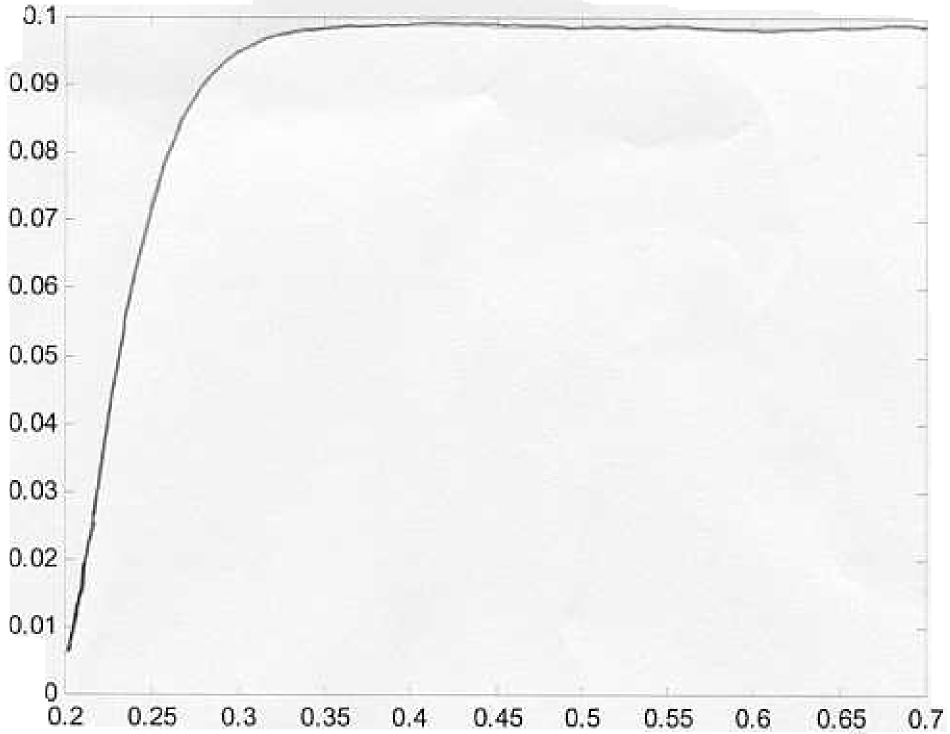


figure 5

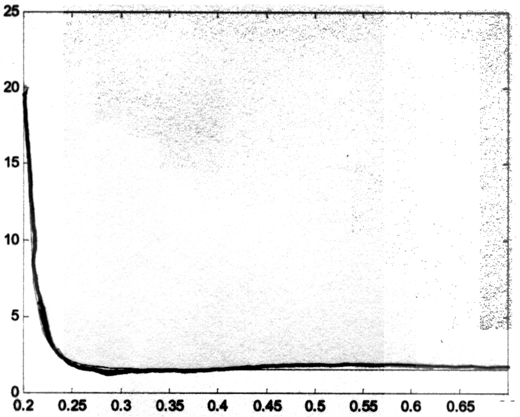
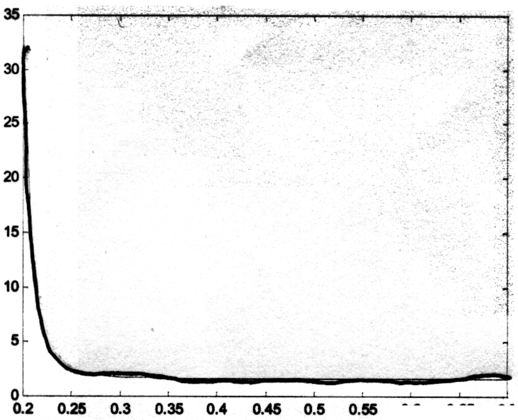


figure 6



figure

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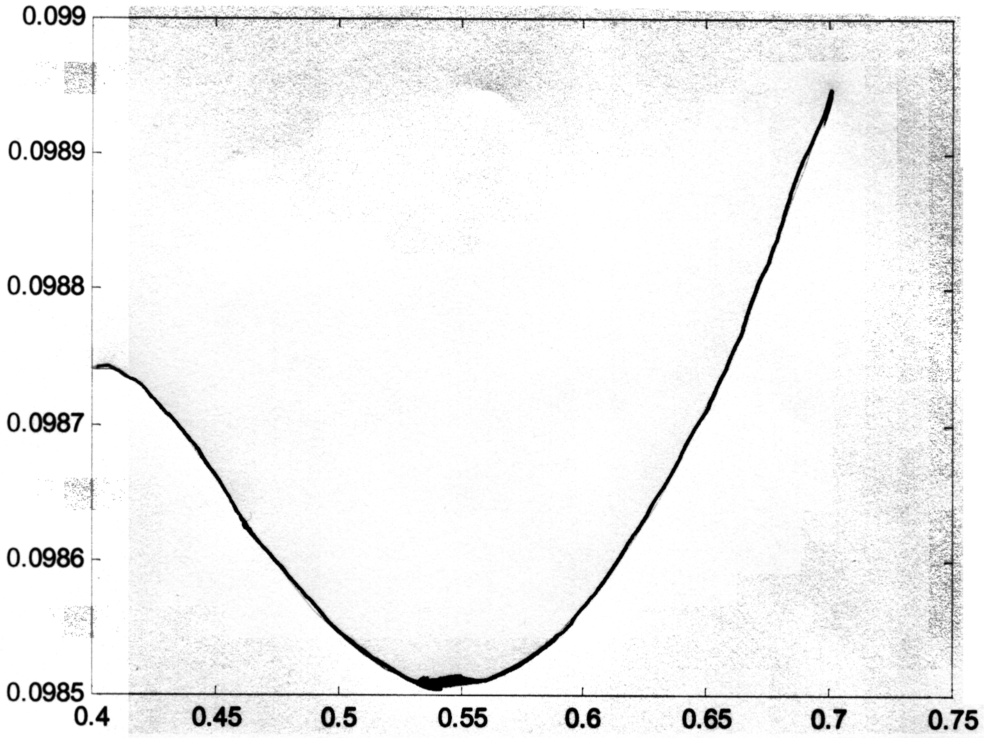


figure 8

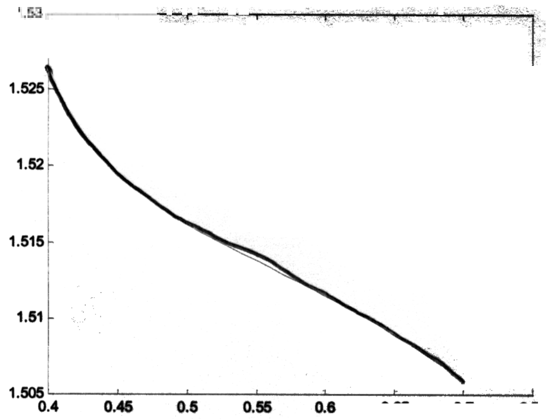
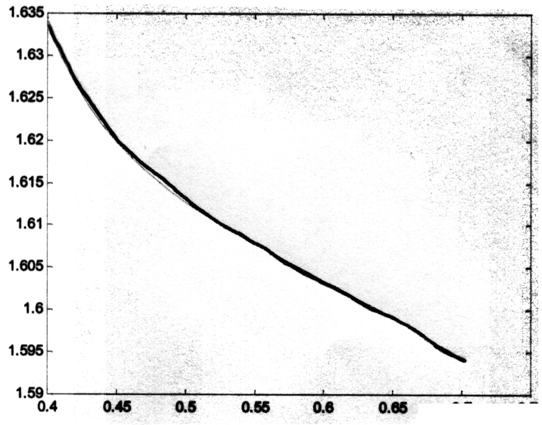


figure 9



figure