2.71

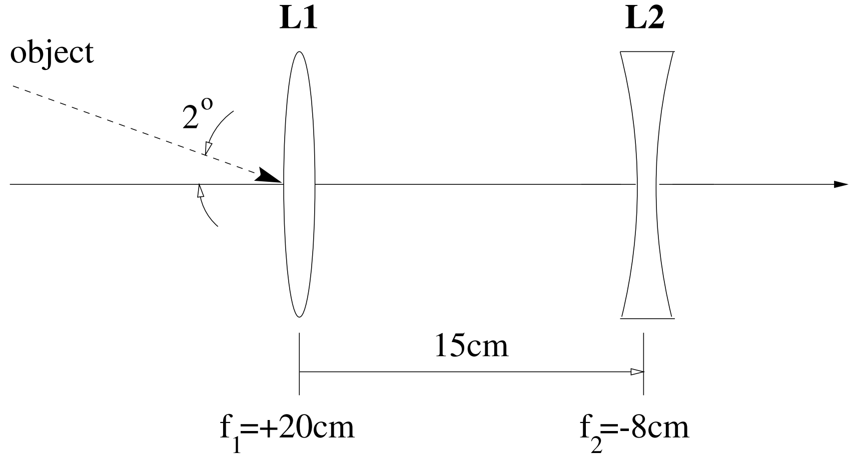
Quiz 1

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

2.71 Optics **Fall '05**

QUIZ 1 Wednesday, Oct. 12, 2005

1. **(60%)** The optical instrument shown below is a "telephoto lens." It consists of a combination of two thin lenses L1, L2 of focal lengths f_1 and f_2, respectively. The schematic is not drawn to scale.



1.a) Locate the principal planes of this telephoto lens and determine the effective focal length.

1.b) Find the image size of a very distant object subtending angle of $2°$ with respect to the telephoto axis.

1.c) Determine the distance from L2 to the image plane.

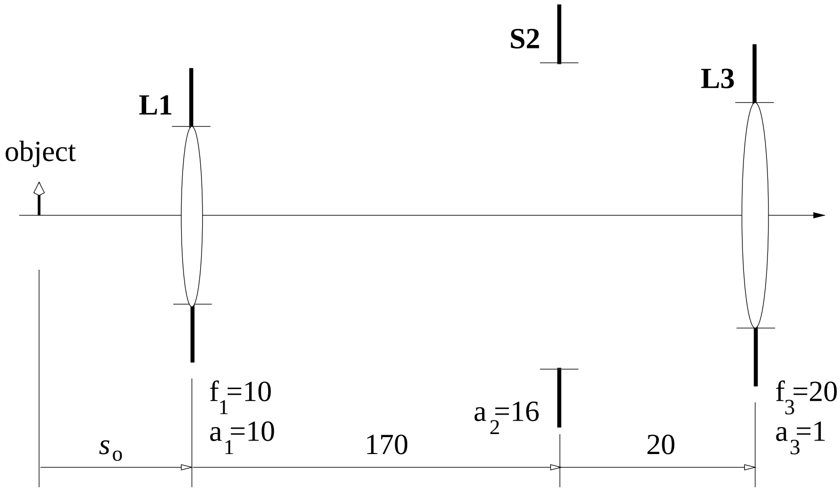
1.d) Suppose that two stops are placed in this system as follows: the first stop is placed at the rim of L1, with radius 5cm; the second stop is placed at the image plane location of question (c), with radius 2cm. Which is the aperture stop and which is the field stop?

1.e) Given the stops of question (d), what is the (angular) field of view?

1.f) If we were to replace the given telephoto by a single positive lens with equal magnifying power, how far would the positive lens have to be located from the image plane? Based on this result, can you justify the purpose of using a telephoto lens (*i.e.*, a combination of a positive and a negative lens as shown above) instead of a single positive lens?

PLEASE TURN OVER!

2. **(40%)** The optical instrument shown below, consisting of lenses L1, L3 and stop S2, is intended for direct viewing by human observers, with the observer's eye located to the right of L3. The symbols $\{f_1, a_1\}$, $\{f_3, a_3\}$ denote the focal lengths and radii of L1, L3, respectively, and a_2 is the radius of S2. All distance units are in millimeters, and the schematic is not drawn to scale.



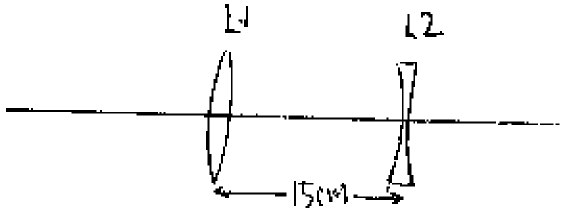
2.a) Determine the object distance s_o so that a human observer's unaccommodated eye may focus the image on the observer's retina.

2.b) What is the best way to use this instrument? Based on your answer, define the instrument's magnifying power (MP) appropriately, and calculate the MP according to your definition.

GOOD LUCK!

Quiz # 1 Solution.

1. 1) a.



The system matrix for the telephoto lens.

$$T = \begin{bmatrix} 1 & \frac{1}{8} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 15 & 1 \end{bmatrix} \begin{bmatrix} 1 & -\frac{1}{20} \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2.875 & -0.0188 \\ 15 & 0.25 \end{bmatrix}$$

$$EFL = \frac{1}{0.0188} = 53.2 \text{ cm}.$$

Propogate a parallel incident ray.

$$\begin{bmatrix} \alpha_{out} \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ BFL & 1 \end{bmatrix} \begin{bmatrix} 2.875 & -0.0188 \\ 15 & 0.25 \end{bmatrix} \begin{bmatrix} 0 \\ h \end{bmatrix}$$

$$= \begin{bmatrix} 2.875 & -0.0188 \\ 2.875 \cdot BFL + 15 & -0.0188 \cdot BFL + 0.25 \end{bmatrix} \begin{bmatrix} 0 \\ h \end{bmatrix}.$$

$$\Rightarrow \quad BFL = \frac{0.25}{0.0188} = 13.3 \text{ cm}.$$

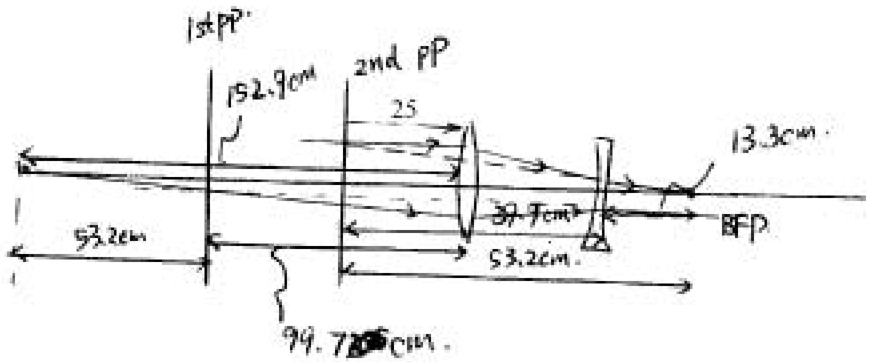
Propogate a ray through FFP.

$$\begin{bmatrix} 0 \\ h_{out} \end{bmatrix} = \begin{bmatrix} 2.875 & -0.0188 \\ 15 & 0.25 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ FFL & 1 \end{bmatrix} \begin{bmatrix} \alpha_{in} \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 2.875 - 0.0188 \cdot FFL & -0.0188 \\ 15 + 0.25FFL & 0.25 \end{bmatrix} \begin{bmatrix} \alpha_{in} \\ 0 \end{bmatrix}$$

$$\Rightarrow \quad FFL = \frac{2.875}{0.0188} = 152.9\text{ cm.}$$

Sketch the PPs and Focal points as shown below



1(b). The intermediate image through L1 is at.
and (c)

$s' = 20$ cm after L1, with height $20 \cdot \frac{2}{180} \cdot \pi = 0.7$ cm.

This forms a virtual object to L2. with

$s = -5$ cm, $f_2 = -8$ cm.

so $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f_2}$

$$\frac{1}{s'} = -\frac{1}{8} + \frac{1}{5}$$

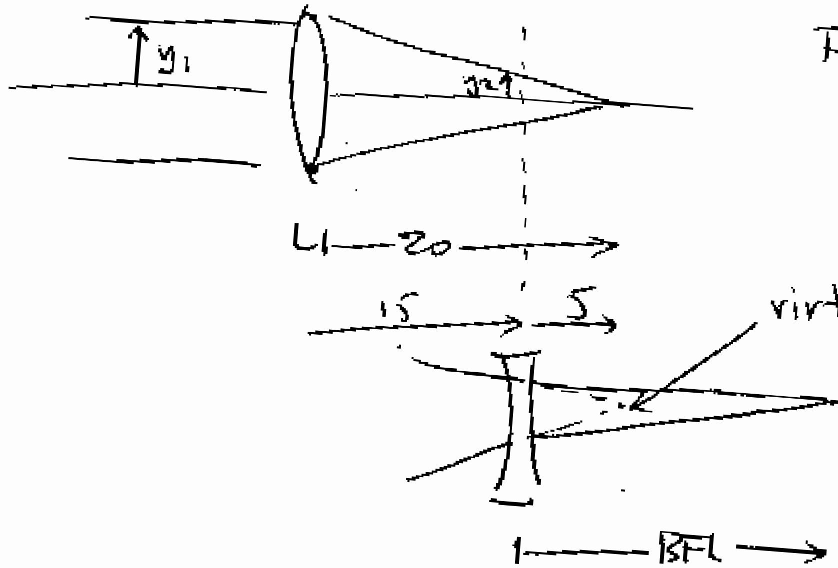
$$= \frac{3}{40}.$$

$s' = 13.3$ cm. after L2.

with height of $0.7 \cdot \frac{13.3}{5} = 1.86$ cm below the optical axis

=

Another way to solve the telephoto (without using matrices):



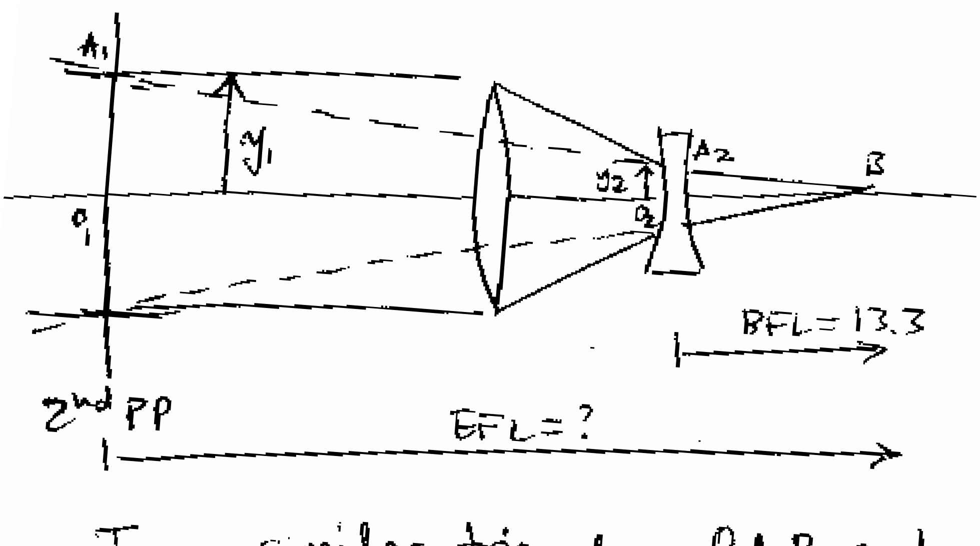
From similar triangles

$$y_2 = y_1 \times \frac{5}{20}$$

virtual object for L2

$$\frac{1}{-5} + \frac{1}{BFL} = \frac{1}{-8} \Rightarrow BFL = 13.3$$

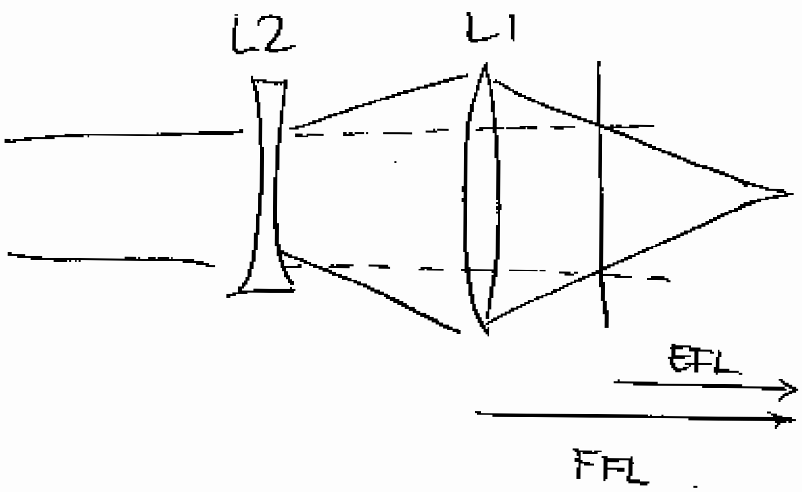
Locate 2nd PP:



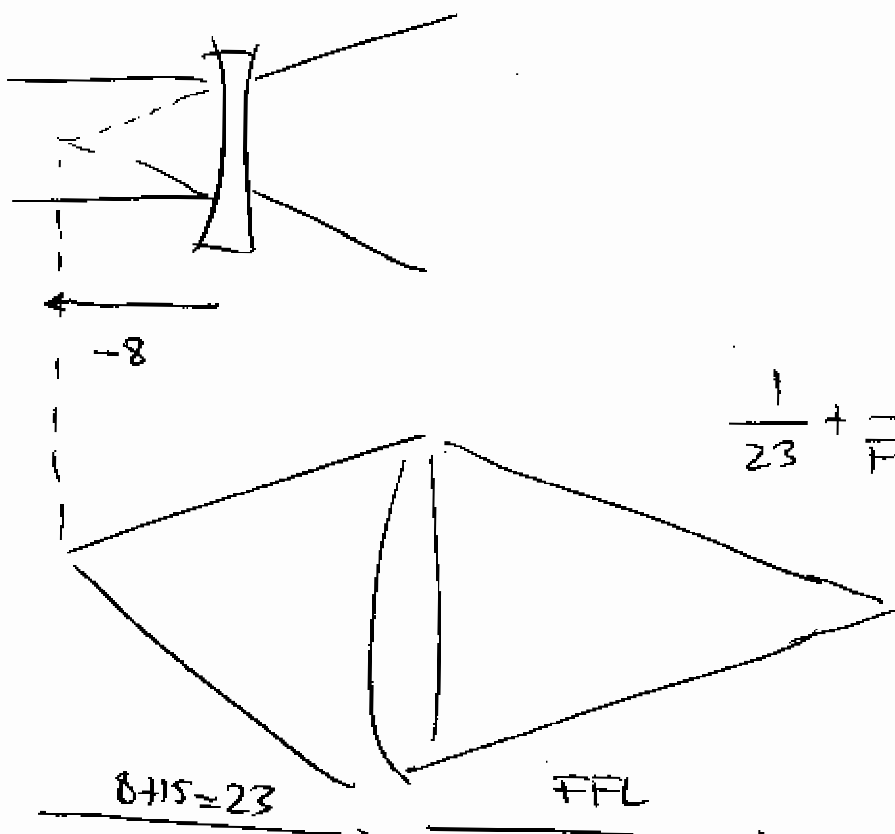
From similar triangles O_1A_1B and O_2A_2B we have

$$\frac{y_1}{EFL} = \frac{y_2}{BFL} \Rightarrow EFL = BFL \times \frac{y_1}{y_2} = 13.3 \times 4 = 53.3.$$

Locate 1st PP: easiest way is to locate 2nd PP of flipped system, i.e.



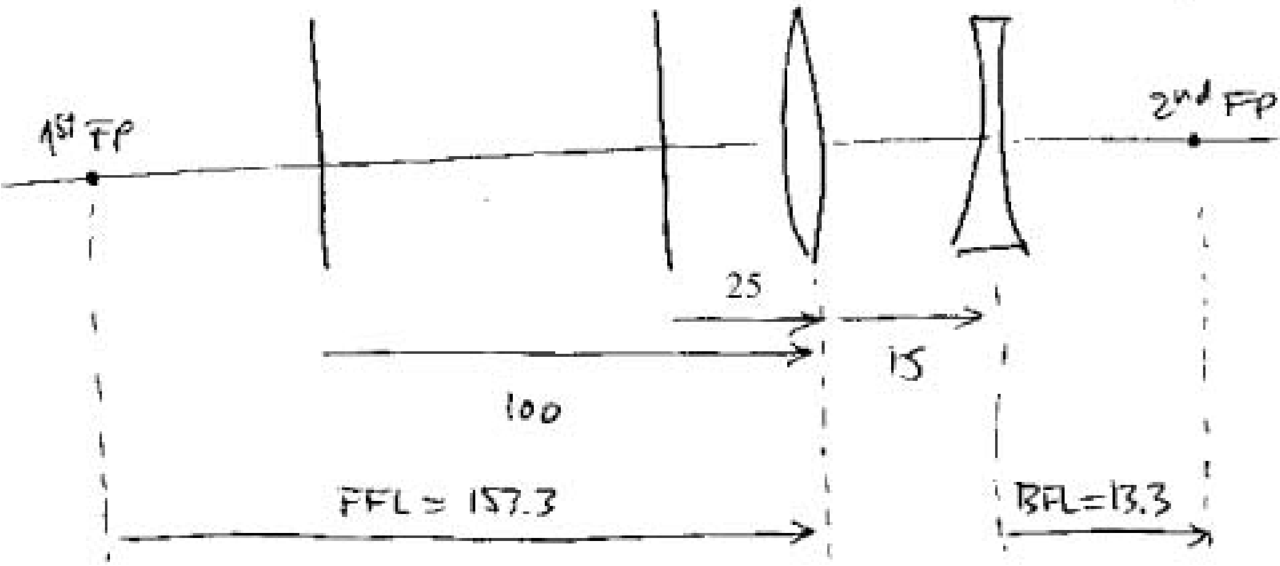
To find the FFL:



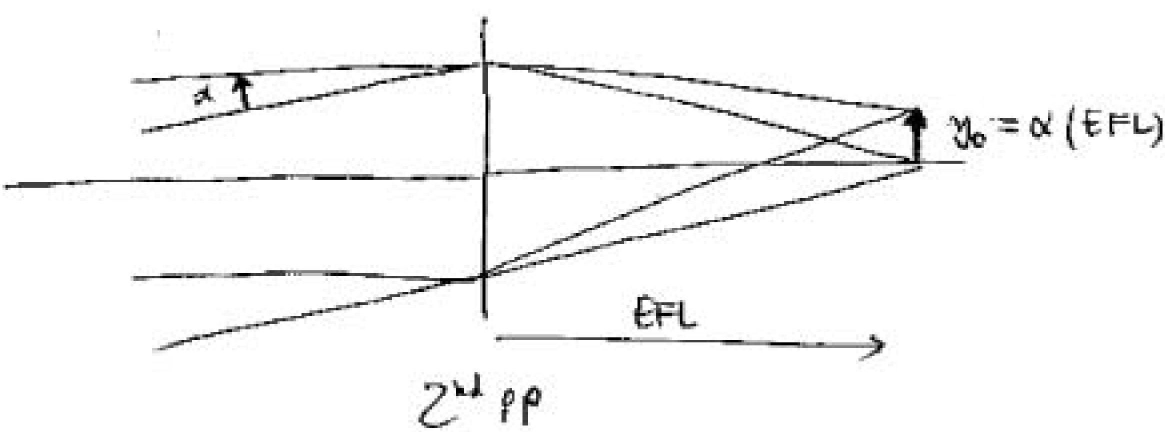
$$\frac{1}{23} + \frac{1}{FFL} = \frac{1}{20} \Rightarrow FFL = 153.3$$

EFL is still 53.3 so 1st PP is 100 units to the left of L1

Finally,



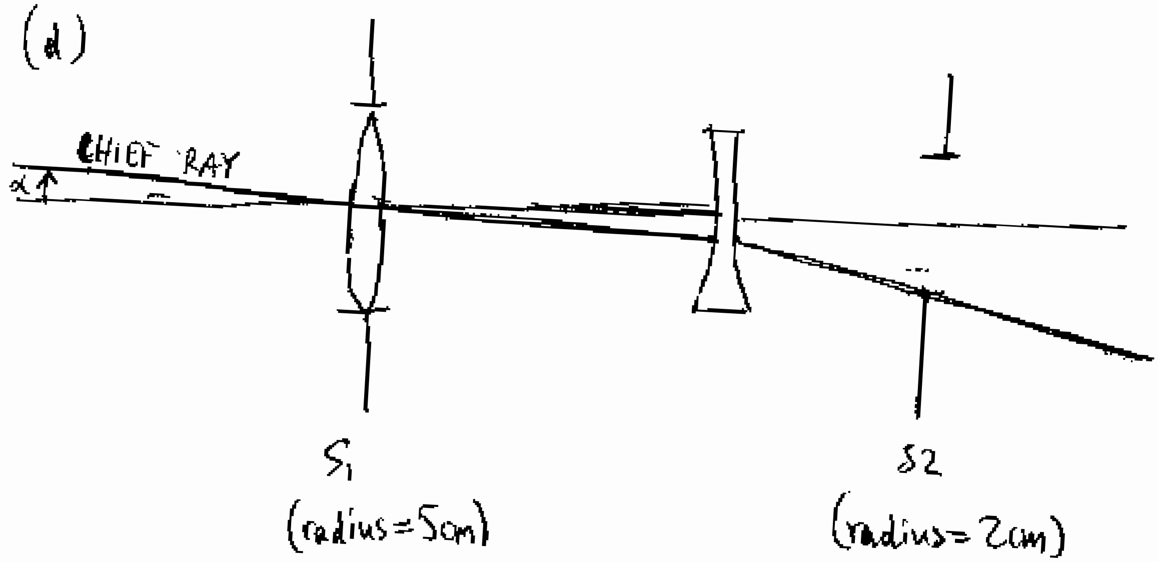
With this in mind we can also easily answer part (b):



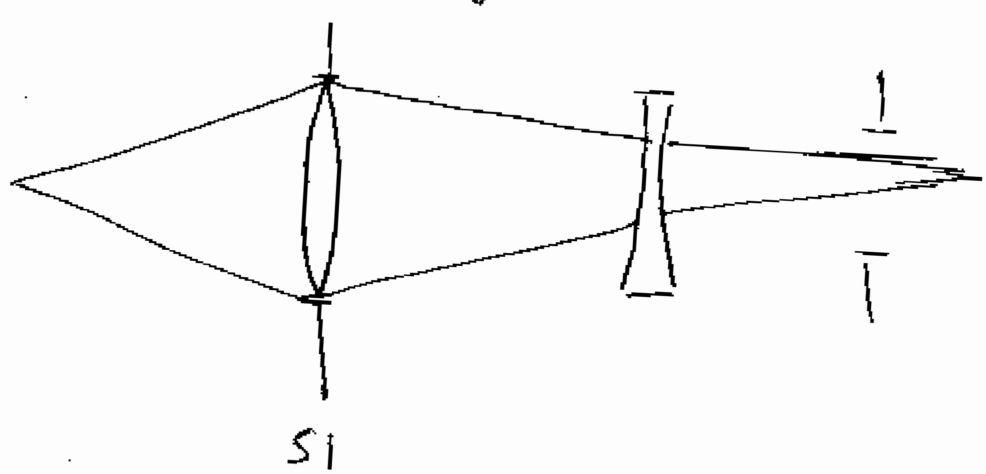
so $$y_1 = \underbrace{\left(2^\circ \times \frac{\pi}{180}\right)}_{\alpha \text{ in radians}} \times 53.3 = 1.86 \text{ cm.}$$

27) only

(d)



S2 is the Field Stop, because it limits the size of the image (and the angular acceptance of objects located at infinity)

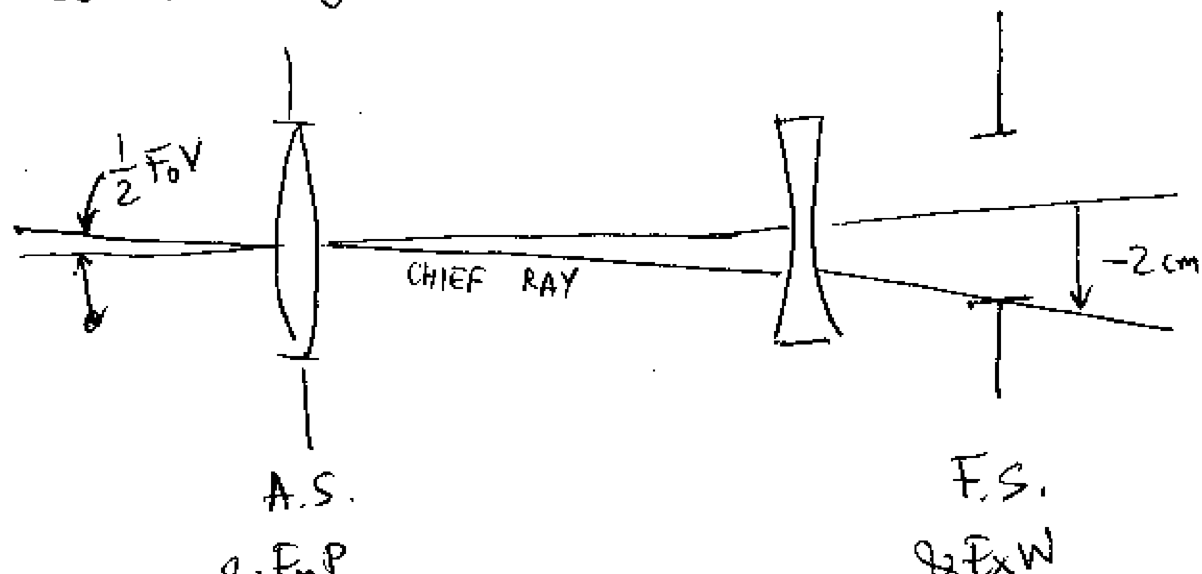


S1 limits the acceptance angle for on-axis remote objects (including objects at ∞) So S1 is the Aperture Stop

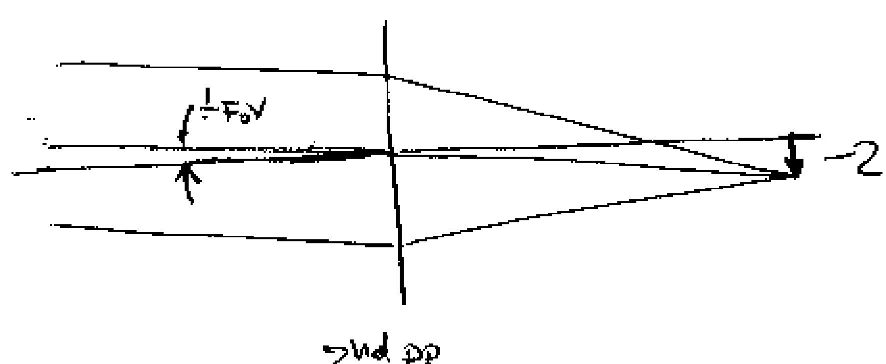
2.71 only

[d] continued.

So the system is:



(e). To find the FoV, we can use the same method as in question (c), except now we are given the Chief Ray's focal point at -2cm:



only slightly larger than the object given in part (c)!

$$\left|\frac{1}{2}(\text{FoV})\right| = \frac{2}{\text{EFL}} = \frac{2}{53.3} = 0.0375 \text{ rad} = \underline{\underline{2.15^\circ}}$$

2.71 (f) → see 2.710 (d) next page

~~1.3~~) If replace with a single lens and equal magnification power. i.e.

2.70 image at the same heigh

1.d

2.71 The $f' = \frac{1.86\text{cm}}{\frac{2}{180}\cdot\pi} = 53.2\ \text{cm}$. $\Leftarrow$ same as EFL.

1.f The lens must be placed 53.2cm before the image.

So obviously, using the telephoto structure reduced the length of a lens w/ long ~~too~~ focal distance.

2.

2.a) To let the human's unaccommodated eye to focus the image on retina, L3 should form a virtual image at infinity. so L1 forms the intermediate image ~~at~~ S2 plane.

As. $\frac{1}{S_o} + \frac{1}{170} = \frac{1}{10}$

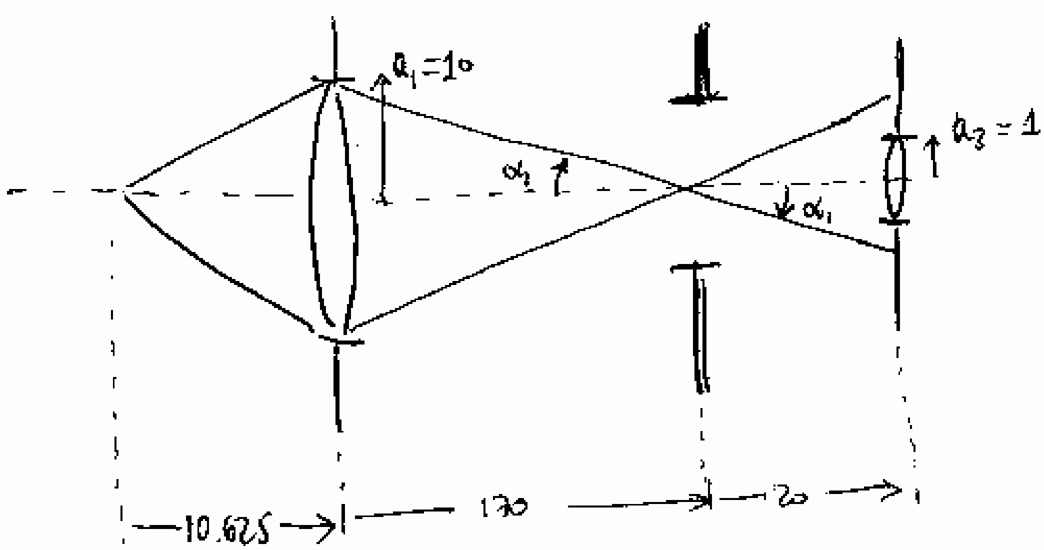
$S_o = 10.625\ \text{mm}$.

2.b) Place a small object at S_o, the intrument acts as a ~~to~~ microscope. The MP is the ratio that ~~the~~ image size formed with intrument on human's retina to the image size formed on the retina with naked eye.

$$MP = M_o \cdot M_e = \frac{170}{10.625} \cdot \frac{\cancel{2\ldots}\ 254}{20} = 203\ X$$

2.c) According to part (a), S2 is at an intermediate image plane, so it is not the aperture stop. [We will prove later that S2 is the field stop]. Potential aperture stops are then the rims of either L1 or L3. The A.S. should be the smallest of the two.

Consider the following schematic:

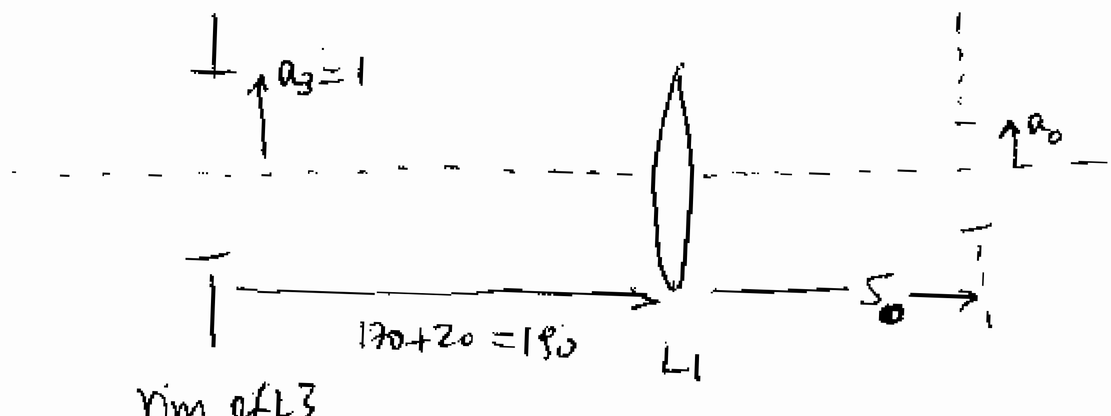


$\alpha_1 = \frac{10}{170}$ is the angle admitted by the rim of L1

but L3 only admits an angle $\frac{1}{20} < \frac{10}{170}$

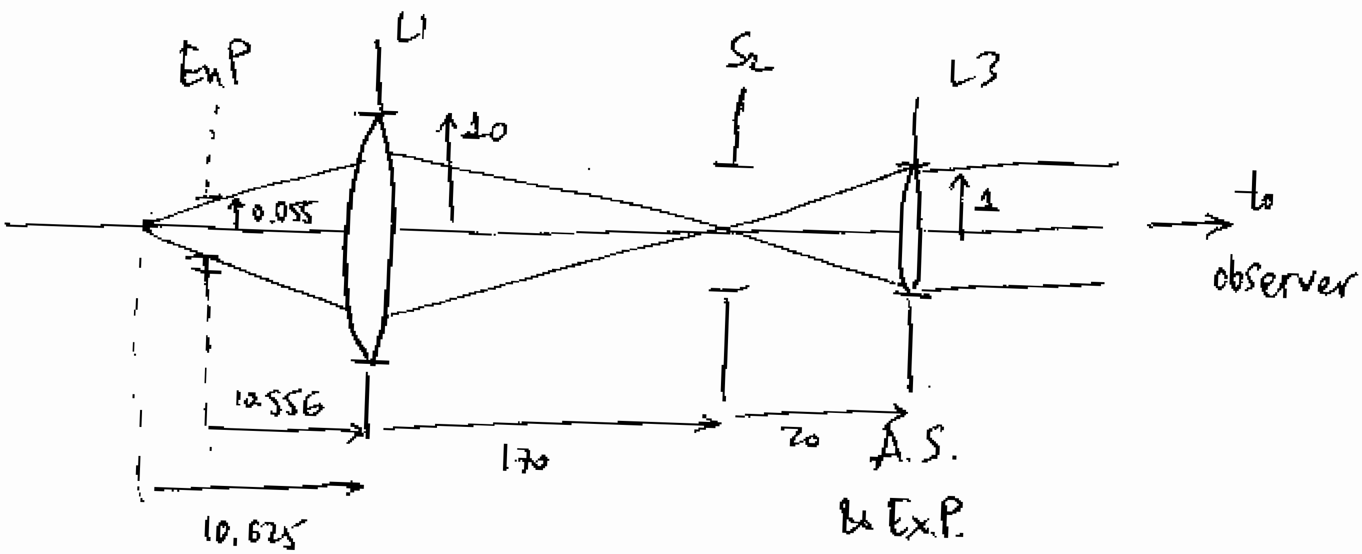
so ~~L3 is~~ the rim of L3 is the A.S.

If we image the rim of L3 through L1, we can find the location and size of the image as follows: (first flip the system so L3 is to the left of L1)



$$\frac{1}{190} + \frac{1}{S_o} = \frac{1}{10} \Rightarrow S_o = 10.556 \text{ mm.} \qquad a_o = a_3 \times \left(-\frac{10.556}{190}\right) = 0.055$$

In the actual system,



The rim of L3 is A.S. and also Ex.P. since there is no optics to its right.

To find the field stop, we need to image S2 through L1. We find that the image S2′ is located exactly at the object plane, at the size of S2′ is

$$16\times\frac{10.625}{170}=1\text{mm}.$$

Now consider a chief ray that goes through the edge of S2′ and the center of the EnP. This ray subtends an angle of arctan[1mm/(10.625-10.556)mm]=86 degrees!! On the other hand, let us consider a chief ray that goes through the edge of L1 and the center of the EnP. Instead, this ray subtends an angle of arctan(10/10.556)=46.5 degrees, which is smaller than the previous one. We see that the stop limiting the field of view is not S2 (or S2′) but L1. Therefore, L1 is the Field Stop. The maximum lateral size of an object that can be viewed is 2 × (10.625-10.556)mm × tan(45.6^0)=0.13mm.

(d) In traditional microscopes, the aperture stop (A.S.) is located at the objective's rim; therefore, the subsequent optics create an image of the A.S. (that is, the Exit Pupil) that is located to the right of the eyepiece. The observer's eye can be comfortably located such that the eye's pupil coincides with the Exit Pupil and the image can be observed without vignetting. In this case, the eyepiece is the A.S. and it is collocated with the Exit Pupil. To avoid vignetting, the eye pupil would have to be adjacent to the eyepiece, which is of course infeasible because (a) the eye pupil is located behind the cornea, and (b) even if the small distance between the cornea and pupil could be neglected, it would be really uncomfortable for the viewer to place his or her eye in contact with the eyepiece. One remedy to this problem is to stop down the objective, i.e. reduce its radius so that it becomes the A.S. instead of the eyepiece; that is not a good solution because, as we will see when we do wave optics, this solution reduces the overall numerical aperture of the system and, hence the resolution of the microscope. A better remedy is to replace the eyepiece with one that has larger radius (assuming we can afford one.) Then again the objective becomes the A.S. as desired.

The schematic below justifies why L1 is the F.S. and also why this system is subject to vignetting.