Imaging properties of three-dimensional pupils

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Abstract: Three-dimensional pupils present new opportunities for optical design, including controlled shift variance, depth selectivity, and dispersion. We present analytical and numerical studies of the impulse response of 3D pupils and discuss implications for computational imaging.

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1. Introduction

The impulse response and transfer function of a system with a thin transparency modulation at the pupil plane is a classical component of every introductory Optics textbook. Following Goodman [1], the impulse response is the Fourier transform of the thin transparency transmission function, and the optical system is shift invariant within the limits of ideal thin lenses and negligible vignetting. In this paper we consider the response of an optical system where the pupil is three-dimensional (3D), i.e. it occupies a finite volume in the vicinity of the Gaussian pupil plane. A classical case of such a 3D pupil is a volume hologram, but in general any 3D distribution of refractive and absorptive elements interacting with the optical field constitutes a 3D pupil. Our treatment includes the scalar and weak diffraction approximations, but 3D pupils can be generalized to include sub-wavelength and large index contrast elements, such as photonic crystals; however, these are beyond the scope of the present analysis.

3D pupils are the most general form of optical processing for generalized imaging systems. As the subsequent analysis shows, the 3D pupil impulse response is strongly shift variant, and can be shaped with more degrees of freedom than traditional 2D pupils. This is particularly useful when the optimization of a computational imaging system requires arbitrary shaping of the optical field presented to the opto-electronic converter (detector). As evidence of the power of 3D optics for imaging, we have previously shown volume holograms to be useful as non-scanning hyperspectral imagers at long working distances [2]. The present analysis is useful for quantifying image quality in these systems, and also for optimizing systems that include post-processing, e.g. Viterbi estimation [3].

2. Coherent response of a 3D pupil

The system we have chosen for this analysis is the “4-F system” composed of two Fourier lenses in tandem, as shown in Figure 1. This choice is convenient because it has a clearly defined Fourier plane, and the results can be easily generalized for other systems. The focal lengths are \(f_1, f_2\), respectively. The spatially coherent input field is \(p(x, y)\), and our objective is to determine the output field \(q(x', y')\) as function of the 3D pupil function \(\varepsilon(x^*, y^*, z^*)\). Without loss of generality, we assume that \(\varepsilon(x^*, y^*, z^*)\) is a phase-only pupil, and that the modulation it imposes on the field is weak so that the 1st-order Born approximation applies to field propagation inside the volume of the 3D pupil. The field generated by \(p(x,y)\) in the vicinity of the shared focal plane in the absence of the 3D pupil is...
\[ P(x^*, y^*, z^*) = \exp\left\{ i2\pi \frac{z^*}{\lambda} \right\} \iint dx dy \phi(x, y) \exp\left\{ i2\pi \frac{xx^* + yy^*}{\lambda f_1} - i\pi \frac{x^2 + y^2}{\lambda f_1^2} \right\}, \] (1)

The 3D pupil modifies the field in the vicinity of the focal plane as

\[ g(x^*, y^*, z^*) = \mathcal{E}(x^*, y^*, z^*) \cdot P(x^*, y^*, z^*). \] (2)

The output field is obtained by propagating the field indicated by (2) to the right-most edge of the 3D pupil element, and Fourier transforming the result. We then obtain for the output field the expression

\[ q(x', y') = G \left[ \frac{x'}{\lambda f_2'}, \frac{y'}{\lambda f_2'}, \frac{1}{\lambda} \left( 1 - \frac{x'^2 + y'^2}{2f_2'^2} \right) \right], \] (3)

where \( G(u,v,w) \) denotes the 3D Fourier transform of \( g(x^*, y^*, z^*) \).

### 3. Effects of shift variance, defocus, and dispersion

Several interesting conclusions follow from this treatment, and we summarize the most salient ones here:

- The output field is a parabolic-shape manifold of the 3D Fourier transform \( G(u,v,w) \), as shown in Figure 2. The manifold shape is a consequence of the ideal quadratic nature of the lenses; it can be changed by aberrating the lenses, for example.

- The result is strongly shift-variant because of the 3rd argument in the Fourier transform relationship (3). The shift variance is a direct consequence of the fact that we are constrained to observe the output field on a surface, even though the scatterer \( \mathcal{E}(x^*, y^*, z^*) \) is volumetric. An example of shift-variant response is shown in Fig. 3.

- The 3rd argument in (3) is also the reason for the Bragg selectivity effect in volume holograms. Examples of Bragg selectivity manifesting itself in response to angular deviation of the reference beam, defocus of the reference beam, and change in wavelength combined with defocus are shown in Figures 4, 5, and 6 on the next page.

The numerical simulations in Figure 3 was conducted with a hologram of thickness \( L=2\times10^3\lambda \), lateral aperture \( a=10^3\lambda \), and focal lengths \( f_1=f_2=2.5\times10^3\lambda \), and it is independent of the specific way the hologram was recorded (i.e., it holds true for any Bragg matched hologram that produces a point-like diffraction pattern.) The simulations in Figure 4-6 were for \( L=10^3\lambda \), \( a=10^3\lambda \), \( f_1=f_2=4\times10^3\lambda \), for holograms recorded as interference patterns of two plane waves whose \( \mathbf{k} \)-vectors both lay on the \( x^*z^* \)-plane. The signal beam was on-axis signal and the reference was off-axis at angle \( \theta_c=0.25\text{rad} \).
Note the peculiar nature of the defocused response in Figure 5, which is due to the shift-variant hologram response. Instead of the familiar Fresnel-rippled disk with the Poisson spot in the middle, we observe that for relatively large values of defocus the response is masked by a sinc-like aperture due to Bragg mismatch. The vertical orientation of the slit is determined by the degeneracy direction of this particular hologram, i.e. along the $y''$ axis. The same holds true for the wavelength-shifted response shown in Figure 6, except the slit shifts as function of defocus.

4. Conclusions and future work

We have presented an analytical formulation for computing the response of volume holograms, and in general 3D pupils, to coherent input illumination. The spatially incoherent response can also be computed using the same formulation in straightforward manner but it was omitted here due to space limitations. The resulting formulae are easily amenable to parameterization. The latter will facilitate the inverse computation of the 3D pupil according to optimization criteria on image quality or, more generally, task-specific optimization. The results are approximate because of the need to use the 1st order Born approximation, but a concurrent numerical effort using rigorous diffraction theory for strong gratings of a general nature is also underway [4]. This research was funded by the DARPA/MTO Montage program and AFOSR/MURI on Autonomous Navigation in 3D Adversarial Environments. The author is grateful to A. Sinha, K. Tian, W. Sun, X. Zao, T. Shih, and J. A. Dominguez-Caballero for invaluable discussions and clarifications.

References