Axial imaging necessitates loss of lateral shift invariance

Andy Stein, Arnab Sinha, and George Barbastathis

Mechanical Engineering Department & Micro-Photonics Center
Massachusetts Institute of Technology, Cambridge, MA 02139 USA

{astein,arnab,gbarb}@mit.edu

Abstract: We conjecture that the lateral shift invariance of an imaging system must be limited if axial imaging capability is desired. We develop shift invariance and depth resolution metrics and demonstrate the trade-off in simple representative systems.

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Known imaging systems capable of resolving object structure along the axial dimension (e.g. confocal microscopes, interferometers, binocular vision systems) usually exhibit limited shift invariance; *i.e.*, they possess a location-dependent impulse response. For example, the confocal microscope [1], shown in Figure 1, achieves optical sectioning via two means: (1) active illumination is focused on a specific object point, and (2) a pinhole placed at the detector plane rejects all out-of-focus light coming from the object. Clearly, both features severely limit the shift invariance of the system. If the pinhole is gradually opened up from its ideal zero diameter, the amount of lateral image information allowed through proportionally increases, whereas the depth-resolving capability decreases. In the limit of the pinhole becoming as wide as the field of view, axial imaging capability is essentially eliminated.

We conjecture that axial imaging and depth resolution are coupled in any imaging system. Thus, by measuring the amount of shift invariance one should be able to estimate the depth resolution of an imaging system. Perhaps more interesting is the possibility of trading shift invariance off for depth resolution when designing an imaging instrument. Among optical elements, volume holograms provide the capability of "tuning" the shift invariance at will. For example, this property has been studied in the context of holographic correlators for optical pattern recognition [2]. Recently, it was shown that a volume holographic matched filter can replace the pinhole of a confocal microscope to provide depth selectivity [3]. Here, we are interested in quantifying the shift invariance *vs.* depth resolution trade-off for general optical systems. For this purpose, we define metrics S and Δz of shift invariance and axial resolution, respectively, such that they can be applied to general imaging systems. We then apply the definitions to a diffraction-limited confocal microscope, and show that shift invariance and depth resolution exhibit opposite trends as the pinhole radius of the system increases.

The <u>axial</u>, longitudinal, or depth direction $\hat{\mathbf{z}}$ (we use all three terms interchangeably) with respect to an imaging system as the direction of an optical axis, if one is defined. If more than one axes can be identified in the system (e.g., in the case of multiple cameras), then we define the "effective axial" direction $\hat{\mathbf{z}}$ as any normalized convex sum of the axes of the system. For example, in the binocular arrangement of Figure 2 with two optical axes $\hat{\mathbf{z}}_1$ and $\hat{\mathbf{z}}_2$, the effective axis may be selected as $\hat{\mathbf{z}} = [\alpha\hat{\mathbf{z}}_1 + (1-\alpha)\,\hat{\mathbf{z}}_2]\,/\{\alpha^2 + (1-\alpha)^2\}^{1/2}$ for any $\alpha \in [0,1]$. The choice $\alpha = 0.5$ is perhaps the most natural, but a different choice may be better suited for a specific application. We will ignore pathological situations, e.g. two cameras facing in opposite

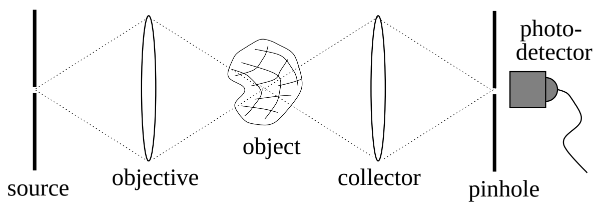

Fig. 1. Confocal microscope.

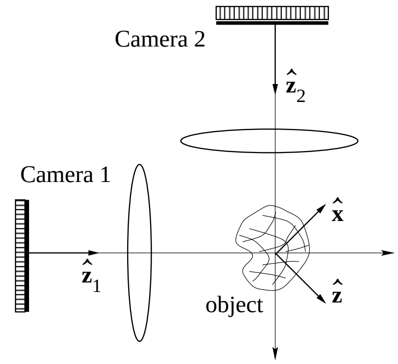

Fig. 2. Binocular imaging.

directions ($\hat{\mathbf{z}}_1 = -\hat{\mathbf{z}}_2$ in Fig. 2). Once $\hat{\mathbf{z}}$ has been defined, the lateral coordinates $\hat{\mathbf{x}}$, $\hat{\mathbf{y}}$ are selected freely provided that $(\hat{\mathbf{x}}, \hat{\mathbf{y}}, \hat{\mathbf{z}})$ form an orthonormal triad. We also assume that the image data are acquired by a detector (or combination of detectors, as in Fig. 2) and denote by (x', y') the coordinates at the detector plane. In multi-camera systems, such as the binocular one of Figure 2, the detector spaces would have to be "concatenated" into a single plane.

Let $h(x, y; x', y')$ denote the lateral intensity impulse response of an arbitrary imaging system at a fixed depth z_0. We define the function

$$S(x,y) = \frac{\iint_{-\infty}^{+\infty} \left[h\left(x,y;x',y'\right) - h\left(0,0;x'-x,y'-y\right)\right]^2 \mathrm{d}x'\mathrm{d}y'}{\iint_{-\infty}^{+\infty} h^2\left(0,0;x'-x,y'-y\right)\mathrm{d}x'\mathrm{d}y'}. \qquad (1)$$

The shift invariance metric is then defined as the distance Δr from the origin required for S to reach 90% of its peak value, as shown in Figure 3. It is easy to see that Δr tends to infinity for a perfectly shift-invariant system. On the other hand, consider an infinitessimally small pinhole in the geometrical optics approximation. Such a system exhibits severe shift variance; consistently, its metric Δr approaches the value 0. For finite pinhole size and diffraction-limited imaging, Δr takes intermediate values, as we show later. Using the above definitions, it is straightforward to compute Δr for other systems such the binocular one of Figure 2. Most interferometers would have very small Δr values because of the ambiguity resulting from repeating fringes.

A simple way to measure axial imaging capability is the use of the uncertainty Δz in determining the axial location of a point source around the reference depth z_0. This may depend on several factors, primarily detection noise but also image quantization (e.g., in the case of a binocular system) etc. Both metrics Δr and Δz generally depend on z_0; complete analysis of this phenomenon is beyond the scope of this paper, however.

Consider again the confocal microscope system of Figure 1, equipped with a scanning mechanism which allows it to acquire 3D data. This system is exceptional in that the shift variance and resolution do not depend on the depth z_0; this is because the system has a focal plane and the image data are acquired one point at a time. This property simplifies the understanding of the trade-offs that we want to discuss here.

The shift invariance data in Figure 4 were generated from (1) using the in-focus lateral intensity impulse response of a lens, squared (to account for confocal illumination)

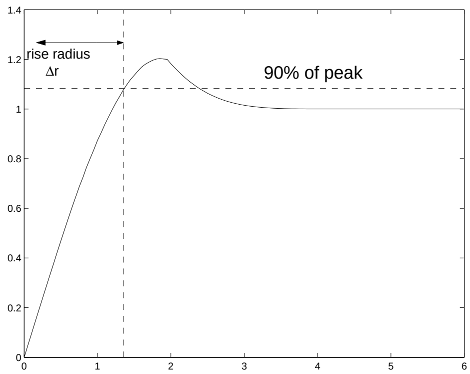

Fig. 3. How to compute Δr.

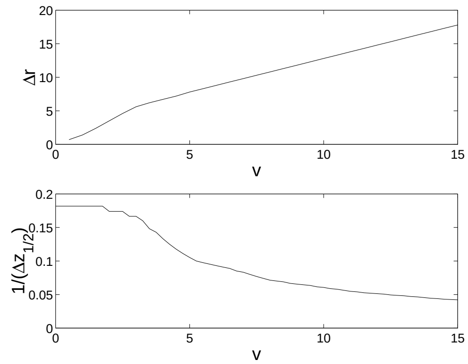

Fig. 4. Shift invariance and depth resolution.

and multiplied by the pinhole mask (radius d) as

$$h(x,y;x',y') = \left[\frac{2\,\mathrm{J}_1\left(\frac{2\pi}{\lambda(\mathrm{NA})}\sqrt{(x-x')^2+(y-y')^2}\right)}{\left(\frac{2\pi}{\lambda(\mathrm{NA})}\sqrt{(x-x')^2+(y-y')^2}\right)} \right]^2 \mathrm{circ}\left(\frac{\sqrt{(x')^2+(y')^2}}{d}\right). \quad (2)$$

The depth resolution data of Figure 4 were generated using the integrated intensity derivation of [4, sec. 8.8.3]. The resolution Δz was defined as the full width at half maximum (FWHM) point. This is slightly pessimistic because it does not account for the improved signal-to-noise ratio at large pinhole diameters. With this caveat, the loss of shift invariance as depth selectivity improves is apparent from Figure 4. The horizontal axis variable in both plots is the normalized pinhole radius $v = 2\pi d/\lambda(\mathrm{NA})$.

The competition among shift invariance and depth resolution is clear from the data of Figure 4. It is also evident in many imaging systems that evolved in Nature. For example, humans can tolerate limited depth selectivity in favor of relatively large domain of shift invariance. This is because humans' cognitive capabilities can compensate depth and shape perception from other cues, such as object size, shading, texture and general knowlegde. We believe that similar exchanges can be applied to the design of artificial "smart" imaging systems which trade aspects of their image quality to maximize overall performance in cognitive tasks.

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References and links

1. M. Minsky, "Microscopy apparatus", US Patent 3,013,467, granted 1961; T. Wilson, editor, *Confocal microscopy*, Academic Press, 1990; J. K. Stevens, L. R. Mills, and J. E. Trogadis, editors, *Three-dimensional confocal microscopy: volume investigation of biological systems*, Academic Press, 1994.
2. M. Levene, G. J. Steckman, and D. Psaltis, "Method for controlling the shift invariance of optical correlators," *Appl. Opt.* **38**(2):394-398, 1999.
3. G. Barbastathis, M. Balberg, and D. J. Brady, "Confocal microscopy with a volume holographic filter", *Opt. Lett.*, **24**(12):811–813, 1999.
4. M. Born and E. Wolf, *Principles of optics*, Pergamon Presss, 7th edition, 1998.