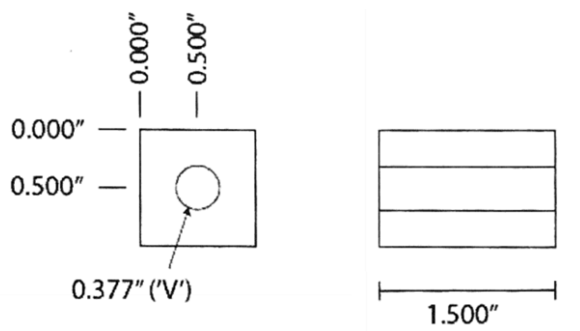
MIT 2.810 Manufacturing Processes and Systems
Homework 3 Solutions
Machining

Revised October 15, 2015

Problems 1 & 2 are based on the flashlight project from lab. Please refer to your lab booklet or to Figures 1 & 2 for dimensions.

Problem 1. Drilling
Consider the drilling of Feature 3 (Thru Hole) on the flashlight head (Figure 1). Assume the feed of the drilling operation (i.e., the distance the drill penetrates per unit revolution) is f = 0.01 in/rev.
Note: Drilling is covered in Section 23.5 of Kalpakjian, 7th ed.



Recommended tool: V drill bit.
Recommended speed: 1200 RPM.

Figure 1. Thru hole step for Problem 1.

a) Calculate the material removal rate (in in³/s or mm³/s).

From the description and diagram of the process step, we know the diameter of the drill (0.377 in) and its rotational velocity (1200 RPM). We can calculate the material removal rate by referring to the formula in Section 23.5.2 (Kalpakjian, 7th ed).

Rotational velocity: $\Omega = 1200\frac{rev}{min} = 20\frac{rev}{s}$

Material removal rate:

$$MRR = \frac{\pi D^2 \Omega f}{4} = \frac{\pi(0.377\ in)^2\left(20\frac{rev}{s}\right)\left(0.01\frac{in}{rev}\right)}{4} = 0.022\frac{in^3}{s}$$

b) Use the Taylor Tool Life equation to estimate the tool life (in minutes) for this operation. Use the table below to pick appropriate values of C and n (the flashlight material is aluminum and the tools are high-speed steel). How many such holes would you be able to drill before the tool needs to be replaced?

		C			
		Nonsteel Cutting		Steel Cutting	
Tool Material	***n***	**m/min**	**(ft/min)**	**m/min**	**ft/min**
Plain carbon tool steel	0.1	70	(200)	20	60
High-speed steel	0.125	120	(350)	70	200
Cemented carbide	0.25	900	(2700)	500	1500
Cermet	0.25			600	2000
Coated carbide	0.25			700	2200
Ceramic	0.6			3000	10,000

Compiled from [4], [9], and other sources.
The parameter values are approximated for turning at feed = 0.25 mm/rev (0.010 in/rev) and depth = 2.5 mm (0.100 in). Nonsteel cutting refers to easy-to-machine metals such as aluminum, brass, and cast iron. Steel cutting refers to the machining of mild (unhardened) steel. It should be noted that significant variations in these values can be expected in practice.

Table 1. Representative values of n and C in the Taylor tool life equation for selected tool materials (from M.P. Groover, Fundamentals of Modern Manufacturing, 5th ed.)

Cutting speed: $v = \Omega\pi D = \left(1200\frac{rev}{min}\right)(0.377\ in)\pi = \left(20\frac{rev}{s}\right)(0.377\ in)\pi = 23.69\frac{in}{s} = 118.44\frac{ft}{min}$

Taylor tool life: $T = \left(\frac{C}{v}\right)^{\frac{1}{n}}$

From the table, for high-speed steel tooling and aluminum (non-steel) work material, C = 350 ft/min, n = 0.125. Plug in v from above.

$$T = \left(\frac{350\frac{ft}{min}}{118.44\frac{ft}{min}}\right)^{\frac{1}{0.125}} = 5{,}815\ min = 96.9\ hours$$

The drilling time for one hole (1.5 inches deep):

$$T_m = \frac{L}{f_r} = \frac{1.5\ in}{0.2\frac{in}{s}} = 7.5\ s$$

Number of holes that can be drilled before replacing the tool:

$$\frac{T}{T_m} = \frac{5815\ \ min \times 60\frac{s}{min}}{7.5\ s} = 46{,}520$$

c) Estimate the power (in hp or W) required for this operation. The specific energy requirements in cutting operations (Table 21.2 in Kalpakjian, 7th ed.) can be applied to drilling processes.

From Table 21.2, the specific energy for aluminum alloys is given as 0.15-0.4 hp*min/in^3. The actual value will depend on a number of process factors, but we can roughly estimate it as the average of the given range: $u_s = 0.275$ hp*min/in^3.

Since $1\ hp = 33{,}000\ \frac{ft \cdot lb}{min}$, we can convert: $u_s = \ 0.275\frac{hp\cdot min}{in^3} \times 33{,}000\frac{ft\cdot lb}{min} = 9{,}075\ \frac{ft\cdot lb}{in^3}$

$$P = MRR \times u_s = \left(0.022\frac{in^3}{s}\right)\left(9{,}075\frac{ft\cdot lb}{in^3}\right) = 200\frac{ft\cdot lb}{s} = 0.36\ hp$$

d) Estimate the rise in temperature at the tool-chip interface during the drilling (in F or ˚C). How close is it to the melting point of the workpiece material? How close is the adiabatic temperature rise to the melting point?

For aluminum:
- the density is 2700 kg/m^3 = 0.0975 lb/in^3
- the specific heat is 896 J/kg*K = 0.214 Btu/lb*F = 166.5 ft*lb/lb*F
- the thermal diffusivity is $\alpha = \frac{k}{\rho C}$ = 0.107 in^2/s
- the melting point is approximately 1220 °F = 660 °C

From lecture slides, the temperature rise at the tool-chip interface is equal to:

$$\Delta T = \frac{0.4u_s}{\rho C}\left(\frac{vf}{\alpha}\right)^{0.33}$$

$$\Delta T = \frac{0.4\left(9075\frac{ft\cdot lb}{in^3}\right)}{\left(0.0975\frac{lb}{in^3}\right)\left(166.5\frac{ft*lb}{lb*F}\right)}\left(\frac{\left(23.69\frac{in}{s}\right)\left(0.01\frac{in}{rev}\right)}{0.107\frac{in^2}{s}}\right)^{0.33} = 291\ \text{°F} = 162\ \text{°C}$$

In adiabatic conditions, the temperature rise would be:

$$\Delta T_{adiab} = \frac{u_s}{\rho C} = \frac{\left(9075\frac{ft\cdot lb}{in^3}\right)}{\left(0.0975\frac{lb}{in^3}\right)\left(166.5\frac{ft*lb}{lb*F}\right)} = 559\ \text{°F} = 311\ \text{°C}$$

The temperature rise at the tool-chip interface would be ~25% of the melting point temperature (in Celsius). The adiabatic temperature rise would be ~47% of the melting point temperature.

e) If you were fabricating the flashlight from Stainless Steel instead of Aluminum, what would be (1) the tool life, (2) the power requirement, and (3) the temperature rise?

Assuming, hypothetically, that other settings stay the same, the answers would be:

(1) Taylor tool life:
From Table 1 above, for high-speed steel tooling and steel work material, C = 200 ft/min, n = 0.125.

$$T = \left(\frac{200\frac{ft}{min}}{118.44\frac{ft}{min}}\right)^{\frac{1}{0.125}} = 66.1\ min = 1.1\ hours$$

(2) Power requirement:

From Table 21.2, the specific energy for cutting stainless steel is given as 0.8-1.9 hp*min/in^3. Again, estimating with the mean value 1.35 hp*min/in^3,

$$u_s = 1.35\frac{hp \cdot min}{in^3} \times 33{,}000\frac{ft \cdot lb}{min} = 44{,}550\ \frac{ft \cdot lb}{in^3}$$

$$P = MRR \times u_s = \left(0.022\frac{in^3}{s}\right)\left(44{,}550\frac{ft \cdot lb}{in^3}\right) = 980\frac{ft \cdot lb}{s} = 1.78\ hp$$

(3) Temperature rise:

For stainless steel (AISI 304):
- the density is 7900 kg/m^3 = 0.285 lb/in^3
- the specific heat is 477 J/kg*K = 0.114 Btu/lb*F = 88.7 ft*lb/lb*F
- the thermal diffusivity is $\alpha = \frac{k}{\rho C}$ = 3.98*10^{-6} m^2/s = 0.00617 in^2/s
- melting point T_m = 1670 K = 1397 °C

$$\Delta T = \frac{0.4u_s}{\rho C}\left(\frac{vf}{\alpha}\right)^{0.33} = \frac{0.4\left(44{,}550\frac{ft \cdot lb}{in^3}\right)}{\left(0.285\frac{lb}{in^3}\right)\left(88.7\frac{ft * lb}{lb * F}\right)}\left(\frac{\left(23.69\frac{in}{s}\right)\left(0.01\frac{in}{rev}\right)}{0.00617\frac{in^2}{s}}\right)^{0.33} = 2346°\text{F} = 1304\ °\text{C}$$

Note that this temperature rise is extremely high, because the above answers assume that feed and rotational speed are the same as the ones used for aluminum. In reality, you would switch to a lower feed rate and RPM, which would reduce the (vf) term in the equation for ΔT (as well as the the v and the MRR terms in the equations for Taylor tool life and Power). Per Table 23.12 (Kalpakjian, 7th ed.), the recommended feed and the rotational speed for stainless steel workpieces should be about several times lower than for aluminum workpieces (at drill sizes ~0.5 in). Our drill size is 0.377 in, so you could estimate a new adjusted feed and adjusted rotational speed (both lower than for aluminum) from that table and re-do the calculations using those values.

Problem 2. Turning

Consider Step 2 (Turn Shoulder) in machining the flashlight handle (Figure 2). Assume you are using a feed f = 0.015 in/rev and a spindle speed Ω = 900 RPM.

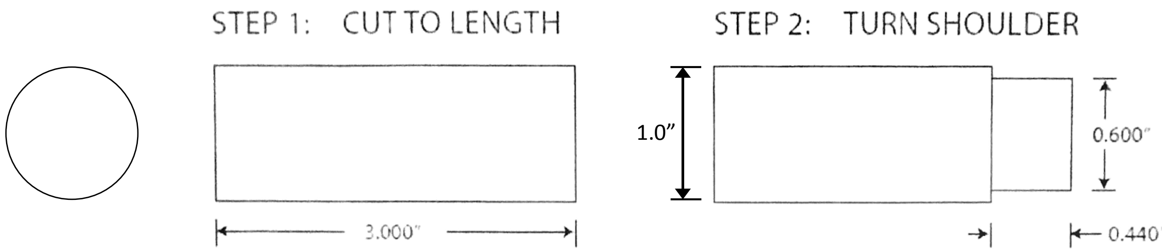


Figure 2. Turning step for Problem 2.

a) Calculate the cutting time (in s) and the material removal rate (in in³/s or mm³/s) for this cut, assuming you remove all of the material with one pass (i.e., depth of cut d = (1”-0.6”)/2 = 0.2”). Approximately how long did it take you to make this cut in the lab?

$$\Omega = 900\,\frac{rev}{min} = 15\,\frac{rev}{s}$$

Using equations from Section 23.2 in Kalpakjian (7th ed):

$$T_m = \frac{L}{f\Omega} = \frac{0.440\ in}{\left(0.015\,\frac{in}{rev}\right)\left(15\,\frac{rev}{s}\right)} = 1.96\ s$$

$$D_{avg} = \frac{D_{initial} + D_{final}}{2} = \frac{1\ in + 0.6\ in}{2} = 0.8\ in$$

$$MRR = \pi\Omega D_{avg} f d = \pi\left(15\,\frac{rev}{s}\right)(0.8\ in)\left(0.015\,\frac{in}{rev}\right)(0.2\ in) = 0.113\,\frac{in^3}{s}$$

b) Estimate the power (hp or W) required for this turning operation. What changes could you make to the process if you needed to decrease the power requirement?

Assume the same specific energy as before in Problem 1 (c): $u_s = 0.275$ hp*min/in³. Then the power needed by the operation is:

$$P = MRR \times u_s = 0.113\,\frac{in^3}{s} \times 60\,\frac{s}{min} \times 0.275\,\frac{hp \cdot min}{in^3} = 1.86\ hp$$

You could lower the power requirement by decreasing the feed or using a larger positive rake angle.

c) Assume the lathe is equipped with a 5-hp electric motor and has a mechanical efficiency of 80%. Estimate the maximum feed that can be used before the lathe begins to stall.

The maximum mechanical power is $P_{max} = 0.8(5\ hp) = 4\ hp$

$$MRR_{max} = \frac{P_{max}}{u_s} = \frac{4\ hp}{0.275\ \frac{hp \cdot min}{in^3}} = 14.55\frac{in^3}{min} = 0.24\frac{in^3}{s}$$

$$f_{max} = \frac{MRR_{max}}{\pi \Omega D_{avg} d} = \frac{0.24\frac{in^3}{s}}{\pi\left(15\frac{rev}{s}\right)(0.8\ in)(0.2\ in)} = 0.032\frac{in}{rev}$$

d) Estimate the cutting force F_c (in N or lb) during the cut.

Since $1\ hp = 33{,}000\ \frac{ft \cdot lb}{min} = 396{,}000\frac{in \cdot lb}{min}$, the power is $P = 1.86\ hp = 61{,}380\ \frac{ft \cdot lb}{min} = 736{,}560\frac{in \cdot lb}{min}$

$$F_c = \frac{P}{v}$$

$$v = \Omega \pi D_{avg} = \pi\left(15\frac{rev}{s}\right)(0.8\ in) = 37.7\frac{in}{s} = 188\frac{ft}{min}$$

$$F_c = \frac{P}{v} = \frac{\left(61{,}380\ \frac{ft \cdot lb}{min}\right)}{\left(188\frac{ft}{min}\right)} = 326\ lb$$

e) We are interested in possible deflections caused by the cutting forces acting on the rod. Estimate the elastic twist angle when the tool engages at the very tip of the cantilevered workpiece (assume 2 inches of the workpiece extends outside the collet). The shear modulus of Aluminum is $G = 3.77 \times 10^6$ psi (= 26 GPa).
(*Hint:* You will need to find the torque and the polar moment of inertia for the rod.)

$$P = Torque \times 2\pi\Omega$$

$$Torque = \frac{P}{2\pi\Omega} = \frac{736{,}560\frac{in \cdot lb}{min}}{2\pi\left(900\frac{rev}{min}\right)} = 130.3\ in \cdot lb$$

The area moment of inertia for a round rod:

$$I_z = \frac{\pi\left(\frac{D}{2}\right)^4}{2} = \frac{\pi(1\ in)^4}{32} = 0.098\ in^4$$

The twist angle can be found using the torque T and the shear modulus G:

$$\theta = \frac{TL}{GI_z} = \frac{(130.3\ in \cdot lb)(2\ in)}{\left(3.77 \times 10^6\frac{lb}{in^2}\right)(0.098\ in^4)} = 0.0007\ rad$$

Problem 3. Angles and Forces in Cutting

a) For orthogonal cutting, please verify that the following expression for shear stress holds:

$$\tau = \frac{F_c \sin\phi \cos\phi - F_t \sin^2\phi}{t_0 w}$$

where the angles and forces are illustrated below, t_0 is the depth of cut and w is the width of the cut.

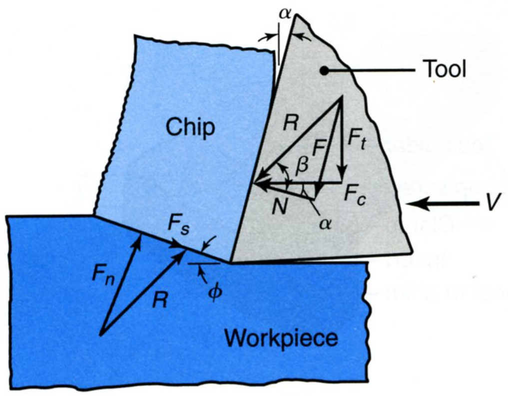


Figure 3. Orthogonal cutting model.

Answer:
The shear stress, τ, is equal to the shear force divided by the shear area:

$$\tau = \frac{F_s}{A_s}$$

The shear force is:

$$F_s = F_c \cos\phi - F_t \sin\phi$$

The shearing takes place along a length ($t_0/\sin\phi$), along the entire width, w, of the cut. Substituting these values in the expression for τ, we get:

$$\tau = \frac{F_c \cos\phi - F_t \sin\phi}{\frac{t_0 w}{\sin\phi}} = \frac{F_c \sin\phi \cos\phi - F_t \sin^2\phi}{t_0 w}$$

See Figure 21.3 and Section 21.3 in Kalpakjian (6th ed.) for further details.

b) Assuming that the shear angle adjusts to minimize the cutting force, derive the following expression:

$$\phi = 45° + \frac{\alpha}{2} - \frac{\beta}{2}$$

Answer:

$$\tau = \frac{1}{t_0 w}(F_c \sin\phi\cos\phi - F_c \tan(\beta-\alpha)\sin^2\phi)$$

$$\frac{\mathrm{d}}{\mathrm{d}\phi}\left(\sin\phi\cos\phi - \tan(\beta-\alpha)\sin^2\phi\right) = 0$$

$$-\sin^2\phi + \cos^2\phi - \tan(\beta-\alpha)\times 2\sin\phi\cos\phi = 0$$

$$\tan(2\phi) = \frac{1}{\tan(\beta-\alpha)}$$

But,

$$\tan(2\phi) = \cot(\pi/2 - 2\phi)$$

Therefore,

$$90 - 2\phi = \beta - \alpha$$

$$\phi = 45 + \alpha/2 - \beta/2\,.$$

This result, the so-called Merchant equation, gives one a good sense for how the shear angle would vary with changes in rake angle and friction. However, because the actual shear strength of a material may vary (e.g., with strain rate and temperature), this equation should be viewed as an approximate relationship rather than an exact one.

c) Cutting forces were measured during the machining of a steel bar and found to be F_c = 520 N and F_t = 189 N. The rake angle on the tool is 10º. Based on this information, calculate: (1) the coefficient of friction at the tool-chip interface, (2) the shear angle.

Answer:

(1) The coefficient of friction can be calculated using just the given values.
The rake angle is $\alpha = 10° = 0.175$ radians.
From Kalpakjian (7th ed.), Equation 21.14:

$$\mu = \frac{F}{N} = \frac{F_t + F_c\tan\alpha}{F_c - F_t\tan\alpha} = \frac{189 + 520\tan 0.175}{520 - 189\tan 0.175} = \frac{189 + 520\tan 0.175}{520 - 189\tan 0.175} = 0.577$$

(2) In Figure 21.11 (a), you can see a right triangle formed by sides F, N, and R. In that triangle, the angle β between sides N and R is the friction angle, and $\tan\beta = \frac{F}{N}$. Since $\mu = \frac{F}{N}$ as well (from equation above):

$$\tan\beta = \mu = 0.577$$
$$\beta = \tan^{-1} 0.577 = 0.52\,rad \;\approx 30°$$

Now we can use the Merchant equation to solve for the shear angle:

$$\phi = 45° + \frac{10}{2} - \frac{30}{2} = 35°$$

Problem 4. Process Plan

Consider the drawing of a part called a rocker arm shown below. The tolerance on the three 1-inch diameter holes is ±0.001 in. For all other dimensions the tolerance is ±0.007 in. The material is 1018 steel with a density of 0.3 lb/in^3 (8.9 g/cm^3).

a) Assuming you start with bar stock with the nominal dimensions of the cross-section for this part (you may adjust this slightly), please write down a process plan (i.e., the machine, tool, and operation for each step, as well as any re-fixturing when needed) to make this part using a band saw and a single CNC milling machine.
(*Note:* Estimates of the times required to perform these operations using manual machining can be found in the *Simplified Time Estimation Booklet for Basic Machining Operations*.)

b) Now consider higher volume production of this part and suggest alternatives to increase the production rate and decrease the cycle time.

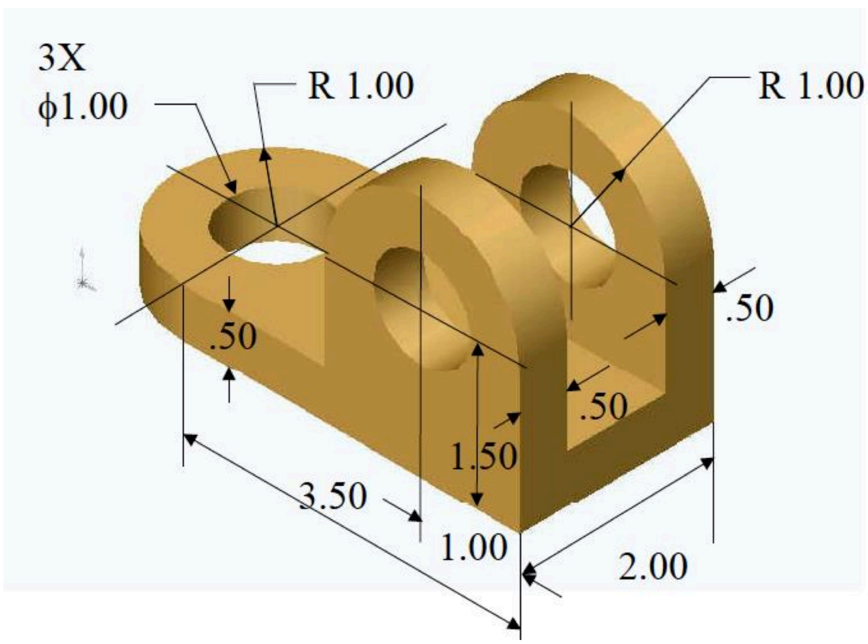


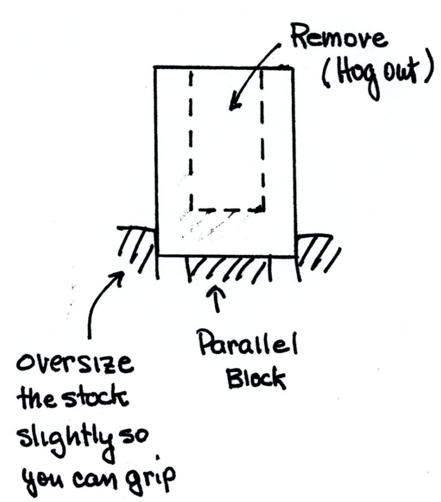
Figure 4. Rocker arm part isometric drawing.

Answer:

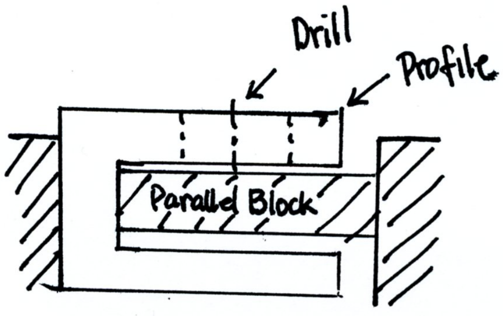
(a) Machines: horizontal saw, vertical CNC milling machine.
Stock: 2" x 2.75" Cross Section steel stock (oversized by ¼" to allow easier fixturing in the mill).

Process step	Machine	Operation	Tool
1	Horizontal saw	Cut to 4.6"	
2	Vertical Mill	Fixture as shown (setup #1)	
2.1	Vertical Mill	Face end(s)	½" end mill
2.2	Vertical Mill	Hog-out	½" end mill
2.3	Vertical Mill	Profile	½" end mill
2.4	Vertical Mill	Drill	Center drill
2.5	Vertical Mill	Drill	Pilot ½"
2.6	Vertical Mill	Drill	Pilot 63/64"
2.7	Vertical Mill	Drill	Ream
3.	Vertical Mill	Re-fixture on side (setup #2)	
3.1	Vertical Mill	Profile	½" end mill
3.2	Vertical Mill	Drill	Center drill
3.3	Vertical Mill	Drill	Pilot ½"
3.4	Vertical Mill	Drill	Pilot 63/64"
3.5	Vertical Mill	Drill	Ream
4.	Vertical Mill	Re-fixture on other side	
4.1	Vertical Mill	Profile	½" end mill
4.2	Vertical Mill	Drill	Center drill
4.3	Vertical Mill	Drill	Pilot ½"
4.4	Vertical Mill	Drill	Pilot 63/64"
4.5	Vertical Mill	Drill	Ream
5.0	Vertical Mill	Re-fixture (setup #3)	
5.1	Vertical Mill	Face bottom	1/2" end mill

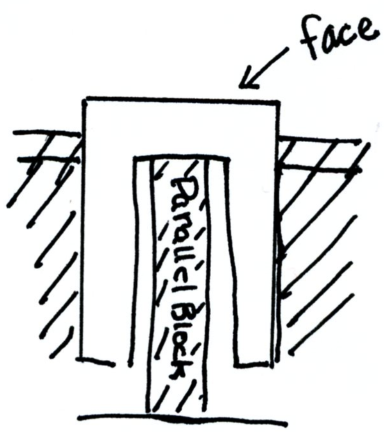
Setup #1: First set up for hogging out, profiling and drilling 1" hole in base.



Setup #2: Re-fixture to profile and drill 1" hole, flip and do other side.



Setup #3. Refixture to remove excess material on bottom.



(b) Alternatives for higher-volume production:
- The bulk of the time to produce the rocker arm is in the "hog out" step (2.2). You could consider starting with a C-shaped extrusion instead, which would all but eliminate the rough machining part of this step. The extrusion per lb would be more costly than the block, but fewer lbs would be needed.
- Consider using form drills; this eliminates all the tool changes to make the holes.
- Consider rotary axes (which can be retrofit to machine tools) to automatically reorient the part and eliminate refixturing.
- Loosen up tolerances and surface finish to reduce the finishing steps.
- Consider a machine tool with a faster tool changer.
- Design a cell with simple machines.