Sheet Metal Forming

2.810

D. Cooper

“Sheet Metal Forming” Ch. 16 Kalpakjian
“Design for Sheetmetal Working”, Ch. 9 Boothroyd, Dewhurst and Knight
Examples - sheet metal formed
Sheet metal stamping/drawing – car industry

- 90 million cars and commercial vehicles produced worldwide in 2014

Diagram:
- Female die
- Metal sheet
- Blank holder (ring) on cushion
- Compressible cushion
- Male die (punch/post)
Stamping Auto body panels

- 3 to 5 dies each
- Prototype dies ~ $50,000
- Production dies ~ $0.75-1

- Forming dies
- Trimming station
- Flanging station
Objectives

By the end of today you should be able to...

...**describe** different forming processes, when they might be used, and **compare** their production rates, costs and environmental impacts

...**calculate** forming forces, **predict** part defects (tearing, wrinkling, dimensional inaccuracy), and **propose** solutions

...**explain** current developments: opportunities and challenges
LMP Shop

Brake press

Finger brake
Technology – a brief review

Material drawn into shape

• Conventional drawing/stamping – expensive tooling, no net thinning, quick

• Hydro-forming – cheap tooling, no net thinning, slow, high formability

Material stretched into shape

• Stretch forming – very cheap tooling, net thinning, slow, low formability

• Super-plastic forming – cheap tooling, net thinning, expensive sheet metal, slow, very high formability

Forming Speed

20-1000pts/hr

7-13cycles/hr

3-8pts/hr

0.3-4pts/hr
Drawing – expensive tooling, no net thinning, quick

Deep-drawing

Shallow-drawing (stamping)
Deep-drawing

Blank holder helps prevent wrinkling and reduces springback

Blank holder not necessary if blank diameter / blank thickness is less than 25-40. Smaller values for deeper forming.
Blank holder force: forming window

Depth of draw

Blankholder force

Wrinkling

Tearing

Window for forming
Deep Drawing of drinks cans

Hosford and Duncan (can making): http://www.chymist.com/Aluminum%20can.pdf

1. Blanking

2. Deep drawing

3. Redrawing

4. Ironing

5. Domeing

6. Necking

7. Seaming

FIGURE 16.31 The metal-forming processes involved in manufacturing a two-piece aluminum beverage can.
Hydro-forming – cheap tooling, no net thinning, slow(ish), high formability

Low volume batches
Hydro-forming – cheap tooling, no net thinning, slow(ish), high formability

Low volume batches
Hydro-forming – cheap tooling, no net thinning, slow, high formability

Small flexforming tool made by additive manufacturing
**Stretch forming** – very cheap tooling, net thinning, slow, low formability, sheet metal up to 15mx9m

* source: http://www.cyrilbath.com/sheet_process.html

Low volume batches
Stretch forming: Example parts

Higher aspect ratio, deeper parts
Super-plastic forming – cheap tooling, net thinning, slow, expensive sheet metal, very high formability

Low volume batches, 0.5-0.75 melting temp
Forming forces and part geometry
Tensile test – the Stress-strain diagram

\[ \sigma_y = Y \]
True stress & strain

\[ \varepsilon_{tr} = \ln(1 + \varepsilon_{en}) \]

\[ \sigma_{tr} = \sigma_{en} (1 + \varepsilon_{en}) \]

True stress can be expressed using a power law (Hollomon equation):

\[ \sigma_{tr} = K \varepsilon_{tr}^n \]
Power-Law Expression (Hollomon equation)

\[ \sigma_{tr} = K \varepsilon_{tr}^n \]

Can be re-written: \[ \log(\sigma_{tr}) = n \log(\varepsilon_{tr}) + \log K \]
Power-Law Expression (Hollomon equation)

\[ \sigma_{tr} = K \varepsilon_{tr}^n \]

Can be re-written: \[ \log(\sigma_{tr}) = n \log(\varepsilon_{tr}) + \log K \]
<table>
<thead>
<tr>
<th>Material</th>
<th>K (MPa)</th>
<th>n</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminum</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1100-O</td>
<td>180</td>
<td>0.20</td>
</tr>
<tr>
<td>2024-T4</td>
<td>690</td>
<td>0.16</td>
</tr>
<tr>
<td>5052-O</td>
<td>202</td>
<td>0.13</td>
</tr>
<tr>
<td>6061-O</td>
<td>205</td>
<td>0.20</td>
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<tr>
<td>6061-T6</td>
<td>410</td>
<td>0.05</td>
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<tr>
<td>7075-O</td>
<td>400</td>
<td>0.17</td>
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<tr>
<td>Brass</td>
<td></td>
<td></td>
</tr>
<tr>
<td>70-30, annealed</td>
<td>900</td>
<td>0.49</td>
</tr>
<tr>
<td>85-15, cold rolled</td>
<td>580</td>
<td>0.34</td>
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<tr>
<td>Cobalt-based alloy, heat treated</td>
<td>2070</td>
<td>0.50</td>
</tr>
<tr>
<td>Copper, annealed</td>
<td>315</td>
<td>0.54</td>
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<tr>
<td>Steel</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Low-C, annealed</td>
<td>530</td>
<td>0.26</td>
</tr>
<tr>
<td>1020, annealed</td>
<td>745</td>
<td>0.20</td>
</tr>
<tr>
<td>4135, annealed</td>
<td>1015</td>
<td>0.17</td>
</tr>
<tr>
<td>4135, cold rolled</td>
<td>1100</td>
<td>0.14</td>
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<tr>
<td>4340, annealed</td>
<td>640</td>
<td>0.15</td>
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<tr>
<td>304 stainless, annealed</td>
<td>1275</td>
<td>0.45</td>
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<tr>
<td>410 stainless, annealed</td>
<td>960</td>
<td>0.10</td>
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<tr>
<td>Titanium</td>
<td></td>
<td></td>
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<tr>
<td>Ti-6Al-4V, annealed, 20°C</td>
<td>1400</td>
<td>0.015</td>
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<tr>
<td>Ti-6Al-4V, annealed, 200°C</td>
<td>1040</td>
<td>0.026</td>
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<tr>
<td>Ti-6Al-4V, annealed, 600°C</td>
<td>650</td>
<td>0.064</td>
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<tr>
<td>Ti-6Al-4V, annealed, 800°C</td>
<td>350</td>
<td>0.146</td>
</tr>
</tbody>
</table>
Tensile instability - necking

\[ \sigma = k \varepsilon^n \]

True: \( \Delta \varepsilon = \frac{dL}{L} \)

Nominal:
\[ \sigma = \frac{F}{A_0}, \quad \varepsilon = \frac{L-L_0}{L_0} \]

Neck forms at \( \varepsilon^* \)

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Tensile instability (1-D)

\[ F = \sigma A; \quad \text{so} \quad dF = \sigma dA + A d\sigma = 0 \quad \text{at max load} \]

\[ \frac{d\sigma}{\sigma} = - \frac{dA}{A} = d\varepsilon \]

\[ \frac{d\sigma}{d\varepsilon} = \sigma \]

With \( \sigma = k \varepsilon^n \): \[ \frac{d\sigma}{d\varepsilon} = n k \varepsilon^{n-1} = \sigma = k \varepsilon^n \]

\[ \Rightarrow \quad \varepsilon^* = \eta \]
Useful assumptions

Only interested in plastic effects:
**Perfectly plastic material**
At $Y$, material deforms ('flows') in compression and fails in tension

Interested in elastic and plastic effects:
**Elastic-perfectly plastic material**
In 1-D, \( \sigma = K \varepsilon^n \) assuming perfectly plastic, yielding at: \( \sigma = Y \)

In 3-D, \( \sigma_{\text{eff}} = K \varepsilon^n_{\text{eff}} \) assuming perfectly plastic, yielding at:

\[ \sigma_{\text{eff}} = Y \]
### 3D Yield Criteria

<table>
<thead>
<tr>
<th><strong>Tresca:</strong> Yielding occurs at a maximum shear stress</th>
<th><strong>Von Mises:</strong> Yielding at maximum distortion strain energy</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Effective stress (in principal directions):</strong></td>
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</tr>
<tr>
<td>( \sigma_{\text{eff}} = \left[ \sigma_i - \sigma_j \right]_{\text{max}}, \quad i \neq j )</td>
<td>( \sigma_{\text{eff}} = \sqrt{\frac{1}{2} \left[ \left( \sigma_2 - \sigma_3 \right)^2 + \left( \sigma_3 - \sigma_1 \right)^2 \right]} )</td>
</tr>
<tr>
<td><strong>Yield criterion:</strong></td>
<td><strong>Yield criterion:</strong></td>
</tr>
<tr>
<td>( \sigma_{\text{eff}} = Y )</td>
<td>( \sigma_{\text{eff}} = Y )</td>
</tr>
<tr>
<td>( \tau_{\text{max}} = k = \frac{Y}{2} )</td>
<td>( Y = \sqrt{3}k )</td>
</tr>
<tr>
<td><strong>Effective strain:</strong></td>
<td><strong>Effective strain:</strong></td>
</tr>
<tr>
<td>( \varepsilon_{\text{eff}} = \left( \varepsilon_i \right)_{\text{max}} )</td>
<td>( \varepsilon_{\text{eff}} = \sqrt{\left( \frac{1}{2} \right) \left( \varepsilon_1^2 + \varepsilon_2^2 + \varepsilon_3^2 \right)} )</td>
</tr>
</tbody>
</table>
Shearing

$F = 0.7 \times T \times L \times (UTS)$

$T = $ Sheet Thickness
$L = $ Total length Sheared
$UTS = $ Ultimate Tensile Strength of material

Shear press - LMP Shop
2-6 EFFECTIVE STRESS

With either yield criterion, it is useful to define an effective stress denoted as $\bar{\sigma}$ which is a function of the applied stresses. If the magnitude of $\bar{\sigma}$ reaches a critical value, then the applied stress state will cause yielding; in essence, it has reached an effective level. For the von Mises criterion,

$$\bar{\sigma} = \frac{1}{\sqrt{2}} \left[ (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 \right]^{1/2}$$  \hspace{1cm} (2-16)

while for the Tresca criterion,

$$\bar{\sigma} = \sigma_1 - \sigma_3 \quad \text{where} \quad \sigma_1 > \sigma_2 > \sigma_3$$  \hspace{1cm} (2-17)

Yielding occurs when $\sigma_{\text{effective}} = Y$

Material taken from *Metal Forming*, by Hosford and Caddell
2-7 EFFECTIVE STRAIN

Effective strain is defined such that the incremental work per unit volume is

\[ dw = \sigma \, d\varepsilon = \sigma_1 \, d\varepsilon_1 + \sigma_2 \, d\varepsilon_2 + \sigma_3 \, d\varepsilon_3 \]  \hspace{1cm} (2-18)

For the von Mises criterion, the effective strain is given by

\[ d\varepsilon = \frac{\sqrt{2}}{3} \left[ (d\varepsilon_1 - d\varepsilon_2)^2 + (d\varepsilon_2 - d\varepsilon_3)^2 + (d\varepsilon_3 - d\varepsilon_1)^2 \right]^{1/2} \]  \hspace{1cm} (2-19)

which may be expressed in a simpler form as

\[ d\varepsilon = \left[ \frac{2}{3} (d\varepsilon_1^2 + d\varepsilon_2^2 + d\varepsilon_3^2) \right]^{1/2} \]  \hspace{1cm} (2-20)

If the straining is proportional (with a constant ratio of \( d\varepsilon_1 : d\varepsilon_2 : d\varepsilon_3 \)), the total effective strain may be expressed in terms of the total strains as

\[ \bar{\varepsilon} = \left[ \frac{2}{3} (\varepsilon_1^2 + \varepsilon_2^2 + \varepsilon_3^2) \right]^{1/2} \]  \hspace{1cm} (2-21)

If the strain path is not constant, \( \bar{\varepsilon} \) must be found from a path integral of \( d\varepsilon \). In this case

\[ \bar{\Omega} = K \bar{\varepsilon}^n \]

Material taken from Metal Forming, by Hosford and Caddell
3D Yield Effective stress

Tresca predicts ‘flow’ for lower stresses than von Mises

\[ \sigma_3 = 0 \]
Forming Limit Diagrams

Figure 15-8  Strips of varying width are stretched to obtain different $\varepsilon_1/\varepsilon_2$ ratios.

Excluded: $\varepsilon_1$ must be $> \varepsilon_2$
Tensile test

\[ \varepsilon_1 = -2\varepsilon_2 \]

\[ \varepsilon_1 = n = \text{necking} \]
Pure Shear
\[ \varepsilon_1 = -\varepsilon_2 \]
Stretch forming: Forming force

\[ F = \frac{Y_S + UTS}{2} \times A \]

- \( F \) = stretch forming force (lbs)
- \( Y_S \) = material yield strength (psi)
- \( UTS \) = ultimate tensile strength of the material (psi)
- \( A \) = Cross-sectional area of the workpiece (in²)
Forces needed to bend sheet metal
Bending

\[ \Delta L = (L - L_0) = (\rho + y) \theta - \rho \theta = y \theta \]

\[ \epsilon = \frac{\Delta L}{L_0} = \frac{y \theta}{\rho \theta} = \frac{y}{\rho} \]

\[ \epsilon_{\text{max}} = \frac{h/2}{R + h/2} = \frac{1}{2R + h/2} + 1 \]

Figure Coordinate system for analysis of bending.
Stress distribution through the thickness of the part

Elastic

Elastic-plastic

Fully plastic

Fully Plastic Moment, \( M = Y \left( \frac{b \cdot h}{2} \right) \left( \frac{h}{2} \right) = Ybh^{2}/4 \)
Balance external and internal moments

Ybh^2/4 = FL/4 = M_{max}

F = bh^2Y/L
Bending Force Requirement

\[ F = \frac{LT^2}{W} (UTS) \]

\( T \) = Sheet Thickness  \\
\( W \) = Width of Die Opening  \\
\( L \) = Total length of bend  \\
(\text{into the page})  \\
\( UTS \) = Ultimate Tensile Strength of material

Note: the notation used in the text \((L, W)\) differs from that used in the previous development \((b, L)\).
LMP Shop

Brake press

Finger brake
What shape have we created?
Steel versus aluminum...

Strength ($\sigma_y$) versus Stiffness ($E$)

- Mild steel (33,000psi) & Al. 5052$_{H32}$ (33,000psi)
- Al. 5052$_{H32}$ (10.6E6psi)

Mild steel (30E6psi)
Steel versus aluminum...

Strength ($\sigma_y$) versus Stiffness ($E$)

Mild steel (33,000 psi) & Al. 5052$_{H32}$ (33,000 psi)

Mild steel (30E6 psi)
Low spring back

Al. 5052$_{H32}$ (10.6E6 psi)
High spring back
Steel versus aluminum...

Strength ($\sigma_y$) versus Stiffness (E)

- --- Mild steel (33,000 psi) & Al. 5052$^\text{H32}$ (33,000 psi)
- --- Al. 2024$^\text{T3}$ (50,000 psi)

Mild steel (30E6 psi)
- --- Low spring back

Al. 2024$^\text{T3}$ & 5052$^\text{H32}$ (10.6E6 psi)
- --- High spring back
Steel versus aluminum...

**Strength ($\sigma_y$) versus Stiffness ($E$)**

- **Al. 2024$_{T3}$** (50,000psi)
  - High spring back
- **Mild steel** (33,000psi) & **Al. 5052$_{H32}$** (33,000psi)
  - Low spring back
- **Mild steel** (30E6psi)
  - Low spring back
- **Al. 2024$_{T3}$ & 5052$_{H32}$** (10.6E6psi)
  - High spring back
Springback note R in the figure below is mislabeled, should go to the centerline of the sheet.

Springback: \( \frac{R_i}{R_f} = 4 \left( \frac{R_i Y}{ET} \right)^3 - 3 \left( \frac{R_i Y}{ET} \right) + 1 \)
1. Assume plane sections remain plane:
   \[ e_y = -\frac{y}{r} \]  \hspace{1cm} (1)

2. Assume elastic-plastic behavior for material

\[ \sigma = E e \quad e < \varepsilon \psi \]

\[ \sigma = \sigma_Y \quad e \geq e \]
Bending Moment – Curvature

\[ M = \frac{EI}{1/r} \]

\[ M_Y \]

\[ 1/r_Y \]

\[ 1/R_1 \]

\[ 1/R_0 \]

\[ 1/r \]

Loading

Unloading
3. We want to construct the following Bending Moment “M” vs. curvature “1/ρ” curve

Springback is measured as

Permanent set is

$$M = \frac{M_Y}{(1/\rho)_y} \left[ \frac{1}{\rho} - \frac{1}{R_1} \right]$$

$$1/R_0 - 1/R_1$$

$$1/R_1$$
4. Stress distribution through the thickness of the beam

Elastic

Elastic-plastic

Fully plastic
5. \( M = \int_A \sigma y \, dA \)

Elastic region

\[
M = \int \sigma y \, dA = -E \int \frac{y^2}{\rho} \, dA = -\frac{EI}{\rho}
\] (3)

At the onset of plastic behavior

\[
\sigma = -\frac{y}{\rho} E = -\frac{h}{2\rho} E = -Y
\] (4)

This occurs at

\[
\frac{1}{\rho} = \frac{2Y}{hE} = \frac{1}{\rho_Y}
\] (5)

Substitution into eqn (3) gives us the moment at on-set of yield, \( M_Y \)

\[
M_Y = -\frac{EI}{\rho_Y} = EI \frac{2Y}{hE} = 2IY/h
\] (6)

After this point, the \( M \) vs \( 1/r \) curve starts to “bend over.” Note from \( M=0 \) to \( M=M_Y \) the curve is linear.
In the elastic – plastic region

\[ M = \int \sigma y b dy = 2 \int_{y_y}^{h/2} Y by dy + 2 \int_{0}^{y_y} \frac{y}{y_Y} Y by dy \]

\[ = 2 Y b \frac{y^2}{2} \bigg|_{y_y}^{h/2} + 2 \frac{Y}{y_Y} \frac{b}{3} y^3 \bigg|_{0}^{y_y} \]

\[ = Y b \left( \frac{h^2}{4} - y_y^2 \right) + \frac{2}{3} y_y^2 Y b \]

\[ M = \frac{b h^2}{4} Y \left[ 1 - \frac{1}{3} \left( \frac{y_y}{h/2} \right)^2 \right] \]  

(7)

Note at \( y_y = h/2 \), you get on-set at yield, \( M = M_Y \)
And at \( y_y = 0 \), you get fully plastic moment, \( M = 3/2 M_Y \)
To write this in terms of $M$ vs $1/\rho$ rather than $M$ vs $y_y$, note that the yield curvature $(1/\rho)_y$ can be written as (see eqn (1))

$$\frac{1}{\rho_y} = \frac{\epsilon_y}{h/2} \quad (8)$$

Where $\epsilon_y$ is the strain at yield. Also since the strain at $y_y$ is $-\epsilon_y$, we can write

$$\frac{1}{\rho} = \frac{\epsilon_y}{y_y} \quad (9)$$

Combining (8) and (9) gives

$$\frac{y_y}{h/2} = \frac{(1/\rho)_y}{1/\rho} \quad (10)$$
Substitution into (7) gives the result we seek:

\[ M = \frac{3}{2} M_Y \left[ 1 - \frac{1}{3} \left( \frac{1/\rho}{1/\rho_Y} \right)^2 \right] \]  

(11)

\[ M = \frac{M_Y}{(1/\rho)_Y} \left[ \frac{1}{\rho} - \frac{1}{R_1} \right] \]  

(12)
Now, eqn’s (11) and (12) intersect at $1/\rho = 1/R_0$

Hence,

$$\frac{M_Y}{(1/\rho)_Y} \left[ \frac{1}{R_0} - \frac{1}{R_1} \right] = \frac{3}{2} M_Y \left[ 1 - \frac{1}{3} \left( \frac{(1/\rho)_Y}{1/R_0} \right)^2 \right]$$

Rewriting and using $(1/\rho)_Y = 2Y/hE$ (from a few slides back), we get

$$\left[ \frac{1}{R_0} - \frac{1}{R_1} \right] = 3 \frac{Y}{hE} - 4R_0^2 \left( \frac{Y}{hE} \right)^3 \quad (13)$$
R_0 = R_1

\[ \frac{1}{R_0} - \frac{1}{R_1} = 3 \frac{Y}{hE} - 4R_0^2 \left( \frac{Y}{hE} \right)^3 \]

Thickness, h = 0.0625 in.
Methods to reduce springback

• Smaller Y/E
• Larger thickness
• Over-bending
• Stretch forming
• “coining” or bottoming the punch
Pure Bending

Bending & Stretching

tension

compression

Fully plastic

Bending & Stretching
Stretch forming: can we achieve a strain of 0.035 at A?

Sheet thickness 1 mm, $\mu=0.1$

Material: $\sigma=520\varepsilon^{0.18}$ MPa
Can we achieve a strain of 0.035 at A?

Sheet thickness 1mm, $\mu=0.1$

Material: $\sigma=520\varepsilon^{0.18}\text{MPa}$

$F_A = 0.001 \times 520 \times (0.035)^{0.18} = 284\text{kN/m}$

$F_B = F_A \times \exp(0.1 \times 0.25) = 292\text{kN/m}$

$F_C = F_B \times \exp(0.1 \times 1.05) = 323\text{kN/m}$

Max allowable force
$= 0.001 \times 520 \times (0.18)^{0.18} = 381\text{kN/m}$

Grips
Punch
R=8m, $\theta=0.25$
R'=0.1m, $\theta=1.25$

Capstan equation
Friction and the capstan equation

Typical stamping lubricants:
- Oil-based lubricants
- Aqueous lubricants
- Soaps and greases
- Solid films

\[ T_{load} = T_{hold} \times \exp(\mu \theta) \]
Research opportunities and challenges: reducing cost and environmental impacts
Energy & cost: Stamping alum car hoods

- Final part = 5.4kgs
- Total number of parts made = 400
- Die material: cast and machined zinc alloy

Energy. 2.3GJ/pt. Stamping alum. car hoods. 5.4kgO/P. (400pts)

Sheet metal scrapped in factory = 44%

Cost. 136USD/pt

60 Ton Discrete Die Press (LMP - Hardt)

6 feet
The Shape Control Concept
Stretch Forming with Reconfigurable Tool @ Northrop Grumman
Flexible Forming at Ford
Conventional Spinning

Flexible Spinning

Sheet metal forming in a low carbon future?
See the wonderful...
http://www.withbotheyesopen.com

Extra slides – just for fun
Surface finish defects

- Orange peel effect
- Lüders bands
Material embodied energy: **Aluminum primary production**

Milford et al. (2011): Whole supply chain

**Process step**

- al-making
- Ingot casting
- Sawing
- Scalping
- Preheating
- Hot rolling
- Cold rolling
- Annealing
- Slitting
- Blanking

**MJ/Kg output**

- Perfect yield
Material embodied energy: **Aluminum primary production**

![Material embodied energy: Aluminum primary production](image)

-Milford et al. (2011): Whole supply chain-

-MIT Institute of Technology-
Material embodied energy: **Aluminum primary production**

![Graph showing material embodied energy for aluminum in the primary production process.](image)

- **Milford et al. (2011): Whole supply chain**
  - *Perfect yield* and *Real yield* are shown for each step.

<table>
<thead>
<tr>
<th>Process step</th>
<th>MJ/Kg output</th>
</tr>
</thead>
<tbody>
<tr>
<td>Al-making</td>
<td>100</td>
</tr>
<tr>
<td>Ingot casting</td>
<td>150</td>
</tr>
<tr>
<td>Sawing</td>
<td>200</td>
</tr>
<tr>
<td>Scalping</td>
<td>250</td>
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<tr>
<td>Preheating</td>
<td>300</td>
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<tr>
<td>Hot rolling</td>
<td>350</td>
</tr>
<tr>
<td>Cold rolling</td>
<td>400</td>
</tr>
<tr>
<td>Annealing</td>
<td>450</td>
</tr>
<tr>
<td>Slitting</td>
<td>500</td>
</tr>
<tr>
<td>Blanking</td>
<td>550</td>
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</table>

- **10kg liquid al.**
- **4kg car door**

**Material embodied energy:**
- **Aluminum primary production**

**MJ/Kg output**

**Perfect yield** and **Real yield**

**Massachusetts Institute of Technology**