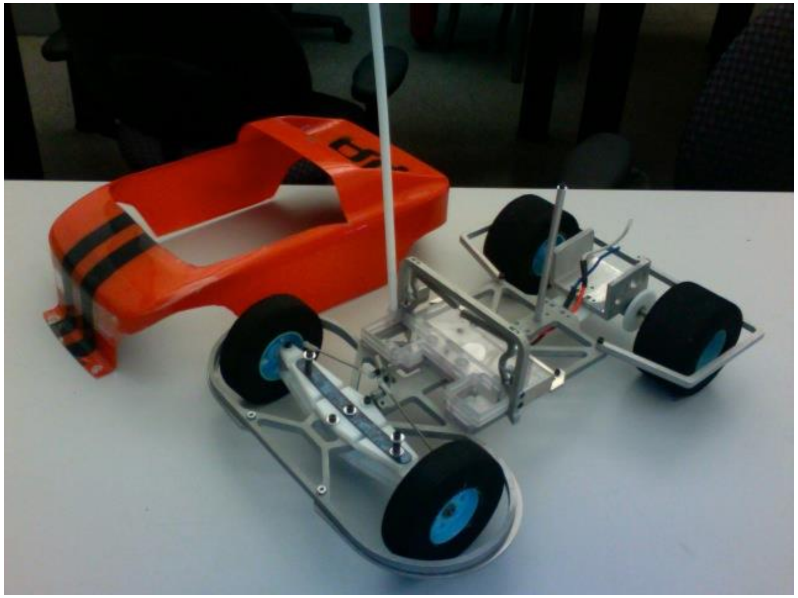
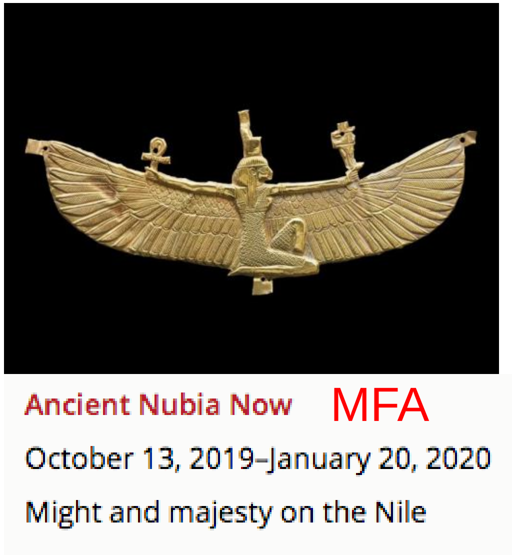


Casting & AM



Ancient Nubia Now MFA

October 13, 2019–January 20, 2020

Might and majesty on the Nile

2.810

T. Gutowski

Outline

1. Quick Review: Sand Casting, Die Casting
2. Basics: Phase Change, Shrinkage, Heat Transfer
3. Design Issues, patterns and tools
4. Additive (laser metal powder bed) Manufacturing

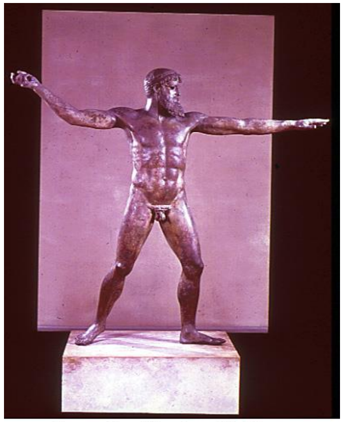
Casting since about 3200 BCE…

Etruscan casting with runners
circa 500 BCE

China circa 3000BCE

Lost wax jewelry from Greece
circa 300 BCE

Bronze age to iron age

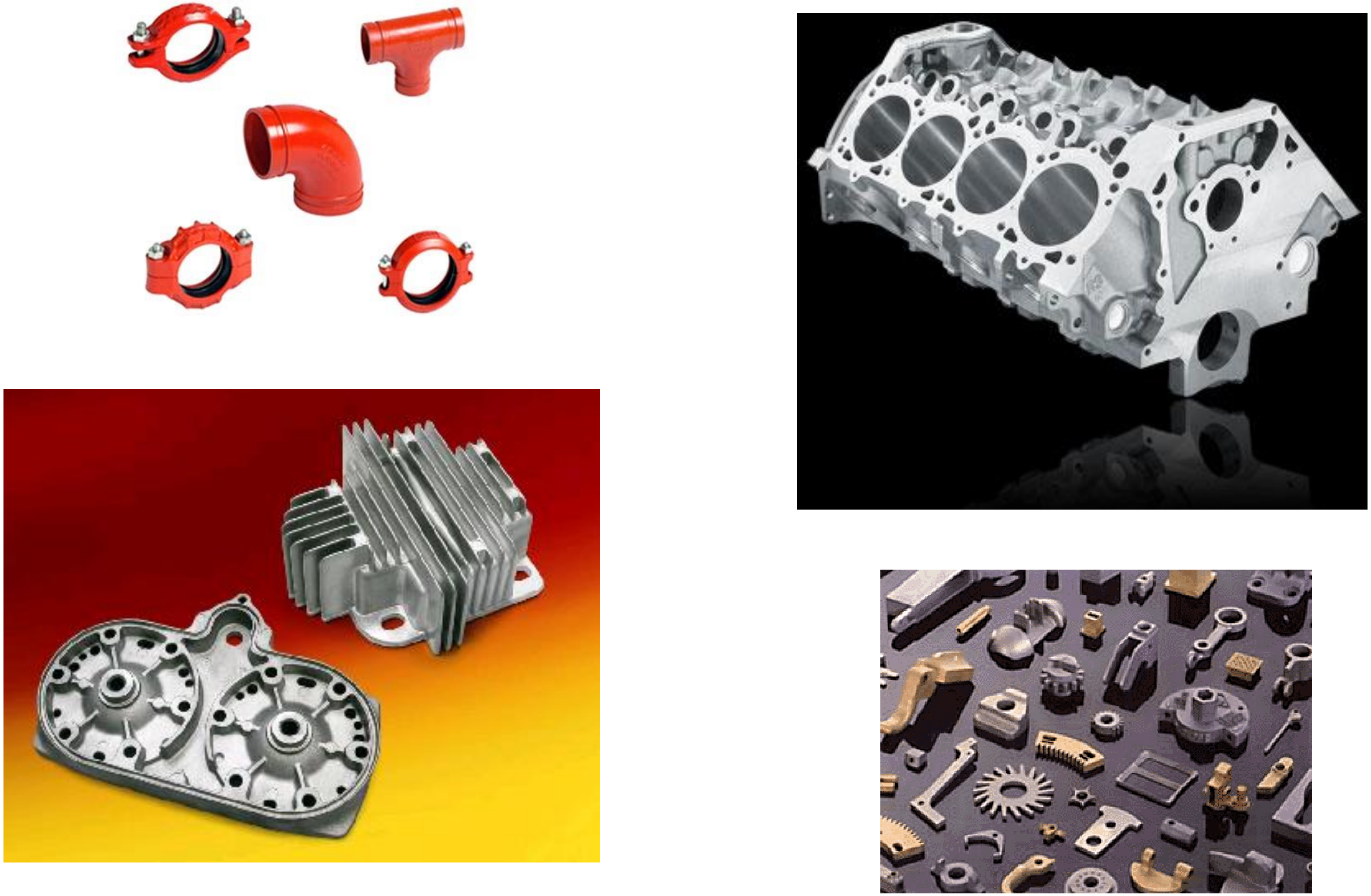


Bronze statue of Zeus from Artemision,
ca. 460 BC

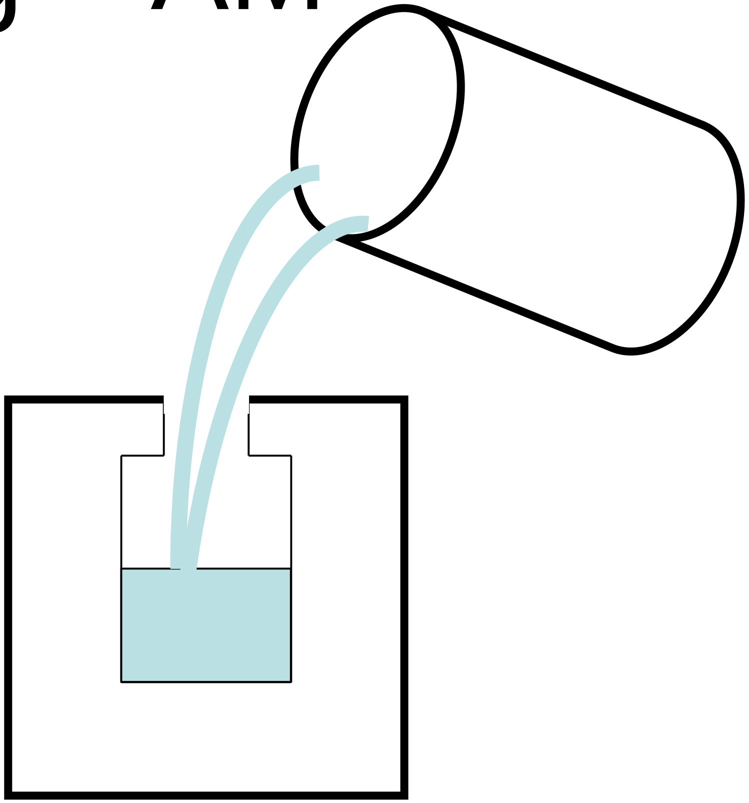
Ancient Greece; bronze statue casting circa 450BCE

Iron works in early Europe, e.g. cast iron cannons from England circa 1543

Cast Parts



Casting + AM



Readings;

1. *Kalpakjian, Chapters 10, 11, 12*
2. *Booothroyd, Ch 10, 21*
3. *Flemings "Heat Flow in Solidification"*
4. *Quinlan, …Hart JIE 2017*

Note: a good heat transfer reference can be found by
Profs John Lienhard online http://web.mit.edu/lienhard/www/ahtt.html

Casting Methods

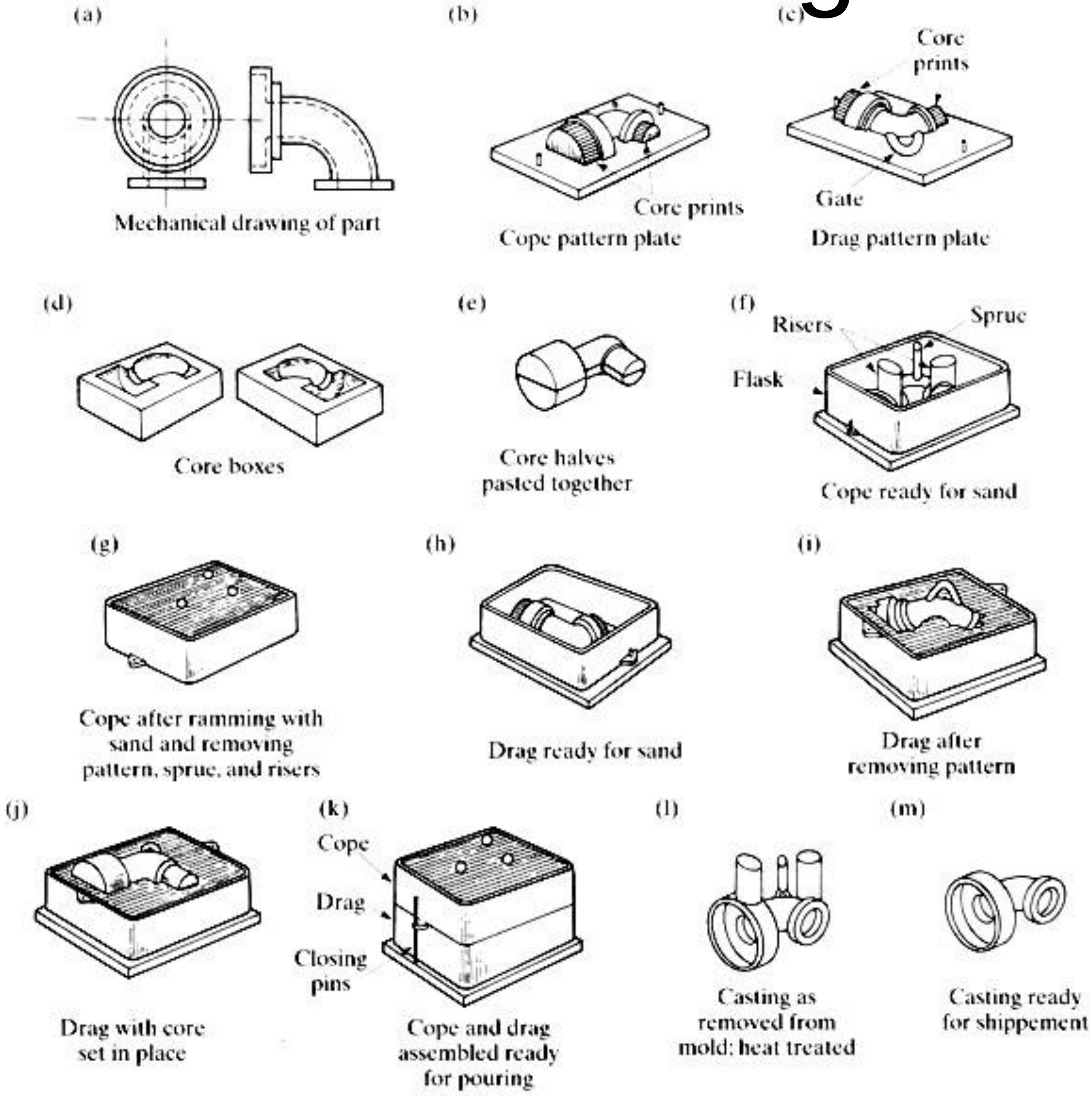
- Sand Casting

High Temperature Alloy,
Complex Geometry,
Rough Surface Finish

- Die Casting

High Temperature Alloy,
Moderate Geometry,
Smooth Surface

Sand Casting



(a) Mechanical drawing of part

(b) Cope pattern plate

(c) Drag pattern plate

(d) Core boxes

(e) Core halves pasted together

(f) Cope ready for sand

(g) Cope after ramming with sand and removing pattern, sprue, and risers

(h) Drag ready for sand

(i) Drag after removing pattern

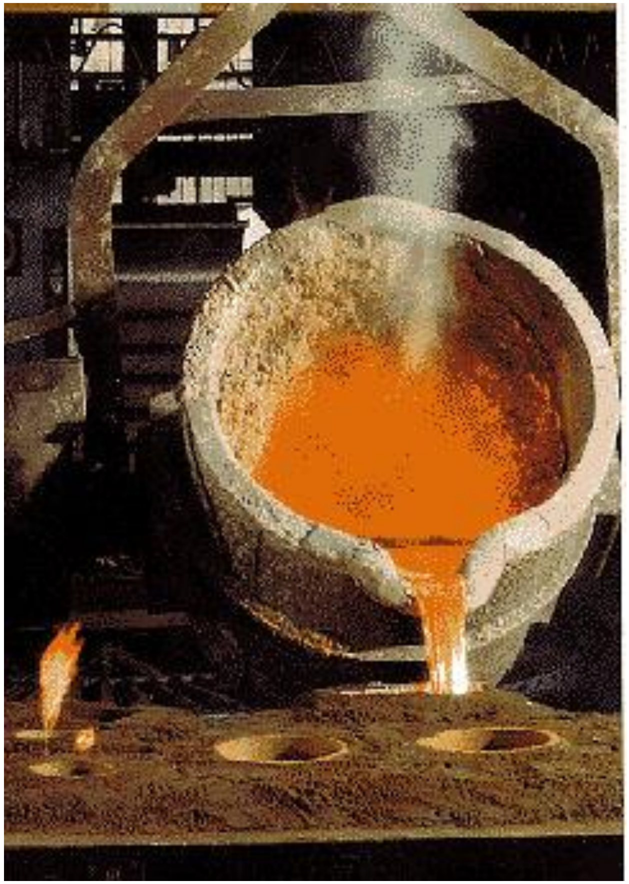
(j) Drag with core set in place

(k) Cope and drag assembled ready for pouring

(l) Casting as removed from mold; heat treated

(m) Casting ready for shippement

Sand Casting



Description: Tempered sand is packed into wood or metal pattern halves, removed form the pattern, and assembled with or without cores, and metal is poured into resultant cavities. Various core materials can be used. Molds are broken to remove castings. Specialized binders now in use can improve tolerances and surface finish.

Metals: Most castable metals.

Size Range: Limitation depends on foundry capabilities. Ounces to many tons.

Tolerances:
- Non-Ferrous ± 1/32″ to 6″
- Add ± .003″ to 3″, ± 3/64″ from 3″ to 6″.
- Across parting line add ± .020″ to ± .090″ depending on size.
- (Assumes metal patterns)

Surface Finish:
- Non-Ferrous: 150-350 RMS
- Ferrous: 300-700RMS

Minimum Draft Requirements:
- 1° to 5°
- Cores: 1° to 1 1/2°

Normal Minimum Section Thickness:
- Non-Ferrous: 1/8″ - 1/4″
- Ferrous: 1/4″ - 3/8″

Ordering Quantities: All quantities

Normal Lead Time:
- Samples: 2-10 weeks
- Production 2-4 weeks A.S.A.

Sand Casting Mold Features

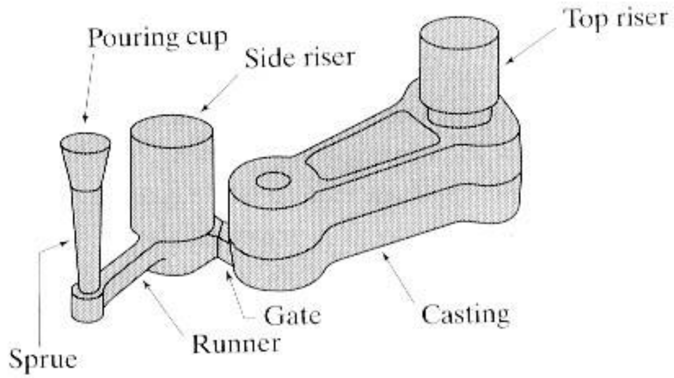


FIGURE 10.7 Schematic illustration of a typical riser-gated casting. Risers serve as reservoirs, supplying molten metal to the casting as it shrinks during solidification. See also Fig. 11.4. *Source*: American Foundrymen's Society.

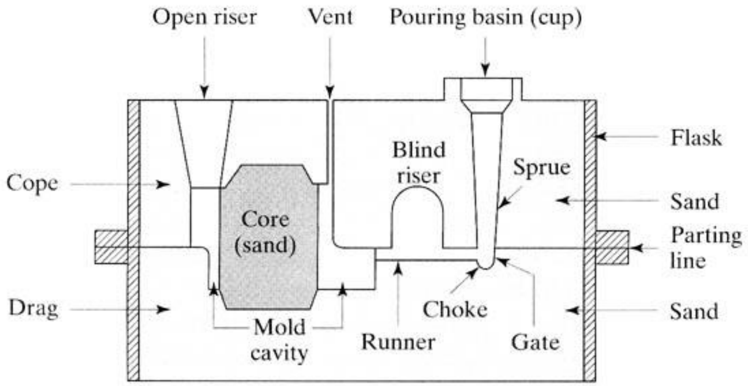


FIGURE 11.4 Schematic illustration of a sand mold, showing various features.

Vents, which are placed in molds to carry off gases produced when the molten metal comes into contact with the sand in the molds and core. They also exhaust air from the mold cavity as the molten metal flows into the mold.

Videos from Mass & Burlington Foundries

Production sand casting

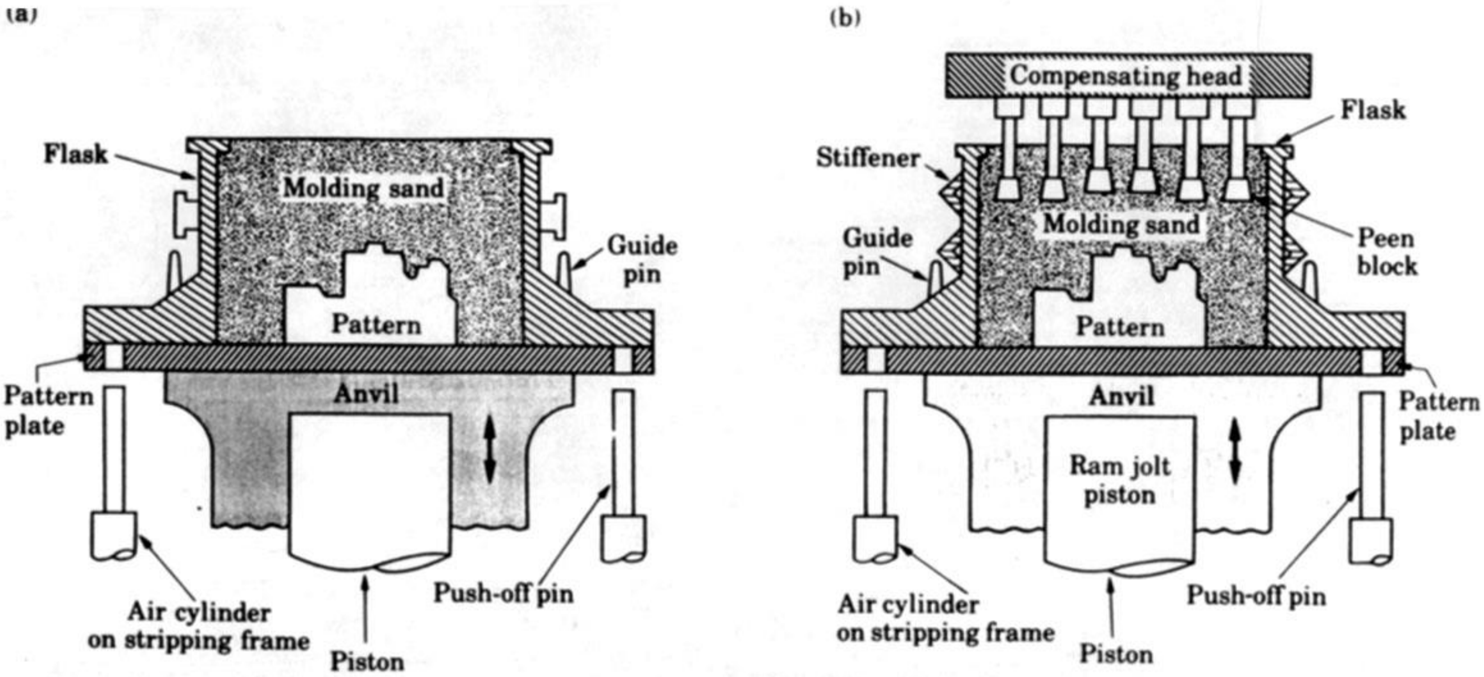


FIGURE 11.8

(a) Schematic illustration of a jolt-type mold-making machine. (b) Schematic illustration of a mold-making machine which combines jolting and squeezing.

Die Casting

Description: Molten metal is injected, under pressure, into hardened steel dies, often water cooled. Dies are opened, and castings are ejected.

Metals: Aluminum, Zinc, Magnesium, and limited Brass.

Size Range: Not normally over 2 feet square. Some foundries capable of larger sizes.

Tolerances:
- Al and Mg ± .002″/in.
- Zinc ± .0015″/in.
- Brass ± .001″/in.
- Add ± .001″ to ± .015″ across parting line depending on size

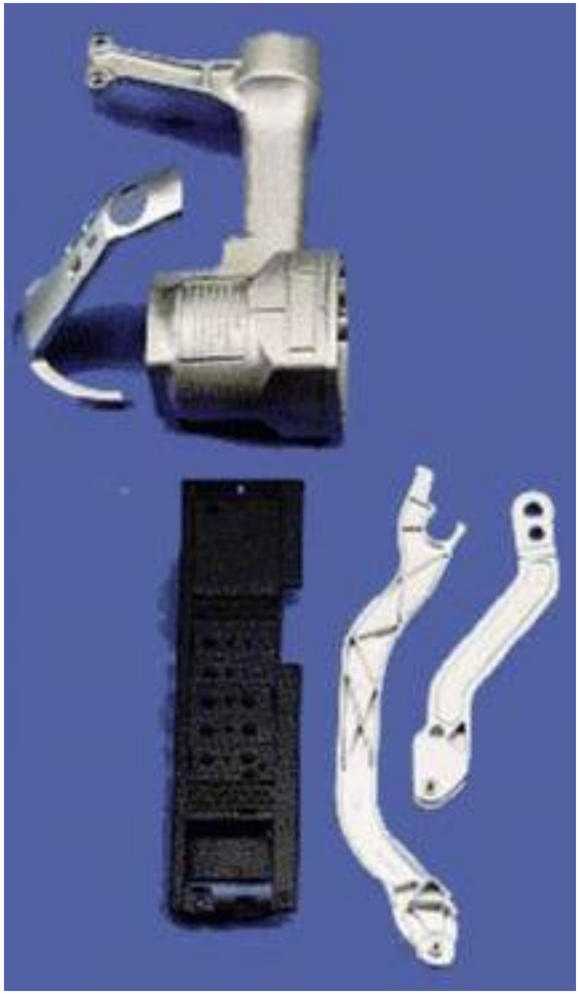
Surface Finish: 32-63RMS

Minimum Draft Requirements:
- Al & Mg: 1° to 3°
- Zinc: 1/2° to 2°
- Brass: 2° to 5°

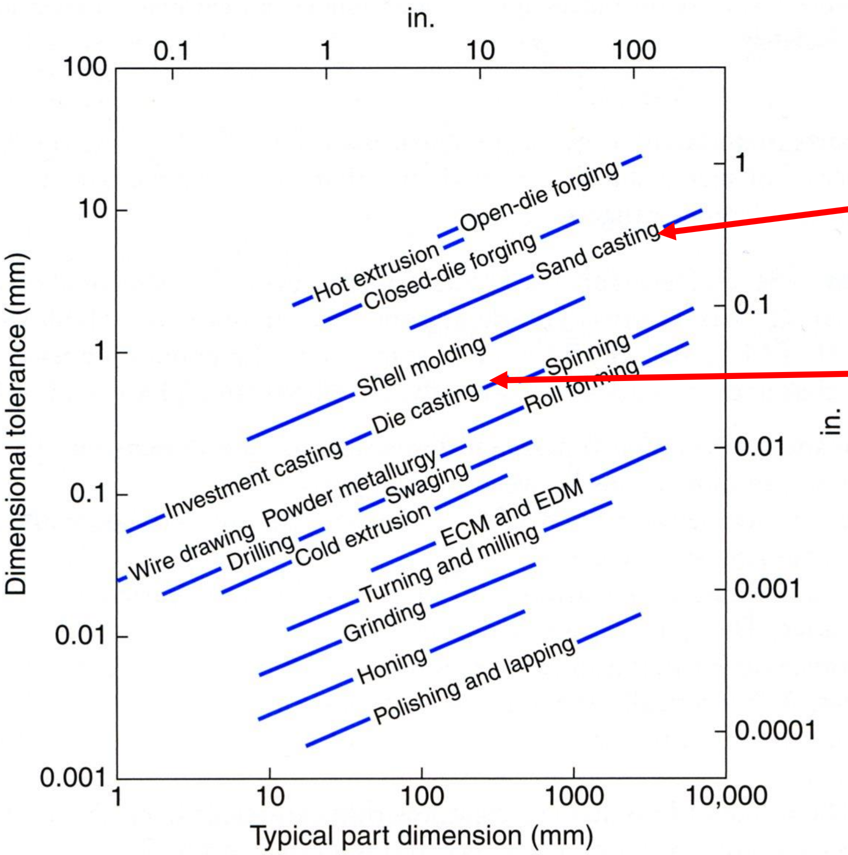
Normal Minimum Section Thickness:
- Al & Mg: .03″ Small Parts: .06″ Medium Parts
- Zinc: .03″ Small Parts: .045″ Medium Parts
- Brass: .025″ Small Parts: .040″ Medium Parts

Ordering Quantities:
- Usually 2,500 and up.

Normal Lead Time:
- Samples: 12-20 weeks
- Production: ASAP after approval.



Process variation/tolerance



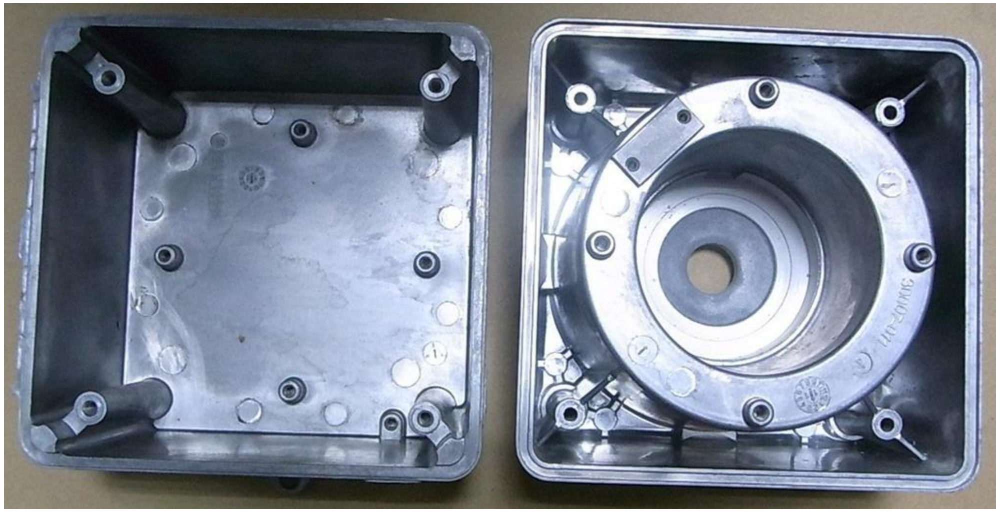
Sand casting

Die casting

FIGURE 35.20 Dimensional tolerances as a function of part size for various manufacturing processes; note that because many factors are involved, there is a broad range for tolerances.

Kalpakjian

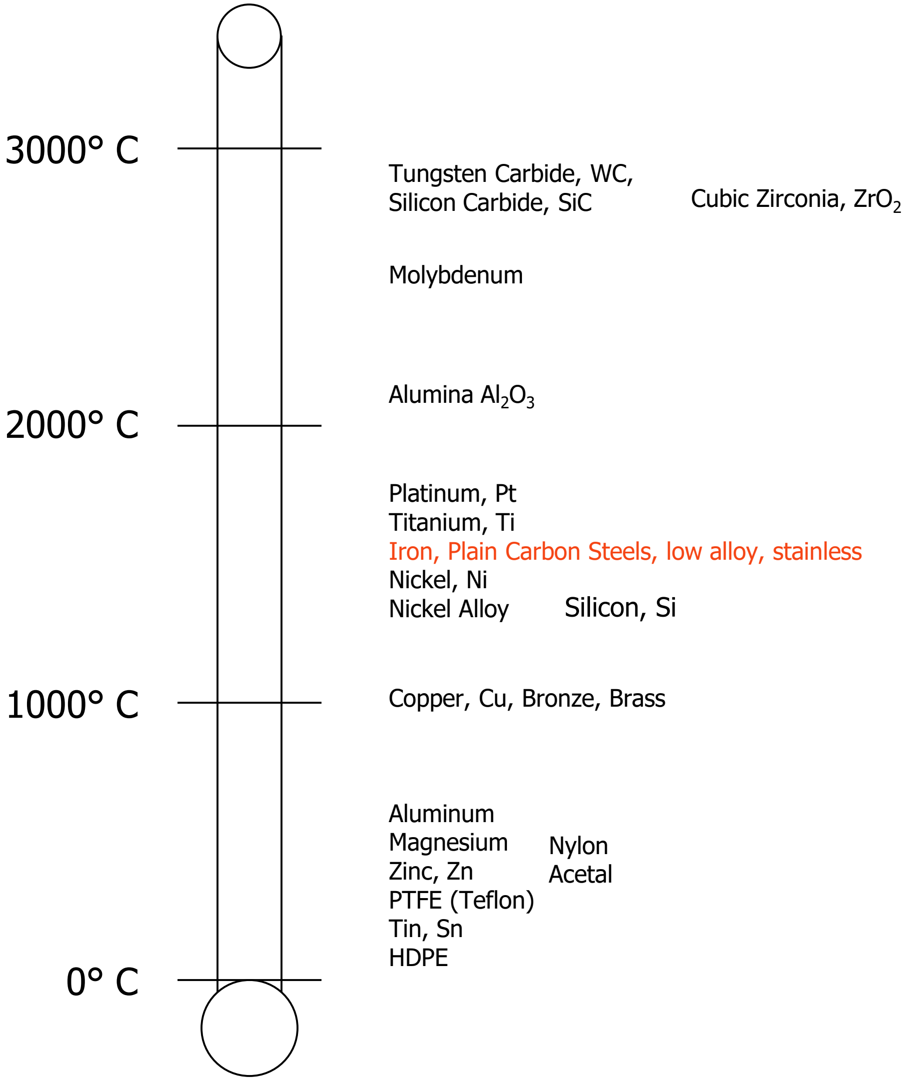
Die cast parts & runners



High Melt Temperature

- Reactivity
  - with air, mold mat’ls,
- Gas solubility
  - H_2 gas in Al
- Safety
  - Metal fires, e.g. Mg

$$Mg + 2H_2O \rightarrow Mg(OH)_2 + H_2$$



3000° C

Tungsten Carbide, WC,
Silicon Carbide, SiC

Cubic Zirconia, ZrO_2

Molybdenum

Alumina Al_2O_3

2000° C

Platinum, Pt
Titanium, Ti
Iron, Plain Carbon Steels, low alloy, stainless
Nickel, Ni
Nickel Alloy

Silicon, Si

1000° C

Copper, Cu, Bronze, Brass

Aluminum
Magnesium
Zinc, Zn
PTFE (Teflon)
Tin, Sn
HDPE

Nylon
Acetal

0° C

Phase Change & Shrinkage

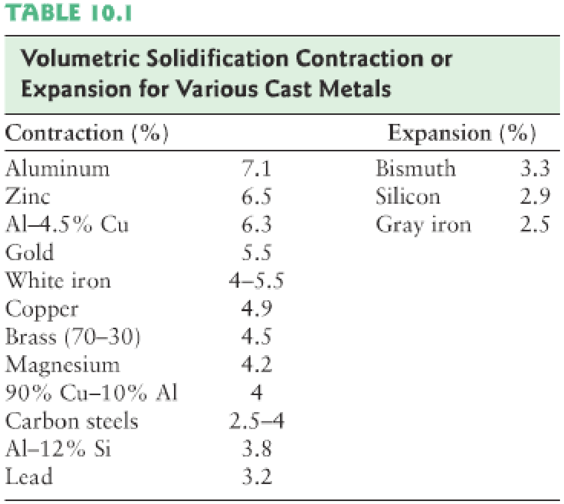
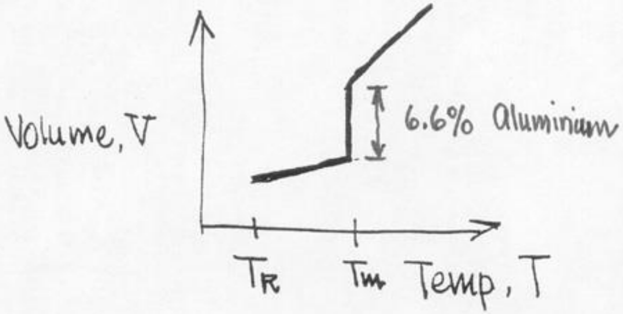
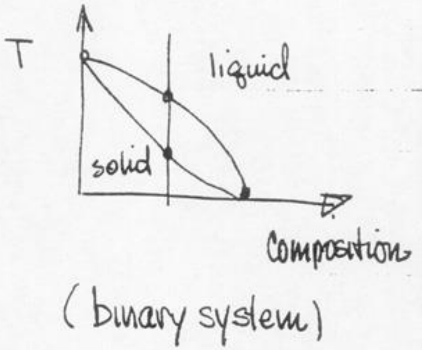


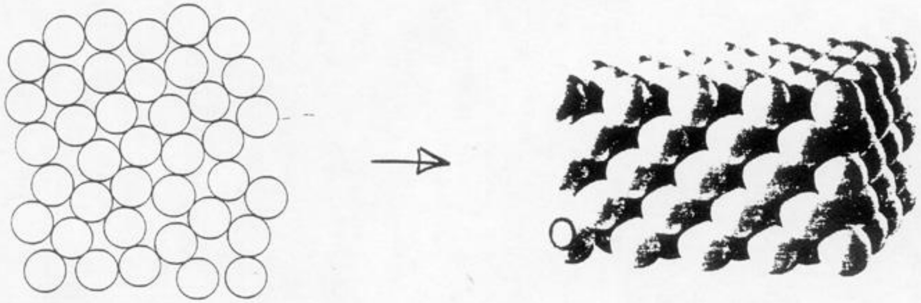
TABLE 10.1

Volumetric Solidification Contraction or Expansion for Various Cast Metals

Contraction (%)		Expansion (%)	
Aluminum	7.1	Bismuth	3.3
Zinc	6.5	Silicon	2.9
Al–4.5% Cu	6.3	Gray iron	2.5
Gold	5.5		
White iron	4–5.5		
Copper	4.9		
Brass (70–30)	4.5		
Magnesium	4.2		
90% Cu–10% Al	4		
Carbon steels	2.5–4		
Al–12% Si	3.8		
Lead	3.2		

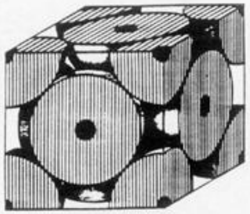


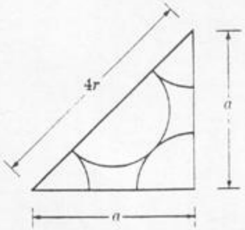




liquid metal

face-centered cubic metal





$$a_{fcc} = 4r/\sqrt{2}$$

$$a_{bcc} = 4r/\sqrt{3}$$

Solidification of a binary alloy

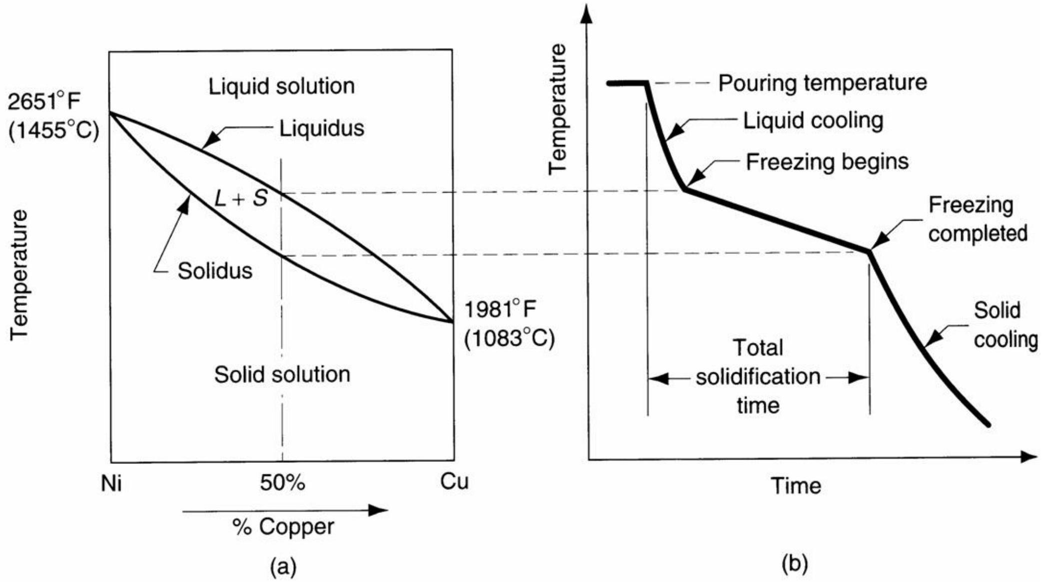


FIGURE 12.5 (a) Phase diagram for a copper–nickel alloy system and (b) associated cooling curve for a 50%Ni–50%Cu composition during casting.

Composition change during solidification

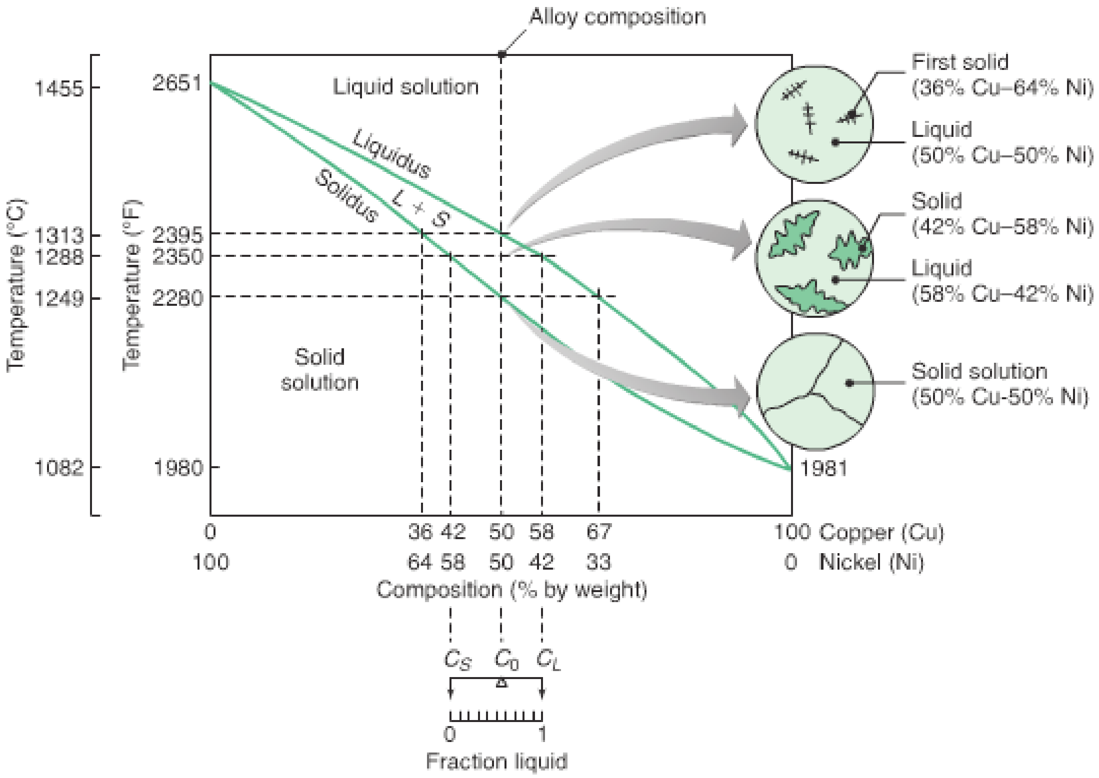


FIGURE 4.5 Phase diagram for nickel–copper alloy system obtained at a slow rate of solidification. Note that pure nickel and pure copper each have one freezing or melting temperature. The top circle on the right depicts the nucleation of crystals. The second circle shows the formation of dendrites (see Section 10.2). The bottom circle shows the solidified alloy, with grain boundaries.

Pb-Sn phase diagram

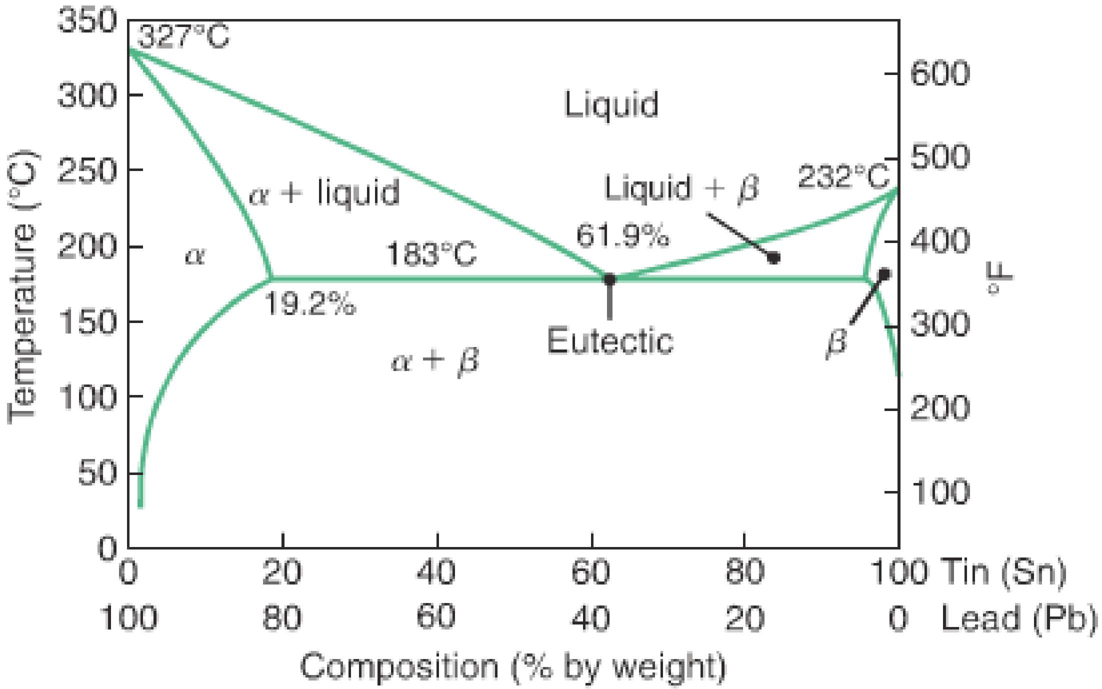


FIGURE 4.7 The lead–tin phase diagram. Note that the composition of the eutectic point for this alloy is 61.9% Sn–38.1% Pb. A composition either lower or higher than this ratio will have a higher liquidus temperature.

Solidification

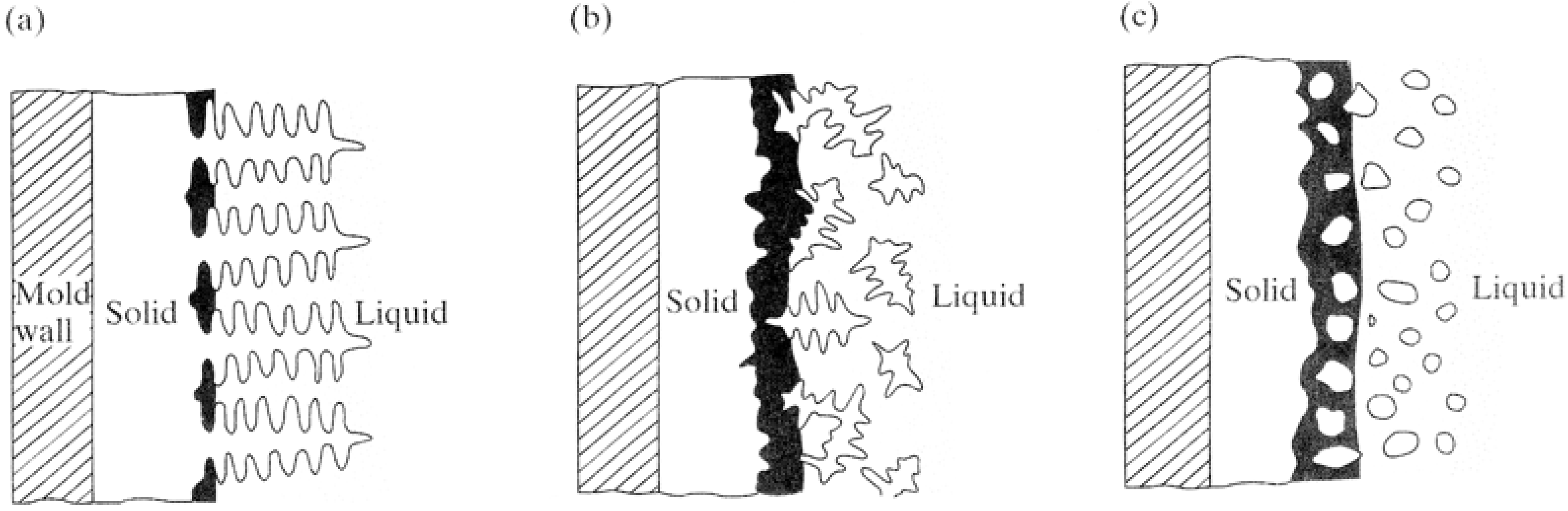
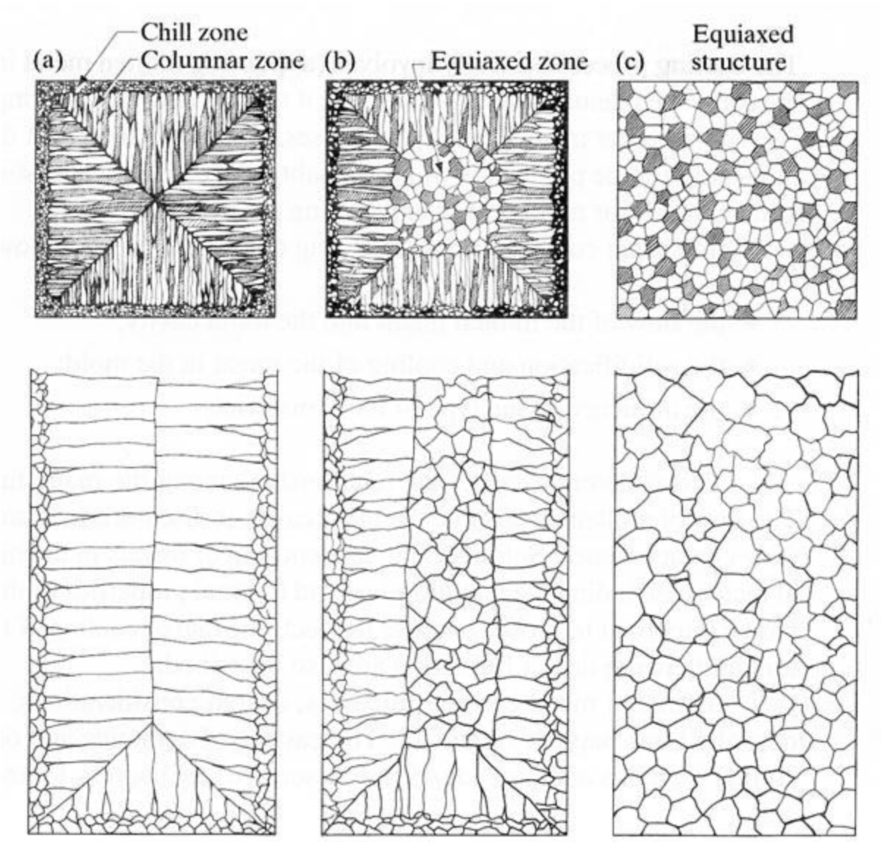


FIGURE 10.5 Schematic illustration of three basic types of cast structures:(a) columnar dendritic; (b) equiaxed dendritic; and (c) equiaxed nondendritic. *Source*: D. Apelian.

http://www.its.caltech.edu/~atomic/snowcrystals/

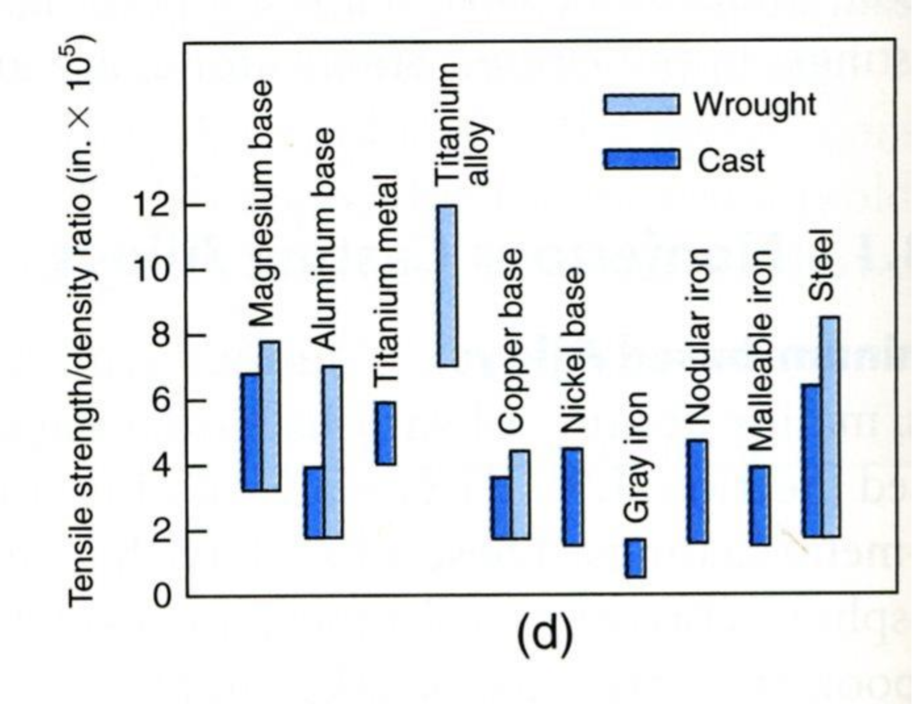
Dendrite growth in metals- lower surface energy crystallographic planes are favored, producing tree like structures if not disturbed. See next slide for explanation of a, b & c

Cast structures



Schematic illustration of three cast structures solidified in a square mold: (a) pure metals; (b) solid solution alloys; and c) structure obtained by using nucleating agents. *Source*: G. W. Form, J. F. Wallace, and A. Cibula

Properties of castings



e.g. Compare elongation of carbon steels (4-36%)Table 5.3, with cast irons (0-18%) Table 12.3 (Kalpakjian & Schmid 7th)

How long does it take to solidify?

Thickness ~ 30 cm

Thickness ~ 0.5 cm

Sand Casting – Solidification Time

$$t_s \approx \left(\frac{V}{A} \right)^2$$

Chvorinov’s Rule
p.247 your text

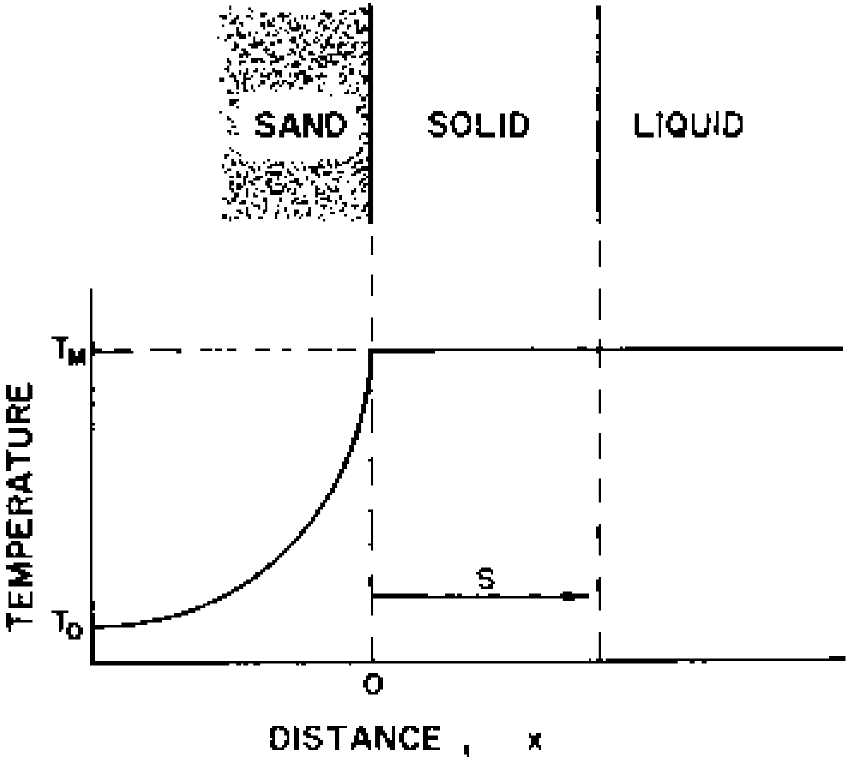


FIGURE 1-6
Approximate temperature profile in solidification of a pure metal poured at its melting point against a flat, smooth mold wall.

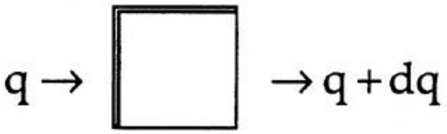
Ref: Mert Flemings “Solidification Processing”

Thermal Conductivity “k” (W/m·K)

$$q = -k\frac{dT}{dx}$$

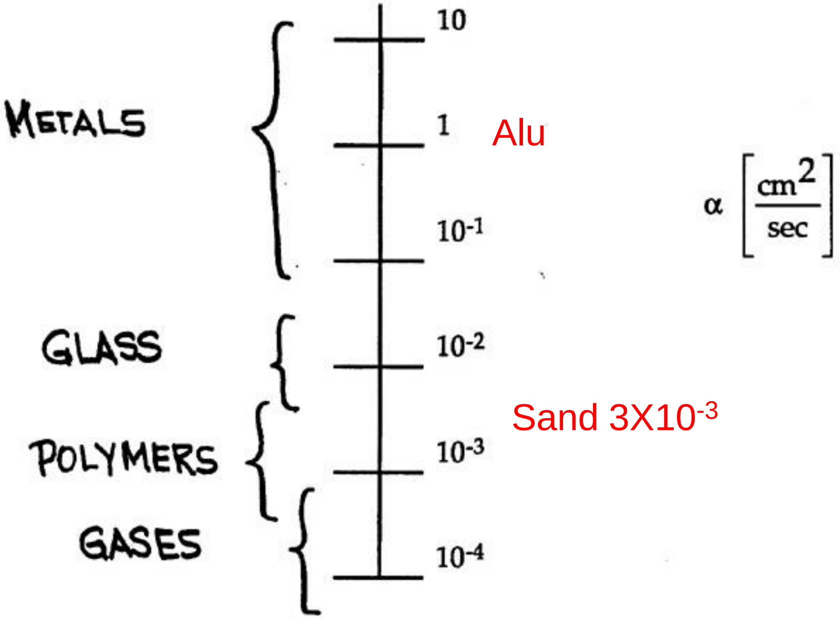
Copper	394
Aluminum	222
Iron	29
Sand	0.61
PMMA	0.20
PVC	0.16

Transient Heat Transfer



$$\boxed{\rho C_p \frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial x^2}}$$

$k \neq k(x)$



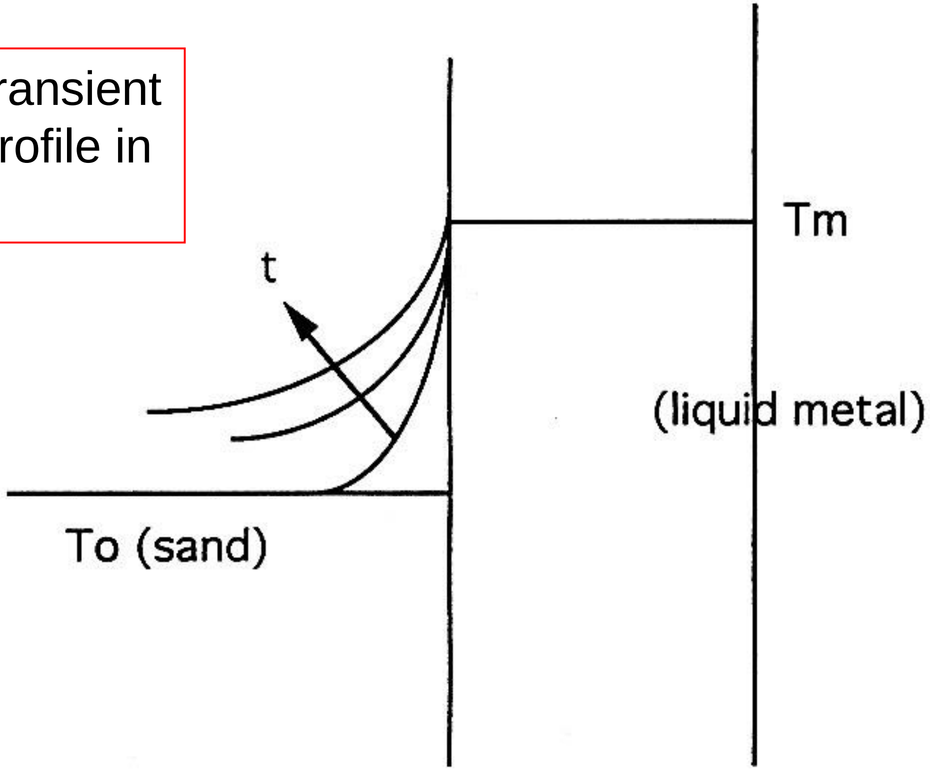
$$\alpha = \frac{k}{\rho C_p}$$

Sand Casting (see Flemings)

Define new variable $\zeta = x / \sqrt{\alpha t}$

Use $\theta = \dfrac{T - T_M}{T_o - T_M}$

We seek the transient temperature profile in the sand.



Sand Casting (see Flemings)

Ordinary differential eq'u

$$\frac{d^2\theta}{d\zeta^2} = -\frac{\zeta}{2}\frac{d\theta}{d\zeta}$$

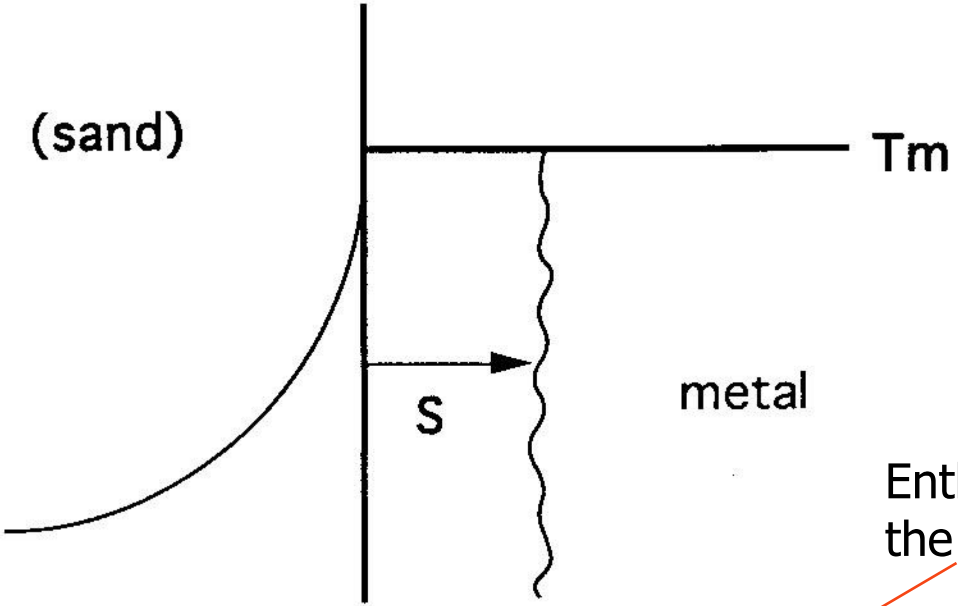
i.c. $\theta = 1$ at $\zeta = \infty$ At t=0, $T=T_o$ everywhere

b.c. $\theta = 0$ at $\zeta = 0$ At x=0, $T=T_m$ always

$$\boxed{\theta = \mathrm{erf}\left(-\frac{\zeta}{2}\right)}$$

This will allow us to calculate the heat lost by the metal at the boundary with the sand tooling

Solidification Time



Heat required to solidify to distance "s"

$$= A \cdot s \cdot \rho \cdot H$$

Enthapy of
the phase change/wt

Rate eq'n (per unit area)

$$\rho H \frac{ds}{dt} = -\dot{q} = k\left(\frac{\partial T}{\partial x}\right)_{x=0}$$

Use Flemings
result here

Solidification Time (cont.)

this leads to

$$s = \frac{2}{\sqrt{\pi}}\left(\frac{T_M - T_o}{\rho_M H_M}\right)\sqrt{K_s \rho_s C_{p_s} t}$$

let $s = \frac{V}{A}$ $\boxed{t = C\left(\frac{V}{A}\right)^2}$ Chvorinov's rule

The constant “C” (in this case not heat capacity) is determined by experiment.

Several references suggest that values range:

C ~ 2 to 4 min/cm^2 (with most data for iron and steel)

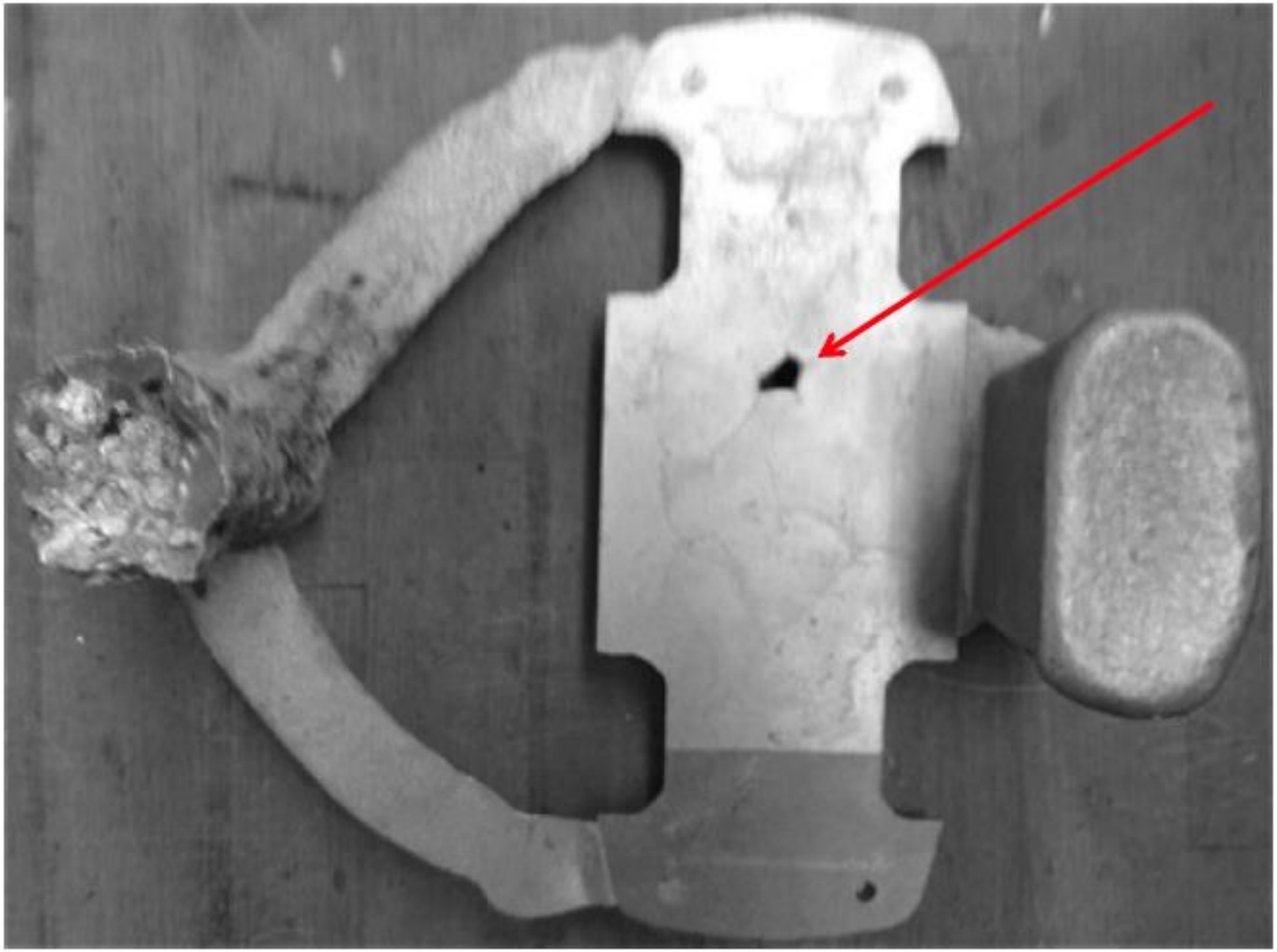
How long does it take to solidify?

Order of magnitude estimate using half thickness, & C = 3.3 min/cm^2

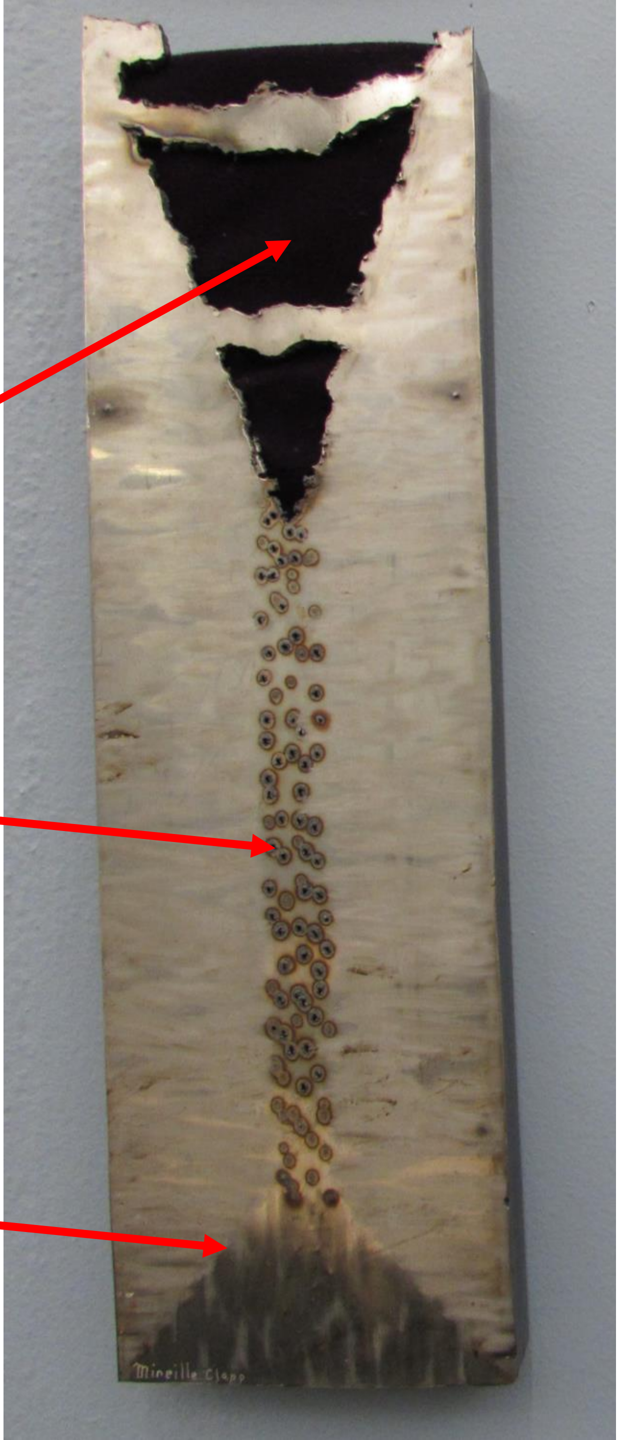
Thickness ~ 30 cm
Solidification time = 3.3 $(30/2)^2$ [min] ~ 12 hrs

Thickness ~ 0.5 cm
t = 3.3 $(0.5/2)^2$ [min] ~ 12 sec

What happened?



Can you explain these Solidification features?



Picture taken from the Chipman Room

Heat Transfer – Die Casting

$$t_s \approx \left(\frac{V}{A}\right)^1$$

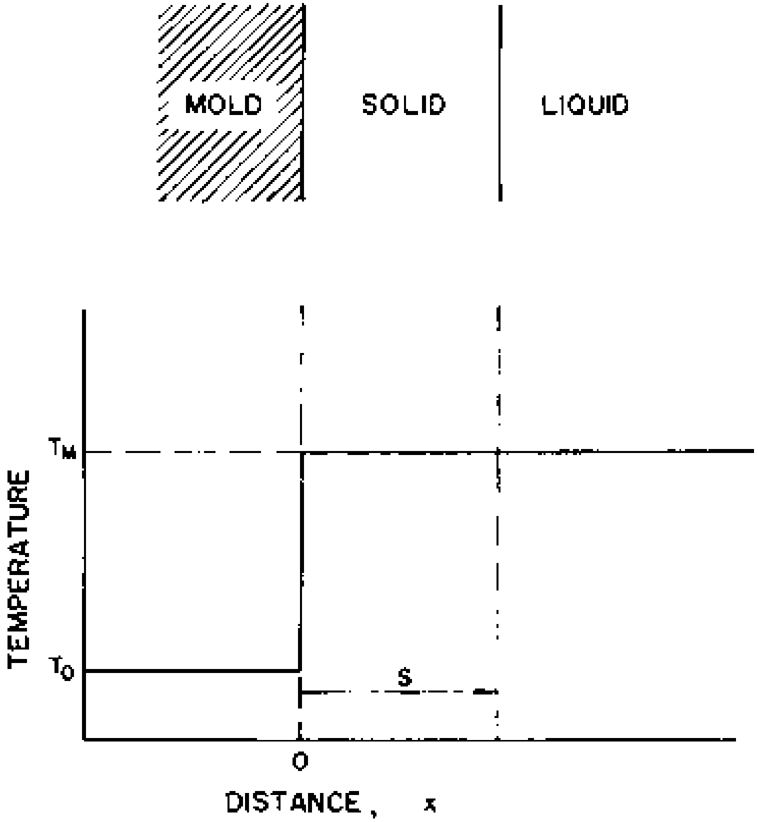
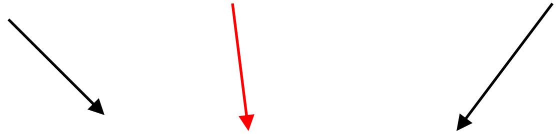


FIGURE 1-9
Temperature profile during solidification against a large flat mold wall with mold-metal interface resistance controlling.

Film Coefficients "h" W/m^2·K

$$Q = -hA(T_{high} - T_{low})$$

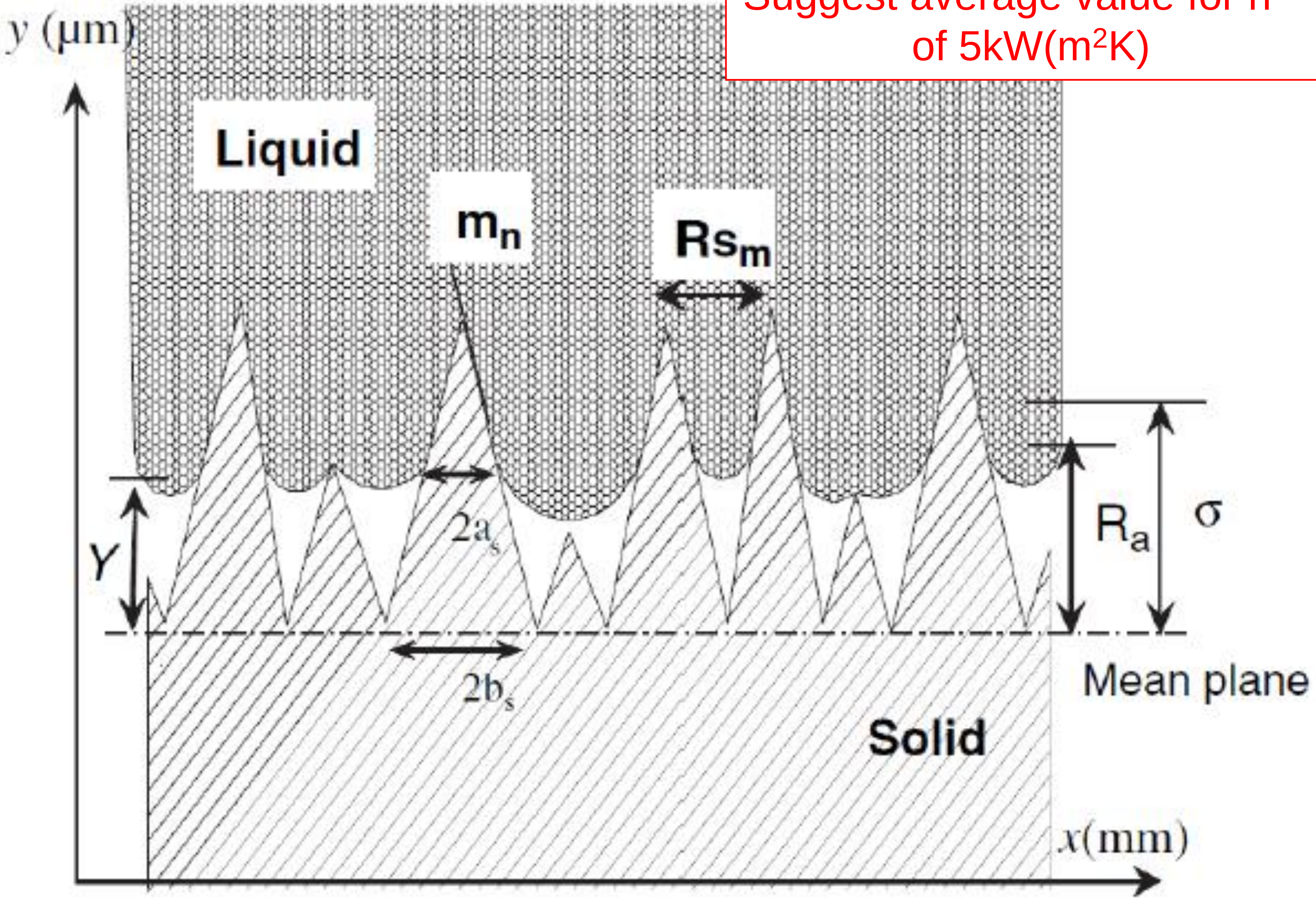
Carbon coating high pressure low pressure polished die



Typical die casting	1,000 - 10,000
Natural convection	1 - 10
Flowing air	10 - 50

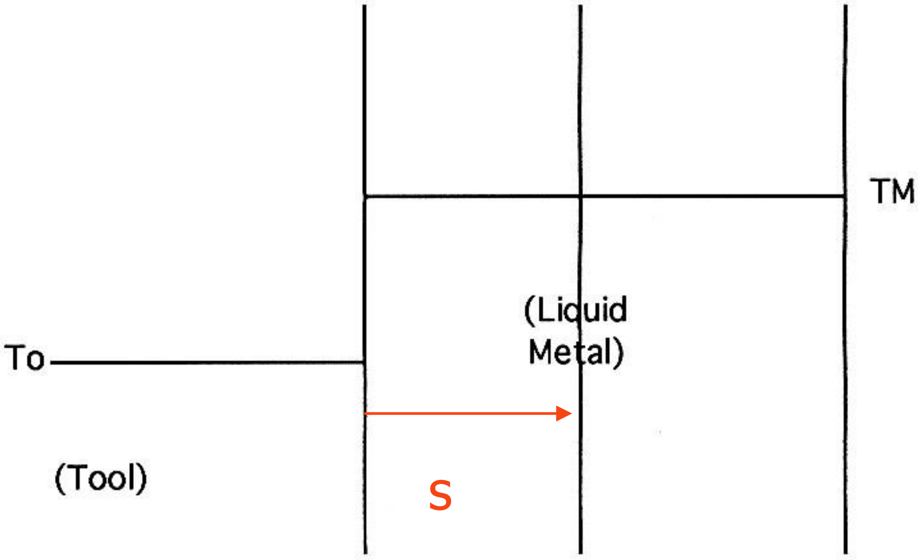
Die casting contact resistance

see Boothroyd Ch 10, p446-447
Suggest average value for h
of 5kW(m^{2}K)



A. Hamasaiid, G. Dour, T. Loulou c, M.S. Dargusch; A predictive model for the evolution of the thermal conductance at the casting–die interfaces in high pressure die casting, International Journal of Thermal Sciences 49 (2010) 365–372

Die Casting Solidification Time



Time to form solid part

$$\dot{q} = -\bar{h}A(T_M - T_o) = \rho_M H_M A \frac{ds}{dt}$$

$$\boxed{t = \frac{\rho_M H_M}{\bar{h}(T_M - T_o)} \frac{V}{A}}$$

Also need to cool casting to below T_M

to eject $\rightarrow T_{eject}$

and will inject at $T_{inject} > T_M$.

Time to cool part from melt temp to the ejection temperature.

$$mC\frac{dT}{dt} = -Ah\left(T - T_o\right)$$

Let, $\theta = T - T_o$

$$\frac{1}{\theta}\frac{d\theta}{dt} = -\frac{Ah}{mC}$$

Integration yields…

$$t = \frac{-mC}{Ah}\ln\frac{\Delta\theta_f}{\Delta\theta_i}$$

For two sided cooling this gives

$$\boxed{t = \frac{\rho \cdot C \cdot w}{2h} ln \frac{(T_M - T_W)}{(T_{eject} - T_W)}}$$

Note C= heat capacity, h = film coefficient, w = part width, T_M = melt temp, T_w =wall temp

Time to solidify + cool using superheat temperature T_{sp}

For thin sheets of thickness "w", including phase change

$\Delta\theta_i = T_i + \Delta T_{sp} - T_{mold}$

$\Delta T_{sp} = H/C$

$\Delta\theta_f = T_{eject} - T_{mold}$

$$t = \frac{w\rho C}{2h} \ln\left(\frac{T_{inject+\Delta T_{sp}} - T_{mold}}{T_{eject} - T_{mold}} \right)$$

Same as eq'n 10.29 Boothroyd

"sp" means superheat
C is heat capacity
H is latent heat of fusion
h is film transfer coef

Approximations,
$t \approx 0.42$ sec/mm x w_{max} (Zn)
$t \approx 0.47$ sec/mm x w_{max} (Al)
$t \approx 0.63$ sec/mm x w_{max} (Cu)
$t \approx 0.31$ sec/mm x w_{max} (Mg)

Ref Boothroyd, Dewhurst, Knight p 447

Pattern and Tooling Design

- For pattern and tooling design issues see Boothroyd, Dewhurst, & Knight:
- Ch 10 for die casting
- Ch 12 for sand casting, and the following

  5 slides

Pattern Design suggestions

Figure 7.2.24 Identifying hot spots in castings by using outward projecting arrows of length half the casting thickness. Where arrows overlap, hot spots may develop. (Courtesy of Meehanite Metal Corp.)

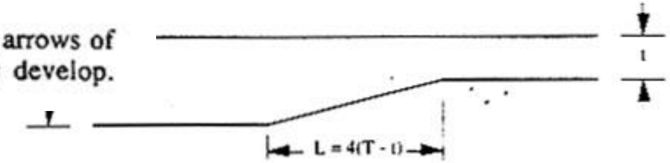
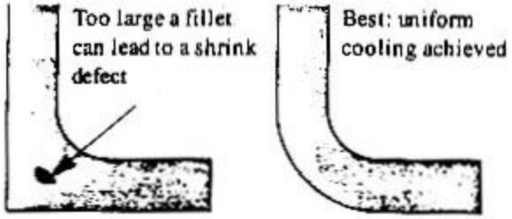


Figure 7.2.28 Avoid abrupt section changes. (Courtesy of Meehanite Metal Corp.)



s. (Courtesy of Meehanite Metal Corp.)

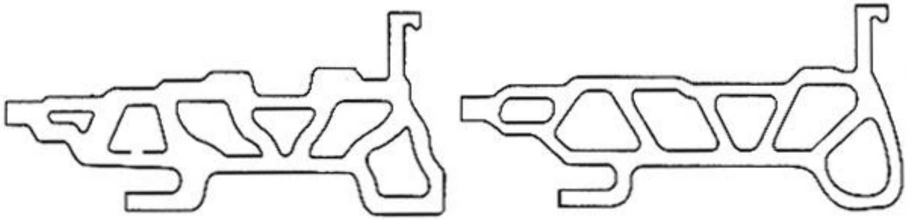


Figure 7.2.29 Design for uniform thickness in sections. (Courtesy of Meehanite Metal Corp.)

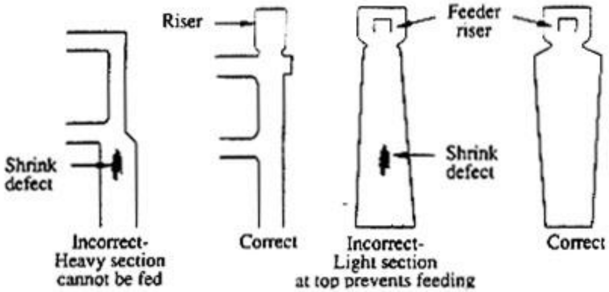
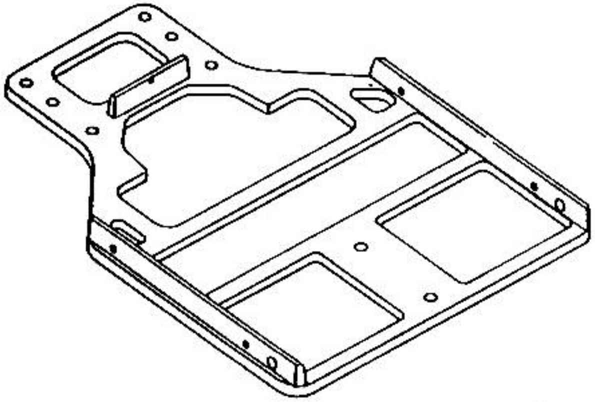


Figure 7.2.30 More intersection details. (Courtesy of Meehanite Metal Corp.)

Pattern Design Issues (Alum)

- Shrinkage Allowance: 1.3%
- Machining Allowance: 1/16” = 1.6 mm
- Minimum thickness: 3/16” = 5 mm
- Parting Line: even
- Draft Angle: 3 to 5%
- Thickness: even



Pattern Design

Table 12.1

Normal Shrinkage Allowance for Some Metals Cast in Sand Molds

Metal	Percent
Gray cast iron	0.83 – 1.3
White cast iron	2.1
Malleable cast iron	0.78 – 1.0
Aluminum alloys	1.3
Magnesium alloys	1.3
Yellow brass	1.3 – 1.6
Phosphor bronze	1.0 – 1.6
Aluminum bronze	2.1
High-manganese steel	2.6

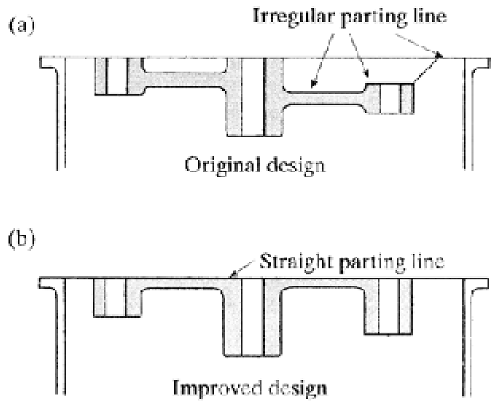
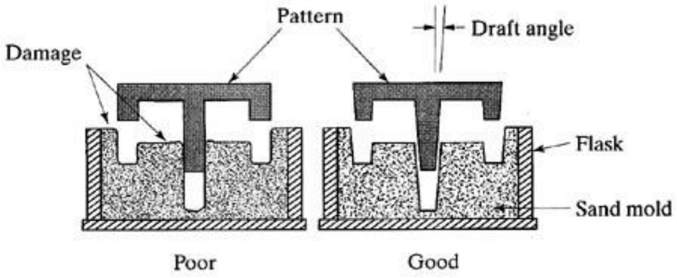
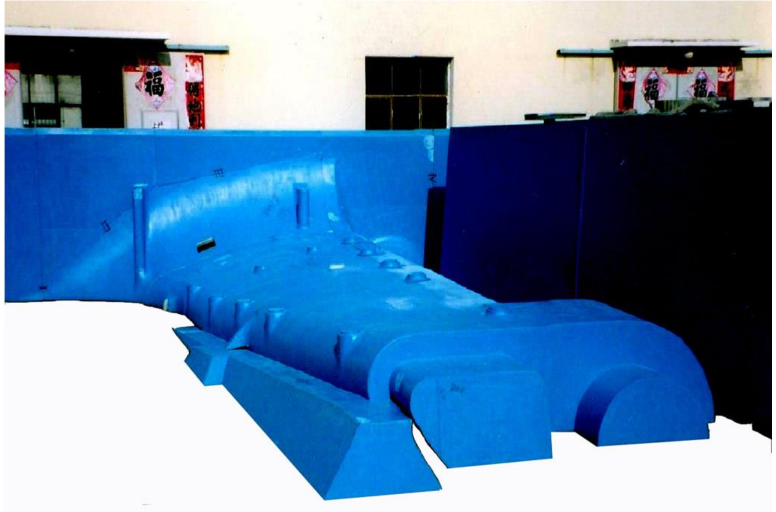


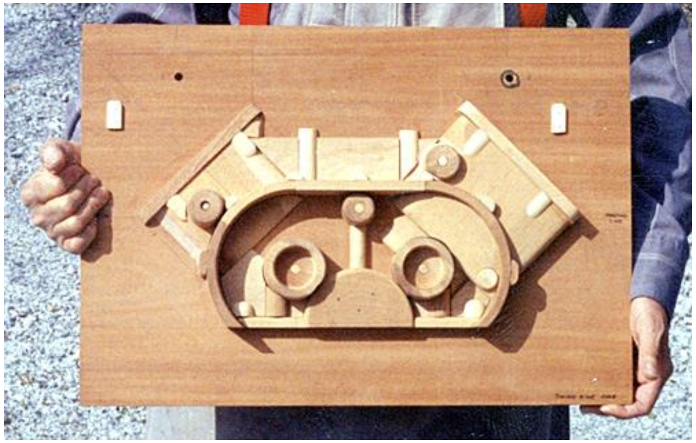
FIGURE 12.5 Redesign of a casting by making the parting line straight to avoid defects. *Source: Steel Casting Handbook*, 5th ed. Steel Founders' Society of America, 1980. Used with permission.

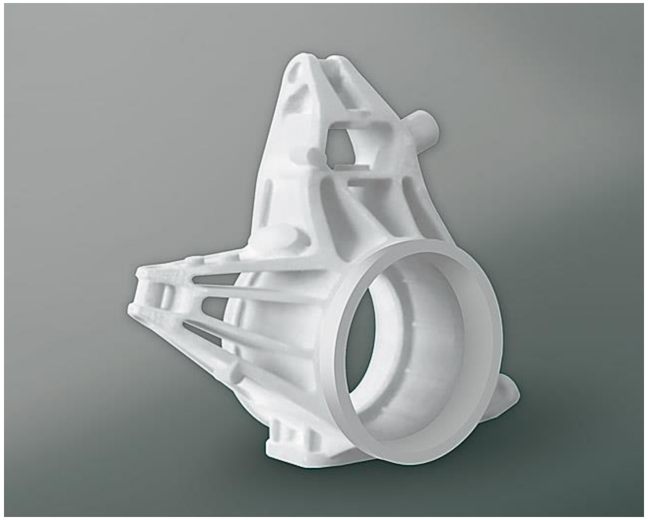
FIGURE 11.7 Taper on patterns for ease of removal from the sand mold.



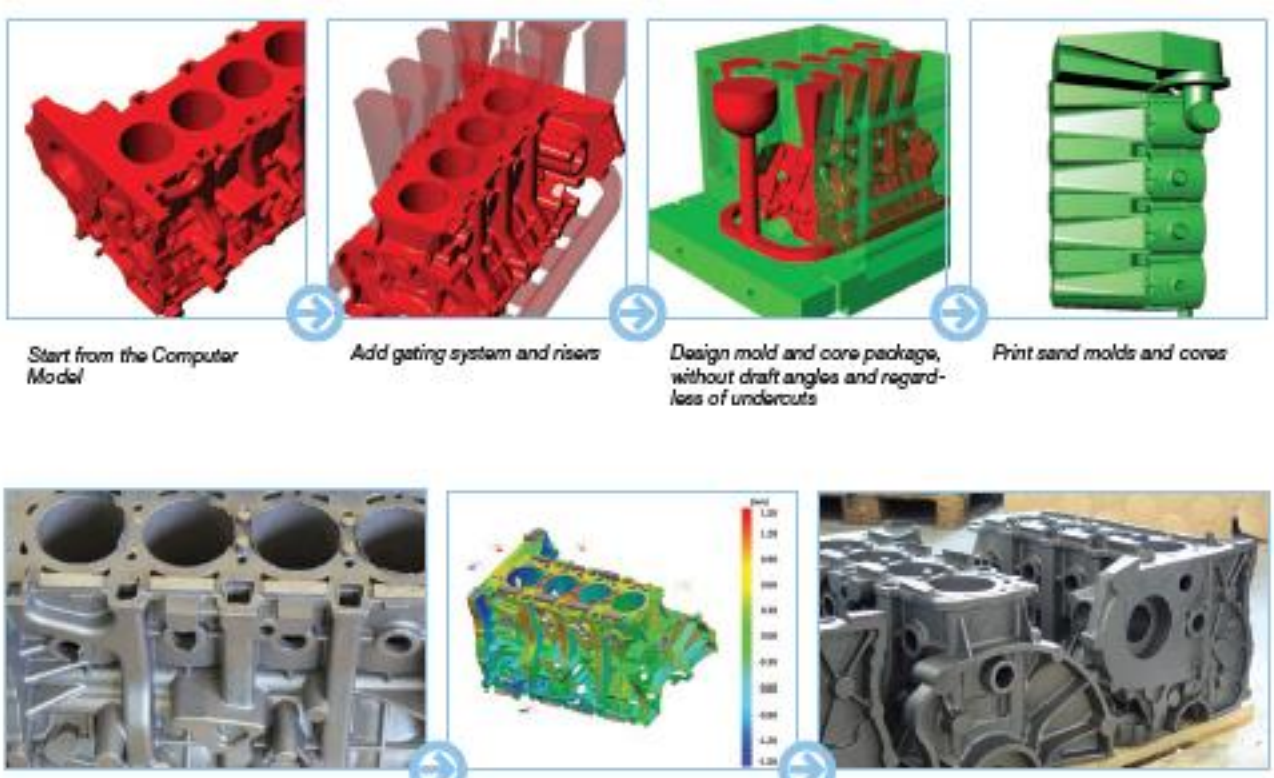
Pattern materials







Digital Sand Casting:
Print molds or parts?



Start from the Computer Model

Add gating system and risers

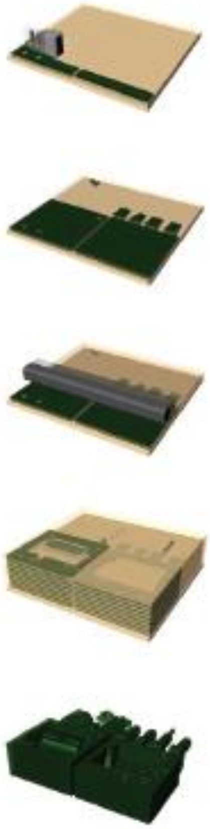
Design mold and core package, without draft angles and regardless of undercuts

Print sand molds and cores

Casting is poured & ready

Casting is inspected

Parts are ready to be dispatched



1. Print
Selectively dispense binder using inkjet printing technology

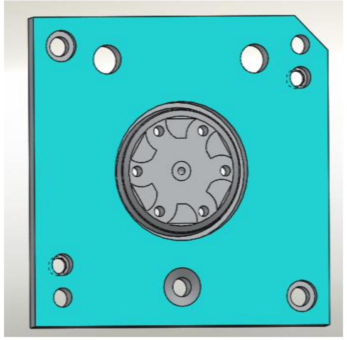
2. New layer
The build platform is lowered by a set increment.

3. Spread
Spreads a new layer of pre-mixed molding sand.

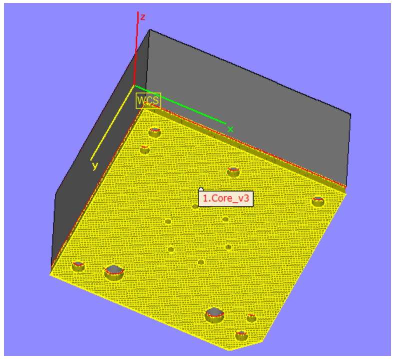
4. Repeat
Repeat Steps 1-3, until the molds or cores are built.

5. Finishing
Unbound sand is removed. Metal parts can be cast.

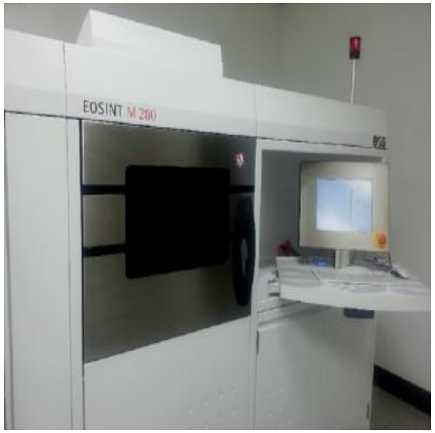
AM (Laser Metal Powder Bed) Steps to produce tooling



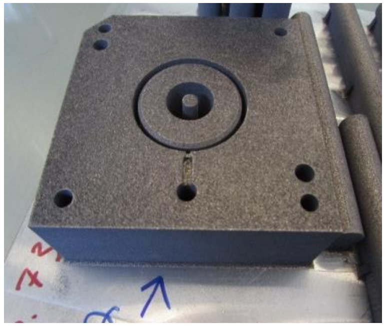
CAD file



Support structure generation

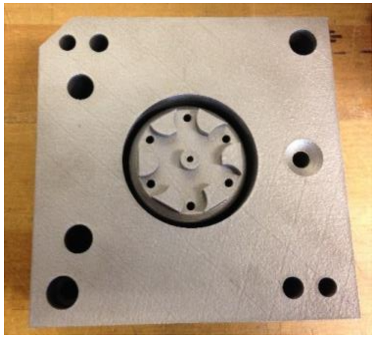


Printing: EOS M280



Printed part on plate, stress relieve

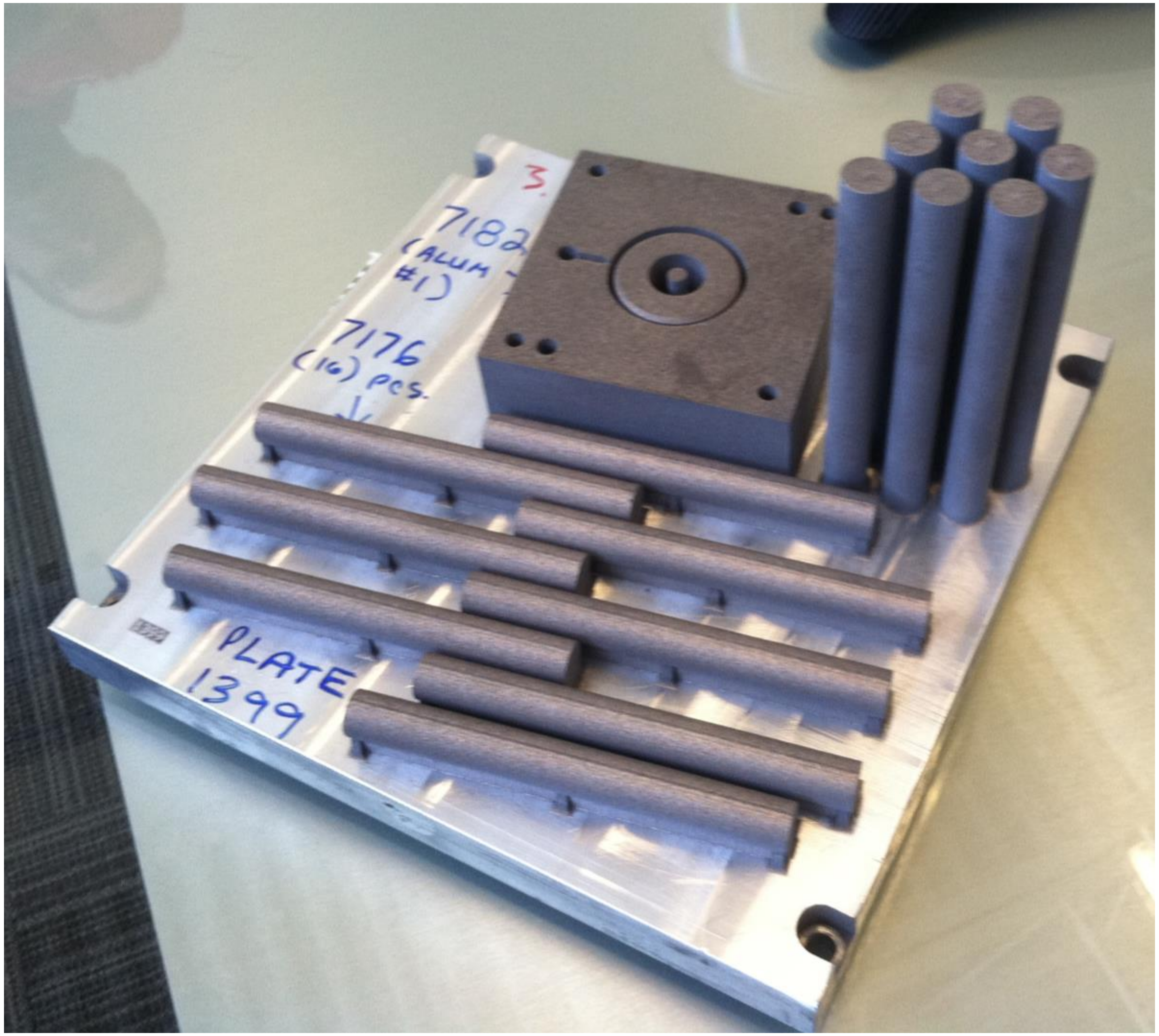
Sawing (or wire EDM) and hand tool removal of support structure



3D printed part

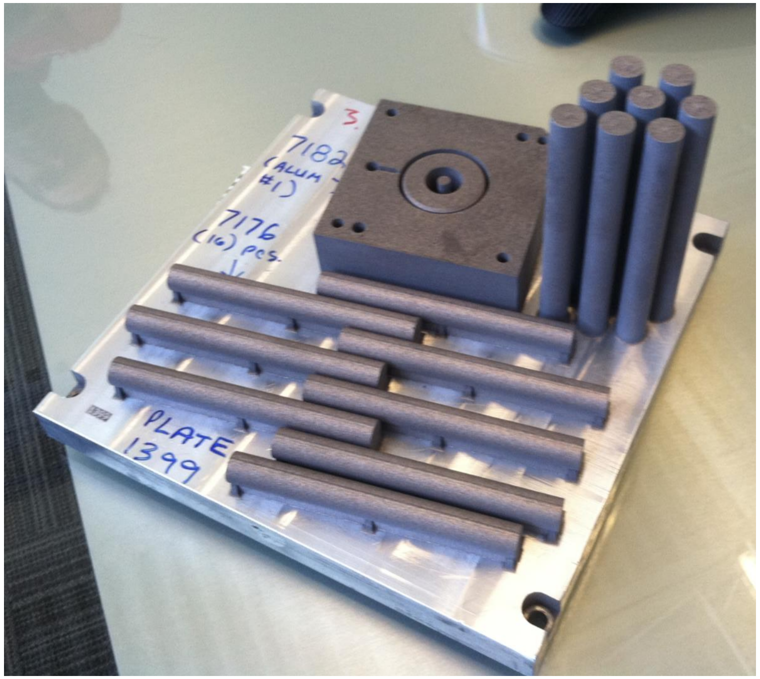
Printed steel & aluminum tools

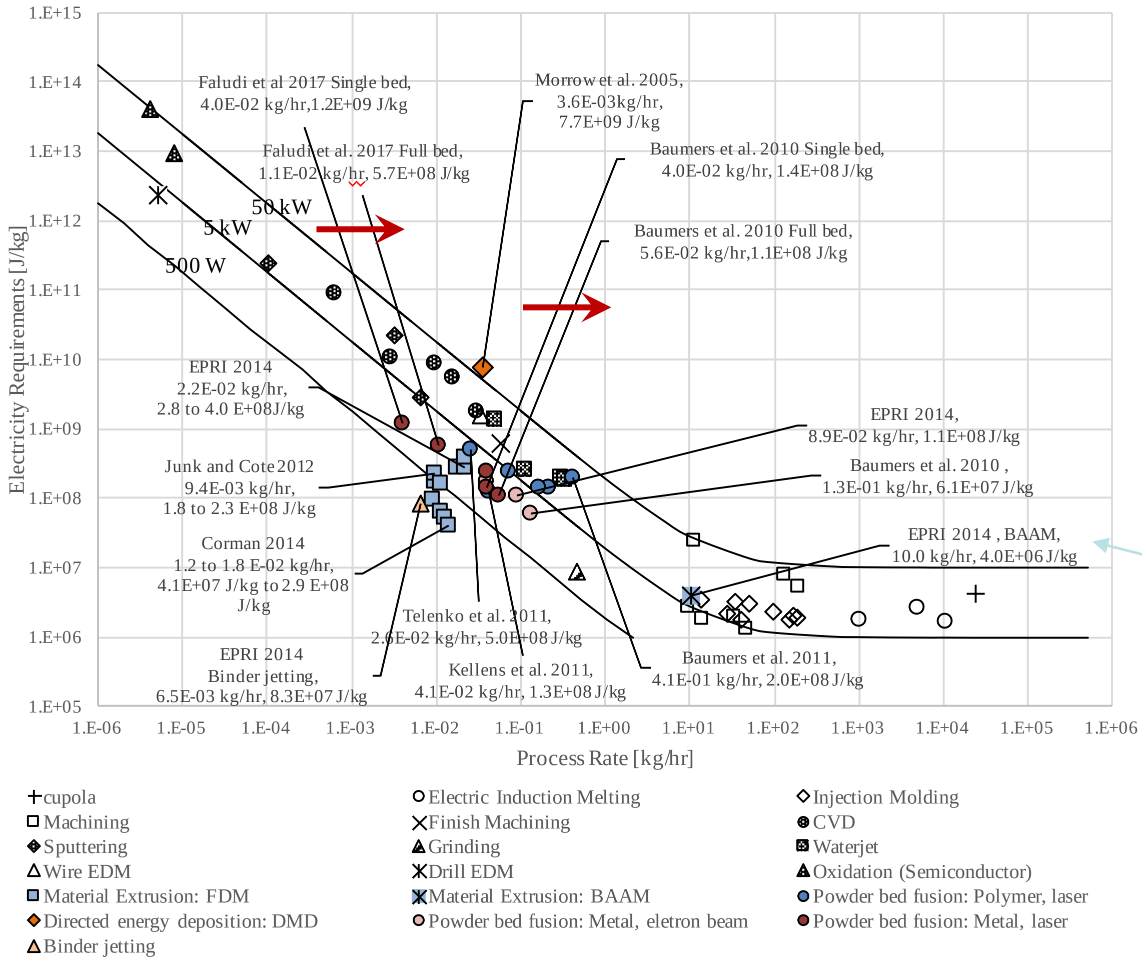
Actual Build



Laser Metal Powder Issues

1. Tolerances
2. Quality
3. Surface Finish
4. Production Rate





Measured 3D Printing energy intensity (J/kg)
Vs Print rate (kg/hr) for AM
(metals in red,
plastics in blue)
overlaid on conventional
manufacturing processes.

BAAM technology
More later…

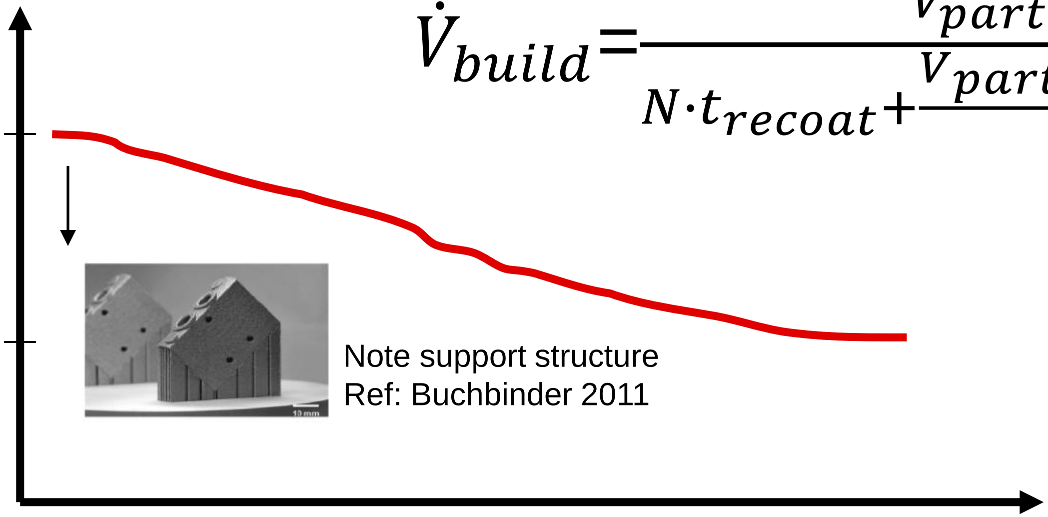
(Gutowski et al 2017 JIE)

Strategies for improved rate

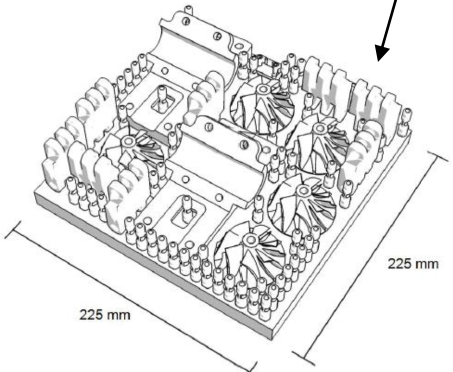
- Powder delivery- full bed, orientation
- Larger laser (tuned)
- Heated chamber
- Multiple lasers/heads
- Change basic mechanism – EBM, Direct Metal,…

Simple Build Rate model for Powders

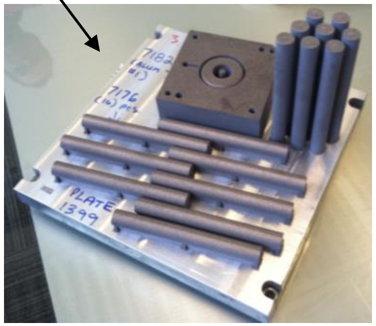
$$\dot{V}_{build} = \frac{V_{part}}{N \cdot t_{recoat} + \frac{V_{part} + V_{support}}{\dot{V}_{scan}}}$$



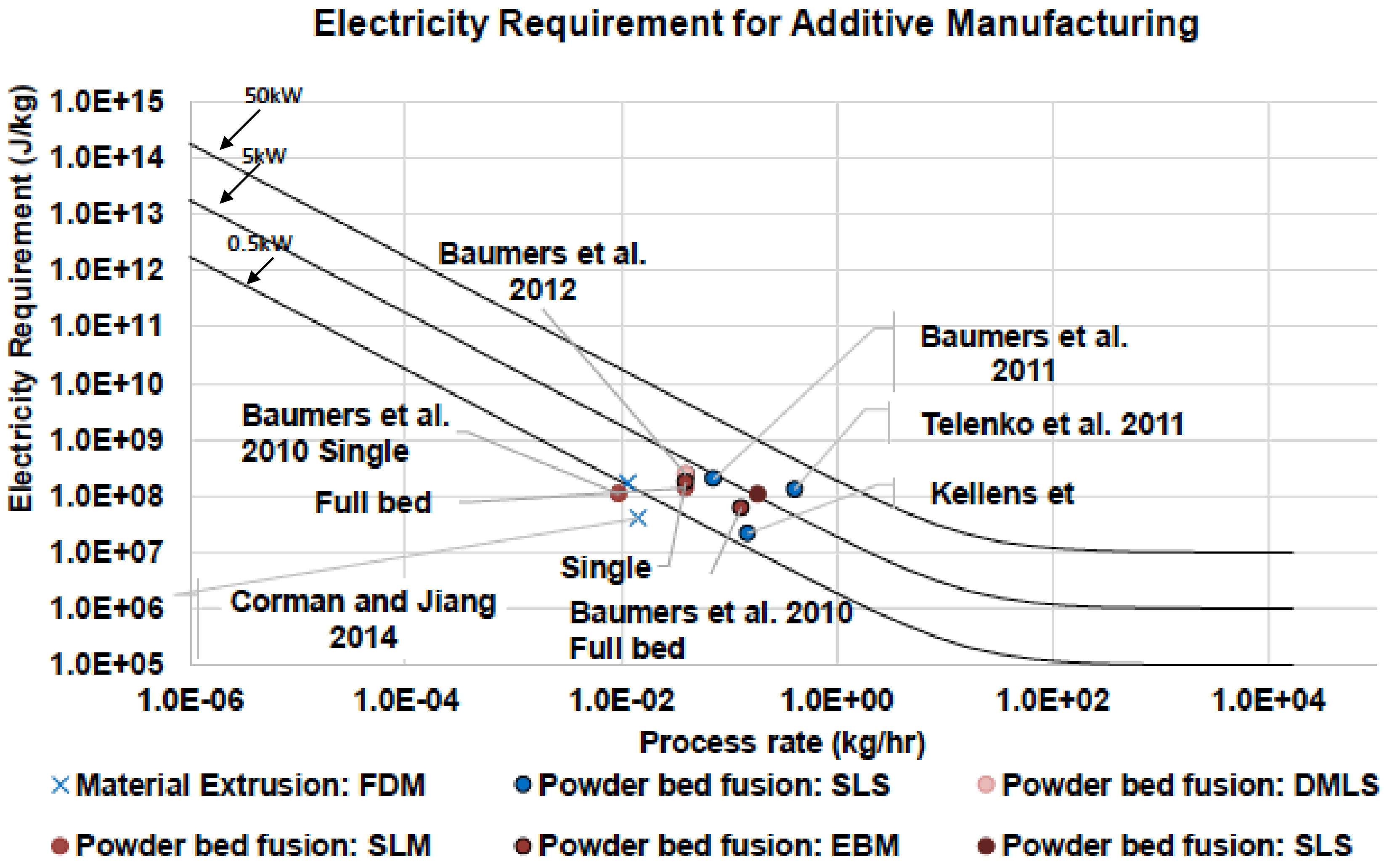
Note support structure
Ref: Buchbinder 2011



Baumers 2012



Hockey stick: Single and full bed (measured data)

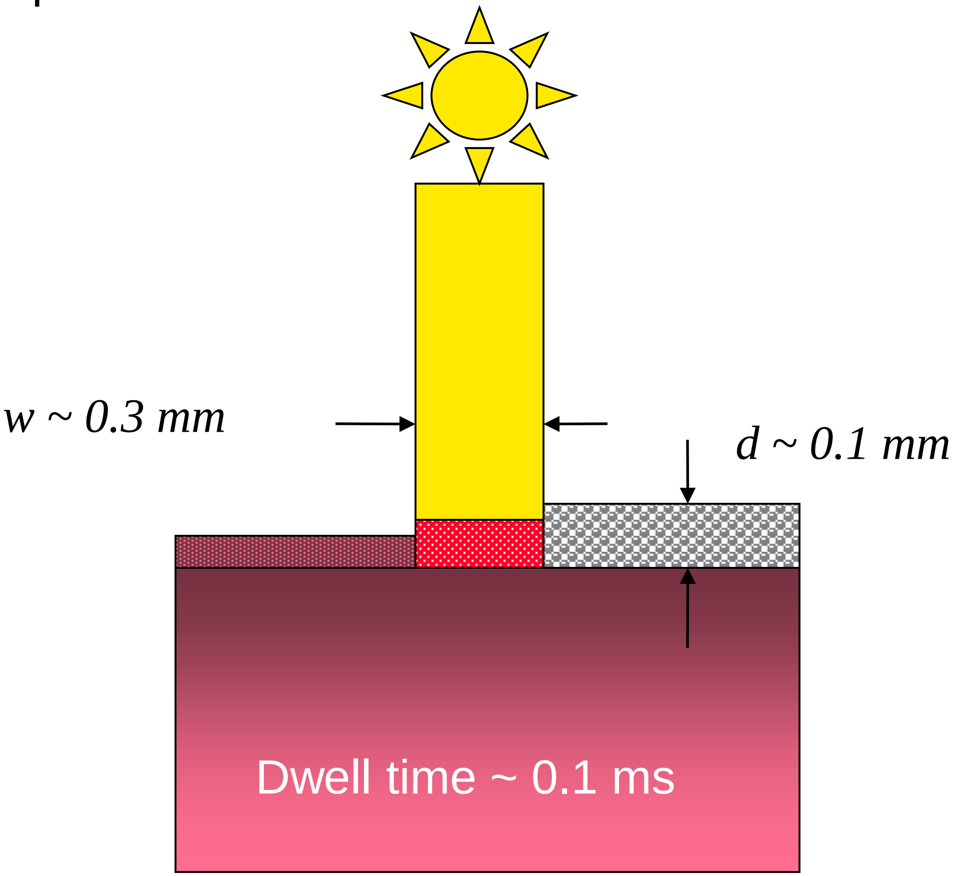


Laser melting of powders

Common laser power:
Polymers ~ 50 W (CO_2 10.6μm)
Metals ~ 200 W (Nd:YAG 1.06μm)
Absorption ~ 0.6

Laser scan speeds
~ .1 to 1 m/s

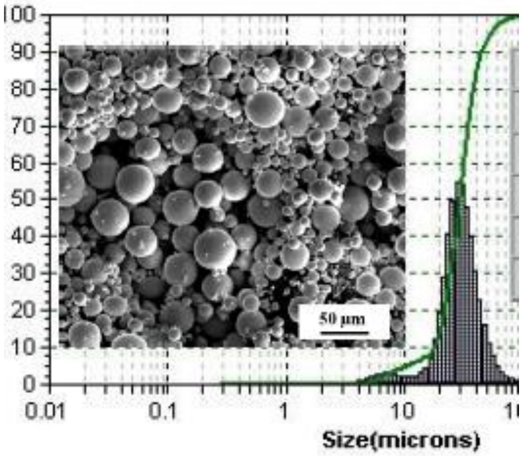
“max” build rate:
30mm^3/s
= 108 cm^3/hr



w ~ 0.3 mm

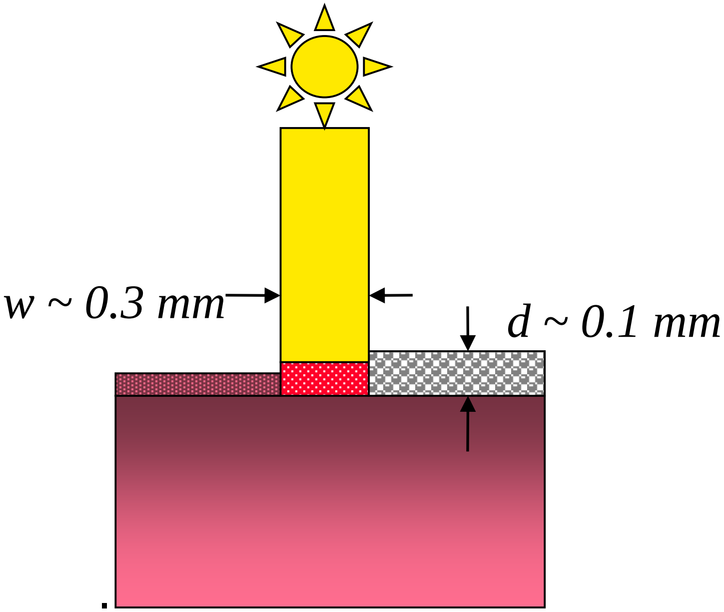
d ~ 0.1 mm

Dwell time ~ 0.1 ms



Ref Sheng Jiang M.S. 2015

Adiabatic Rate Efficiency



$$\dot{m}_{adiabatic} = \frac{\alpha P}{c \cdot \Delta T + \gamma}$$

α = laser/material absorption coefficient

$$(0 \leq \alpha \leq 1)$$

P = laser power (W)

c = average specific heat (J/(kg K))

$\Delta T = T_{melt} - T_{start}$ (K)

γ = enthalpy of melting (J/kg)

$$\eta_{adiabatic} = \frac{\dot{m}_{actual}}{\dot{m}_{adiabatic}}$$

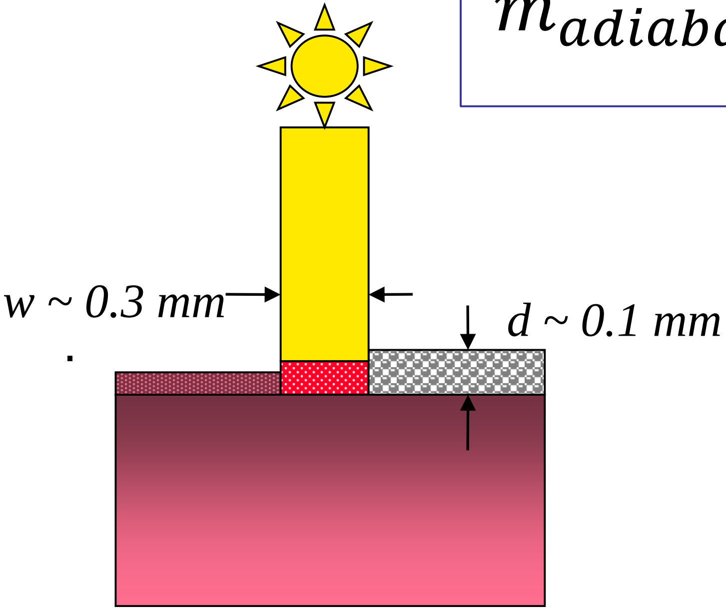
Improvement Strategies

Tune laser

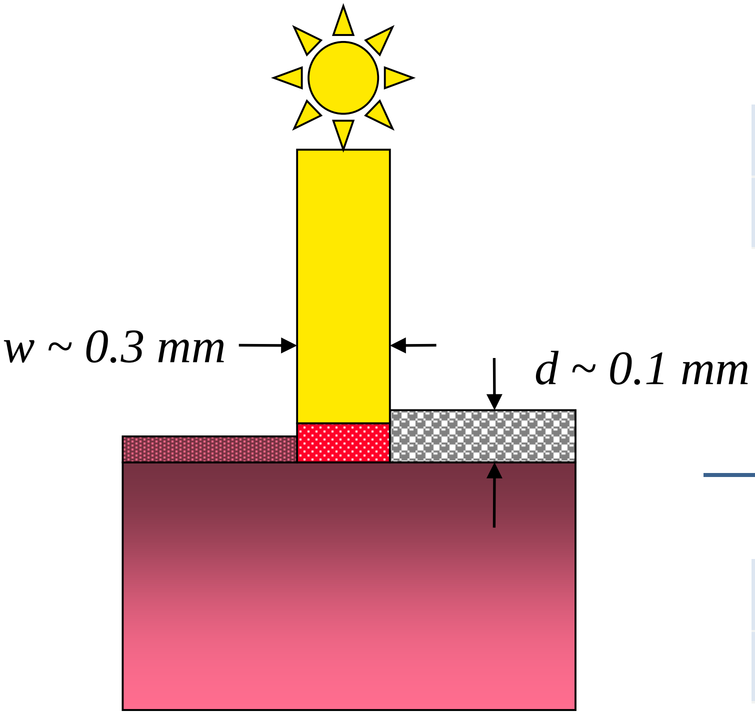
Increase power

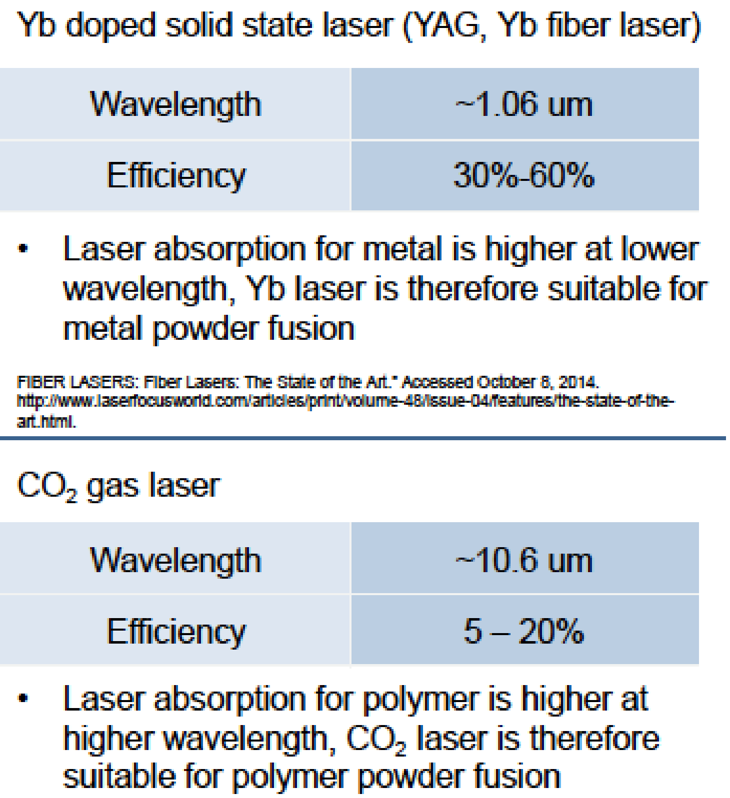
$$\dot{m}_{adiabatic} = \frac{\alpha P}{c \cdot \Delta T + \gamma}$$

Heat bed



Laser selection





Yb doped solid state laser (YAG, Yb fiber laser)

Wavelength	~1.06 um
Efficiency	30%-60%

- Laser absorption for metal is higher at lower wavelength, Yb laser is therefore suitable for metal powder fusion

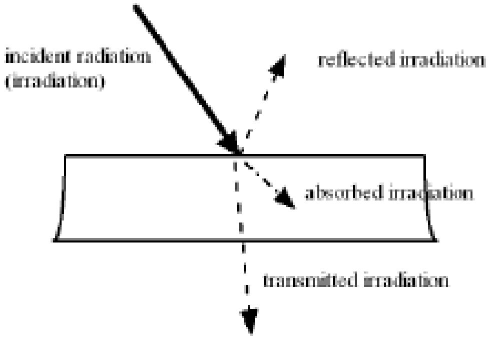
FIBER LASERS: Fiber Lasers: The State of the Art." Accessed October 8, 2014. http://www.laserfocusworld.com/articles/print/volume-48/issue-04/features/the-state-of-the-art.html.

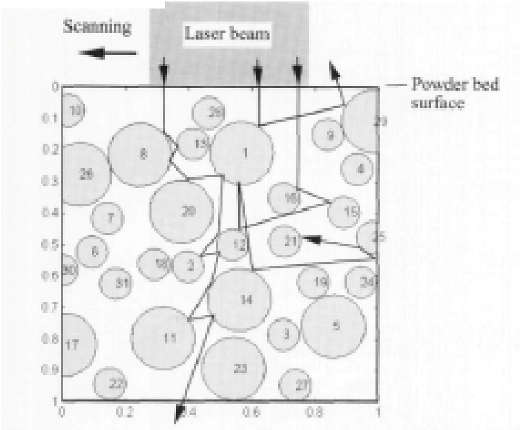
CO_2 gas laser

Wavelength	~10.6 um
Efficiency	5 – 20%

- Laser absorption for polymer is higher at higher wavelength, CO_2 laser is therefore suitable for polymer powder fusion

Absorptivity of powders





Kruth, J.p., X. Wang, T. Laoui, and L. Froyen. "Lasers and Materials in Selective Laser Sintering." Assembly Automation 23, no. 4 (December 1, 2003)
Alkahari et al. Thermal Conductivity of Metal Powder and Consolidated Material fabricated vis Selective Laser Melting, 2012

Laser beam, when directed onto powder surface, is generally absorbed better as it can bounce among particles multiple times and gets absorbed each time

Material	Absorption rate
Fe (solid)	30%
Fe (Powder)	64%
Cu (solid)	2-10%
Cu (Powder)	59%

Solid vs powder metal material laser absorption (Nd-YAG laser, 1.06 um)

Tolochko, Nikolay K., Yurii V. Khlopkov, Sergei E. Mozzharov, Michail B. Ignatiev, Tahar Laoui, and Victor I. Titov. "Absorptance of Powder Materials Suitable for Laser Sintering." Rapid Prototyping Journal 6, no. 3 (September 1, 2000): 155–61. doi:10.1108/13552540010337029.

Extinction coefficient

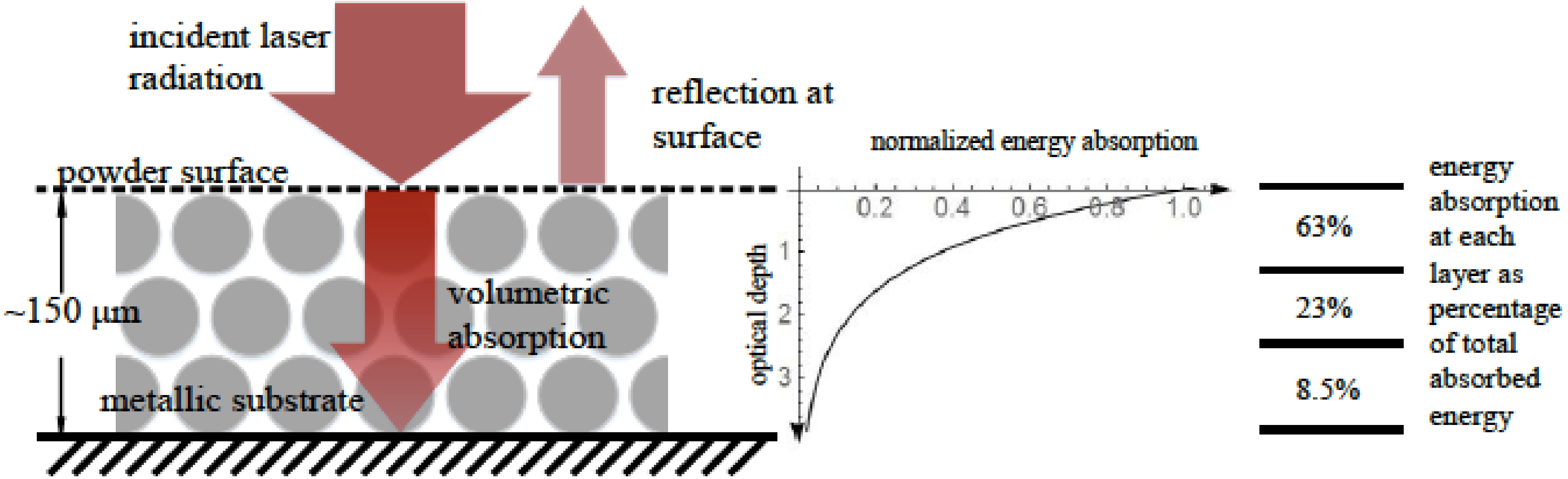
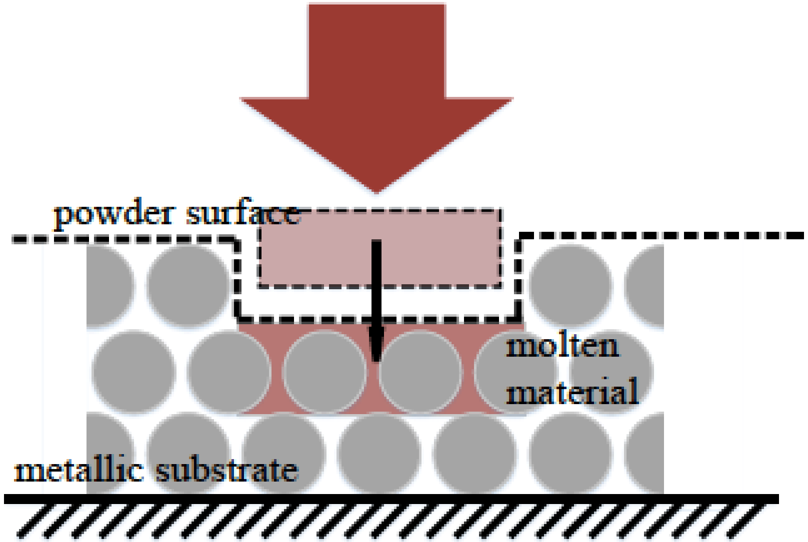
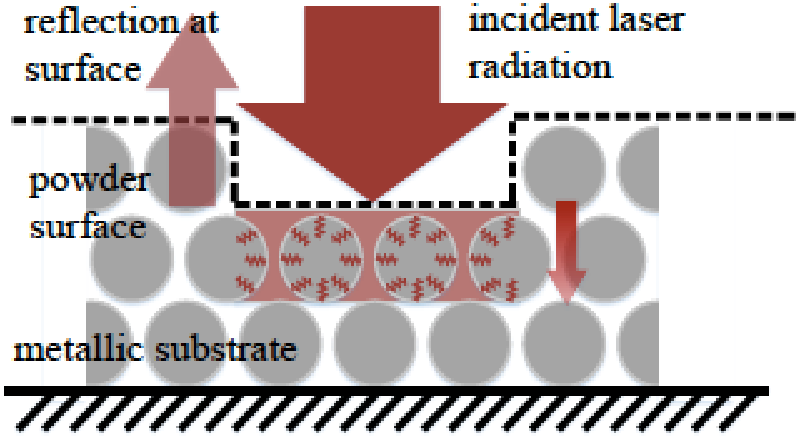


Figure 4.11. Volumetric heat absorption in powder bed (optical depth = β×z)

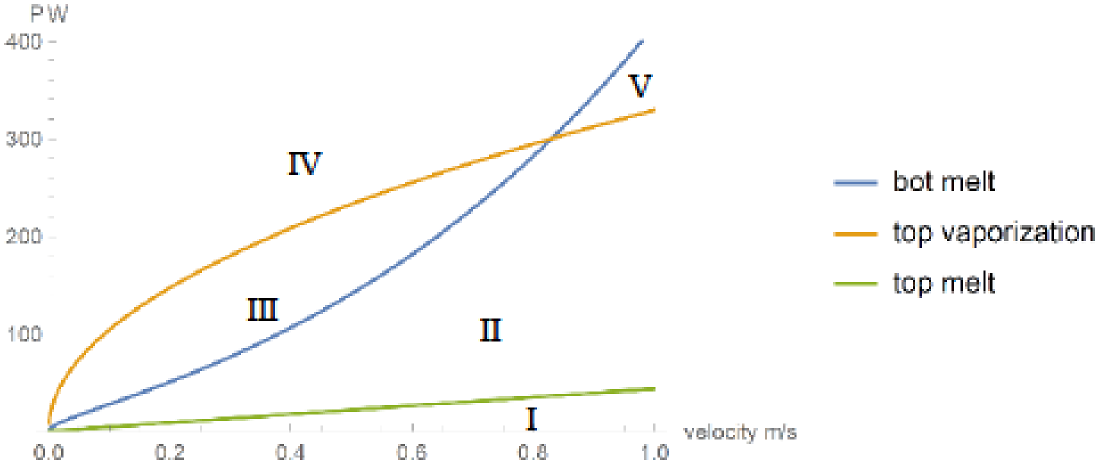
Capillarity & Vaporization





Process window for one-dimensional constant heat flux model

Ref Sheng Jiang M.S. 2015



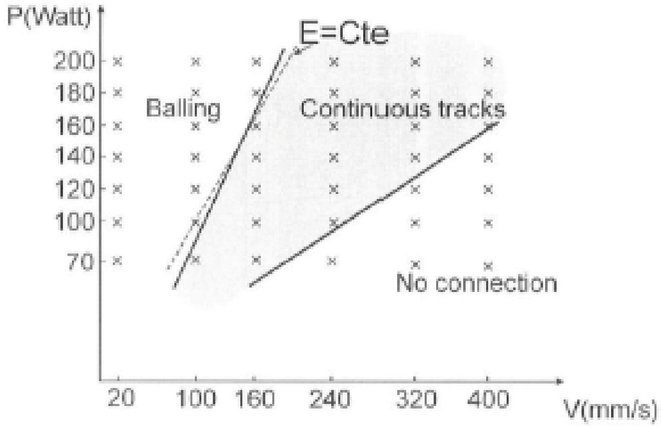
Process window plot consists of five sections

- I: laser is moving too fast, all the material remains solid phase
- II: laser is moving too fast, top layer material melted, bottom layer remains solid
- III: process window where all the material are melt with no vaporization
- IV: laser moves slow with given power, the top surface material starts vaporization while the bottom surface is melted
- V: laser moves slow with given power, the top surface starts vaporization before bottom surface starts melting

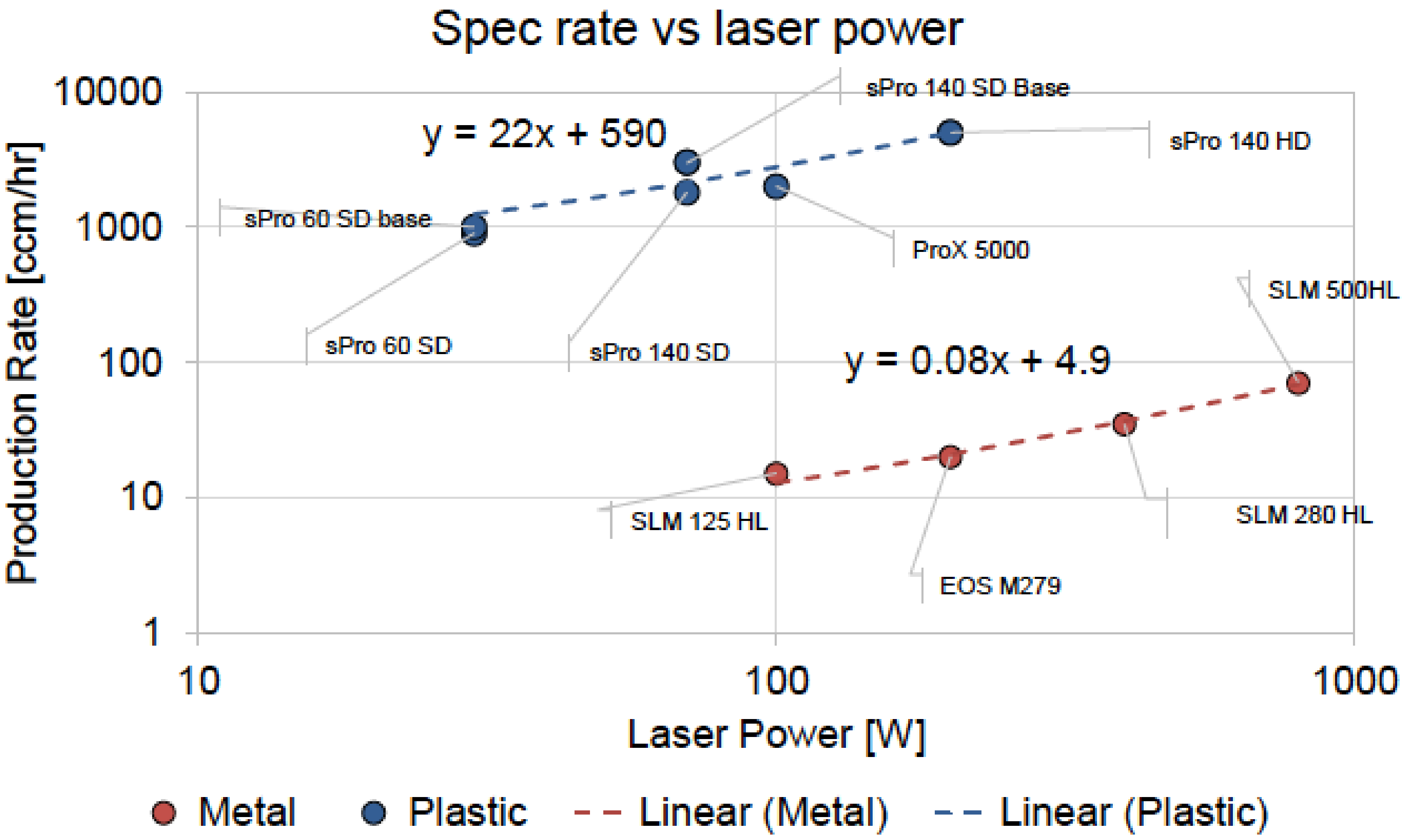
Parameters for iron powder processes:

Parameter	Value	Reference
Heat capacity [J/(kg-ºC]	444	Online: http://www2.ucdsb.on.ca/tiss/stretton/database/Specific_Heat_Capacity_Table.html
Melting temperature [ºC]	1538	Online http://physics.info/heat-latent/
Thermal conductivity [W/(m-ºC)]	20	Carson et al. 2005
Latent heat [J/kg]	247,000	Online http://physics.info/heat-latent/
Laser material absorption rate	0.4	Tolochko et al. 2000
Layer thickness [µm]	150,200,250	
Laser focus spot diameter [µm]	200	

Figure 18 SLM process window for an iron based powder mixture



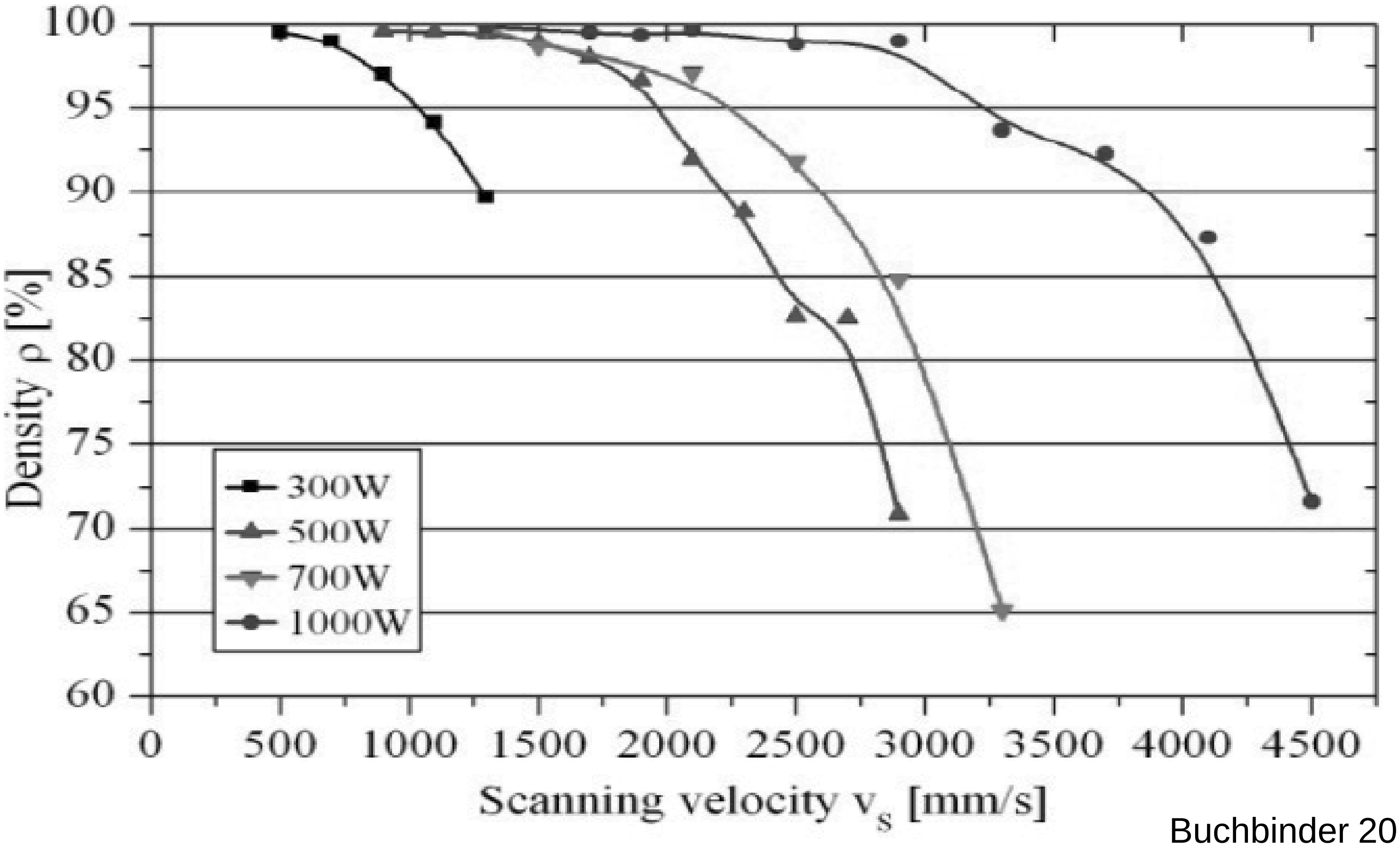
Source: University of Leuven



From manufacturer’s spec sheets

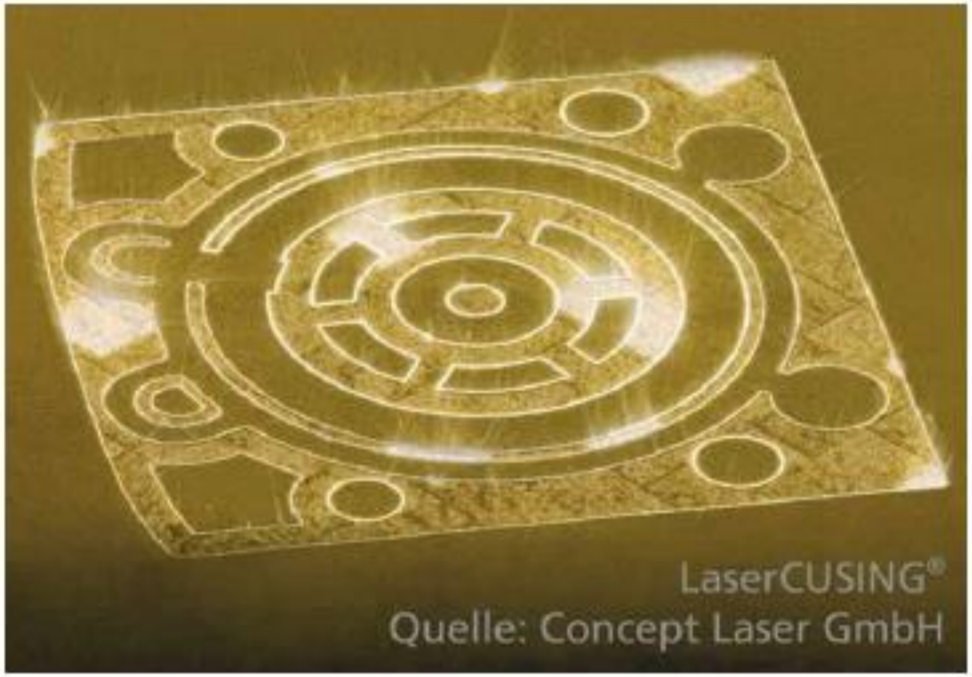
How to make the process fasted

1. More powerful laser(s), 2. Heated Chamber



Buchbinder 2011

New developments with multiple lasers



Concept laser, Sep 19th, 2014

On same part

http://additivemanufacturing.com/2014/09/19/concept-laser-to-present-unmissable-innovations-at-euromold-2014/

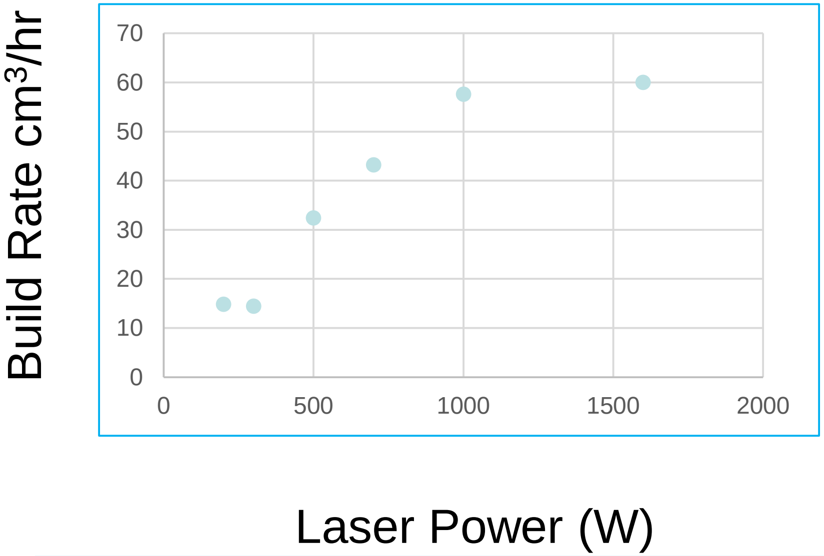
Euromold takes place between
25th – 28th November in Frankfurt/Main.

SLM 500 HL, 4 laser system

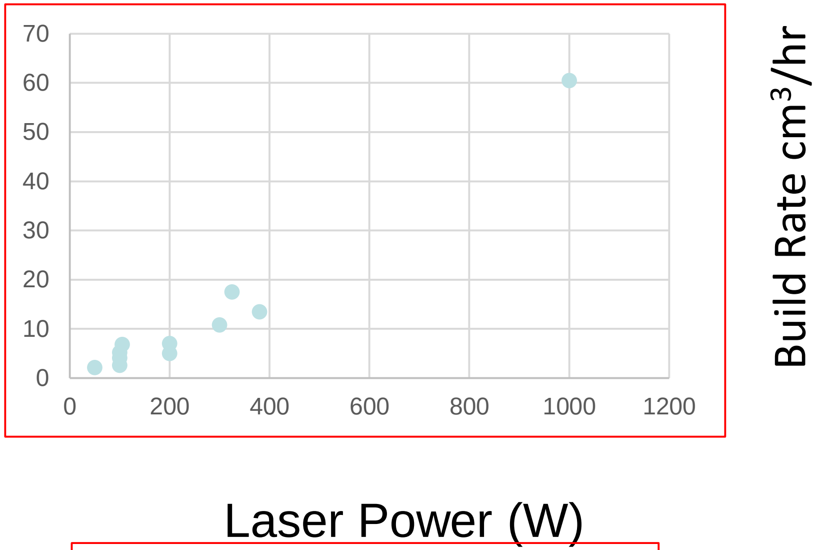
On different parts

http://lane-conference.org/downloads/IndustrialContributions/LANE2014_Wiesner_Multi-Laser_Selective_Laser.pdf

Increase in build rate with laser power

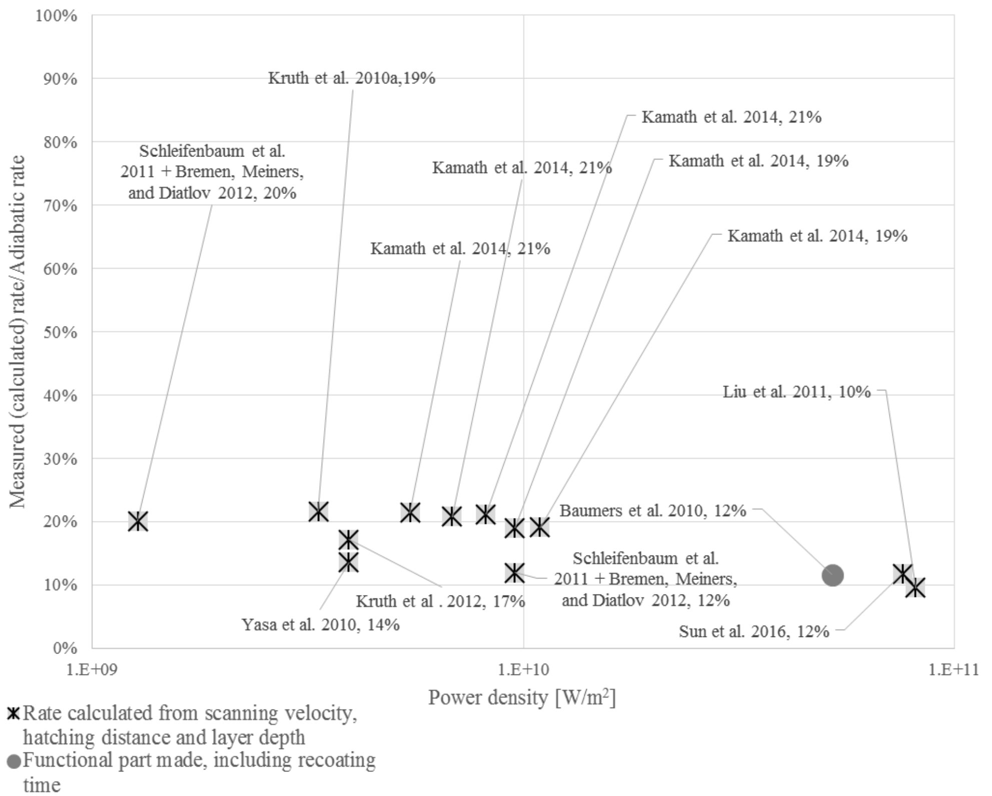


Measured print rates
for aluminum powders (AlSi10Mg)
Vs laser power



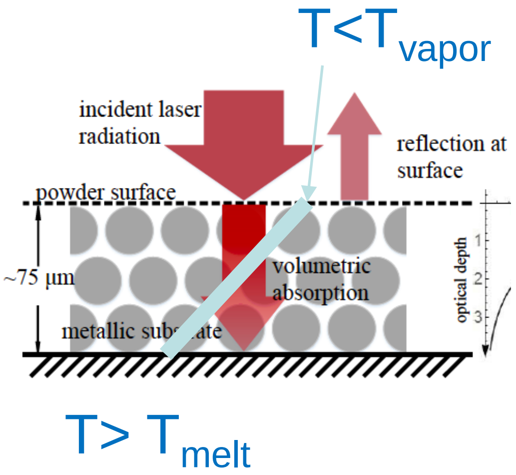
Measured print rates
for steel powders (mostly
316 SS)
Vs laser power

(Gutowski et al 2017 JIE)



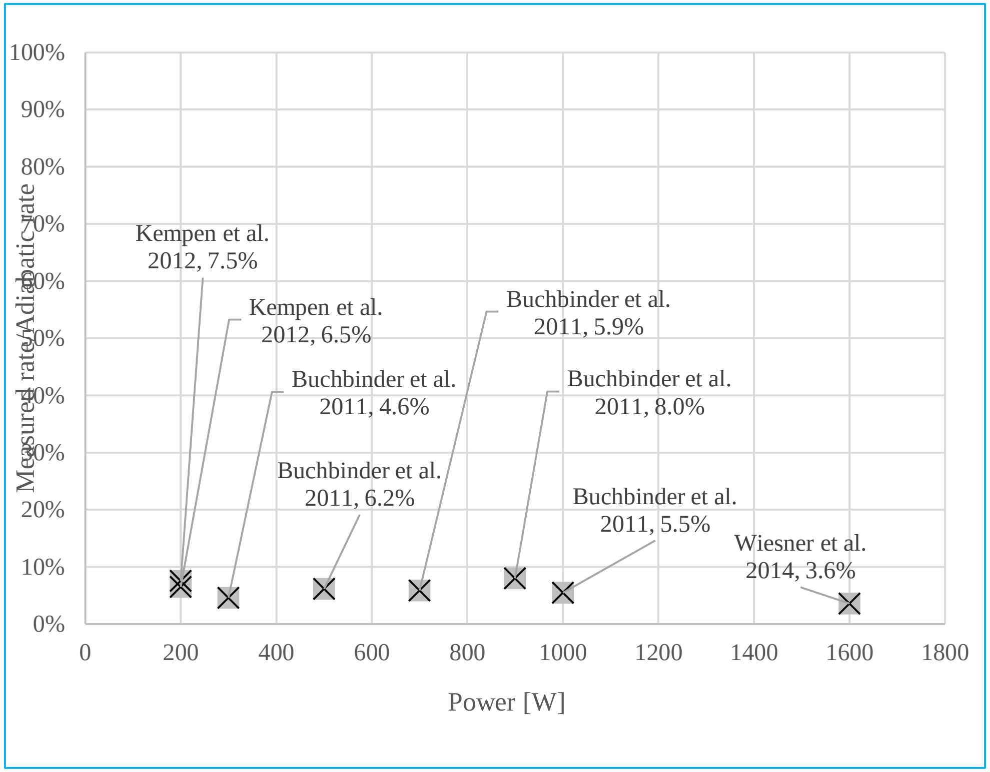
Measured rate/adiabatic rate Vs laser power intensity for steel powders for different additive equipment using larger lasers and defocusing.

$$\dot{m}_{adiabatic} = \frac{\alpha P}{c \cdot \Delta T + \gamma}$$

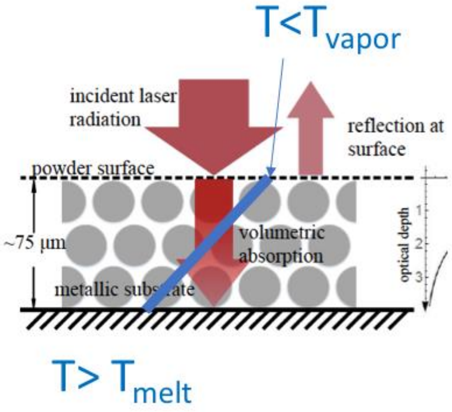


$$\eta_{adiabatic} = \frac{\dot{m}_{actual}}{\dot{m}_{adiabatic}}$$

(Gutowski et al 2017 JIE)



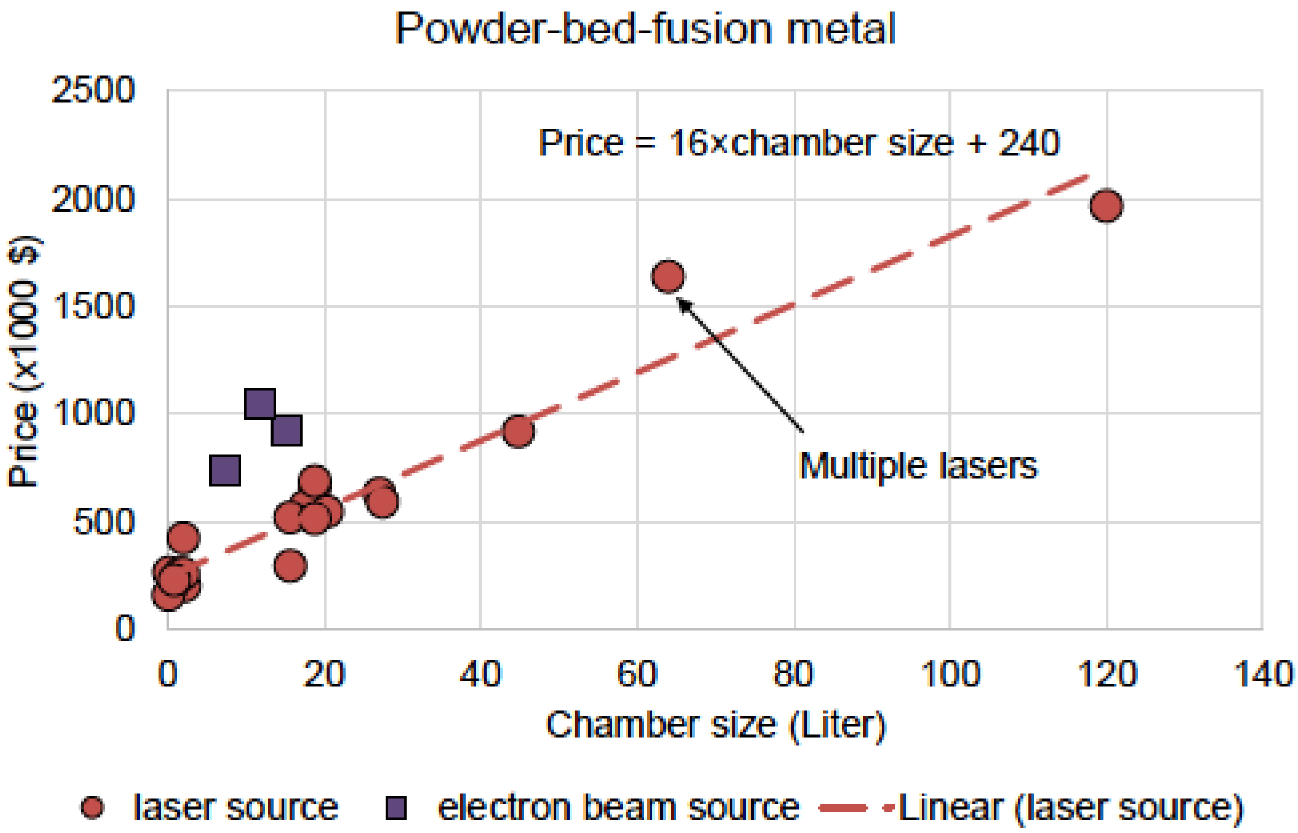
$$\dot{m}_{adiabatic} = \frac{\alpha P}{c \cdot \Delta T + \gamma}$$



$$\eta_{adiabatic} = \frac{\dot{m}_{actual}}{\dot{m}_{adiabatic}}$$

Measured rate/adiabatic rate Vs laser power for aluminum powders for different additive equipment using various improvement strategies.

(Gutowski et al 2017 JIE)



- Price and chamber size correlates linearly
- Most machines have chamber size around 1L and 20L
- Biggest chamber size machine is Concept Laser Xl1000R at 120L
- Systems with chamber volume bigger than 40L are all from Germany
- EBM machines are more expensive than Laser pbf machine at same chamber size

Wohlers 2014

Summary

- Digital to Part is a reality, opening up all kinds of creative opportunities, but
- As a Mfg Process rate remains an issue
  - Increase adsorption
  - Split laser beam
  - Change heating mechanism (EBM, Direct Metal P/M extrusion and sinter)

stratasys
stratasys | F120

Videos from Mass & Burlington Foundries

Adiabatic Rate Efficiency

$$\dot{m}_{adiabatic} = \frac{\alpha P}{c \cdot \Delta T + \gamma}$$

α = laser/material absorption coefficient

$$(0 \leq \alpha \leq 1)$$

P = laser power (W)

c = average specific heat (J/(kg K))

$\Delta T = T_{melt} - T_{start}$ (*K*)

γ = enthalpy of melting (J/kg)

$$\eta_{adiabatic} = \frac{\dot{m}_{actual}}{\dot{m}_{adiabatic}}$$

.

Energy Efficiency

$$\eta_{energy} = \frac{m(c(T_f - T_c) + \gamma + c(T_c - T_{amb}))}{\alpha P t \cdot \frac{1}{\eta_{laser}} \cdot \frac{1}{\eta_{grid}} + mc(T_c - T_{amb}) \cdot \frac{1}{\eta_{chamber}} \cdot \frac{1}{\eta_{grid}}}$$

$$\eta_{energy} = \frac{1 + \frac{(T_c - T_{amb})}{(T_f - T_c) + {}^{\gamma}/_{c}}}{\frac{1}{\eta_{adiabatic}} \cdot \frac{1}{\eta_{laser}} \cdot \frac{1}{\eta_{grid}} + \frac{(T_c - T_{amb})}{(T_f - T_c) + {}^{\gamma}/_{c}} \cdot \frac{1}{\eta_{chamber}} \cdot \frac{1}{\eta_{grid}}}$$

$$\eta_{energy} \cong \eta_{adiabatic} \cdot \eta_{laser} \cdot \eta_{grid}$$

(Gutowski et al 2017 JIE)