Casting & AM

2.810
T. Gutowski

Ancient Nubia Now MFA
October 13, 2019-January 20, 2020
Might and majesty on the Nile
Outline

1. Quick Review: Sand Casting, Die Casting
2. Basics: Phase Change, Shrinkage, Heat Transfer
3. Design Issues, patterns and tools
4. Additive (laser metal powder bed) Manufacturing
Casting since about 3200 BCE…

Etruscan casting with runners circa 500 BCE

China circa 3000 BCE

Lost wax jewelry from Greece circa 300 BCE
Bronze age to iron age

Ancient Greece; bronze statue casting circa 450BCE

Iron works in early Europe, e.g. cast iron cannons from England circa 1543
Casting + AM

Readings:
1. Kalpakjian, Chapters 10, 11, 12
2. Boothroyd, Ch 10, 21
3. Flemings “Heat Flow in Solidification”
4. Quinlan, …Hart JIE 2017

Note: a good heat transfer reference can be found by Profs John Lienhard online http://web.mit.edu/lienhard/www/ahtt.html
Casting Methods

- **Sand Casting**
  High Temperature Alloy, Complex Geometry, Rough Surface Finish

- **Die Casting**
  High Temperature Alloy, Moderate Geometry, Smooth Surface
Sand Casting

(a) Mechanical drawing of part
(b) Cope pattern plate
(c) Core prints
(d) Core boxes
(e) Core halves pasted together
(f) Risers
(g) Cope after ramming with sand and removing pattern, sprue, and risers
(h) Drag ready for sand
(i) Drag after removing pattern
(j) Drag with core set in place
(k) Cope and drag assembled ready for pouring
(l) Casting as removed from mold; heat treated
(m) Casting ready for shipment
Sand Casting

Description: Tempered sand is packed into wood or metal pattern halves, removed from the pattern, and assembled with or without cores, and metal is poured into resultant cavities. Various core materials can be used. Molds are broken to remove castings. Specialized binders now in use can improve tolerances and surface finish.

Metals: Most castable metals.

Size Range: Limitation depends on foundry capabilities. Ounces to many tons.

Tolerances:
- Non-Ferrous ± 1/32” to 6”
- Add ± .003” to 3”, ± 3/64” from 3” to 6”.
- Across parting line add ± .020” to ± .090” depending on size.
  (Assumes metal patterns)

Surface Finish:
- Non-Ferrous: 150-350 RMS
- Ferrous: 300-700 RMS

Minimum Draft Requirements:
- 1° to 5°
- Cores: 1° to 1 1/2°

Normal Minimum Section Thickness:
- Non-Ferrous: 1/8” - 1/4”
- Ferrous: 1/4” - 3/8”

Ordering Quantities: All quantities

Normal Lead Time:
- Samples: 2-10 weeks
- Production 2-4 weeks A.S.A.
Sand Casting Mold Features

**FIGURE 10.7** Schematic illustration of a typical riser-gated casting. Risers serve as reservoirs, supplying molten metal to the casting as it shrinks during solidification. See also Fig. 11.4. Source: American Foundrymen’s Society.

**FIGURE 11.4** Schematic illustration of a sand mold, showing various features.

Vents, which are placed in molds to carry off gases produced when the molten metal comes into contact with the sand in the molds and core. They also exhaust air from the mold cavity as the molten metal flows into the mold.
Videos from Mass & Burlington Foundries
Production sand casting

FIGURE 11.8
(a) Schematic illustration of a jolt-type mold-making machine. (b) Schematic illustration of a mold-making machine which combines jolting and squeezing.
Die Casting

Description: Molten metal is injected, under pressure, into hardened steel dies, often water cooled. Dies are opened, and castings are ejected.

Metals: Aluminum, Zinc, Magnesium, and limited Brass.

Size Range: Not normally over 2 feet square. Some foundries capable of larger sizes.

Tolerances:
- Al and Mg ± .002”/in.
- Zinc ± .0015”/in.
- Brass ± .001”/in.
- Add ± .001” to ± .015” across parting line depending on size

Surface Finish: 32-63RMS

Minimum Draft Requirements:
- Al & Mg: 1° to 3°
- Zinc: 1/2° to 2°
- Brass: 2° to 5°

Normal Minimum Section Thickness:
- Al & Mg: .03” Small Parts: .06” Medium Parts
- Zinc: .03” Small Parts: .045” Medium Parts
- Brass: .025” Small Parts: .040” Medium Parts

Ordering Quantities:
- Usually 2,500 and up.

Normal Lead Time:
- Samples: 12-20 weeks
- Production: ASAP after approval.
Process variation/tolerance

**Figure 35.20** Dimensional tolerances as a function of part size for various manufacturing processes; note that because many factors are involved, there is a broad range for tolerances.
Die cast parts & runners
High Melt Temperature

• Reactivity
  • with air, mold mat’ls,

• Gas solubility
  • $H_2$ gas in Al

• Safety
  • Metal fires, e.g. Mg

\[ Mg + 2H_2O \rightarrow Mg(OH)_2 + H_2 \]

- $3000^\circ C$
  - Tungsten Carbide, WC,
    - Silicon Carbide, SiC
  - Cubic Zirconia, ZrO$_2$

- $2000^\circ C$
  - Molybdenum

- $1000^\circ C$
  - Alumina $\text{Al}_2\text{O}_3$
  - Platinum, Pt
    - Titanium, Ti
    - Iron, Plain Carbon Steels, low alloy, stainless
    - Nickel, Ni
    - Nickel Alloy
    - Silicon, Si
  - Copper, Cu, Bronze, Brass

- $0^\circ C$
  - Aluminum
  - Magnesium
  - Zinc, Zn
  - PTFE (Teflon)
  - Tin, Sn
  - HDPE
Phase Change & Shrinkage

**TABLE 10.1**

| Volumetric Solidification Contraction or Expansion for Various Cast Metals |
|---|---|
| Contraction (%) | Expansion (%) |
| Aluminum | 7.1 | Bismuth | 3.3 |
| Zinc | 6.5 | Silicon | 2.9 |
| Al-4.5% Cu | 6.3 | Gray iron | 2.5 |
| Gold | 5.5 |  |
| White iron | 4-5.5 |  |
| Copper | 4.9 |  |
| Brass (70-30) | 4.5 |  |
| Magnesium | 4.2 |  |
| 90% Cu-10% Al | 4 |  |
| Carbon steels | 2.5-4 |  |
| Al-12% Si | 3.8 |  |
| Lead | 3.2 |  |
Solidification of a binary alloy

![Phase diagram for a copper-nickel alloy system and associated cooling curve for a 50% Ni-50% Cu composition during casting.](image)

FIGURE 12.5 (a) Phase diagram for a copper-nickel alloy system and (b) associated cooling curve for a 50% Ni-50% Cu composition during casting.
Composition change during solidification

**FIGURE 4.5** Phase diagram for nickel-copper alloy system obtained at a slow rate of solidification. Note that pure nickel and pure copper each have one freezing or melting temperature. The top circle on the right depicts the nucleation of crystals. The second circle shows the formation of dendrites (see Section 10.2). The bottom circle shows the solidified alloy, with grain boundaries.
FIGURE 4.7  The lead–tin phase diagram. Note that the composition of the eutectic point for this alloy is 61.9% Sn–38.1% Pb. A composition either lower or higher than this ratio will have a higher liquidus temperature.
Solidification

Dendrite growth in metals- lower surface energy crystallographic planes are favored, producing tree like structures if not disturbed. See next slide for explanation of a, b & c

http://www.its.caltech.edu/~atomic/snowcrystals/
Cast structures

Schematic illustration of three cast structures solidified in a square mold: (a) pure metals; (b) solid solution alloys; and c) structure obtained by using nucleating agents. *Source:* G. W. Form, J. F. Wallace, and A. Cibula
Properties of castings

e.g. Compare elongation of carbon steels (4-36%) Table 5.3, with cast irons (0-18%) Table 12.3 (Kalpakjian & Schmid 7th)
How long does it take to solidify?

Thickness ~ 30 cm

Thickness ~ 0.5 cm
Sand Casting – Solidification Time

$$t_s \approx \left( \frac{V}{A} \right)^2$$

Chvorinov’s Rule
p.247 your text

**FIGURE 1-6**
Approximate temperature profile in solidification of a pure metal poured at its melting point against a flat, smooth mold wall.

Ref: Mert Flemings “Solidification Processing”
Thermal Conductivity “k” (W/m·K)

\[ q = -k \frac{dT}{dx} \]

Copper: 394
Aluminum: 222
Iron: 29
Sand: 0.61
PMMA: 0.20
PVC: 0.16
Transient Heat Transfer

\[ q \rightarrow \begin{array} \end{array} \rightarrow \begin{array} \end{array} q + dq \]

\[ \rho C_p \frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial x^2} \]

\[ k \neq k(x) \]

α = \frac{k}{\rho C_p}

Metals

Glass

Polymers

Gases

\[ \alpha \left[ \frac{cm^2}{sec} \right] \]

Sand 3 \times 10^{-3}

Aluminum
Sand Casting (see Flemings)

Define new variable

\[ \zeta = \frac{x}{\sqrt{\alpha t}} \]

Use

\[ \theta = \frac{T - T_M}{T_o - T_M} \]

We seek the transient temperature profile in the sand.
Sand Casting (see Flemings)

Ordinary differential eq'us

$$\frac{d^2 \theta}{d\zeta^2} = -\frac{\zeta}{2} \frac{d\theta}{d\zeta}$$

i.c.  \( \theta = 1 \) at \( \zeta = \infty \)

b.c.  \( \theta = 0 \) at \( \zeta = 0 \)

At \( t=0, T=T_0 \) everywhere

At \( x=0, T=T_m \) always

This will allow us to calculate the heat lost by the metal at the boundary with the sand tooling

\[ \theta = \text{erf} \left( -\frac{\zeta}{2} \right) \]
Solidification Time

Heat required to solidify to distance "s"

\[ = A \cdot s \cdot \rho \cdot H \]

Rate eq'n (per unit area)

\[ \rho H \frac{ds}{dt} = -\dot{q} = k \left( \frac{\partial T}{\partial x} \right)_{x=0} \]

Use Flemings result here

Enthapy of the phase change/wt
Solidification Time (cont.)

this leads to

\[ s = \frac{2}{\sqrt{\pi}} \left( \frac{T_M - T_o}{\rho M H_M} \right) \sqrt{K_s \rho_s C_p s t} \]

let \( s = \frac{V}{A} \)

\[ t = C \left( \frac{V}{A} \right)^2 \]

Chvorinov's rule

The constant “C” (in this case not heat capacity) is determined by experiment.

Several references suggest that values range:

\[ C \sim 2 \text{ to } 4 \text{ min/cm}^2 \] (with most data for iron and steel)
How long does it take to solidify?

Order of magnitude estimate using half thickness, & $C = 3.3 \text{ min/cm}^2$

Thickness $\sim 30 \text{ cm}$
Solidification time $= 3.3 (30/2)^2 \text{ [min]} \sim 12 \text{ hrs}$

Thickness $\sim 0.5 \text{ cm}$
$t = 3.3 (0.5/2)^2 \text{ [min]} \sim 12 \text{ sec}$
What happened?
Can you explain these Solidification features?

Picture taken from the Chipman Room
Heat Transfer – Die Casting

\[ t_s \approx \left( \frac{V}{A} \right)^1 \]

**FIGURE 1-9**
Temperature profile during solidification against a large flat mold wall with mold-metal interface resistance controlling.
Film Coefficients \( "h" \) W/m\(^2\)·K

\[
Q = -hA(T_{high} - T_{low})
\]

- **Typical die casting**: 1,000 - 10,000
- **Natural convection**: 1 - 10
- **Flowing air**: 10 - 50

Carbon coating: **high pressure** low pressure polished die
Die casting contact resistance

see Boothroyd Ch 10, p446-447
Suggest average value for $h$ of $5\text{kW(m}^2\text{K)}$
Die Casting
Solidification Time

Time to form solid part

\[ \dot{q} = -\bar{h}A(T_M - T_o) = \rho_M H_M A \frac{ds}{dt} \]

\[ t = \frac{\rho_M H_M V}{\bar{h}(T_M - T_o) A} \]

Also need to cool casting to below \( T_M \) to eject \( T_{\text{eject}} \)

and will inject at \( T_{\text{inject}} > T_M \).
Time to **cool part from melt temp to the ejection temperature.**

\[ mC \frac{dT}{dt} = -Ah(T - T_o) \]

Let, \( \theta = T - T_o \)

\[ \frac{1}{\theta} \frac{d\theta}{dt} = -\frac{Ah}{mC} \]

Integration yields...

\[ t = \frac{-mC}{Ah} \ln \frac{\Delta \theta_f}{\Delta \theta_i} \]

For two sided cooling this gives

\[ t = \frac{\rho \cdot C \cdot w}{2h} \ln \frac{(T_M - T_w)}{(T_{eject} - T_w)} \]

Note \( C = \) heat capacity, \( h = \) film coefficient, \( w = \) part width, \( T_M = \) melt temp, \( T_w = \) wall temp
Time to solidify + cool using superheat temperature $T_{sp}$

For thin sheets of thickness “w”, including phase change

\[ \Delta \theta_i = T_i + \Delta T_{sp} - T_{mold} \]
\[ \Delta T_{sp} = \frac{H}{C} \]
\[ \Delta \theta_f = T_{eject} - T_{mold} \]

"sp" means superheat
C is heat capacity
H is latent heat of fusion
h is film transfer coef

Approximations,
\[ t \approx 0.42 \text{ sec/mm x } w_{max} \text{ (Zn)} \]
\[ t \approx 0.47 \text{ sec/mm x } w_{max} \text{ (Al)} \]
\[ t \approx 0.63 \text{ sec/mm x } w_{max} \text{ (Cu)} \]
\[ t \approx 0.31 \text{ sec/mm x } w_{max} \text{ (Mg)} \]

Ref Boothroyd, Dewhurst, Knight p 447
Pattern and Tooling Design

• For pattern and tooling design issues see Boothroyd, Dewhurst, & Knight:
  • Ch 10 for die casting
  • Ch 12 for sand casting, and the following 5 slides
Pattern Design suggestions

Figure 7.2.24 Identifying hot spots in castings by using outward projecting arrows of length half the casting thickness. Where arrows overlap, hot spots may develop. (Courtesy of Meehanite Metal Corp.)

Figure 7.2.28 Avoid abrupt section changes. (Courtesy of Meehanite Metal Corp.)

Figure 7.2.29 Design for uniform thickness in sections. (Courtesy of Meehanite Metal Corp.)

Figure 7.2.30 More intersection details. (Courtesy of Meehanite Metal Corp.)
Pattern Design Issues (Alum)

- Shrinkage Allowance: 1.3%
- Machining Allowance: 1/16” = 1.6 mm
- Minimum thickness: 3/16” = 5 mm
- Parting Line: even
- Draft Angle: 3 to 5%
- Thickness: even
Table 12.1
Normal Shrinkage Allowance for Some Metals Cast in Sand Molds

<table>
<thead>
<tr>
<th>Metal</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gray cast iron</td>
<td>0.83 – 1.3</td>
</tr>
<tr>
<td>White cast iron</td>
<td>2.1</td>
</tr>
<tr>
<td>Malleable cast iron</td>
<td>0.78 – 1.0</td>
</tr>
<tr>
<td>Aluminum alloys</td>
<td>1.3</td>
</tr>
<tr>
<td>Magnesium alloys</td>
<td>1.3</td>
</tr>
<tr>
<td>Yellow brass</td>
<td>1.3 – 1.6</td>
</tr>
<tr>
<td>Phosphor bronze</td>
<td>1.0 – 1.6</td>
</tr>
<tr>
<td>Aluminum bronze</td>
<td>2.1</td>
</tr>
<tr>
<td>High-manganese steel</td>
<td>2.6</td>
</tr>
</tbody>
</table>
Pattern materials
Digital Sand Casting: Print molds or parts?

1. **Print**
   - Selectively dispense binder using inkjet printing technology.

2. **New layer**
   - The build platform is lowered by a set increment.

3. **Spread**
   - Spreads a new layer of pre-mixed molding sand.

4. **Repeat**
   - Repeat Steps 1-3, until the molds or cores are built.

5. **Finishing**
   - Unbound sand is removed. Metal parts can be cast.

- Start from the Computer Model
- Add gating system and risers
- Design mold and core package, without draft angles and regardless of undercuts
- Print sand molds and cores
- Casting is poured & ready
- Casting is inspected
- Parts are ready to be dispatched
AM (Laser Metal Powder Bed) Steps to produce tooling

1. CAD file
2. Support structure generation
3. Printing: EOS M280
4. Printed part on plate, stress relieve
5. Sawing (or wire EDM) and hand tool removal of support structure
6. 3D printed part
Printed steel & aluminum tools
Laser Metal Powder Issues

1. Tolerances
2. Quality
3. Surface Finish
4. Production Rate
Measured 3D Printing energy intensity (J/kg) Vs Print rate (kg/hr) for AM (metals in red, plastics in blue) overlaid on conventional manufacturing processes.

BAAM technology
More later...

(Gutowski et al 2017 JIE)
Strategies for improved rate

• Powder delivery - full bed, orientation
• Larger laser (tuned)
• Heated chamber
• Multiple lasers/heads
• Change basic mechanism – EBM, Direct Metal,…
Simple Build Rate model for Powders

\[
\dot{V}_{\text{build}} = \frac{V_{\text{part}}}{N \cdot t_{\text{recoat}} + \frac{V_{\text{part}} + V_{\text{support}}}{V_{\text{scan}}}}
\]

- Scan rate
- Recoating rate

Note support structure
Ref: Buchbinder 2011

Full print chamber
Under utilized bed

Baumers 2012
Hockey stick: Single and full bed (measured data)
Laser melting of powders

Common laser power:
- Polymers ~ 50 W (CO$_2$ 10.6μm)
- Metals ~ 200 W (Nd:YAG 1.06μm)
Absorption ~ 0.6

Laser scan speeds
- ~ .1 to 1 m/s

“max” build rate:
- 30mm$^3$/s
  - = 108 cm$^3$/hr

Ref: Sheng Jiang M.S. 2015
Adiabatic Rate Efficiency

\[ \dot{m}_{adiabatic} = \frac{\alpha P}{c \cdot \Delta T + \gamma} \]

\( \alpha = \) laser/material absorption coefficient 
(0 \leq \alpha \leq 1)

\( P = \) laser power (W)

\( c = \) average specific heat (J/(kg K))

\( \Delta T = T_{melt} - T_{start} \) (K)

\( \gamma = \) enthalpy of melting (J/kg)

\[ \eta_{adiabatic} = \frac{\dot{m}_{actual}}{\dot{m}_{adiabatic}} \]
**Improvement Strategies**

- Tune laser
- Increase power
- Heat bed

\[ \dot{m}_{\text{adiabatic}} = \frac{\alpha P}{c \cdot \Delta T + \gamma} \]
Laser selection

**Yb doped solid state laser (YAG, Yb fiber laser)**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wavelength</td>
<td>~1.06 um</td>
</tr>
<tr>
<td>Efficiency</td>
<td>30%-60%</td>
</tr>
</tbody>
</table>

- Laser absorption for metal is higher at lower wavelength, Yb laser is therefore suitable for metal powder fusion.

**CO₂ gas laser**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wavelength</td>
<td>~10.6 um</td>
</tr>
<tr>
<td>Efficiency</td>
<td>5 – 20%</td>
</tr>
</tbody>
</table>

- Laser absorption for polymer is higher at higher wavelength, CO₂ laser is therefore suitable for polymer powder fusion.
Absorptivity of powders

Laser beam, when directed onto powder surface, is generally absorbed better as it can bounce among particles multiple times and gets absorbed each time.

<table>
<thead>
<tr>
<th>Material</th>
<th>Absorption rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fe (solid)</td>
<td>30%</td>
</tr>
<tr>
<td>Fe (Powder)</td>
<td>64%</td>
</tr>
<tr>
<td>Cu (solid)</td>
<td>2-10%</td>
</tr>
<tr>
<td>Cu (Powder)</td>
<td>59%</td>
</tr>
</tbody>
</table>

Solid vs powder metal material laser absorption (Nd-YAG laser, 1.06 um)

Extinction coefficient

Figure 4.11. Volumetric heat absorption in powder bed (optical depth = \( \beta \times z \))
Capillarity & Vaporization
Process window for one-dimensional constant heat flux model

Process window plot consists of five sections:

- I: laser is moving too fast, all the material remains solid phase
- II: laser is moving too fast, top layer material melted, bottom layer remains solid
- III: process window where all the material are melt with no vaporization
- IV: laser moves slow with given power, the top surface material starts vaporization while the bottom surface is melted
- V: laser moves slow with given power, the top surface starts vaporization before bottom surface starts melting

Parameters for iron powder processes:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Heat capacity [J/(kg·°C)]</td>
<td>444</td>
<td>Online: <a href="http://www2.ucdsb.on.ca/tiss/stretton/database/Specific_Heat_Capacity_Table.html">http://www2.ucdsb.on.ca/tiss/stretton/database/Specific_Heat_Capacity_Table.html</a></td>
</tr>
<tr>
<td>Melting temperature [°C]</td>
<td>1538</td>
<td>Online <a href="http://physics.info/heat-latent/">http://physics.info/heat-latent/</a></td>
</tr>
<tr>
<td>Thermal conductivity [W/(m·°C)]</td>
<td>20</td>
<td>Carson et al. 2005</td>
</tr>
<tr>
<td>Laser material absorption rate</td>
<td>0.4</td>
<td>Tolochko et al. 2000</td>
</tr>
<tr>
<td>Layer thickness [μm]</td>
<td>150,200,250</td>
<td></td>
</tr>
<tr>
<td>Laser focus spot diameter [μm]</td>
<td>200</td>
<td></td>
</tr>
</tbody>
</table>
Spec rate vs laser power

- Metal
- Plastic
- Linear (Metal)
- Linear (Plastic)

From manufacturer’s spec sheets
How to make the process fasted

1. More powerful laser(s), 2. Heated Chamber

Buchbinder 2011
New developments with multiple lasers

Euromold takes place between 25th – 28th November in Frankfurt/Main.

Concept laser, Sep 19th, 2014
On same part

SLM 500 HL, 4 laser system
On different parts


Increase in build rate with laser power

Measured print rates for aluminum powders (AlSi10Mg) vs laser power

Measured print rates for steel powders (mostly 316 SS) vs laser power

(Gutowski et al 2017 JIE)
Measured rate/adiabatic rate Vs laser power intensity for **steel powders** for different additive equipment using larger lasers and defocusing.

\[ \dot{m}_{\text{adiabatic}} = \frac{\alpha P}{c \cdot \Delta T + \gamma} \]

\[ \eta_{\text{adiabatic}} = \frac{\dot{m}_{\text{actual}}}{\dot{m}_{\text{adiabatic}}} \]

(Gutowski et al 2017 JIE)
Measured rate/adiabatic rate Vs laser power for aluminum powders for different additive equipment using various improvement strategies.

\[ \dot{m}_{\text{adiabatic}} = \frac{\alpha P}{c \cdot \Delta T + \gamma} \]

\[ \eta_{\text{adiabatic}} = \frac{\dot{m}_{\text{actual}}}{\dot{m}_{\text{adiabatic}}} \]
Price and chamber size correlates linearly.

Most machines have chamber size around 1L and 20L.

Biggest chamber size machine is Concept Laser Xl1000R at 120L.

Systems with chamber volume bigger than 40L are all from Germany.

EBM machines are more expensive than Laser pbf machine at same chamber size.
Summary

• Digital to Part is a reality, opening up all kinds of creative opportunities, but

• As a Mfg Process rate remains an issue
  – Increase adsorption
  – Split laser beam
  – Change heating mechanism (EBM, Direct Metal P/M extrusion and sinter)
Meet the Stratasys F120 3D printer.

Twice as fast as the competition. Not twice the price.

Free Sample Part
Videos from Mass & Burlington Foundries
\[
\dot{m}_{\text{adiabatic}} = \frac{\alpha P}{c \cdot \Delta T + \gamma}
\]

\(\alpha\) = laser/material absorption coefficient  
\((0 \leq \alpha \leq 1)\)

\(P\) = laser power (W)

\(c\) = average specific heat (J/(kg K))

\(\Delta T = T_{\text{melt}} - T_{\text{start}}\) (K)

\(\gamma\) = enthalpy of melting (J/kg)

\[\eta_{\text{adiabatic}} = \frac{\dot{m}_{\text{actual}}}{\dot{m}_{\text{adiabatic}}}\]

\[
\eta_{\text{energy}} = \frac{m(c(T_f - T_c) + \gamma + c(T_c - T_{\text{amb}}))}{\alpha P t \cdot \frac{1}{\eta_{\text{laser}}} \cdot \frac{1}{\eta_{\text{grid}}} + mc(T_c - T_{\text{amb}}) \cdot \frac{1}{\eta_{\text{chamber}}} \cdot \frac{1}{\eta_{\text{grid}}}}
\]

\[
\eta_{\text{energy}} \approx \frac{1}{\eta_{\text{adiabatic}}} \cdot \frac{1}{\eta_{\text{laser}}} \cdot \frac{1}{\eta_{\text{grid}}} + \frac{(T_c - T_{\text{amb}})}{(T_f - T_c) + \frac{\gamma}{c}} \cdot \frac{1}{\eta_{\text{chamber}}} \cdot \frac{1}{\eta_{\text{grid}}}
\]

\[
\eta_{\text{energy}} \approx \eta_{\text{adiabatic}} \cdot \eta_{\text{laser}} \cdot \eta_{\text{grid}}
\]

(Gutowski et al 2017 JIE)